# Schema Mapping Discovery from Data Instances

Georg Gottlob, Pierre Senellart

*Free University of Bozen–Bolzano*, 2 February 2011

# Different sources organize the same data differently

| | | |
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Querying websites using compact skeletons - all 11 versions »
A Rajaraman, **JD Ullman** - Journal of Computer and System Sciences, 2003 - Elsevier
Several commercial applications, such as online comparison shopping and process automation, require integrating information that is scattered across multiple websites or XML documents. Much research has been devoted to this problem, **...**
Cited by 13 - Related Articles - Web Search

**[BOOK]** Wprowadzenie do teorii automatów, jezyków i obliczen
JE Hopcroft, **JD Ullman**, B Konikowska - 2003 - Wydaw. Naukowe PWN
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Improving the efficiency of database-system teaching - all 3 versions »
**JD Ullman** - Proceedings of the 2003 ACM SIGMOD international conference …, 2003 - portal.acm.org
ABSTRACT The education industry has a very poor record of produc- tivity gains. In this brief article, I outline some of the ways the teaching of a college course in database systems could be made more ecient, and sta time used **...**
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A survey of new directions in database systems - all 5 versions »
**JD Ullman** - Database Systems for Advanced Applications, 2003.(DASFAA …, 2003 - ieeexplore.ieee.org
A survey of new directions in database systems. Ullman, JD Stanford University; This paper appears in: Database Systems for Advanced Applications, 2003. (DASFAA 2003). Proceedings. Eighth International **...**
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# Motivation

## Context

- Multiple data sources containing information about similar entities, with some redundancy (e.g., sources of the deep Web).
- Several different ways to present this information, i.e., several different schemata.
- No a priori information about (some of) these schemata.

How to know the relationships between these schemata, by just looking at the instances?

Other way to see this problem: Match operator on schema mappings, in the setting of data exchange.



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# Problem definition

## Problem

Given two (relational) database instances $I$ and $J$ with different schemata, what is the optimal description $\Sigma$ of $J$ knowing $I$ (with $\Sigma$ a finite set of formulas in some logical language)?

What does optimal implies:

- Conciseness of description.
- Validity of facts predicted by $I$ and $\Sigma$.
- All facts of $J$ explained by $I$ and $\Sigma$.

(Note the asymmetry between $I$ and $J$; context of data exchange where $J$ is computed from $I$ and $\Sigma$).

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# Outline

# Source-to-target tuple-generating dependencies

## Definition (Source-to-target tgd)

First-order formula of the form:

$$\forall \mathbf{x}\, \varphi(x) \rightarrow \exists \mathbf{y}\, \psi(x, y)$$

with:

- $\varphi$ conjunction of source relation atoms;
- $\psi$ conjunction of target relation atoms;
- all variables of x bound in $\varphi$.

## Example

$$\forall x_1 \forall x_2\, R_1(x_1, x_2) \wedge R_2(x_2) \rightarrow \exists y\, R'(x_1, y)$$

# Particular tgds

Two ways of having simpler tgds:

- Disallow existential quantifiers on the right hand-side: full tgds.
- Disallow cycles on both left- and right-hand sides: acyclic tgds. (Classical notion of acyclicity on hypergraphs extending the basic notion of acyclicity on graphs.)

## Examples

$\forall x_1 \forall x_2 \forall x_3\ R_1(x_1, x_2) \wedge R_2(x_2, x_3) \wedge R_3(x_3, x_1) \rightarrow R'(x_1)$ is cyclic (and full).

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$\mathcal{L}_{\text{tgd}}$: arbitrary source-to-target tgds;

$\mathcal{L}_{\text{full}}$: full tgds;

$\mathcal{L}_{\text{acyc}}$: acyclic tgds;

$\mathcal{L}_{\text{facyc}}$: full and acyclic tgds.

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# Pertinence of a set of tgds

## Example

| R |
|---|
| a |
| b |
| c |
| d |

| R′ | |
|---|---|
| a | a |
| b | b |
| c | a |
| d | d |
| g | h |

$\Sigma_0 = \varnothing$

$\Sigma_1 = \{\forall x\ R(x) \rightarrow R'(x,x)\}$

$\Sigma_2 = \{\forall x\ R(x) \rightarrow \exists y\ R'(x,y)\}$

$\Sigma_3 = \{\forall x_1 \forall x_2\ R(x_1) \wedge R(x_2) \rightarrow R'(x_1,x_2)\}$

$\Sigma_4 = \{\exists y_1 \exists y_2\ R'(y_1,y_2)\}$

# Pertinence of a set of tgds

## Example

| $R$ |
|---|
| a |
| b |
| c |
| d |

| $R'$ | |
|---|---|
| a | a |
| b | b |
| c | a |
| d | d |
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$$\Sigma_4 = \{\exists y_1 \exists y_2\ R'(y_1,y_2)\}$$

# Idea

- Size of a formula: number of occurrences of variables and constants.
- Cost of a schema mapping $\Sigma$: Size of the minimum repair of $\Sigma$ that is valid and explains all facts of $J$.
- Types of repairs considered:
  - "fix" a universal quantifier by adding conditions ($x = a$ or $x \neq a$);
  - "fix" an existential quantifier by giving corresponding constants ($\tau(\mathbf{x}) \rightarrow y = a$ with $\tau$ a conjunction of conditions on universally quantified variables);
  - add ground facts to the target instance.
- The problem is then to find a schema mapping of minimal cost.

