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Screening fifth forces in generalized Proca theories

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For a massive vector field with derivative self-interactions, the breaking of the gauge invariance allows the propagation of a longitudinal mode in addition to the two transverse modes. We consider generalized Proca theories with second-order equations of motion in a curved space-time and study how the longitudinal scalar mode of the vector field gravitates on a spherically symmetric background. We show explicitly that cubic-order self-interactions lead to the suppression of the longitudinal mode through the Vainshtein mechanism. Provided that the dimensionless coupling of the interaction is not negligible, this screening mechanism is sufficiently efficient to give rise to tiny corrections to gravitational potentials consistent with solar-system tests of gravity. We also study the quartic interactions with the presence of nonminimal derivative coupling with the Ricci scalar and find the existence of solutions where the longitudinal mode completely vanishes. Finally, we discuss the case in which the effect of the quartic interactions dominates over the cubic one and show that local gravity constraints can be satisfied under a mild bound on the parameters of the theory.

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I. INTRODUCTION

The construction of theories beyond General Relativity (GR) is motivated not only by the ultraviolet completion of gravity but also by the accumulating observational evidence of the late-time cosmic acceleration. If we modify gravity from GR, however, additional degrees of freedom (DOF) generally arise [1–5]. To keep the theories healthy, these new DOF should give rise to neither ghosts nor instabilities. If the equations of motion are of second order, the lack of higher-order derivatives forbids the propagation of further dangerous DOF associated with Ostrogradski instabilities [6]. In the presence of one scalar degree of freedom, it is known that Horndeski theories [7] are the most general scalar-tensor theories with second-order equations of motion in curved space-times. Independently of the original work, the same action was rederived by extending the so-called Galileon action ("scalar Galileons") [8,9] to curved space-time with the second-order property maintained [10–16].

In 1976, Horndeski derived the most general action of an Abelian vector field with a nonminimal coupling yielding second-order equations of motion, under the assumption that the Maxwell equations are recovered in the flat space-time [17]. The cosmology and the stability of such Horndeski vector-tensor theories were recently studied in Refs. [18,19]. There have been attempts for constructing theories of Abelian vector fields analogous to scalar Galileons [20–22]. If we try to preserve the $U(1)$ gauge invariance for one vector field and stick to second-order equations of motion, there exists a no-go theorem stating that the Maxwell kinetic term is the only allowed interaction [23,24]. However, dropping the $U(1)$ gauge invariance allows us to generate nontrivial terms associated with "vector Galileons" [25,26] (see also Refs. [27–32] for related works).

In relativistic field theory, it is well known that introduction of the mass term for a Maxwell vector field breaks the $U(1)$ gauge invariance. In this massive vector Proca theory, there is one propagating degree of freedom in the longitudinal direction besides two DOF corresponding to the transverse polarizations. In the presence of derivative interactions like those appearing for Galileons, it is natural to ask whether they do not modify the number of DOF in Proca theory. In Ref. [25], one of the authors derived a generalized Proca action for a vector field A^μ with second-order equations of motion on curved space-times. The analysis based on the Hessian matrix showed that only three DOF propagate as in the original Proca theory [25,31]. The action has nonminimal derivative couplings to the Ricci scalar R and the Einstein tensor $G_{\mu\nu}$, whose structure is similar to that in scalar Horndeski theories. In fact, taking the limit $A^\mu \to \nabla^\mu \pi$, the resulting action for the scalar field π reproduces that of scalar Galileons with suitable choices of free functions [25,26].

It was shown in Refs. [26,27] that a subclass of these generalized Proca theories can lead to the self-acceleration of the Universe. If we apply these theories to the present cosmic acceleration, not only a viable cosmic expansion history could be realized but also the gravitational interaction similar to GR could be recovered inside the solar system. In this paper, the issue of how the vector field gravitates in the presence of derivative self-interactions is addressed on the spherically symmetric space-time with a matter source. We first show that the transverse components of the spatial vector A^i vanish on the spherically symmetric background by imposing their regularities at the origin. Hence the longitudinal scalar component is the only relevant contribution to A^i in addition to the time component of A^μ.

We study how the longitudinal propagation affects the behavior of gravitational potentials in the presence of the vector Galileon interactions. We shall consider two cases: (i) the self-interacting Lagrangian $\mathcal{L}_3 = \beta_3 X \nabla_\mu A^\mu$ exists, and (ii) the nonminimal derivative coupling $\beta_4 X^2 R$ is taken into account in the Lagrangian $\mathcal{L}_4$ in addition to $\mathcal{L}_3$. We show that, due to derivative self-interactions, the screening mechanism of the longitudinal mode can be at work. This leads to the suppression of the propagation of the fifth force in such a way that the theories are consistent with local gravity constraints. This is analogous to the Vainshtein mechanism [33] for scalar Galileons [8,34–37], but the property of screened solutions exhibits some difference due to the nontrivial coupling between the longitudinal mode and the time component of A^μ.

This paper is organized as follows. In Sec. II we present the action of the generalized Proca theories in the presence of a matter source and derive the equations of motion up to the Lagrangian $\mathcal{L}_4$ on general curved backgrounds. In Sec. III we obtain the equations of motion on the spherically symmetric background (with coefficients given in the Appendix). In Sec. IV we derive the vector field profiles in the presence of the Lagrangian $\mathcal{L}_3$ both analytically and numerically and compute corrections to leading-order gravitational potentials induced by the longitudinal scalar. In Sec. V we study the cases in which the contribution of the Lagrangian $\mathcal{L}_4$ dominates over that of $\mathcal{L}_3$ and also obtain analytic field profiles as well as gravitational potentials. Section VI is devoted to conclusions.

II. GENERALIZED PROCA THEORIES

We begin with the generalized Proca theories described by the four-dimensional action,

$$S = \int d^4x \sqrt{-g}(\mathcal{L} + \mathcal{L}_m), \qquad \mathcal{L} = \mathcal{L}_F + \sum_{i=2}^{5} \mathcal{L}_i, \tag{2.1}$$

where g denotes the determinant of the metric tensor $g_{\mu\nu}$, $\mathcal{L}_m$ the matter Lagrangian, and $\mathcal{L}_F = -(1/4)F_{\mu\nu}F^{\mu\nu}$ is the standard kinetic term of the vector field A_μ with $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$ (∇_μ is the covariant derivative operator). The Lagrangians $\mathcal{L}_i$ encode the nontrivial derivative interactions [25]

$$\mathcal{L}_2 = G_2(X), \tag{2.2}$$

$$\mathcal{L}_3 = G_3(X)\nabla_\mu A^\mu, \tag{2.3}$$

$$\begin{aligned}\mathcal{L}_4 = {} & G_4(X)R + G_{4,X}(X)[(\nabla_\mu A^\mu)^2 + c_2 \nabla_\rho A_\sigma \nabla^\rho A^\sigma \\ & - (1+c_2)\nabla_\rho A_\sigma \nabla^\sigma A^\rho],\end{aligned} \tag{2.4}$$

$$\begin{aligned}\mathcal{L}_5 = {} & G_5(X)G_{\mu\nu}\nabla^\mu A^\nu - \frac{1}{6}G_{5,X}(X)[(\nabla_\mu A^\mu)^3 \\ & - 3d_2 \nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\rho A^\sigma - 3(1-d_2)\nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho \\ & + (2-3d_2)\nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma + 3d_2 \nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla_\gamma A^\sigma],\end{aligned} \tag{2.5}$$

where R is the Ricci scalar, $G_{\mu\nu}$ is the Einstein tensor, $G_{2,3,4,5}$ as well as c_2, d_2 are arbitrary functions of

$$X \equiv -\frac{1}{2}A_\mu A^\mu, \tag{2.6}$$

and $G_{i,X} \equiv \partial G_i/\partial X$. Note that we could have allowed any contractions of the vector field A_μ with $F_{\mu\nu}$ and $F^*_{\mu\nu}$ (with F^* being the dual of F) in the function G_2, for instance in the form of $A_\mu A_\nu F^{\mu\rho}F^\nu_\rho, \ldots$, etc., or contractions between the vector field and the Einstein tensor $G_{\mu\nu}A^\mu A^\nu$, since they do not contain any time derivative applying on the temporal component of the vector field, but for the purpose of our present analysis of screened solutions we shall simply assume $G_2(X)$.

The Lagrangians $\mathcal{L}_{2,3,4,5}$ given above keep the equations of motion up to second order. They can be constructed from the Lagrangian [25,26]

$$\begin{aligned}\tilde{\mathcal{L}}_{i+2} = {} & -\frac{1}{(4-i)!}G_{i+2}(X)\mathcal{E}_{\alpha_1\cdots\alpha_i\gamma_{i+1\cdots 4}} \\ & \times \mathcal{E}^{\beta_1\cdots\beta_i\gamma_{i+1\cdots 4}}\nabla_{\beta_1}A^{\alpha_1}\cdots\nabla_{\beta_i}A^{\alpha_i},\end{aligned} \tag{2.7}$$

where $i = 0, 1, 2, 3$, and $\mathcal{E}_{\mu_1\mu_2\mu_3\mu_4}$ is the antisymmetric Levi-Civita tensor. For $i=0$ and $i=1$, we have that $\mathcal{L}_2 = \tilde{\mathcal{L}}_2$ and $\mathcal{L}_3 = \tilde{\mathcal{L}}_3$, respectively. For $i = 2, 3$, besides the terms $\tilde{\mathcal{L}}_4$ and $\tilde{\mathcal{L}}_5$, there are other Lagrangians $\bar{\mathcal{L}}_4$ and $\bar{\mathcal{L}}_5$, respectively, derived by exchanging the indices in Eq. (2.7), e.g., $-(1/2)F_4(X)\mathcal{E}_{\alpha_1\alpha_2\gamma_3\gamma_4}\mathcal{E}^{\beta_1\beta_2\gamma_3\gamma_4}\nabla_{\beta_1}A_{\beta_2}\nabla^{\alpha_1}A^{\alpha_2}$ for $i=2$ and $-F_5(X)\mathcal{E}_{\alpha_1\alpha_2\alpha_3\gamma_4}\mathcal{E}^{\beta_1\beta_2\beta_3\gamma_4}\nabla_{\beta_1}A^{\alpha_1}\nabla_{\beta_2}A^{\alpha_2}\nabla^{\alpha_3}A_{\beta_3}$ for $i=3$, where $F_4(X)$ and $F_5(X)$ are arbitrary functions of X. Since $\mathcal{L}_4 = \tilde{\mathcal{L}}_4 + \bar{\mathcal{L}}_4$ and $\mathcal{L}_5 = \tilde{\mathcal{L}}_5 + \bar{\mathcal{L}}_5$, the coefficients c_2 and d_2 appearing in Eqs. (2.4) and (2.5) correspond to $c_2 = F_4(X)/G_4(X)$ and

$d_2 = F_5(X)/G_5(X)$, respectively. Throughout this paper, we assume that c_2 and d_2 are constants. In Eqs. (2.4) and (2.5) the nonminimal coupling terms $G_4(X)R$ and $G_5(X)G_{\mu\nu}\nabla^\mu A^\nu$ are included to guarantee that the equations of motion are of second order [25].

