Scintillation index of optical plane wave propagating through non Kolmogorov moderate-strong turbulence

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ABSTRACT

An optical plane wave propagating through atmospheric turbulence is affected by irradiance fluctuations known as scintillation. The scintillation index of an optical wave in strong turbulence can be analyzed by extended Rytov theory, which uses filter functions to eliminate the effect of cell turbulence sizes that do not contribute to scintillation, and it already has been calculated by Kolmogorov's power spectral density model. However several experiments showed that Kolmogorov theory is sometimes incomplete to describe atmospheric turbulence properly. In this paper, for a horizontal path, we use extended Rytov theory to carry out plane wave scintillation index analysis in non Kolmogorov strong turbulence. We do it using a non Kolmogorov power spectrum which uses a generalized exponent factor and a generalized amplitude factor. Although our final expressions for the scintillation have been obtained by extended Rytov theory, which is necessary to adopt in strong turbulence conditions, they reduce to the proper results also in weak turbulence.

Keywords: Atmospheric propagation, non Kolmogorov spectrum, scintillation, strong optical turbulence

1. INTRODUCTION

Kolmogorov's power spectrum of refractive index fluctuations is widely accepted and has been applied extensively in studies of optical and radio wave propagation in the atmosphere. Unfortunately, recent experiments indicated that turbulence in the upper troposphere and stratosphere deviates from predictions of the Kolmogorov model [5][6][9][10]. Some anomaly behavior [3] seems to occur when the atmosphere is extremely stable because under such condition the turbulence is no longer homogeneous in three dimensions since the vertical component is suppressed. It has been shown [4] that for such two dimensional turbulence, coherent vortices can develop that reduce the rate of the energy cascade from larger to smaller scales. As a result Kolmogorov turbulence will not develop. In addition, anisotropy in stratospheric turbulent inhomogeneities has been experimentally investigated [5][7][8][11][12].
In this paper we use a theoretical spectrum model defined by [13]

$$\Phi_n(\kappa,\alpha) = A(\alpha)\cdot \tilde{C}_n^2 \cdot \kappa^{-\alpha}, \quad \kappa > 0,\ 3 < \alpha < 4 \tag{1}$$

where $\tilde{C}_n^2 = \beta \cdot C_n^2$ is a generalized structure parameter with units $\mathrm{m}^{3-\alpha}$; Here, β is a constant equal to unity when $\alpha = 11/3$, but otherwise has units $\mathrm{m}^{-\alpha+11/3}$; $A(\alpha)$ is defined by

$$A(\alpha) = \frac{1}{4\pi^2}\Gamma(\alpha-1)\cos\left(\frac{\alpha\pi}{2}\right), \quad 3 < \alpha < 4 \tag{2}$$

and the symbol $\Gamma(x)$ in the last expression is the gamma function.
This non Kolmogorov spectrum reduces to one of Kolmogorov only for $\alpha = 11/3$.

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The exponent α can assume all the values between the range 3 to 4. Using this new spectrum, for a horizontal path, we analyzed the impact of the exponent's variation on the scintillation index in moderate strong turbulence. Although our final expressions for the scintillation will be obtained by extended Rytov theory [1], which is necessary to adopt in strong turbulence conditions, they reduce to the proper results also in weak turbulence.

2. SCINTILLATION INDEX AND EXTENDED RYTOV THEORY

The scintillation index is one of the most important parameters that is usually used to characterize laser beam propagation trough random media. It is denoted by

$$\sigma_I^2 = \frac{\left\langle I^2 \right\rangle}{\left\langle I \right\rangle^2} - 1 \tag{3}$$

where I is the optical field irradiance and the brackets, $\langle \ \rangle$, denote an ensemble, or long spatial distance, average. For a plane wave model along a horizontal path and Kolmogorov turbulence, the scintillation index is given by Rytov variance which is defined by

$$\sigma_R^2 = 1.23 \cdot C_n^2 \cdot k^{\frac{7}{6}} \cdot L^{\frac{11}{6}} \tag{4}$$

where the index-of-refraction parameter C_n^2 is a measure of refractive index fluctuations, k denotes the optical wave number, and L is the propagation path.

In the more general case of Kolmogorov spectrum, we have recently defined in [13] a non Kolmogorov Rytov variance by

$$\tilde{\sigma}_{Rytov}^2(\alpha) = 1.23 \cdot \tilde{C}_n^2 \cdot k^{3-\frac{\alpha}{2}} \cdot L^{\frac{\alpha}{2}}, \quad 3 < \alpha < 4 \tag{5}$$

The expression (5) reduces to (4) for $\alpha = 11/3$.