# Example cost computation

## Example

| $R$ |
|---|
| a |
| b |
| c |
| d |

| $R'$ | |
|---|---|
| a | a |
| b | b |
| c | a |
| d | d |
| g | h |

$$\forall x\ R(x) \rightarrow R'(x, x)$$

Predicted $R'$

| | |
|---|---|
| a | a |
| b | b |
| c | c |
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# Example cost computation

Example

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| a |
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| a | a |
| b | b |
| c | a |
| d | d |
| g | h |

$$\forall x\ R(x) \wedge x \neq c \rightarrow R'(x,x)$$

Predicted $R'$

| | |
|---|---|
| a | a |
| b | b |
| d | d |

# Example cost computation

## Example

| $R$ |
|---|
| a |
| b |
| c |
| d |

| $R'$ | |
|---|---|
| a | a |
| b | b |
| c | a |
| d | d |
| g | h |

$$\forall x\ R(x) \land x \neq c \to R'(x,x)$$

$$R'(c,a)$$

Predicted $R'$

| | |
|---|---|
| a | a |
| b | b |
| c | a |
| d | d |

# Example cost computation

## Example

| $R$ |
| --- |
| a |
| b |
| c |
| d |

| $R'$ | |
| --- | --- |
| a | a |
| b | b |
| c | a |
| d | d |
| g | h |

$$\forall x\ R(x) \land x \neq c \rightarrow R'(x, x)$$

$R'(c, a)$

$R'(g, h)$

Predicted $R'$

| | |
| --- | --- |
| a | a |
| b | b |
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# Example cost computation

## Example

| $R$ |
| --- |
| a |
| b |
| c |
| d |

| $R'$ | |
| --- | --- |
| a | a |
| b | b |
| c | a |
| d | d |
| g | h |

$$\forall x\ R(x) \land x \neq c \rightarrow R'(x, x)$$

$$\exists y_1 \exists y_2\ R'(y_1, y_2) \land y_1 = c \land y_2 = a$$

$$\exists y_1 \exists y_2\ R'(y_1, y_2) \land y_1 = g \land y_2 = h$$

Predicted $R'$

| | |
| --- | --- |
| a | a |
| b | b |
| c | c |
| d | d |
| g | h |

# Example cost computation

Example

| $R$ |
| --- |
| a |
| b |
| c |
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| $R'$ | |
| --- | --- |
| a | a |
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$$\exists y_1 \exists y_2\ R'(y_1, y_2) \wedge y_1 = c \wedge y_2 = a$$
$$\exists y_1 \exists y_2\ R'(y_1, y_2) \wedge y_1 = g \wedge y_2 = h$$

Cost: 17

Predicted $R'$

| | |
| --- | --- |
| a | a |
| b | b |
| c | c |
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# Problems considered

Decision problems of interest:

**Cost:** Is the cost of a given schema mapping less than $K$?

**Optimality:** Is a given schema mapping optimal?

Complexity? Algorithms?

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# Outline

# Behavior for simple operators

Consider the elementary operators of the relational algebra:

- Projection
- Intersection
- Selection (conjunction of atomic conditions)
- Cross Product
- Join (on a given attribute)

**Theorem**

*For any elementary operator $\gamma$, the tgd naturally associated with $\gamma$ is optimal with respect to $(I, \gamma(I))$ (or $(\gamma(J), J)$), under some basic assumptions.*

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# Examples of naturally associated tgds

## Examples

| | Condition | $I$ and $J$ | Optimal tgd |
|---|---|---|---|
| Projection | $I \neq \varnothing$<br>$\pi_1(J) \cap \pi_2(J) = \varnothing$,<br>$\|\pi_1(J)\| \geqslant 2$ | $J = \pi_1(I)$<br>$I = \pi_1(J)$ | $R(x, y) \rightarrow R'(x)$<br>$R(x) \rightarrow \exists y\ R'(x, y)$ |
| Selection | $\|\sigma_\varphi(I)\| \geqslant \frac{\mathsf{size}(\varphi)+2}{3}$<br>$\sigma_\varphi(J) \neq \varnothing$ | $J = \sigma_\varphi(I)$<br>$I = \sigma_\varphi(J)$ | $R(x) \rightarrow R'(x)$<br>$R(x) \rightarrow R'(x)$ |
| Product | $R_1^I \neq \varnothing$, $R_2^I \neq \varnothing$<br>${R'_1}^J \neq \varnothing$, ${R'_2}^J \neq \varnothing$ | $J = R_1^I \times R_2^I$<br>$I = {R'_1}^J \times {R'_2}^J$ | $R_1(x) \wedge R_2(y) \rightarrow R'(x, y)$<br>$R(x, y) \rightarrow R'_1(x) \wedge R'_2(y)$ |