The Proca Lagrangian corresponds to the functions $G_2 = m^2X$ and $G_{3,4,5} = 0$, where m corresponds to the mass of the vector field. The generalized Proca theories given by Eq. (2.1) generally break the $U(1)$ gauge symmetry. It is possible to restore the gauge symmetry by introducing a Stueckelberg field π [38], as $A_\mu \to A_\mu + \partial_\mu \pi$. To zeroth order in A_μ, we can extract the longitudinal mode of the vector field [25]. For the functional choices $G_2 = X$, $G_3 = X$ and $G_4 = X^2$, $G_5 = X^2$, this procedure gives rise to the scalar covariant Galileon Lagrangian originally derived in Refs. [8,9] by imposing the Galilean symmetry $\partial_\mu\pi \to \partial_\mu\pi + b_\mu$ in flat space-time. The dependence on the parameters c_2 and d_2 present in Eqs. (2.4) and (2.5) disappears for the Stueckelberg field π. In fact, the terms multiplied by the coefficients c_2 and d_2 are proportional to $G_{4,X}F_{\mu\nu}F^{\mu\nu}$ and $G_{5,X}[(\nabla_\lambda A^\lambda)F_{\mu\nu}F^{\mu\nu}/2 + \nabla_\mu A_\nu \nabla^\nu A_\rho F^{\rho\mu}]$, respectively, which are both expressed in terms of $F_{\mu\nu}$ [25,31].

In the following we focus on theories given by the action (2.1) up to the Lagrangian $\mathcal{L}_4$. We do not consider the Lagrangian $\mathcal{L}_5$ due to its complexity, but we leave such an analysis for a future work. We define the energy-momentum tensor of the matter Lagrangian $\mathcal{L}_m$ as

$$T^{(m)}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}. \tag{2.8}$$

Assuming that matter is minimally coupled to gravity, there is the continuity equation

$$\nabla^\mu T^{(m)}_{\mu\nu} = 0. \tag{2.9}$$

Variation of the action (2.1) with respect to $g^{\mu\nu}$ and A_ν leads to

$$\delta S = \int d^4x\sqrt{-g}\left[\left(\mathcal{G}_{\mu\nu} - \frac{1}{2}T^{(m)}_{\mu\nu}\right)\delta g^{\mu\nu} + \mathcal{A}^\nu \delta A_\nu\right], \tag{2.10}$$

where

$$\mathcal{G}_{\mu\nu} \equiv \frac{\delta\mathcal{L}}{\delta g^{\mu\nu}} - \frac{1}{2}g_{\mu\nu}\mathcal{L}, \qquad \mathcal{A}^\nu \equiv \frac{\delta\mathcal{L}}{\delta A_\nu}. \tag{2.11}$$

The equation of motion of the gravity sector on general curved space-times is given by

$$\mathcal{G}_{\mu\nu} = \frac{1}{2}T^{(m)}_{\mu\nu}, \tag{2.12}$$

with

$$\mathcal{G}_{\mu\nu} = \mathcal{G}^{(F)}_{\mu\nu} + \sum_{i=2}^{4}\mathcal{G}^{(i)}_{\mu\nu}. \tag{2.13}$$

Here each term comes from the standard kinetic term and the Lagrangians (2.2)–(2.4), as

$$\mathcal{G}^{(F)}_{\mu\nu} = \frac{1}{4}g_{\mu\nu}(\nabla_\rho A_\sigma \nabla^\rho A^\sigma - \nabla_\rho A_\sigma \nabla^\sigma A^\rho) - \frac{1}{2}[\nabla_\rho A_\mu \nabla^\rho A_\nu + \nabla_\mu A_\rho \nabla_\nu A^\rho - 2\nabla_\rho A_{(\nu}\nabla_{\mu)}A^\rho], \tag{2.14}$$

$$\mathcal{G}^{(2)}_{\mu\nu} = -\frac{1}{2}g_{\mu\nu}G_2 - \frac{1}{2}G_{2,X}A_\mu A_\nu, \tag{2.15}$$

$$\mathcal{G}^{(3)}_{\mu\nu} = -\frac{1}{2}G_{3,X}[A_\mu A_\nu \nabla_\rho A^\rho + g_{\mu\nu}A^\lambda A_\rho \nabla_\lambda A^\rho - 2A_\rho A_{(\mu}\nabla_{\nu)}A^\rho], \tag{2.16}$$

$$\begin{aligned}\mathcal{G}^{(4)}_{\mu\nu} = {} & G_4 G_{\mu\nu} - \frac{1}{2}G_{4,X}A_\mu A_\nu R \\ & + \frac{1}{2}G_{4,X}g_{\mu\nu}[(\nabla_\rho A^\rho)^2 - (2+c_2)\nabla_\rho A_\sigma \nabla^\rho A^\sigma + (1+c_2)\nabla_\rho A_\sigma \nabla^\sigma A^\rho - 2A_\rho \Box A^\rho + 2A^\rho \nabla_\rho \nabla_\sigma A^\sigma] \\ & + G_{4,X}[(1+c_2)\nabla_\mu A_\rho \nabla_\nu A^\rho - \nabla_\rho A^\rho \nabla_{(\mu}A_{\nu)} - (1+2c_2)\nabla_\rho A_{(\nu}\nabla_{\mu)}A^\rho + (1+c_2)\nabla_\rho A_\mu \nabla^\rho A_\nu \\ & + A_\rho \nabla_{(\mu}\nabla_{\nu)}A^\rho - A^\rho \nabla_\rho \nabla_{(\mu}A_{\nu)} + A_{(\nu}\Box A_{\mu)} - 2A_{(\nu}\nabla_{\mu)}\nabla_\sigma A^\sigma + A_{(\mu}\nabla_\rho \nabla_{\nu)}A^\rho] \\ & - \frac{1}{2}G_{4,XX}\{A_\mu A_\nu[(\nabla_\rho A^\rho)^2 + c_2\nabla_\rho A_\sigma \nabla^\rho A^\sigma - (1+c_2)\nabla_\rho A_\sigma \nabla^\sigma A^\rho] + 2A_\rho A_\sigma \nabla_\mu A^\rho \nabla_\nu A^\sigma \\ & - 2A_\alpha \nabla_\rho A^\alpha[A^\rho \nabla_{(\mu}A_{\nu)} - A_{(\nu}\nabla_{\mu)}A^\rho - A_{(\nu}\nabla^\rho A_{\mu)} - 2g_{\mu\nu}A^{[\rho}\nabla^{\sigma]}A_\sigma] - 4A_\alpha(\nabla_\sigma A^\sigma)A_{(\nu}\nabla_{\mu)}A^\alpha\},\end{aligned} \tag{2.17}$$

where $\nabla_{(\mu}A_{\nu)} \equiv (\nabla_\mu A_\nu + \nabla_\nu A_\mu)/2$ and $A^{[\rho}\nabla^{\sigma]}A_\sigma \equiv (A^\rho \nabla^\sigma A_\sigma - A^\sigma \nabla^\rho A_\sigma)/2$. The equation of motion for the vector field A_ν corresponds to $\mathcal{A}^\nu = 0$, i.e.,

$$\nabla_\mu F^{\mu\nu} - G_{2,X}A^\nu + 2G_{3,X}A^{[\mu}\nabla^{\nu]}A_\mu - RG_{4,X}A^\nu - 2G_{4,X}[\nabla^\nu\nabla_\mu A^\mu + c_2\Box A^\nu - (1+c_2)\nabla^\mu\nabla^\nu A_\mu]$$
$$- G_{4,XX}[A^\nu\{(\nabla_\mu A^\mu)^2 + c_2\nabla_\rho A_\sigma\nabla^\rho A^\sigma - (1+c_2)\nabla_\rho A_\sigma\nabla^\sigma A^\rho\}$$
$$- 2A_\rho\nabla^\nu A^\rho\nabla_\mu A^\mu - 2c_2A_\rho\nabla^\mu A^\rho\nabla_\mu A^\nu + 2(1+c_2)A_\rho\nabla^\mu A^\rho\nabla^\nu A_\mu] = 0. \quad (2.18)$$

In GR we have $G_4 = M_{\rm pl}^2/2$, where $M_{\rm pl}$ is the reduced Planck mass, so $\mathcal{G}_{\mu\nu}^{(4)}$ simply reduces to $(M_{\rm pl}^2/2)G_{\mu\nu}$. Existence of the vector field with derivative self-couplings induces additional gravitational interactions with matter through Eq. (2.12). We shall study whether such a fifth force can be suppressed in local regions with a matter source.

III. EQUATIONS OF MOTION ON THE SPHERICALLY SYMMETRIC BACKGROUND

We derive the equations of motion on the spherically symmetric and static background described by the line element

$$ds^2 = -e^{2\Psi(r)}dt^2 + e^{2\Phi(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (3.1)$$

where $\Psi(r)$ and $\Phi(r)$ are the gravitational potentials that depend on radius r from the center of sphere. For the matter Lagrangian $\mathcal{L}_m$, we consider the perfect fluid with the energy-momentum tensor $T^\mu_\nu = {\rm diag}(-\rho_m, P_m, P_m, P_m)$, where ρ_m is the energy density and P_m is the pressure. Then, the matter continuity equation (2.9) reads

$$P_m' + \Psi'(\rho_m + P_m) = 0, \quad (3.2)$$

where a prime represents a derivative with respect to r.