Rytov variance in Kolmogorov turbulence can be used to establish the scintillation regime; for example the weak scintillation regime is defined by this index value being less than or equal to unity.

However we can not use expression (5) to establish if we are in weak or strong turbulence also in non Kolmogorov case because scintillation index (3) assumes the same value of (5) only for $\alpha = 11/3$.

In fact, the scintillation index for plane wave, without aperture averaging and in weak turbulence, is given by [13]

$$\sigma_{I\_pl}^2(\alpha) = -6.5 \cdot \pi^2 \cdot A(\alpha) \cdot \Gamma\left(1 - \frac{\alpha}{2}\right) \cdot \frac{1}{\alpha} \cdot \sin\left(\alpha \cdot \frac{\pi}{4}\right) \cdot \tilde{\sigma}_{Rytov}^2(\alpha) \tag{6}$$

Hence, to use expression (3) to establish if we are in weak or strong turbulence in the non Kolmogorov case, we redefine a non Kolmogorov Rytov variance by

$$\tilde{\sigma}_R^2(\alpha) = \sigma_{I\_pl}^2(\alpha) = \tilde{\sigma}_{Rytov}^2(\alpha) \cdot R(\alpha) \tag{7}$$

where $R(\alpha) = -6.5 \cdot \pi^2 \cdot A(\alpha) \cdot \Gamma\left(1 - \frac{\alpha}{2}\right) \cdot \frac{1}{\alpha} \cdot \sin\left(\alpha \cdot \frac{\pi}{4}\right)$ (8)

Note that for Kolmogorov case, $R(11/3) = 1$.

By increasing L or $\tilde{C}_n^2$, or both, and depending from alpha value, one will eventually break into moderate to strong turbulence and consequently the focusing properties that are prevalent in weak regime deteriorates as the scintillation index increases. We use the extended Rytov theory [1], thus we assume that the field irradiance can be expressed as $I = X \cdot Y$, where X represents large-scale (refraction properties) and Y denotes small-scale (diffraction properties) fluctuations. By assuming both of these fluctuations are statistically independent from each other and that $\langle I \rangle = 1$, eq.(1) takes the following form

$$\sigma_I^2(\alpha) = \langle X^2 \rangle \langle Y^2 \rangle - 1 = \left(1 + \sigma_X^2(\alpha)\right)\left(1 + \sigma_Y^2(\alpha)\right) - 1 = \sigma_X^2(\alpha) + \sigma_Y^2(\alpha) + \sigma_X^2(\alpha)\sigma_Y^2(\alpha) \tag{9}$$

where
$\sigma_X^2(\alpha) = \exp(\sigma_{\ln X}^2) - 1$ and $\sigma_Y^2(\alpha) = \exp(\sigma_{\ln Y}^2) - 1$ are the respective variances of X and Y. Expressing these variances in terms of log-irradiance allows us to rewrite eq. (9) as

$$\sigma_I^2(\alpha) = \exp\left[\sigma_{\ln X}^2(\alpha) + \sigma_{\ln Y}^2(\alpha)\right] - 1 \tag{10}$$

where

$$\sigma_{\ln X}^2(\alpha) = 8\pi^2 k^2 \int_0^L \int_0^\infty \kappa \cdot \Phi_n(\kappa,\alpha) \cdot G_X(\kappa,\alpha)\left[1 - \cos\left(\frac{\kappa^2 z}{k}\right)\right] dk dz \tag{11}$$

$$\sigma_{\ln Y}^2(\alpha) = 8\pi^2 k^2 \int_0^L \int_0^\infty \kappa \cdot \Phi_n(\kappa,\alpha) \cdot G_Y(\kappa,\alpha)\left[1 - \cos\left(\frac{\kappa^2 z}{k}\right)\right] dk dz \tag{12}$$

The non Kolmogorov spectrum appearing in (11) and (12) is defined by (1).
The large scale and small scale filter functions appearing in (11) and (12) are defined, respectively, by

$$G_X(\kappa,\alpha) = \exp\left(-\frac{\kappa^2}{\kappa_X^2}\right), \tag{13}$$

$$G_Y(\kappa,\alpha) = \frac{\kappa^\alpha}{\left(\kappa^2 + \kappa_Y^2\right)^{\frac{\alpha}{2}}}. \tag{14}$$