# The Polynomial Hierarchy

| | |
|---|---|
| P | polynomial deterministic algorithm |
| NP | polynomial non-deterministic algorithm |
| coNP | complement NP |
| DP | problems expressible as the conjunction of a NP and a coNP problem |
| $\Sigma_2^P$ | polynomial non-deterministic with $\Sigma_1^P$ oracle |
| $\Pi_2^P$ | complement $\Sigma_2^P$ |
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Union of all these classes: PH $\subseteq$ PSPACE, the polynomial hierarchy.

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# (Combined) Complexity Results

| | $\mathcal{L}_{\text{tgd}}$ | $\mathcal{L}_{\text{full}}$ |
|---|---|---|
| Cost | $\Sigma_3^P$, $\Pi_2^P$-hard | $\Sigma_2^P$, DP-hard |
| Optimality | $\Pi_4^P$, DP-hard | $\Pi_3^P$, DP-hard |

| | $\mathcal{L}_{\text{acyc}}$ | $\mathcal{L}_{\text{facyc}}$ |
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# Vertex-Cover in $r$-partite $r$-uniform hypergraph

Vertex-Cover: find a set of vertices of minimal size that cover all (hyper)edges in a (hyper)graph.

- NP-complete for general (hyper)graphs.
- PTIME for bipartite graphs (Kőnig's theorem).

**Lemma**

*Vertex-Cover is NP-complete for $r$-partite $r$-uniform hypergraphs for $r \geqslant 3$.*

$r$-partite: partition of the set of vertices into $r$ sets, with no hyperedge spanning two vertices of the same set.

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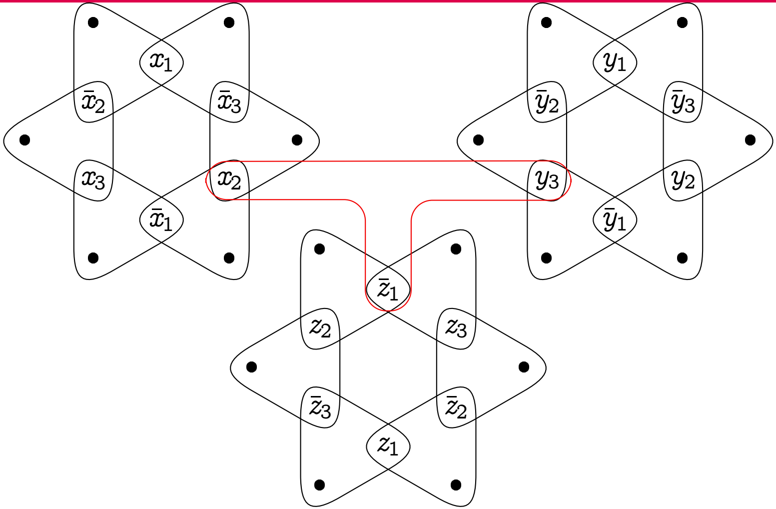
# Encoding of 3-SAT




$\neg z \lor x \lor y$

# Cost is NP-hard for $\mathcal{L}_{\text{facyc}}$

Reduction from Vertex-Cover in 3-partite 3-uniform hypergraphs.

Without $x = a$ repairs on the left-hand side of a tgd:

- $R(x_1, x_2, x_3) \rightarrow R'(x_1)$
- Source instance: hypergraph
- Target instance: empty

Cost: size of the tgd plus twice the minimum size of a vertex cover.

With $x = a$ repairs: a little more difficult, but feasible!



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With $x = a$ repairs: a little more difficult, but feasible!

# Cost is NP-hard for $\mathcal{L}_{\text{facyc}}$

Reduction from Vertex-Cover in 3-partite 3-uniform hypergraphs.

Without $x = a$ repairs on the left-hand side of a tgd:

- $R(x_1, x_2, x_3) \to R'(x_1)$
- Source instance: hypergraph
- Target instance: empty

Cost: size of the tgd plus twice the minimum size of a vertex cover.

With $x = a$ repairs: a little more difficult, but feasible!

# Outline

# Extension to Relational Calculus

- Definition of repairs can be extended to relational calculus.
- Same definition of cost, optimality.
- Cost is not recursive (but co-r.e.).
- Computability of Optimality: open (!).

# Other Cost Functions

Why not counting the number of tuples to add or remove in $J$?

... because it can be exponential in the size of the schema mapping!

Why not counting the number of tuples to add or remove in $I$ or $J$?

... because selections are not captured!



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- Link with Inductive Logic Programming?
- Heuristics?
- Approximation algorithms?
- Generalization of acyclicity?

Merci.