We write the vector field A^μ in the form

$$A^\mu = (\phi, A^i), \quad (3.3)$$

where $i = 1, 2, 3$. From Helmholtz's theorem, we can decompose the spatial components A^i into the transverse and longitudinal modes, as

$$A_i = A_i^{(T)} + \nabla_i\chi, \quad (3.4)$$

where $A_i^{(T)}$ obeys the traceless condition $\nabla^i A_i^{(T)} = 0$ and χ is the longitudinal scalar. On the spherically symmetric configuration, it is required that the θ and φ components of $A_i^{(T)}$ (i.e., $A_2^{(T)}$ and $A_3^{(T)}$) vanish. Then, the traceless condition gives the following relation:

$$A_1^{(T)\prime} + \frac{2}{r}A_1^{(T)} - \Phi' A_1^{(T)} = 0, \quad (3.5)$$

whose solution is given by

$$A_1^{(T)} = C\frac{e^\Phi}{r^2}, \quad (3.6)$$

where C is an integration constant. For the regularity of $A_1^{(T)}$ at $r = 0$, we require that $C = 0$. This discussion shows that the transverse vector $A_i^{(T)}$ vanishes, so we only need to focus on the propagation of the longitudinal mode, i.e., $A_i = \nabla_i\chi$. Then, the components of A^μ on the spherical coordinate (t, r, θ, φ) are given by

$$A^\mu = (\phi(r), e^{-2\Phi}\chi'(r), 0, 0). \quad (3.7)$$

The (0, 0), (1, 1) and (2, 2) components of Eq. (2.12) reduce, respectively, to[1]

$$\mathcal{C}_1\Psi'^2 + \left(\mathcal{C}_2 + \frac{\mathcal{C}_3}{r}\right)\Psi' + \left(\mathcal{C}_4 + \frac{\mathcal{C}_5}{r}\right)\Phi' + \mathcal{C}_6 + \frac{\mathcal{C}_7}{r} + \frac{\mathcal{C}_8}{r^2} = -e^{2\Phi}\rho_m, \quad (3.8)$$

$$\mathcal{C}_9\Psi'^2 + \left(\mathcal{C}_{10} + \frac{\mathcal{C}_{11}}{r}\right)\Psi' + \mathcal{C}_{12} + \frac{\mathcal{C}_{13}}{r} + \frac{\mathcal{C}_{14}}{r^2} = e^{2\Phi}P_m, \quad (3.9)$$

$$\mathcal{C}_{15}\Psi'' + \mathcal{C}_{16}\Psi'^2 + \mathcal{C}_{17}\Psi'\Phi' + \left(\mathcal{C}_{18} + \frac{\mathcal{C}_3/4 + \mathcal{C}_{15}}{r}\right)\Psi' + \left(-\frac{\mathcal{C}_{13}}{2} + \frac{\mathcal{C}_{19}}{r}\right)\Phi' + \mathcal{C}_{20} + \frac{\mathcal{C}_{21}}{r} = e^{2\Phi}P_m, \quad (3.10)$$

where the coefficients $\mathcal{C}_i$ $(i = 1, 2, \ldots, 21)$ are given in the Appendix. The mass term (2.6) can be decomposed as $X = X_\phi + X_\chi$, where

$$X_\phi \equiv \frac{1}{2}e^{2\Psi}\phi^2, \qquad X_\chi \equiv -\frac{1}{2}e^{-2\Phi}\chi'^2. \quad (3.11)$$

The $\nu = 0$ and $\nu = 1$ components of Eq. (2.18) reduce, respectively, to

$$\mathcal{D}_1(\Psi'' + \Psi'^2) + \mathcal{D}_2\Psi'\Phi' + \left(\mathcal{D}_3 + \frac{\mathcal{D}_4}{r}\right)\Psi' + \left(\mathcal{D}_5 + \frac{\mathcal{D}_6}{r}\right)\Phi' + \mathcal{D}_7 + \frac{\mathcal{D}_8}{r} + \frac{\mathcal{D}_9}{r^2} = 0, \quad (3.12)$$

[1] We note that the (0, 1) component of Eq. (2.12) reduces to the same form as Eq. (3.13).

$$\mathcal{D}_{10}\Psi'^2 + \left(\mathcal{D}_{11} + \frac{\mathcal{D}_{12}}{r}\right)\Psi' + \mathcal{D}_{13} + \frac{\mathcal{D}_{14}}{r} + \frac{\mathcal{D}_{15}}{r^2} = 0, \tag{3.13}$$

where we introduced the short-cut notations for convenience:

$$\begin{aligned}
&\mathcal{D}_1 = 2\phi(2c_2G_{4,X} - 1), \qquad \mathcal{D}_2 = 2\phi[1 - 2c_2(G_{4,X} + 2X_\chi G_{4,XX})],\\
&\mathcal{D}_3 = \phi\chi' G_{3,X} - \phi'[3 - 2c_2(3G_{4,X} + 2X_\phi G_{4,XX})] - 4c_2 e^{-2\Phi}\phi\chi'\chi'' G_{4,XX},\\
&\mathcal{D}_4 = 4\phi(2c_2G_{4,X} - 2X_\chi G_{4,XX} - 1), \qquad \mathcal{D}_5 = -\phi\chi' G_{3,X} + \phi'[1 - 2c_2(G_{4,X} + 2X_\chi G_{4,XX})],\\
&\mathcal{D}_6 = 4\phi(G_{4,X} + 2X_\chi G_{4,XX}),\\
&\mathcal{D}_7 = e^{2\Phi}\phi G_{2,X} + \phi\chi'' G_{3,X} - \phi''(1 - 2c_2G_{4,X}) + c_2(e^{2\Psi}\phi\phi'^2 - 2e^{-2\Phi}\phi'\chi'\chi'')G_{4,XX},\\
&\mathcal{D}_8 = 2\phi\chi' G_{3,X} - 2\phi'(1 - 2c_2G_{4,X}) + 4e^{-2\Phi}\phi\chi'\chi'' G_{4,XX}, \qquad \mathcal{D}_9 = -2\phi[(1 - e^{2\Phi})G_{4,X} + 2X_\chi G_{4,XX}],\\
&\mathcal{D}_{10} = 8c_2 e^{-2\Phi}\chi' X_\phi G_{4,XX}, \qquad \mathcal{D}_{11} = 2(X_\chi - X_\phi)G_{3,X} + 4c_2 e^{2\Psi-2\Phi}\phi\phi'\chi' G_{4,XX},\\
&\mathcal{D}_{12} = 4e^{-2\Phi}\chi'[G_{4,X} + 2(X_\chi - X_\phi)G_{4,XX}], \qquad \mathcal{D}_{13} = -\chi' G_{2,X} - e^{2\Psi}\phi\phi' G_{3,X} + c_2 e^{2\Psi-2\Phi}\phi'^2\chi' G_{4,XX},\\
&\mathcal{D}_{14} = 4X_\chi G_{3,X} - 4e^{2\Psi-2\Phi}\phi\phi'\chi' G_{4,XX}, \qquad \mathcal{D}_{15} = -2\chi'[(1 - e^{-2\Phi})G_{4,X} - 2e^{-2\Phi}X_\chi G_{4,XX}].
\end{aligned} \tag{3.14}$$

Among the six equations of motion (3.2), (3.8)–(3.10), (3.12), and (3.13), five of them are independent. For a given density profile ρ_m of matter, solving five independent equations of motion leads to the solutions to P_m, Ψ, Φ, ϕ, χ with appropriate boundary conditions.

For the consistency with local gravity experiments within the solar system, we require that the gravitational potentials Ψ and Φ need to be close to those in GR. In GR without the vector field A^μ, we have $G_2 = G_3 = 0$, $G_4 = M_{\rm pl}^2/2$ and $\phi = 0 = \chi'$, so Eqs. (3.8) and (3.9) read

$$\frac{2M_{\rm pl}^2}{r}\Phi'_{\rm GR} - \frac{M_{\rm pl}^2}{r^2}(1 - e^{2\Phi_{\rm GR}}) = e^{2\Phi_{\rm GR}}\rho_m, \tag{3.15}$$

$$\frac{2M_{\rm pl}^2}{r}\Psi'_{\rm GR} + \frac{M_{\rm pl}^2}{r^2}(1 - e^{2\Phi_{\rm GR}}) = e^{2\Phi_{\rm GR}}P_m. \tag{3.16}$$

Since $\Phi_{\rm GR}$ and $\Psi_{\rm GR}$ would be the leading-order contributions to gravitational potentials under the operation of the screening mechanism, we first derive their solutions inside and outside a compact body. We assume that the change of ρ_m occurs rapidly at the distance r_*, so that the matter density can be approximated as $\rho_m(r) \simeq \rho_0$ for $r < r_*$ and $\rho_m(r) \simeq 0$ for $r > r_*$. This configuration is equivalent to that of the Schwarzschild interior and exterior solutions. For $r < r_*$, integration of Eq. (3.2) leads to $P_m = -\rho_m + \mathcal{C}e^{-\Psi(r)}$, where $\mathcal{C}$ is an integration constant known by imposing the condition $P_m(r_*) = 0$.

Matching the interior and exterior solutions of Ψ and Φ at $r = r_*$ with appropriate boundary conditions (at $r = 0$ and $r \to \infty$), the gravitational potentials inside and outside the body are given by

$$\begin{aligned}
e^{\Psi_{\rm GR}} &= \frac{3}{2}\sqrt{1 - \frac{\rho_0 r_*^2}{3M_{\rm pl}^2}} - \frac{1}{2}\sqrt{1 - \frac{\rho_0 r^2}{3M_{\rm pl}^2}},\\
e^{\Phi_{\rm GR}} &= \left(1 - \frac{\rho_0 r^2}{3M_{\rm pl}^2}\right)^{-1/2},
\end{aligned} \tag{3.17}$$

for $r < r_*$, and

$$e^{\Psi_{\rm GR}} = \left(1 - \frac{\rho_0 r_*^3}{3M_{\rm pl}^2 r}\right)^{1/2}, \qquad e^{\Phi_{\rm GR}} = \left(1 - \frac{\rho_0 r_*^3}{3M_{\rm pl}^2 r}\right)^{-1/2}, \tag{3.18}$$

for $r > r_*$. In the following, we employ the weak gravity approximation under which $|\Psi|$ and $|\Phi|$ are much smaller than 1, i.e.,

$$\Phi_* \equiv \frac{\rho_0 r_*^2}{M_{\rm pl}^2} \ll 1. \tag{3.19}$$

This condition means that the Schwarzschild radius of the source $r_g \approx \rho_0 r_*^3/M_{\rm pl}^2$ is much smaller than r_*. Then, the solutions (3.17) and (3.18) reduce, respectively, to

$$\Psi_{\rm GR} \simeq \frac{\rho_0}{12M_{\rm pl}^2}(r^2 - 3r_*^2), \qquad \Phi_{\rm GR} \simeq \frac{\rho_0 r^2}{6M_{\rm pl}^2}, \tag{3.20}$$

for $r < r_*$, and

$$\Psi_{\rm GR} \simeq -\frac{\rho_0 r_*^3}{6M_{\rm pl}^2 r}, \qquad \Phi_{\rm GR} \simeq \frac{\rho_0 r_*^3}{6M_{\rm pl}^2 r}, \tag{3.21}$$

for $r > r_*$. For the theories with the action (2.1), the vector field interacts with gravity through the derivative terms Ψ'',

Ψ', Φ' in Eqs. (3.12) and (3.13). The leading-order contributions of such gravitational interactions follow from the derivatives $\Psi''_{\rm GR}$, $\Psi'_{\rm GR}$, $\Phi'_{\rm GR}$ of the GR solutions (3.17) and (3.18). Then, we can integrate Eqs. (3.12) and (3.13) to obtain the solutions to ϕ and χ'. The next-to-leading order corrections to Ψ and Φ can be derived by substituting the solutions of ϕ and χ' into Eqs. (3.8) and (3.9). In Secs. IV and V we apply this procedure to concrete theories.