The quantities κ_X and κ_Y represent cutoff spatial frequencies that eliminate mid range scale size effects under moderate-to-strong fluctuations. Because the low-pass and high-pass spatial frequency cutoffs appearing in the filter functions (13) and (14) are directly related to the correlation width and scattering disk of the propagating optical wave [1], we assume at any distance L into the random medium that exists an effective scattering disk L/kl_X and an effective correlation width l_Y related, respectively, to the cutoff wave numbers according to

$$\frac{L}{k \cdot l_X} = \frac{1}{\kappa_X} \cong \begin{cases} \sqrt{\frac{L}{k}}, \tilde{\sigma}_R^2(\alpha) << 1 \\ \frac{L}{k\rho_0(\alpha)}, \tilde{\sigma}_R^2(\alpha) >> 1 \end{cases} \tag{15}$$

$$l_Y = \frac{1}{\kappa_Y} \cong \begin{cases} \sqrt{\frac{L}{k}}, \tilde{\sigma}_R^2(\alpha) << 1 \\ \rho_0(\alpha), \tilde{\sigma}_R^2(\alpha) >> 1 \end{cases} \tag{16}$$

3. SCINTILLATION INDEX IN SATURATION REGIME

In the saturation regime, such is for high value of the Rytov variance, the scintillation index for an unbounded plane wave can be expressed as [1]

$$\sigma_I^2(\alpha) = 1 + 32\pi^2 k^2 L \int_0^1 \int_0^\infty \kappa \cdot \Phi_n(\kappa,\alpha) \cdot \sin^2\left[\frac{L\kappa^2}{2k} w(\xi,\xi)\right] \cdot$$
$$\times \exp\left\{-\int_0^1 D_S\left[\frac{L\kappa}{k} \cdot w(\tau,\xi)\right] d\tau\right\} d\kappa d\xi, \tilde{\sigma}_R^2 >> 1 \tag{17}$$

where τ is a normalized distance variable and the exponential function acts like a low-pass spatial filter defined by the plane wave structure function of phase $D_S(\rho)$. The function $w(\tau,\xi)$ is defined by

$$w(\tau,\xi) = \begin{cases} \tau, \tau < \xi \\ \xi, \tau > \xi \end{cases} \tag{18}$$

The low-pass spatial filter in (17) ensures that $L\kappa^2/k << 1$, which represents a geometrical optics approximation. Under the assumption that the inner scale of turbulence is smaller than the spatial coherence radius of the optical plane wave, than based on our non Kolmogorov spectrum (12) with structure function $D_S(\rho)$ given by [14]

$$D_S(\rho) = 4\pi^2 A(\alpha) \tilde{C}_n^2 k^2 L \cdot \left[-\frac{\Gamma\left(1-\frac{\alpha}{2}\right)}{\Gamma\left(\frac{\alpha}{2}\right)} \cdot \left(\frac{\rho^2}{4}\right)^{\frac{\alpha}{2}-1}\right], \tag{19}$$

it follows that

$$\int_0^1 D_S\left[\frac{L\kappa}{k} w(\tau,\xi)\right] d\tau$$
$$= -\frac{\pi^2}{1.23} \cdot 4^{2-\frac{\alpha}{2}} \cdot \frac{\Gamma\left(1-\frac{\alpha}{2}\right)}{\Gamma\left(\frac{\alpha}{2}\right)} \cdot \frac{A(\alpha)}{R(\alpha)} \cdot \tilde{\sigma}_R^2 \cdot \left(\frac{L}{k}\right)^{\frac{\alpha}{2}-1} \kappa^{\alpha-2} \cdot \xi^{\alpha-2} \cdot \left(1 + \frac{2-\alpha}{\alpha-1}\xi\right) \tag{20}$$

Also, the sine function in (17) may be approximated by its leading term, which yields

$$\sin^2\left(\frac{L\kappa^2\xi}{2k}\right)\cong\frac{L^2\kappa^4\xi^2}{4k^2} \tag{21}$$

and, consequently, (17) leads to

$$\sigma_I^2(\alpha)=1+\frac{\gamma(\alpha)I(\alpha)}{\tilde{\sigma}_R^{\frac{4(4-\alpha)}{\alpha-2}}},\tilde{\sigma}_R^2>>1 \tag{22}$$

where

$$I(\alpha)=\int_0^1\xi^{\alpha-4}\left(1+\frac{2-\alpha}{\alpha-1}\xi\right)^{\frac{\alpha-6}{\alpha-2}}d\xi \tag{23}$$

$$\gamma(\alpha)=\frac{64.2}{\alpha-2}\cdot\left[\frac{A(\alpha)}{R(\alpha)}\right]^{\frac{2(\alpha-4)}{\alpha-2}}\cdot\Gamma\left(\frac{6-\alpha}{\alpha-2}\right)\cdot\left[-\frac{\Gamma\left(1-\frac{\alpha}{2}\right)}{\Gamma\left(\frac{\alpha}{2}\right)}\right]^{\frac{\alpha-6}{\alpha-2}}\cdot2^{\frac{(\alpha-6)\cdot(7-\alpha)}{\alpha-2}} \tag{24}$$

