SCHOTTKY NOISE AND BEAM TRANSFER FUNCTION DIAGNOSTICS*)

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The following is a new version of the first part of Section 3.3 of the report cited in the footnote.

3.3 Bunched-beams signal processing

As all Schottky lines in a frequency interval f_b are correlated (see section 1), it is interesting to sample the beam signal at the revolution frequency. All lines will be folded in the base band giving a much better signal to noise ratio as will be shown in the following.

In the case of a detector with a bandwidth larger than f_b, the Schottky signal appears like a noise burst of length $2\hat{\tau}_m$. Its peak power is the same as that of an unbunched beam having the same line density, i.e. with a total number of particles $N_{eff} = N / 2\hat{\tau}_m f_0$. Therefore, when sampling the Schottky signal at the revolution frequency the resulting signal to noise is the same as if it were given by an unbunched beam of N_{eff} ($N_{eff} >> N$) particles.

With a detector having a slower response, like a travelling wave pick-up, the output RF burst (Fig. 23) can be much longer than the bunch itself ($\tau >> 2\tau_m$), but nevertheless very short compared to $1/f_0$. Take, for instance a transverse detector giving an output voltage:

$$V = 2\, S_\Delta\, e\, \frac{x}{\tau} \cos(\omega_\beta t + \Psi) \cos\omega_{pu} t \tag{63}$$

for a single particle which performs a betatron oscillation $x \cos(\omega_\beta t + \Psi)$. ω_{pu} is the centre frequency of the detector.

The average power, per particle is therefore:

$$\langle v^2 \rangle = S_\Delta{}^2\, e^2\, \frac{x^2}{\tau^2}$$

and for N particles:

$$\langle v^2 \rangle = N\, S_\Delta{}^2\, e^2\, \frac{\langle x^2 \rangle}{\tau^2} \quad . \tag{64}$$

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* New version and addenda to sections of D. Boussard, Schottky Noise and Beam Transfer Function Diagnostics, Proc. CAS, Advanced Accelerator Physics, CERN 87-03 (1987).

The thermal noise power of the amplifier, referred to the input, for an amplifier bandwidth B can be written as:

$$\langle v_{th}^2 \rangle = F\, k_o t_o R_o B \quad .$$

The power signal to noise ratio, during the time interval τ is therefore:

$$\frac{1}{U} = \frac{N\, e^2 S_\Delta^{\,2} \langle x^2 \rangle}{F\, \tau^2\, k_o\, t_o\, R_o\, B} \; . \qquad (65)$$

This is also the signal to noise ratio after sampling. We can select B ($B = B_{opt}$) to optimize 1/U. B_{opt} is the minimum bandwidth for which the useful signal is not reduced significantly. This happens if the rise time of the band limited RF burst is of the order of its length: $1/B \approx \tau$, as illustrated in Fig. 27. More precisely B_{opt} is that of the so called "optimum filter" (radar terminology) for which the impulse response is the time reversed image of the RF burst. With that condition (65) becomes:

$$\frac{1}{U} = \frac{1}{\tau f_o}\, N\, \frac{e^2\, f_o\, S_\Delta^{\,2}\, x^2}{F\, k_o\, t_o\, R_o} \qquad (66)$$

which is the same as for the debunched beam case, except for the enhancement factor $1/\tau f_o$ which can be much larger than unity.[10)]

<u>Addenda to Section 4.2</u>

More generally the transverse kicker sensitivity defined by:

$$K_\perp = \frac{1}{V_k} \frac{\upsilon}{e} \Delta p_\perp \qquad (77.1)$$

where V_K is the voltage applied to the kicker, can be related to the pick-up sensitivity S_Δ of the same structure.

Combining Eqs. (70) and (75), gives:

$$\frac{\upsilon}{e} \Delta p_x = \int_z \frac{\upsilon}{j\omega} \frac{dE_z}{dx}\, dz \; . \qquad (77.2)$$

When applying the reciprocity theorem to a transverse pick-up, we already obtained (Eq. (43), modified for the transverse case):

$$I_1\, V_{out} = \int_z \frac{dE_z}{dx}\, i_b\, \Delta_x\, dz \qquad (77.3)$$

I_1 being the current source producing the field E_z. For a matched pick-up or kicker, I_1 is split equally between the structure and the load resistor, which gives:

$$V_K = I_1 R_o/2 \quad . \tag{77.4}$$

Combining Eqs. (77.2), (77.3), (77.4) and (33) one obtains:

$$K_\perp = -2j \frac{\upsilon}{\omega} \frac{S_\Delta}{R_o} \tag{77.5}$$

which relates pick-up and kicker sensitivities.

For the "TEM" travelling wave kicker, which is the same structure as the one shown in Fig. 15b, combining Eqs. (34) and (A5) gives:

$$K_\perp = -j \frac{\lambda}{\pi} \frac{\sqrt{2}}{h} \left(\sin \frac{2\pi\ell}{\lambda}\right) \tanh \left(\frac{\pi w}{h}\right) \tag{77.6}$$

and finally:

$$K_\perp = -j\, 2 \sqrt{2} \frac{\ell}{h} \tanh \left(\frac{\pi w}{h}\right) \frac{\sin 2\pi\, \ell/\lambda}{2\pi\, \ell/\lambda} \, . \tag{77.7}$$