IV. THEORIES WITH THE CUBIC LAGRANGIAN

Let us first consider theories in which the function G_4 corresponds only to the Einstein-Hilbert term, i.e.,

$$G_4 = \frac{M_{\rm pl}^2}{2}, \tag{4.1}$$

where $M_{\rm pl}$ is the reduced Planck mass. In this case the $G_{4,X}$ term in the Lagrangian $\mathcal{L}_4$ vanishes, but the Lagrangian $\mathcal{L}_3$ gives rise to a nontrivial gravitational interaction with the vector field. The equations of motion (3.12) and (3.13) reduce, respectively, to

$$\begin{aligned}&\frac{1}{r^2}\frac{d}{dr}(r^2\phi') - e^{2\Phi}G_{2,X}\phi - G_{3,X}\phi\frac{1}{r^2}\frac{d}{dr}(r^2\chi')\\&+2\phi(\Psi''+\Psi'^2-\Psi'\Phi') - \left(\phi\chi' G_{3,X} - 3\phi' - \frac{4\phi}{r}\right)\Psi'\\&+(\phi\chi' G_{3,X}-\phi')\Phi' = 0,\end{aligned} \tag{4.2}$$

$$\begin{aligned}&\chi' G_{2,X} + \left(e^{2\Psi}\phi\phi' + \frac{2}{r}e^{-2\Phi}\chi'^2\right)G_{3,X}\\&+(e^{2\Psi}\phi^2 + e^{-2\Phi}\chi'^2)G_{3,X}\Psi' = 0.\end{aligned} \tag{4.3}$$

For concreteness, we shall focus on the theories given by the functions

$$G_2(X) = m^2 X, \qquad G_3(X) = \beta_3 X, \tag{4.4}$$

where m is the mass of the vector field, and β_3 is a dimensionless constant. The choice of $G_3(X)$ given above is related with that of scalar Galileons. In what follows, we obtain analytic solutions to Eqs. (4.2) and (4.3) under the approximation of weak gravity.

A. Analytic vector field profiles

1. Solutions for $r < r_*$

For the distance r smaller than r_*, we substitute the derivatives of Eq. (3.20) into Eqs. (4.2) and (4.3) to derive leading-order solutions to ϕ and χ'. The terms containing $e^{2\Psi}$ and $e^{-2\Phi}$ provide the contributions linear in Ψ and Φ [say, $\Psi\phi\phi' G_{3,X}$ in Eq. (4.3)]. After deriving analytic solutions to ϕ and χ', however, we can show that such terms give rise to contributions much smaller than the leading-order solutions. Hence it is consistent to employ the approximations $e^{2\Psi}\simeq 1$ and $e^{-2\Phi}\simeq 1$ in Eqs. (4.2) and (4.3), such that

$$\begin{aligned}&\frac{d}{dr}(r^2\phi') - m^2 r^2\phi - \beta_3\phi\frac{d}{dr}(r^2\chi')\\&+\frac{\rho_0}{6M_{\rm pl}^2}[6\phi + r(\phi'+\beta_3\chi'\phi)]r^2 \simeq 0,\end{aligned} \tag{4.5}$$

$$m^2\chi' + \beta_3\left(\phi\phi' + \frac{2}{r}\chi'^2 + \frac{\rho_0\phi^2}{6M_{\rm pl}^2}r\right) \simeq 0. \tag{4.6}$$

From Eq. (4.6) it follows that

$$\chi' = \frac{m^2 r}{4\beta_3}\left[-1+\sqrt{1-\frac{8\beta_3^2}{m^4 r}\left(\phi\phi' + \frac{\rho_0\phi^2}{6M_{\rm pl}^2}r\right)}\,\right]. \tag{4.7}$$

The sign of (4.7) has been chosen in such a way that χ' vanishes for $\beta_3/m^2 \to 0$, which can be regarded as the GR limit. Since we are interested in how the screening mechanism is at work in the presence of the Lagrangian $\mathcal{L}_3$ for a very light field (e.g., the vector field associated with the late-time cosmic acceleration), we take another limit $\beta_3/m^2 \to \infty$ in the discussion below. In other words, we focus on the case $m \to 0$ with a nonzero dimensionless coupling β_3. For $\beta_3 > 0$, Eq. (4.7) reduces to

$$\chi' = \sqrt{-\frac{r}{2}\left(\phi\phi' + \frac{\rho_0\phi^2}{6M_{\rm pl}^2}r\right)}. \tag{4.8}$$

For the consistency of Eq. (4.8) we require the condition $\phi\phi' < 0$.

We search for solutions where the scalar potential ϕ does not vary much with respect to r, i.e.,

$$\phi(r) = \phi_0 + f(r), \qquad |f(r)| \ll |\phi_0|, \tag{4.9}$$

where ϕ_0 is a constant and $f(r)$ is a function of r. We also focus on the case where $\phi(r)$ decreases with the growth of r, such that $\phi'(r) < 0$ with $\phi_0 > 0$. In Eq. (4.5) we also neglect the terms $r(\phi' + \beta_3\chi'\phi)$ relative to 6ϕ. The validity of this approximation can be checked after deriving the solutions to ϕ and χ'. Substituting Eq. (4.8) into Eq. (4.5) with Eq. (4.9), we obtain the integrated solution

$$r^2 f' - \beta_3\phi_0^{3/2} r^2\sqrt{-\frac{r}{2}\left(f' + \frac{\rho_0\phi_0 r}{6M_{\rm pl}^2}\right)} + \frac{\rho_0\phi_0}{3M_{\rm pl}^2}r^3 = C, \tag{4.10}$$

where C is a constant. Under the boundary condition $\phi'(0) = 0$, we can fix $C = 0$ and hence

$$f' - \beta_3 \phi_0^{3/2} \sqrt{-\frac{r}{2}\left(f' + \frac{\rho_0 \phi_0 r}{6M_{\rm pl}^2}\right)} = -\frac{\rho_0 \phi_0}{3M_{\rm pl}^2} r. \qquad (4.11)$$

Clearly, there is a solution of the form $f'(r) \propto -r$. Substituting the solution $f(r) = -Br^2$ into Eq. (4.11), we find that the positive constant B, which remains finite in the limit $\beta_3 \to \infty$, is given by

$$B = \frac{\rho_0 \phi_0}{6M_{\rm pl}^2} \mathcal{F}(s_{\beta_3}), \qquad (4.12)$$

where

$$s_{\beta_3} \equiv \frac{3(\beta_3 \phi_0 M_{\rm pl})^2}{4\rho_0}, \qquad (4.13)$$

$$\mathcal{F}(s_{\beta_3}) \equiv (1 + s_{\beta_3})\left(1 - \sqrt{\frac{s_{\beta_3}}{1 + s_{\beta_3}}}\right). \qquad (4.14)$$

Then, we obtain the following analytic field profiles:

$$\phi(r) = \phi_0 \left[1 - \mathcal{F}(s_{\beta_3}) \frac{\rho_0}{6M_{\rm pl}^2} r^2\right], \qquad (4.15)$$

$$\chi'(r) = \sqrt{\frac{\rho_0 \phi_0^2}{6M_{\rm pl}^2}\left[\mathcal{F}(s_{\beta_3}) - \frac{1}{2}\right]} r. \qquad (4.16)$$

As s_{β_3} increases from 0 to ∞, the function $\mathcal{F}(s_{\beta_3})$ decreases from 1 to 1/2. This means that the terms inside the square root of Eq. (4.8) remain positive. Since $\mathcal{F}(s_{\beta_3})\rho_0 r^2/(6M_{\rm pl}^2) \ll 1$ from the condition (3.19) of weak gravity, the solution (4.15) is consistent with the assumption (4.9). In the limit that $s_{\beta_3} \ll 1$, the field profiles (4.15) and (4.16) reduce, respectively, to

$$\phi(r) \simeq \phi_0 \left(1 - \frac{\rho_0}{6M_{\rm pl}^2} r^2\right), \quad \chi'(r) \simeq \sqrt{\frac{\rho_0 \phi_0^2}{12M_{\rm pl}^2}} r, \qquad (4.17)$$

whereas, for $s_{\beta_3} \gg 1$, it follows that

$$\phi(r) \simeq \phi_0 \left(1 - \frac{\rho_0}{12M_{\rm pl}^2} r^2\right), \quad \chi'(r) \simeq \frac{\rho_0}{6\beta_3 M_{\rm pl}^2} r. \qquad (4.18)$$

The amplitude of $\chi'(r)$ in Eq. (4.18) is about $s_{\beta_3}^{-1/2}$ times smaller than that in Eq. (4.17). For a larger coupling $|\beta_3|$, the screening effect is efficient to suppress the propagation of the longitudinal mode. On using the solutions (4.15) and (4.16), we can confirm that the terms $r(\phi' + \beta_3 \chi' \phi)$ in Eq. (4.5) are much smaller than 6ϕ and that the approximations $e^{2\Psi} \simeq 1$ and $e^{-2\Phi} \simeq 1$ employed in Eq. (4.6) are also justified.

2. Solutions for $r > r_*$

Employing the GR solution (3.21) of gravitational potentials in the regime $r > r_*$ and substituting them into Eqs. (4.2) and (4.3), it follows that

$$\frac{d}{dr}(r^2 \phi') - m^2 r^2 \phi - \beta_3 \phi \frac{d}{dr}(r^2 \chi') + \frac{\rho_0 r_*^3}{9M_{\rm pl}^4 r^2}[\rho_0 r_*^3 \phi + 3M_{\rm pl}^2 r^2 (2\phi' - \beta_3 \chi' \phi)] \simeq 0, \qquad (4.19)$$

$$m^2 \chi' + \beta_3 \left(\phi \phi' + \frac{2}{r}\chi'^2 + \frac{\rho_0 \phi^2 r_*^3}{6M_{\rm pl}^2 r^2}\right) \simeq 0. \qquad (4.20)$$

Taking the $m \to 0$ limit and considering the branch $\chi' > 0$, Eq. (4.20) gives the following relation:

$$\chi' = \sqrt{-\frac{r}{2}\left(\phi \phi' + \frac{\rho_0 \phi^2 r_*^3}{6M_{\rm pl}^2 r^2}\right)}. \qquad (4.21)$$

The term $(\rho_0 r_*^3)^2 \phi/(9M_{\rm pl}^4 r^2)$ in Eq. (4.19) is at most Φ_* times as small as the term $\rho_0 \phi/M_{\rm pl}^2$ in Eq. (4.5). Moreover, after deriving the solutions to ϕ and χ', we can confirm that the contributions $3M_{\rm pl}^2 r^2 (2\phi' - \beta_3 \chi' \phi)$ in Eq. (4.19) are at most of the order of $\rho_0 r_*^3 \phi$. Hence it is a good approximation to neglect the terms inside the square bracket of Eq. (4.19). Substituting Eq. (4.21) into Eq. (4.19) with the approximation (4.9) and matching the integrated solution at $r = r_*$ on account of Eq. (4.11), we obtain

$$r^2 \phi' - \beta_3 \phi_0^{3/2} r^2 \sqrt{-\frac{r}{2}\left(\phi' + \frac{\rho_0 \phi_0 r_*^3}{6M_{\rm pl}^2 r^2}\right)} \simeq -\frac{\rho_0 \phi_0 r_*^3}{3M_{\rm pl}^2}. \qquad (4.22)$$

More explicitly, the field derivative ϕ' can be expressed as

$$\phi'(r) = -\frac{\rho_0 \phi_0 r_*^3}{3M_{\rm pl}^2 r^2} \mathcal{F}(\xi), \qquad \xi \equiv s_{\beta_3} \frac{r^3}{r_*^3}. \qquad (4.23)$$

From Eq. (4.21) the longitudinal mode is given by

$$\chi'(r) = \sqrt{\frac{\rho_0 r_*^3 \phi_0^2}{6M_{\rm pl}^2 r}\left[\mathcal{F}(\xi) - \frac{1}{2}\right]}. \qquad (4.24)$$

If $s_{\beta_3} \gg 1$, then $\xi \gg 1$ for $r > r_*$. In this case it follows that

$$\phi'(r) \simeq -\frac{\rho_0 \phi_0 r_*^3}{6M_{\rm pl}^2 r^2}, \qquad \chi'(r) \simeq \frac{\rho_0 r_*^3}{6\beta_3 M_{\rm pl}^2 r^2}. \qquad (4.25)$$

If $s_\beta \lesssim 1$, there is the transition radius r_V at which the r dependence of the longitudinal mode changes.