4. SCINTILLATION INDEX ANALYSIS

In the development of a scintillation model below we select the cutoff frequencies κ_X and κ_Y on the basis of the assumed asymptotic behavior in (15) and (16) as well as established behavior of the scintillation index in this asymptotic regime provided by (22).
In particular, in weak and strong fluctuation regimes, the assumed scintillation index (9) reduces to

$$\sigma_I^2(\alpha)\cong\begin{cases}\sigma_{\ln X}^2(\alpha)+\sigma_{\ln Y}^2(\alpha),\tilde{\sigma}_R^2(\alpha)<<1\\1+2\sigma_{\ln X}^2(\alpha),\tilde{\sigma}_R^2(\alpha)>>1\end{cases} \tag{25}$$

where the second expression in (25) is based on the limiting value $\sigma_Y^2(\alpha)=\exp\left[\sigma_{\ln Y}^2(\alpha)\right]-1\to1,$ or $\sigma_{\ln Y}^2(\alpha)\to\ln2$ in the saturation regime. Thus, we need to determine the large scale and small scale log irradiance so that

$$\begin{aligned}&\sigma_{\ln X}^2(\alpha)+\sigma_{\ln Y}^2(\alpha)=\tilde{\sigma}_R^2(\alpha),\tilde{\sigma}_R^2(\alpha)<<1\\&\sigma_{\ln X}^2(\alpha)=\frac{\gamma(\alpha)I(\alpha)}{2\tilde{\sigma}_R^{\frac{4(4-\alpha)}{\alpha-2}}},\tilde{\sigma}_R^2(\alpha)>>1\end{aligned} \tag{26}$$

4.1 Large-scale log irradiance

For the large-scale log irradiance (11) we can use the geometrical optics approximation, which in this case corresponds to

$$1-\cos\left(\frac{\kappa^2 z}{k}\right)\cong\frac{1}{2}\left(\frac{\kappa^2 z}{k}\right)^2, \kappa << \kappa_X. \tag{27}$$

Consequently, by using the approximation (27) and introducing the parameter changes $\xi=\frac{z}{L}$ and $\eta=\frac{L\kappa^2}{k}$, the large-scale log-irradiance variance reduces to

$$\begin{aligned}\sigma^2_{\ln X}(\alpha) &\cong 8\pi^2 k^2\int_0^L\int_0^\infty \kappa\cdot A(\alpha)\cdot\tilde{C}_n^2\cdot\kappa^{-\alpha}\cdot\exp\left(\frac{\kappa^2}{\kappa_x^2}\right)\frac{1}{2}\left(\frac{\kappa^2 z}{k}\right)^2 dkdz\\ &\cong 2\pi^2 A(\alpha)\tilde{C}_n^2 L^{\frac{\alpha}{2}} k^{3-\frac{\alpha}{2}}\int_0^1\xi^2 d\xi\int_0^\infty\eta^{2-\frac{\alpha}{2}}\exp\left(-\frac{\eta}{\eta_X}\right)d\eta\\ &\cong 5.35\cdot\frac{A(\alpha)}{R(\alpha)}\cdot\Gamma\left(3-\frac{\alpha}{2}\right)\cdot\tilde{\sigma}_R^2\cdot\eta_X^{3-\frac{\alpha}{2}}\end{aligned} \tag{28}$$

where $\eta_X=\frac{L{\kappa_X}^2}{k}$. To determine the cutoff wave number κ_X, we use the asymptotic result (15) according to

$$\frac{1}{\kappa_X^2}=c_1(\alpha)\frac{L}{k}+c_2(\alpha)\left(\frac{L}{k\rho_0(\alpha)}\right)^2 \tag{29}$$