The radius r_V can be identified by the condition $\xi = 1$, i.e.,

$$r_V = \frac{r_*}{s_{\beta_3}^{1/3}}. \tag{4.26}$$

For the distance $r_* < r \ll r_V$ we have $\mathcal{F} \simeq 1$, so the solutions reduce to

$$\phi'(r) \simeq -\frac{\rho_0 \phi_0 r_*^3}{3M_{\rm pl}^2 r^2}, \qquad \chi'(r) \simeq \sqrt{\frac{\rho_0 r_*^3 \phi_0^2}{12 M_{\rm pl}^2 r}}. \tag{4.27}$$

For $r \gg r_V$ we have $\xi \gg 1$ and hence the resulting solutions are given by Eq. (4.25). In this regime, the longitudinal mode $\chi'(r)$ decreases faster than that for $r_* < r \ll r_V$ with a suppressed amplitude. The distance r_V can be regarded as the Vainshtein radius above which $\chi'(r)$ starts to decay quickly. If $|\beta_3|$ obeys the condition $s_{\beta_3} \gg 1$, $\chi'(r)$ is strongly suppressed both inside and outside the body due to the Vainshtein mechanism, see Eqs. (4.18) and (4.25). Meanwhile, for $s_{\beta_3} \lesssim 1$, the screening of the longitudinal mode manifests for the distance $r > r_V$. The fact that the suppression of the longitudinal mode occurs *outside* the radius r_V for small $|\beta_3|$ is a unique feature of vector Galileons.

B. Numerical solutions for the vector field

To confirm the validity of the analytic solutions derived above, we shall numerically solve Eqs. (4.2) and (4.3) coupled with the gravitational equations (3.8)–(3.10). For concreteness we consider the density distribution given by

$$\rho_m(r) = \rho_0 e^{-ar^2/r_*^2}, \tag{4.28}$$

where a is a positive constant of the order of 1. With this profile, the matter density starts to decrease significantly for $r \gtrsim r_*$. For the numerical purpose, it is convenient to introduce the following dimensionless quantities:

$$x = \frac{r}{r_*}, \qquad y = \frac{\phi}{\phi_0}, \qquad z = \frac{\chi'}{\phi_0}, \tag{4.29}$$

where ϕ_0 is the value of ϕ at $r = 0$. In the massless limit with $G_3 = \beta_3 X$, we can express Eqs. (4.2) and (4.3) in the forms

$$\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} - \beta_3 r_* \phi_0 y \left(\frac{dz}{dx} + \frac{2}{x} z\right) + 2y \left[\frac{d^2\Psi}{dx^2} + \left(\frac{d\Psi}{dx}\right)^2 - \frac{d\Psi}{dx}\frac{d\Phi}{dx}\right] - \left(\beta_3 r_* \phi_0 yz - 3\frac{dy}{dx} - \frac{4}{x} y\right)\frac{d\Psi}{dx} + \left(\beta_3 r_* \phi_0 yz - \frac{dy}{dx}\right)\frac{d\Phi}{dx} = 0, \tag{4.30}$$

$$z = e^{\Psi+\Phi}\sqrt{-xy\left(\frac{dy}{dx} + y\frac{d\Psi}{dx}\right)\left(2 + x\frac{d\Psi}{dx}\right)^{-1}}, \tag{4.31}$$

where the quantity $\beta_3 r_* \phi_0$ is related with s_{β_3} as $\beta_3 r_* \phi_0 = \sqrt{4 s_{\beta_3} \Phi_*/3}$. We take the x derivative of Eq. (4.31) and then eliminate the term dz/dx by combining it with Eq. (4.30) to obtain the second-order equation for $y(x)$. To derive the leading-order gravitational potentials $\Phi_{\rm GR}$ and $\Psi_{\rm GR}$, we also solve Eqs. (3.15) and (3.16) with a vanishing pressure P_m. This procedure gives rise to the solutions derived under the weak gravity approximation, e.g., Eq. (3.17). Numerically, we confirmed that the approximation substituting $\Phi_{\rm GR}$ and $\Psi_{\rm GR}$ into Eqs. (4.30) and (4.31) provides practically identical results to those obtained by solving full Eqs. (3.8)–(3.10).

In Fig. 1 we plot the field profile for $\rho_m = \rho_0 e^{-4r^2/r_*^2}$ and $\Phi_* = 10^{-4}$ with two different values of s_{β_3}. The boundary conditions of y and dy/dx around the center of body are chosen to match with Eq. (4.15). As we see in Fig. 1, both $-\phi'(r)$ and $\chi'(r)$ linearly grow in r for the distance smaller than r_*. The left panel of Fig. 1 corresponds to $s_{\beta_3} = 10^{-4}$, in which case the solutions to $\phi(r)$ and $\chi'(r)$ are well described by Eq. (4.17) in the regime $r < r_*$. For s_{β_3} larger than the order of 1, the longitudinal mode $\chi'(r)$ tends to be suppressed according to Eq. (4.18). The right panel of Fig. 1, which corresponds to $s_{\beta_3} = 1$, is the case in which the suppression of $\chi'(r)$ occurs in a mild way for $r < r_*$.

For the distance r larger than r_*, both $-\phi'(r)$ and $\chi'(r)$ start to decrease with the growth of r. When $s_{\beta_3} = 10^{-4}$, the distance r_V is of the order of $10r_*$. Hence the solutions to $\phi'(r)$ and $\chi'(r)$ are given by Eq. (4.27) for $r_* < r \lesssim 10r_*$ and by Eq. (4.25) for $r \gtrsim 10r_*$. In the left panel of Fig. 1, we can confirm that the qualitative behavior of $\chi'(r)$ changes around $r \approx 10r_*$ [i.e., from $\chi'(r) \propto r^{-1/2}$ to $\chi'(r) \propto r^{-2}$]. Note that $|\phi'(r)|$ decreases as $|\phi'(r)| \propto r^{-2}$ for $r > r_*$.

When $s_{\beta_3} = 1$, as seen in the right panel of Fig. 1, we find that there is almost no intermediate regime corresponding to the solution $\chi'(r) \propto r^{-1/2}$ and that the longitudinal mode decreases as $\chi'(r) \propto r^{-2}$ for $r > r_*$. This reflects the fact that, even when $s_{\beta_3} = O(1)$, the quantity ξ

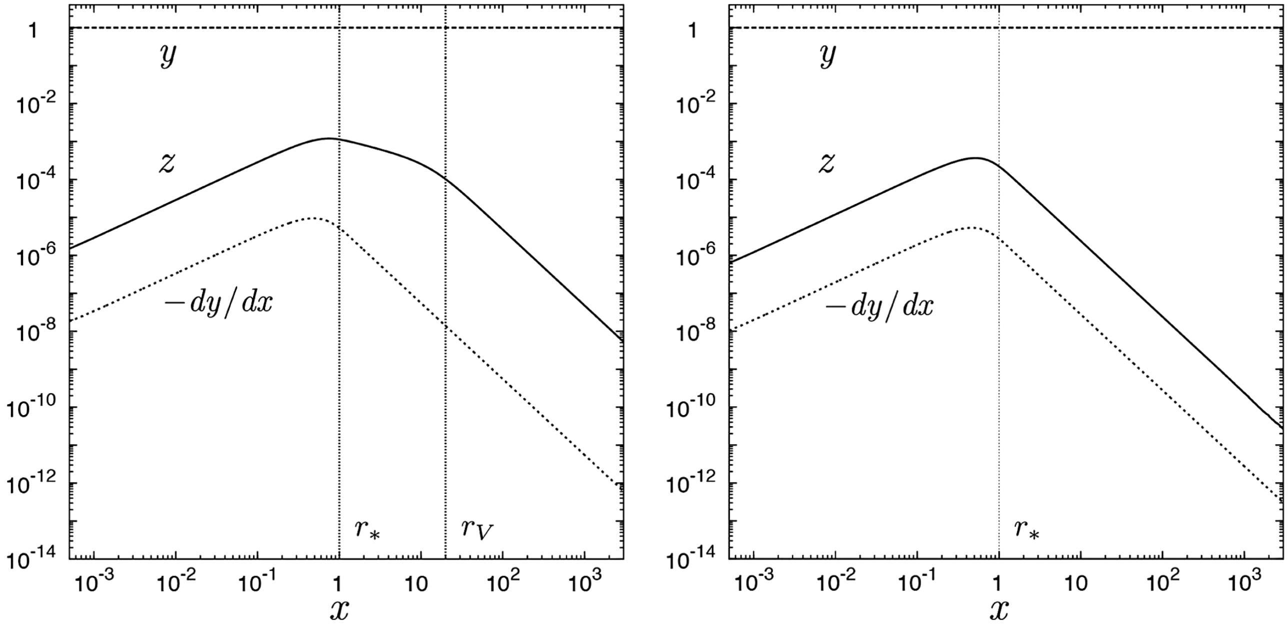

FIG. 1. The numerical solutions to $y = \phi/\phi_0$, $-dy/dx$, and $z = \chi'/\phi_0$ as a function of $x = r/r_*$ for the matter profile $\rho_m = \rho_0 e^{-4r^2/r_*^2}$ with $\Phi_* = 10^{-4}$. Each panel corresponds to $s_{\beta_3} = 10^{-4}$ (left) and $s_{\beta_3} = 1$ (right), respectively. The boundary conditions of Ψ, Φ, y, and dy/dx are chosen to be consistent with Eqs. (3.17) and (4.15) at $x = 10^{-3}$. The vertical lines represent the scales $r = r_*$ and $r_V = 20r_*$ (left panel) and the scale $r = r_*$ (right panel).

in Eq. (4.23) quickly becomes much larger than 1 with the growth of $r(> r_*)$. Then, for $s_{\beta_3} \gtrsim 1$, the solutions in the regime $r > r_*$ are well approximated by Eq. (4.25). For increasing $|\beta_3|$, the suppression for the amplitude of $\chi'(r)$ tends to be more significant outside the body.

In Fig. 1 we also find that $\phi(r)$ stays nearly constant in the whole regime of interest. This is associated with the fact that the r-dependent correction to $\phi(r)$ is at most of the order of $\phi_0\Phi_*$, i.e., much smaller than ϕ_0 under the weak gravity approximation. The numerical solutions to $\phi(r)$ and $\chi'(r)$ are fully consistent with the analytic field profiles derived under the assumption (4.9), so we resort to the analytic solutions for discussing corrections to the leading-order gravitational potentials in Sec. IV C.

C. Corrections to gravitational potentials

We compute the corrections to $\Phi_{\rm GR}$ and $\Psi_{\rm GR}$ induced by the longitudinal propagation of the vector field. Since the leading-order gravitational potentials obey Eqs. (3.15) and (3.16), we can express Eqs. (3.8) and (3.9) in the forms

$$\frac{2M_{\rm pl}^2}{r}\Phi' - \frac{M_{\rm pl}^2}{r^2}(1-e^{2\Phi}) = e^{2\Phi}\rho_m + \Delta_\Phi, \tag{4.32}$$

$$\frac{2M_{\rm pl}^2}{r}\Psi' + \frac{M_{\rm pl}^2}{r^2}(1-e^{2\Phi}) = e^{2\Phi}P_m + \Delta_\Psi, \tag{4.33}$$

where Δ_Φ and Δ_Ψ are correction terms. We are interested in the behavior of gravitational potentials outside a compact object ($r \gtrsim r_*$), so we employ the solutions (4.23) and (4.24) with the leading-order potentials (3.21) to estimate the corrections Δ_Φ and Δ_Ψ. Note that $\phi(r)$ is given by Eq. (4.9) with $|f(r)|$ at most of the order of $\phi_0\Phi_*$.

Let us first consider the case $s_{\beta_3} \gtrsim 1$. Since the solutions to $\phi'(r)$ and $\chi'(r)$ are approximately given by Eq. (4.25), it follows that

$$\Delta_\Phi \simeq \frac{5\Phi_*^2\phi_0^2 r_*^2}{72r^4}, \qquad \Delta_\Psi \simeq -\frac{\Phi_*^2\phi_0^2 r_*^2}{72r^4}, \tag{4.34}$$

where we used the condition $\xi \gg 1$. Integrations of Eqs. (4.32) and (4.33) lead to

$$\Phi(r) \simeq \frac{\Phi_* r_*}{6r}\left[1 - \frac{5\Phi_*}{24}\left(\frac{\phi_0}{M_{\rm pl}}\right)^2 \frac{r_*}{r}\right],$$
$$\Psi(r) \simeq -\frac{\Phi_* r_*}{6r}\left[1 - \frac{\Phi_*}{8}\left(\frac{\phi_0}{M_{\rm pl}}\right)^2 \frac{r_*}{r}\right]. \tag{4.35}$$

To recover the behavior close to GR in the solar system, we require that $\Phi_*(\phi_0/M_{\rm pl})^2(r_*/r) \ll 1$. Under this condition, the post-Newtonian parameter $\gamma \equiv -\Phi/\Psi$ is given by

$$\gamma \simeq 1 - \frac{\Phi_*}{12}\left(\frac{\phi_0}{M_{\rm pl}}\right)^2 \frac{r_*}{r}. \tag{4.36}$$

The local gravity experiments give the bound $|\gamma - 1| < 2.3 \times 10^{-5}$ [39]. For the Sun ($\Phi_* \simeq 10^{-6}$) we have $|\gamma - 1| \simeq 10^{-7}(\phi_0/M_{\rm pl})^2(r_*/r)$, so the experimental bound is well satisfied for $\phi_0 \lesssim M_{\rm pl}$. We also note that the deviation of γ from 1 decreases for larger r.

We proceed to the case $s_{\beta_3} \lesssim 1$. On using the solutions (4.27) for the distance $r_* < r < r_V$, the correction terms in Eqs. (4.32) and (4.33) read

$$\Delta_\Phi \simeq \frac{\sqrt{3\Phi_* r_*}\beta_3 \phi_0^3}{4r^{3/2}}, \quad \Delta_\Psi \simeq \frac{\sqrt{3}(\Phi_* r_*)^{5/2}\beta_3\phi_0^3}{432 r^{7/2}}, \tag{4.37}$$

which means that Δ_Ψ is about $10^{-2}\Phi_*^2(r_*/r)^2$ times as small as Δ_Φ. Integrating Eqs. (4.32) and (4.33), the gravitational potentials are given by

$$\Phi(r) \simeq \frac{\Phi_* r_*}{6r}\left[1+\sqrt{s_{\beta_3}}\left(\frac{\phi_0}{M_{\rm pl}}\right)^2\left(\frac{r}{r_*}\right)^{3/2}\right],$$
$$\Psi(r) \simeq -\frac{\Phi_* r_*}{6r}\left[1-2\sqrt{s_{\beta_3}}\left(\frac{\phi_0}{M_{\rm pl}}\right)^2\left(\frac{r}{r_*}\right)^{3/2}\right], \tag{4.38}$$

where we used s_{β_3} instead of β_3. The correction to $\Psi_{\rm GR}(r) = -\Phi_* r_*/(6r)$ is negligibly small for $\phi_0 \lesssim M_{\rm pl}$. The post-Newtonian parameter can be estimated as

$$\gamma \simeq 1 + 3\sqrt{s_{\beta_3}}\left(\frac{\phi_0}{M_{\rm pl}}\right)^2\left(\frac{r}{r_*}\right)^{3/2}, \tag{4.39}$$

which increases for larger r. The maximum value of $|\gamma-1|$ is reached at the distance $r = r_V$, i.e., $|\gamma - 1|_{\rm max} \simeq 3(\phi_0/M_{\rm pl})^2$. To satisfy the experimental bound of γ at this radius, we require that

$$\phi_0 \lesssim 3\times 10^{-3} M_{\rm pl}. \tag{4.40}$$

For $r > r_V$ the solutions of the vector field change to Eq. (4.25), so the parameter $|\gamma - 1|$ starts to decrease.

The above discussion shows that, when $s_{\beta_3} \gtrsim 1$, the extra gravitational interaction induced by the longitudinal mode $\chi'(r)$ is suppressed due to the Vainshtein mechanism. If $s_{\beta_3} \lesssim 1$, the screening mechanism of the fifth force is at work only at $r > r_V$, so the field value ϕ_0 is constrained as Eq. (4.40) for the consistency with local gravity tests at $r = r_V$. If r_V is larger than the solar-system scale ($r_{\rm solar} \sim 10^{14}$ cm), the upper bound of ϕ_0 gets weaker than Eq. (4.40). For the Sun ($r_* \sim 10^{11}$ cm), we have $r_V > r_{\rm solar}$ for $s_{\beta_3} \lesssim 10^{-9}$. In the limit that $s_{\beta_3} \to 0$ the distance r_V goes to infinity, so there is no upper bound of ϕ_0.

From Eq. (4.13) the following relation holds:

$$\sqrt{s_{\beta_3}} \simeq 2.5\times 10^{45}\beta_3 \frac{\phi_0}{M_{\rm pl}}\sqrt{\frac{1\ {\rm g/cm^3}}{\rho_0}}. \tag{4.41}$$

For the Sun ($\rho_0 \approx 100\ {\rm g/cm^3}$), we have $\sqrt{s_{\beta_3}} \approx 10^{44}\beta_3\phi_0/M_{\rm pl}$. Even if ϕ_0 is much smaller than the order of $M_{\rm pl}$, it is natural to satisfy the condition $s_{\beta_3} \gtrsim 1$ except for a very tiny coupling β_3. In this sense, we can say that the screening mechanism, which occurs for $s_{\beta_3} \gtrsim 1$, is very generic in the presence of a nonvanishing coupling β_3.

V. THEORIES WITH THE CUBIC AND QUARTIC LAGRANGIANS

In this section we study the theories given by the functions

$$G_2(X) = m^2 X, \quad G_3(X) = \beta_3 X, \quad G_4(X) = \frac{M_{\rm pl}^2}{2} + \beta_4 X^2, \tag{5.1}$$

where m, β_3, β_4 are constants (β_4 has a dimension of $[{\rm mass}]^{-2}$). Our interest is how the vector Galileon term $\beta_4 X^2$ modifies the screening mechanism discussed in Sec. IV. For the functions (5.1), the vector field equations of motion (3.12) and (3.13) read

$$\begin{aligned}
&\frac{1}{r^2}\frac{d}{dr}(r^2\phi') - e^{2\Phi}m^2\phi + 2\phi(\Psi'' + \Psi'^2 - \Psi'\Phi') + \left(3\phi' + \frac{4\phi}{r}\right)\Psi' - \phi'\Phi' - \beta_3\phi\left[\frac{1}{r^2}\frac{d}{dr}(r^2\chi') + (\Psi' - \Phi')\chi'\right] \\
&- \frac{2\beta_4 e^{-2\Phi}\phi}{r^2}[4r\chi'\chi'' + e^{2\Psi+2\Phi}\phi^2(e^{2\Phi} - 1 + 2r\Phi') - \chi'^2\{e^{2\Phi} - 3 + 2r(3\Phi' - 2\Psi')\}] \\
&+ \frac{2\beta_4 c_2 e^{-2\Phi}}{r}[2r\chi'\chi''(\phi' + 2\phi\Psi') + \chi'^2\{4\phi\Psi' + [\phi'' + 2\phi(\Psi'' - 3\Psi'\Phi' + \Psi'^2)]r + \phi'(2 - 3r\Phi' + 3r\Psi')\} \\
&- e^{2\Psi+2\Phi}\phi\{r\phi'^2 + \phi\phi'(2 - r\Phi' + 5r\Psi') + \phi(4\phi\Psi' + [\phi'' + 2\phi(\Psi'' - \Psi'\Phi' + \Psi'^2)]r)\}] = 0,
\end{aligned} \tag{5.2}$$

$$\begin{aligned}
&m^2\chi' + \beta_3\left[e^{2\Psi}(\phi\phi' + \phi^2\Psi') + e^{-2\Phi}\chi'^2\left(\frac{2}{r} + \Psi'\right)\right] + \frac{2\beta_4\chi'}{r}\left[e^{2\Psi}\frac{\phi^2}{r}(1 - e^{-2\Phi})\right. \\
&\left. + e^{2\Psi-2\Phi}(4\phi\phi' - c_2 r\phi'^2 - 4c_2\phi\phi' r\Psi' - 4c_2\phi^2 r\Psi'^2 + 2\phi^2\Psi') - e^{-2\Phi}\frac{\chi'^2}{r}(1 - 3e^{-2\Phi} - 6r\Psi' e^{-2\Phi})\right] = 0.
\end{aligned} \tag{5.3}$$

If $\beta_3 = 0$, then there is a solution $\chi' = 0$ to Eq. (5.3). This means that, in the absence of the Lagrangian $\mathcal{L}_3$, the $\beta_4 X^2$ term admits the solution where the longitudinal mode completely vanishes. In what follows, we shall consider the theories with $\beta_3 \neq 0$ and $\beta_4 \neq 0$ by dealing with the coupling β_3 as a small correction to the solution $\chi' = 0$. As we will see below, the longitudinal mode χ' does not completely vanish in such cases.