In order to calculate the scaling constant $c_1(\alpha)$ we use the asymptotic behavior given by (26) under weak fluctuations. Specifically, under weak fluctuations, we tacitly make the assumption that large scale effects account for roughly half of the total scintillation index. However this assumption is somewhat arbitrary, it is not necessarily an optimum choice. Imposing this condition

$$\sigma^2_{\ln X}(\alpha)\cong 5.35\cdot\frac{A(\alpha)}{R(\alpha)}\cdot\Gamma\left(3-\frac{\alpha}{2}\right)\cdot\tilde{\sigma}_R^2\cdot\left(\frac{L\kappa_X^2}{k}\right)^{3-\frac{\alpha}{2}}\cong 0.49\tilde{\sigma}_R^2, \tilde{\sigma}_R^2<<1 \tag{30}$$

we carry out

$$c_1(\alpha)=\left(\frac{0.092\cdot R(\alpha)}{A(\alpha)\Gamma\left(3-\frac{\alpha}{2}\right)}\right)^{\frac{2}{\alpha-6}} \tag{31}$$

In order to calculate the scaling constant $c_2(\alpha)$ we use the asymptotic behavior given by (26) under strong fluctuations. Imposing this condition

$$\sigma^2_{\ln X}(\alpha)\cong 5.35\cdot\frac{A(\alpha)}{R(\alpha)}\cdot\Gamma\left(3-\frac{\alpha}{2}\right)\cdot\tilde{\sigma}_R^2\cdot\left(\frac{L\kappa_X^2}{k}\right)^{3-\frac{\alpha}{2}}\cong\frac{\gamma(\alpha)I(\alpha)}{2{\tilde{\sigma}_R}^{\frac{4(4-\alpha)}{\alpha-2}}}, \tilde{\sigma}_R^2>>1 \tag{32}$$

we carry out

$$c_2(\alpha)=\left(\frac{\gamma(\alpha)I(\alpha)}{10.7\cdot\frac{A(\alpha)}{R(\alpha)}\Gamma\left(3-\frac{\alpha}{2}\right)}\right)^{\frac{2}{\alpha-6}}\cdot\left(\frac{-0.0156\cdot 4^{\frac{\alpha}{2}}\Gamma\left(\frac{\alpha}{2}\right)}{\frac{A(\alpha)}{R(\alpha)}\Gamma\left(1-\frac{\alpha}{2}\right)}\right)^{\frac{2}{\alpha-2}} \tag{33}$$

At this point, replacing (31) and (33) in (29), remembering that $\eta_X=\frac{L\kappa_X{}^2}{k}$, and by using the (28), we deduce the large-scale log-irradiance variance which is given by

$$\sigma^2_{\ln X}(\alpha)\cong\frac{0.49\tilde{\sigma}_R^2}{\left(1+f_X(\alpha)\tilde{\sigma}_R^{\frac{4}{\alpha-2}}\right)^{3-\frac{\alpha}{2}}} \tag{34}$$

where

$$f_X(\alpha)=\left(1.016\cdot\gamma(\alpha)\cdot I(\alpha)\right)^{\frac{2}{\alpha-6}} \tag{35}$$

We plot, in figure 1, the large scale irradiance fluctuations $\sigma_X^2(\alpha)$ as a function of the strength of turbulence for several alpha values. We deduce from figure 1 that, in moderate-strong turbulence, for alpha values higher than 11/3, large scale irradiance fluctuations remarkable increase with respect to Kolmogorov case; instead for alpha values lower than 11/3, large scale irradiance fluctuations decrease with respect to Kolmogorov case.