As for the coupling β_4, the condition under which the term $\beta_4 X^2 R$ in $G_4(X)R$ is subdominant to the Einstein-Hilbert term $M_{\rm pl}^2 R/2$ gives

$$|\beta_4|\phi^4 \ll M_{\rm pl}^2. \tag{5.4}$$

More specifically, we focus on the case in which the coupling β_4 is in the range

$$|\beta_4|\phi^2 \ll 1, \tag{5.5}$$

under which Eq. (5.4) is satisfied for $|\phi| \lesssim M_{\rm pl}$. We also assume that the constant $|c_2|$ is at most of the order of 1.

A. Vector field profiles

To derive analytic solutions to the vector field, we employ the weak gravity approximation ($\Phi_* \ll 1$) and expand Eqs. (5.2) and (5.3) up to first order in Ψ, Φ, and their derivatives. Analogous to the discussion in Sec. IV, we search for the solutions in the form (4.9) with χ'^2 suppressed relative to ϕ^2, i.e.,

$$|r\phi'| \ll |\phi|, \qquad \chi'^2 \ll \phi^2. \tag{5.6}$$

The consistency of these approximations can be checked after deriving analytic solutions of ϕ and χ'. Under this approximation scheme, Eqs. (5.2) and (5.3) reduce, respectively, to

$$(1-2c_2\beta_4\phi^2)\frac{d}{dr}(r^2\phi') - m^2r^2\phi - \beta_3\phi\frac{d}{dr}(r^2\chi') - 4\beta_4\phi\frac{d}{dr}(r\chi'^2) + 2\phi\frac{d}{dr}(r^2\Psi') + \beta_3\phi\chi' r^2(\Phi'-\Psi') - 4\beta_4\phi^3[\Phi + r(\Phi' + 2c_2\Psi' + c_2 r\Psi'')] \simeq 0, \tag{5.7}$$

$$\chi'\left[\frac{4\beta_4}{r}\left\{2\phi\phi' + \frac{\chi'^2}{r} + \frac{\phi^2}{r}(\Phi + r\Psi')\right\} + m^2\right] \simeq -\beta_3\left(\phi\phi' + \frac{2\chi'^2}{r} + \phi^2\Psi'\right). \tag{5.8}$$

In the following, we take the massless limit $m \to 0$. Then, Eq. (5.8) can be expressed as

$$\chi' = -\frac{\beta_3 r}{4\beta_4}\frac{r\phi\phi' + 2\chi'^2 + \phi^2 r\Psi'}{2r\phi\phi' + \chi'^2 + \phi^2(\Phi + r\Psi')}. \tag{5.9}$$

1. *In the regime* $r < r_*$

For the distance $r < r_*$ the leading-order gravitational potentials are given by Eq. (3.20), so Eq. (5.9) reads

$$\chi' = -\frac{\beta_3 r}{4\beta_4}\frac{r\phi\phi' + 2\chi'^2 + \rho_0\phi^2 r^2/(6M_{\rm pl}^2)}{2r\phi\phi' + \chi'^2 + \rho_0\phi^2 r^2/(3M_{\rm pl}^2)}. \tag{5.10}$$

If the condition

$$\chi'^2 \ll r|\phi\phi'| \tag{5.11}$$

is satisfied, Eq. (5.10) reduces to

$$\chi'(r) \simeq -\frac{\beta_3}{8\beta_4}r, \tag{5.12}$$

whose magnitude linearly grows in r. In the limit that $\beta_3 \to 0$, χ' vanishes as expected. Under the assumption (4.9), the field ϕ stays nearly a constant value ϕ_0. On using the solution (5.12), Eq. (5.7) is integrated to give

$$(1-2c_2\beta_4\phi_0^2)r^2\phi' + \frac{\beta_3^2\phi_0 r^3}{16\beta_4} \simeq -\frac{\rho_0\phi_0 r^3}{3M_{\rm pl}^2}[1-2(1+c_2)\beta_4\phi_0^2]. \tag{5.13}$$

Provided that

$$\delta \equiv \frac{3\beta_3^2 M_{\rm pl}^2}{16\beta_4\rho_0} \ll 1, \tag{5.14}$$

the term containing β_3 in Eq. (5.13) is subdominant relative to other terms. Then, we obtain the following solution:

$$\phi'(r) \simeq -\frac{\rho_0\phi_0 r}{3M_{\rm pl}^2}\frac{1-2(1+c_2)\beta_4\phi_0^2}{1-2c_2\beta_4\phi_0^2}, \tag{5.15}$$

which is close to $\phi'(r) \simeq -\rho_0\phi_0 r/(3M_{\rm pl}^2)$. On using this solution with $\Phi_* \ll 1$, it follows that $|r\phi'| \ll |\phi|$ for $r < r_*$. The condition (5.11) translates to

$$\varepsilon \equiv \frac{3\beta_3^2 M_{\rm pl}^2}{64\beta_4^2\rho_0\phi_0^2} \ll 1. \tag{5.16}$$

Since ε is related to δ in Eq. (5.14) as $\delta = 4\varepsilon\beta_4\phi_0^2$, the condition (5.16) is tighter than (5.14) under the assumption (5.5). We also note that, under the condition (5.11) with $|r\phi'| \ll |\phi|$, the second relation of Eq. (5.6) is automatically satisfied.

2. In the regime $r_* < r < r_t$

For $r > r_*$ the leading-order gravitational potentials are given by Eq. (3.21), so integration of Eq. (5.7) leads to

$$(1-2c_2\beta_4\phi_0^2)r^2\phi' - \beta_3\phi_0 r^2\chi' - 4\beta_4\phi_0 r\chi'^2 \simeq -\frac{\rho_0\phi_0 r_*^3}{3M_{\rm pl}^2}, \tag{5.17}$$

whereas Eq. (5.9) reduces to

$$\chi' \simeq -\frac{\beta_3 r}{4\beta_4}\frac{r\phi\phi' + 2\chi'^2 + \rho_0\phi^2 r_*^3/(6M_{\rm pl}^2 r)}{2r\phi\phi' + \chi'^2 + \rho_0\phi^2 r_*^3/(3M_{\rm pl}^2 r)}. \tag{5.18}$$

Unlike Eq. (5.13), the rhs of Eq. (5.17) is constant. For the distance $r < r_*$ the field derivative $|\phi'|$ linearly grows in r as Eq. (5.15), but $|\phi'|$ starts to decrease for $r > r_*$. Meanwhile, as long as the condition (5.11) is satisfied, Eq. (5.18) gives the following solution:

$$\chi'(r) \simeq -\frac{\beta_3}{8\beta_4} r, \tag{5.19}$$

so that $|\chi'|$ continues to grow. Substituting this solution into Eq. (5.17), we obtain

$$\phi'(r) \simeq -\frac{\rho_0\phi_0 r_*^3}{3M_{\rm pl}^2(1-2c_2\beta_4\phi_0^2)r^2}\left(1+\delta\frac{r^3}{r_*^3}\right). \tag{5.20}$$

Under the condition (5.14) the second term in the bracket of Eq. (5.20) is much smaller than 1 around $r = r_*$. Provided that $\delta r^3/r_*^3 \ll 1$, the leading-order solution of Eq. (5.20) decreases for larger r.

Substituting the approximate solutions $\phi \simeq \phi_0$ and $\phi' \simeq -\rho_0\phi_0 r_*^3/(3M_{\rm pl}^2 r^2)$ into Eq. (5.18), it follows that

$$\chi' \simeq -\frac{\beta_3 r}{2\beta_4}\frac{\chi'^2 - \rho_0\phi_0^2 r_*^3/(12M_{\rm pl}^2 r)}{\chi'^2 - \rho_0\phi_0^2 r_*^3/(3M_{\rm pl}^2 r)}. \tag{5.21}$$

The increase of $|\chi'|$ gradually saturates as χ'^2 approaches the value $\rho_0\phi_0^2 r_*^3/(12M_{\rm pl}^2 r)$. We define the transition distance r_t according to the condition $\chi'^2(r_t) = \rho_0\phi_0^2 r_*^3/(12M_{\rm pl}^2 r_t)$. On using the solution (5.19), we obtain

$$r_t = \frac{1}{(4\varepsilon)^{1/3}} r_*, \tag{5.22}$$

which is larger than r_* under the condition (5.16). Around $r = r_t$ the growth of $|\chi'|$ changes to decrease.

3. In the regime $r > r_t$

As $|\chi'|$ decreases for the distance $r > r_t$, the lhs of Eq. (5.8), which is multiplied by the factor $4\beta_4\chi'/r$, becomes subdominant to the rhs, so that we obtain

$$\phi\phi' + \frac{2}{r}\chi'^2 + \frac{\rho_0\phi^2 r_*^3}{6M_{\rm pl}^2 r^2} \simeq 0. \tag{5.23}$$

Since the term $-4\beta_4\phi_0 r\chi'^2$ in Eq. (5.17) is negligible relative to $-\beta_3\phi_0 r^2\chi'$, we have

$$(1-2c_2\beta_4\phi_0^2)r^2\phi' - \beta_3\phi_0 r^2\chi' \simeq -\frac{\rho_0\phi_0 r_*^3}{3M_{\rm pl}^2}. \tag{5.24}$$

Apart from the small difference of the coefficient in front of $r^2\phi'$, the system described by Eqs. (5.23) and (5.24) has the same structure as that studied in Sec. IV. Physically, this means that the effect of the Lagrangian $\mathcal{L}_3$ manifests itself for the distance $r > r_t$.