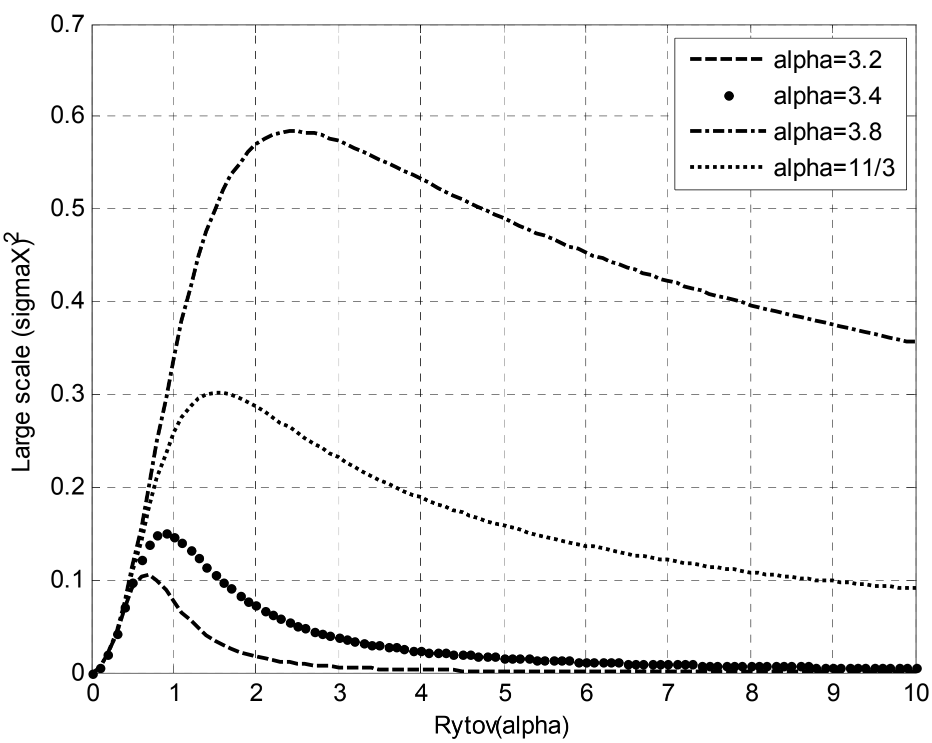

Figure 1- Large scale irradiance fluctuations as a function of the strength of turbulence for several alpha values

4.2 Small-scale log irradiance

In the small-scale log-irradiance variance (12), we use the approximation

$$\int_0^L\int_0^\infty \frac{\kappa}{\left(\kappa^2+\kappa_y^2\right)^{\frac{\alpha}{2}}}\cdot\left[1-\cos\left(\frac{\kappa^2 z}{k}\right)\right]dkdz \cong \int_0^L\int_0^\infty \frac{\kappa}{\left(\kappa^2+\kappa_y^2\right)^{\frac{\alpha}{2}}}dkdz, \kappa_y >> \sqrt{\frac{k}{L}} \tag{36}$$

That is, at high wave numbers $\left(\kappa > \kappa_y >> \sqrt{k/L}\right)$ the integral of the cosine term in (36) with respect to z yields the form $(\sin\eta)/\eta$ which, for large $\eta = L\kappa^2/k$, tends to zero. Hence, using (36) the small-scale-irradiance scintillation (12) leads to

$$\begin{aligned}\sigma_{\ln y}^2(\alpha) &\cong 8\pi^2 k^2 \int_0^L\int_0^\infty \kappa\cdot A(\alpha)\cdot\tilde{C}_n^2\cdot\kappa^{-\alpha}\cdot\frac{\kappa^\alpha}{\left(\kappa^2+\kappa_y^2\right)^{\frac{\alpha}{2}}}\left[1-\cos\left(\frac{\kappa^2 z}{k}\right)\right]dkdz \\ &\cong 8\pi^2\cdot k^2\cdot A(\alpha)\cdot\tilde{C}_n^2\cdot L\cdot\int_0^1 d\xi\int_0^\infty \frac{\kappa}{\left(\kappa^2+\kappa_y^2\right)^{\frac{\alpha}{2}}}dk \\ &\cong 4\pi^2\cdot A(\alpha)\cdot\tilde{C}_n^2\cdot L^{\frac{\alpha}{2}}\cdot k^{3-\frac{\alpha}{2}}\int_0^\infty\left(\eta+\eta_Y\right)^{-\frac{\alpha}{2}}d\eta \\ &\cong 64.2\cdot\frac{A(\alpha)}{R(\alpha)}\cdot\frac{1}{\alpha-2}\cdot\tilde{\sigma}_R^2\cdot\eta_Y^{1-\frac{\alpha}{2}}\end{aligned} \tag{37}$$

where $\eta_Y = \dfrac{L{\kappa_Y}^2}{k}$

Similar to the large-scale case, to determine the cutoff wave number κ_Y, we use the asymptotic results (16) according to

$$\kappa_Y^2 = c_3(\alpha)\frac{k}{L} + c_4(\alpha)\cdot\frac{1}{\rho_0^2(\alpha)} \tag{38}$$

In order to calculate the scaling constant $c_3(\alpha)$ we use the asymptotic behavior given by (25) under weak fluctuations and we tacitly make the assumption that small scale effects account for roughly half of the total scintillation index. However, again, this assumption is somewhat of arbitrary, it is not necessarily an optimum choice. Imposing this condition

$$\sigma_{\ln Y}^2(\alpha) \cong 64.2\cdot\frac{A(\alpha)}{R(\alpha)}\cdot\frac{1}{\alpha-2}\cdot\tilde{\sigma}_R^2\cdot\eta_Y^{1-\frac{\alpha}{2}} \cong 0.51\tilde{\sigma}_R^2, \tilde{\sigma}_R^2 << 1 \tag{39}$$

we carry out

$$c_3(\alpha) = \left(\frac{0.0079\cdot(\alpha-2)\cdot R(\alpha)}{A(\alpha)}\right)^{\frac{2}{2-\alpha}} \tag{40}$$

In order to calculate the scaling constant $c_4(\alpha)$ we use the asymptotic behavior given by (26) under strong fluctuations. Imposing this condition

$$\sigma_{\ln Y}^2(\alpha) \cong 64.2 \cdot \frac{A(\alpha)}{R(\alpha)} \cdot \frac{1}{\alpha - 2} \cdot \tilde{\sigma}_R^2 \cdot \left(\frac{L\kappa_Y^2}{k}\right)^{1-\frac{\alpha}{2}} \cong \ln 2, \tilde{\sigma}_R^2 >> 1 \tag{41}$$

we carry out

$$c_4(\alpha) = \left(\frac{-0.6931 \cdot (\alpha - 2) \cdot \Gamma\left(1 - \frac{\alpha}{2}\right)}{\Gamma\left(\frac{\alpha}{2}\right) \cdot 4^{\frac{\alpha}{2}}}\right)^{\frac{2}{2-\alpha}} \tag{42}$$

At this point, replacing (40) and (42) in (38), remembering that $\eta_Y = \frac{L\kappa_Y{}^2}{k}$, and by using the (37), we deduce the small-scale log-irradiance variance which is given by

$$\sigma_{\ln Y}^2(\alpha) \cong \frac{0.51\tilde{\sigma}_R^2}{\left(1 + f_Y(\alpha)\tilde{\sigma}_R^{\frac{4}{\alpha-2}}\right)^{\frac{\alpha}{2}-1}} \tag{43}$$

where

$$f_Y(\alpha) = [1.3687]^{\frac{2}{2-\alpha}} \tag{44}$$

We plot, in figure 2, the small scale irradiance fluctuations $\sigma_Y^2(\alpha)$ as a function of the strength of turbulence for several alpha values. We deduce from figure 2 that there is an alpha dependence on the small scale irradiance basically for Rytov variance between the range 1 to 6. Inside this range, for alpha values higher than 11/3, the small scale irradiance fluctuations slightly decrease; instead for alpha values lower than 11/3, small scale irradiance fluctuations slightly increase. Outside that range alpha variations do not have remarkable effect on the small scale irradiance fluctuations.

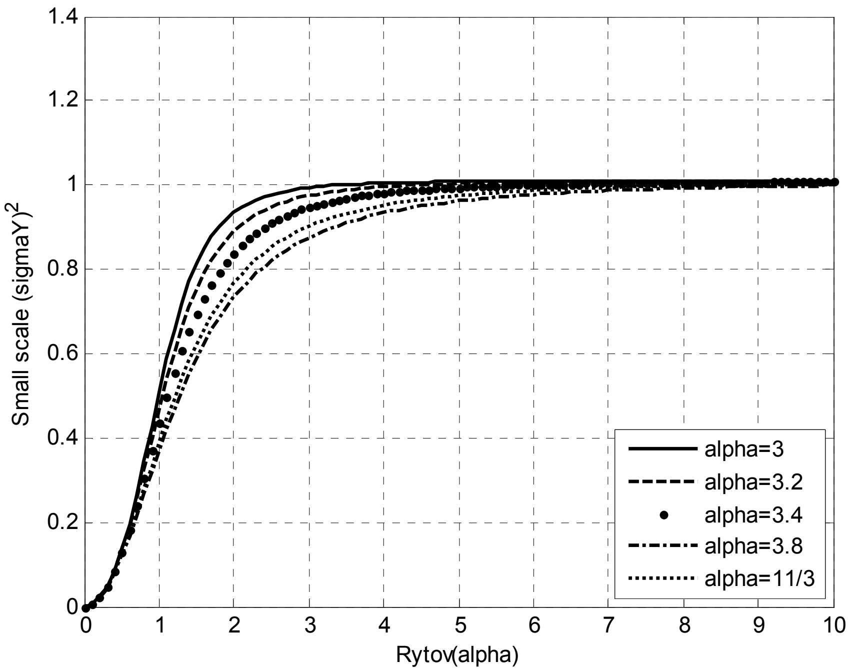

Figure 2- Small scale irradiance fluctuations as a function of the strength of turbulence for several alpha values

4.3 Scintillation index expression

By combining (34) and (43), we see that the scintillation index (9) for plane wave in non Kolmogorov turbulence and in absence of inner scale and outer scale effects is given by

$$\sigma_I^2(\alpha) = \exp\left[\frac{0.49\tilde{\sigma}_R^2}{\left(1+f_X(\alpha)\tilde{\sigma}_R^{\frac{4}{\alpha-2}}\right)^{3-\frac{\alpha}{2}}} + \frac{0.51\tilde{\sigma}_R^2}{\left(1+f_Y(\alpha)\tilde{\sigma}_R^{\frac{4}{\alpha-2}}\right)^{\frac{\alpha}{2}-1}}\right] - 1 \tag{45}$$

We plot, in figure 3, the scintillation index as a function of the strength of turbulence for several alpha values. We deduce from figure 3 that, in moderate-strong turbulence, for alpha values higher than 11/3, scintillation index remarkable increases with respect to the Kolmogorov case; instead for alpha values lower than 11/3, scintillation index decreases with respect to the Kolmogorov case.

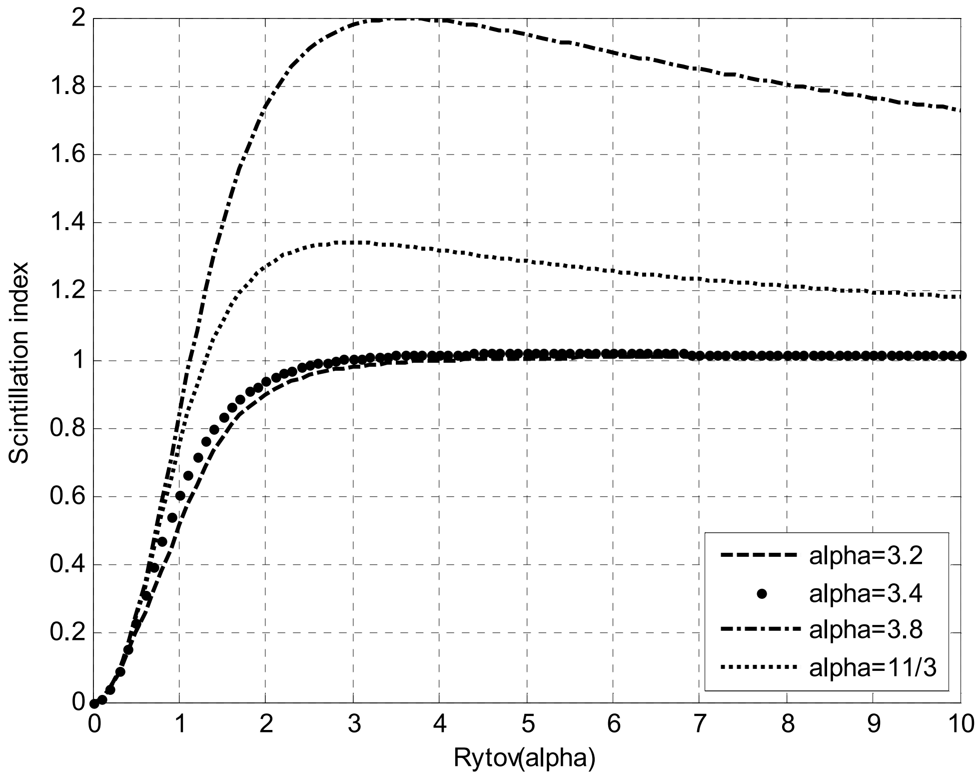

Figure 3- Scintillation index as a function of the strength of turbulence for several alpha values

8. DISCUSSION

It has been shown in moderate-strong turbulence, for a horizontal link, the scintillation index as variations depending on the alpha exponent lead to results somewhat different than obtained with the standard value of Kolmogorov $\alpha = 11/3$. In moderate-strong turbulence, for alpha values higher than 11/3, the scintillation index increases with respect to the Kolmogorov case of 11/3; however, for alpha values lower than 11/3, the scintillation index decreases with respect to the Kolmogorov case. It has been analyzed both the small scale irradiance fluctuations and the large scale irradiance fluctuations. Specifically, the second one gets more impact from a power law spectrum variation and it leads the most contribute on the scintillation index variation. Although our final expressions for the scintillation have been obtained by extended Rytov theory, which is necessary to adopt in strong turbulence conditions, they reduce to the proper results also in weak turbulence.

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