Following the same procedure as that in Sec. IV, we obtain the following field profiles:

$$\phi'(r) = -\frac{\rho_0\phi_0 r_*^3}{3M_{\rm pl}^2 r^2}\mathcal{G}(\eta), \tag{5.25}$$

$$\chi'(r) = \pm\sqrt{\frac{\rho_0\phi_0^2 r_*^3}{6M_{\rm pl}^2 r}\left[\mathcal{G}(\eta) - \frac{1}{2}\right]}, \tag{5.26}$$

where

$$\eta = \frac{s_{\beta_3}}{1-2c_2\beta_4\phi_0^2}\frac{r^3}{r_*^3}, \tag{5.27}$$

$$\mathcal{G}(\eta) = \frac{1+\eta}{1-2c_2\beta_4\phi_0^2}\left[1 - \sqrt{1 - \frac{1+(1-2c_2\beta_4\phi_0^2)\eta}{(1+\eta)^2}}\right]. \tag{5.28}$$

In Eq. (5.28) we have chosen the branch where $\mathcal{G}(\eta)$ does not diverge in the limit that $\eta \to \infty$. If the ratio β_3/β_4 is positive (negative), then χ' has a negative (positive) sign. On using Eq. (5.22), the quantity η can be expressed as

$$\eta = \frac{4\beta_4^2\phi_0^4}{1-2c_2\beta_4\phi_0^2}\frac{r^3}{r_t^3}. \tag{5.29}$$

At the distance $r = r_t$ we have that $\eta \simeq 4\beta_4^2\phi_0^4 \ll 1$, but η increases for larger r. The distance r_v at which η is equivalent to 1 can be estimated as

$$r_v \simeq \frac{1}{(4\beta_4^2\phi_0^4)^{1/3}} r_t. \tag{5.30}$$

For the distance $r_t < r < r_v$, Eqs. (5.25) and (5.26) reduce, respectively, to

$$\phi'(r) \simeq -\frac{\rho_0\phi_0 r_*^3}{3M_{\rm pl}^2(1-2c_2\beta_4\phi_0^2)r^2}, \tag{5.31}$$

$$\chi'(r) \simeq \pm\sqrt{\frac{\rho_0\phi_0^2 r_*^3}{12M_{\rm pl}^2 r}\frac{1+2c_2\beta_4\phi_0^2}{1-2c_2\beta_4\phi_0^2}}. \tag{5.32}$$

The behavior of $\phi'(r)$ is practically unchanged compared to Eq. (5.20). The solution (5.32) smoothly matches with Eq. (5.19) at $r = r_t$ with the amplitude $|\chi'(r_t)| \simeq \sqrt{\rho_0\phi_0^2 r_*^3/(12M_{\rm pl}^2 r_t)}$.

For the distance $r > r_v$ we obtain the following solution:

$$\phi'(r) \simeq -\frac{\rho_0\phi_0 r_*^3}{6M_{\rm pl}^2 r^2}, \tag{5.33}$$

$$\chi'(r) \simeq \pm(1+2c_2\beta_4\phi_0^2)\frac{\rho_0 r_*^3}{6\beta_3 M_{\rm pl}^2 r^2}. \tag{5.34}$$

As in the case of Sec. IV, the behavior of the longitudinal mode changes from $|\chi'| \propto r^{-1/2}$ to $|\chi'| \propto r^{-2}$ around $r = r_v$.

In Fig. 2 we plot an example of the field profile derived by numerically solving the vector-field equations of motion [(5.2) and (5.3)] coupled with the leading-order gravitational equations (3.15) and (3.16). The ratio β_3/β_4 is chosen to be negative in this case, so the sign of χ' is positive. Since $\varepsilon = s_{\beta_3}/(16\beta_4^2\phi_0^4) \simeq 2.4 \times 10^{-4}$, the transition distance r_t corresponds to $r_t \simeq 10r_*$. As estimated from Eqs. (5.12) and (5.19), the numerical simulation of Fig. 2 shows that χ' linearly grows in r up to the distance $r_t \simeq 10r_*$. We also find that the longitudinal mode behaves as $\chi'(r) \propto r^{-1/2}$ for $r_t < r < r_v \simeq 100r_*$ and $\chi'(r) \propto r^{-2}$ for $r > r_v$.

As seen in Fig. 2, the derivative of $\phi(r)$ has the dependence $-\phi'(r) \propto r$ for $r \lesssim r_*$ and $-\phi'(r) \propto r^{-2}$ for $r \gtrsim r_*$. Since $|r\phi'(r)|$ is at most of the order of ϕ_0 for the whole distance range of interest, the field ϕ stays nearly constant around ϕ_0. Thus, our numerical results are fully consistent with the analytic solutions of $\chi'(r)$ and $\phi(r)$.

B. Corrections to gravitational potentials

Let us proceed to the calculations of corrections to gravitational potentials Ψ and Φ induced by the vector field.

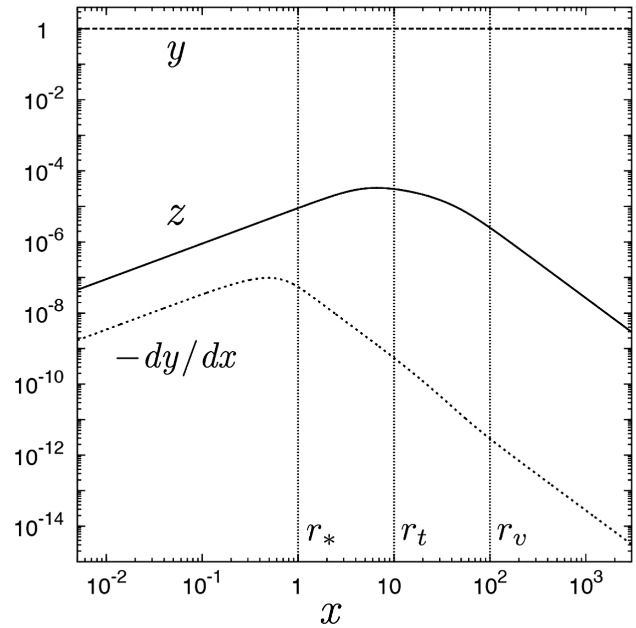

FIG. 2. The numerical solutions to $y = \phi/\phi_0$, $-dy/dx$, and $z = \chi'/\phi_0$ as a function of $x = r/r_*$ for the theories with $s_{\beta_3} = 1.0 \times 10^{-6}$ with $\beta_3 > 0$ and $\beta_4\phi_0^2 = -1.6 \times 10^{-2}$, $c_2 = 1$, and $\phi_0 = 1.0 \times 10^{-3}M_{\rm pl}$. The matter profile is given by $\rho_m = \rho_0 e^{-4r^2/r_*^2}$ with $\Phi_* = 10^{-6}$. The boundary conditions of Ψ, Φ, y, and dy/dx are chosen to be consistent with Eqs. (3.17), (4.9), and (5.15) at $x = 10^{-3}$. The vertical lines stand for the scales $r = r_*$, $r_t = 10r_*$ and $r_v = 100r_*$ respectively.

We shall study the two regimes: (i) $r_* < r < r_t$ and (ii) $r > r_t$, separately.

1. $r_* < r < r_t$

At this distance, the leading-order vector field solutions are given by $\phi \simeq \phi_0$, $\phi' \simeq -\phi_0\Phi_* r_*/(3r^2)$, and $\chi' \simeq -\beta_3 r/(8\beta_4)$, where $\Phi_* = \rho_0 r_*^2/M_{\rm pl}^2$. In Eqs. (3.15) and (3.16) we expand the gravitational potentials Ψ, Φ, and their derivatives up to linear order. The correction terms in Eqs. (4.32) and (4.33) are approximately given by

$$\Delta_\Phi \simeq -\frac{2\beta_4\phi_0^4\Phi_*\varepsilon(\Phi_*\varepsilon x^2+3)}{3r_*^2} - \frac{\phi_0^2\Phi_*^2}{18r_*^2x^4}, \tag{5.35}$$

$$\Delta_\Psi \simeq -\frac{2\beta_4\phi_0^4\Phi_*(5\Phi_*\varepsilon^2x^5+3\varepsilon x^3-3)}{9r_*^2x^3} + \frac{\phi_0^2\Phi_*^2}{18r_*^2x^4}, \tag{5.36}$$

where we have employed the approximation (5.5) and used the parameter ε given by Eq. (5.16) with $\rho_0 = \Phi_* M_{\rm pl}^2/r_*^2$ and $x = r/r_*$. Integrating Eqs. (4.32) and (4.33) with these corrections, the resulting gravitational potentials are

$$\Phi(r) \simeq \frac{\Phi_* r_*}{6r}\left[1 - \frac{2\beta_4\phi_0^4\varepsilon x^3(\Phi_*\varepsilon x^2+5)}{5M_{\rm pl}^2} + \frac{\phi_0^2\Phi_*}{6M_{\rm pl}^2 x}\right], \tag{5.37}$$

$$\Psi(r) \simeq -\frac{\Phi_* r_*}{6r}\left[1+\frac{2\beta_4\phi_0^4(7\Phi_*\varepsilon^2x^5+15\varepsilon x^3+15)}{15M_{\rm pl}^2} + \frac{\phi_0^2\Phi_*}{6M_{\rm pl}^2x}\right]. \quad (5.38)$$

Provided that the corrections to the leading-order gravitational potentials are small, the post-Newtonian parameter $\gamma = -\Phi/\Psi$ reads

$$\gamma \simeq 1-\frac{2\beta_4\phi_0^4(2\Phi_*\varepsilon^2x^5+6\varepsilon x^3+3)}{3M_{\rm pl}^2}. \quad (5.39)$$

At $r = r_*$ the first two terms inside the bracket of Eq. (5.39) are subdominant to the last term, so Eq. (5.39) reduces to

$$\gamma \simeq 1-\frac{2\beta_4\phi_0^4}{M_{\rm pl}^2}. \quad (5.40)$$

The parameter $|\gamma-1|$ increases for larger r and it reaches the maximum value at $r = r_t$, i.e.,

$$\gamma \simeq 1-\frac{3\beta_4\phi_0^4}{M_{\rm pl}^2}. \quad (5.41)$$

Under the condition (5.4), the deviation of γ from 1 is small. From the local gravity bound $|\gamma-1| < 2.3\times10^{-5}$, we obtain

$$|\beta_4|\phi_0^4 < 8\times10^{-6}M_{\rm pl}^2. \quad (5.42)$$

This shows that, as long as the nonzero coupling β_3 obeys Eq. (5.16), the local gravity constraint is satisfied under the condition (5.42) for the distance $r < r_t$.

2. $r > r_t$

For r larger than r_t, we only need to study the behavior of Ψ and Φ in the regime $r_t < r < r_v$ (because $|\gamma-1|$ decreases for $r > r_v$ as we discussed in Sec. IV). The leading-order field solutions for $r_t < r < r_v$ are given by $\phi \simeq \phi_0$, $\phi' \simeq -\phi_0\Phi_* r_*/(3r^2)$ with the two branches of χ', i.e., $\chi' \simeq -\sqrt{\phi_0^2\Phi_* r_*/(12r)}$ for $\beta_3/\beta_4 > 0$ and $\chi' \simeq \sqrt{\phi_0^2\Phi_* r_*/(12r)}$ for $\beta_3/\beta_4 < 0$. By using the relation $\beta_3 = \pm 8\beta_4\phi_0\sqrt{\Phi_*\varepsilon}/(\sqrt{3}r_*)$, the correction terms in Eqs. (4.32) and (4.33) can be expressed independently of the sign of β_3/β_4, as

$$\Delta_\Phi \simeq -\frac{2\beta_4\phi_0^4\Phi_*\sqrt{\varepsilon}}{r_*^2x^{3/2}} - \frac{\phi_0^2\Phi_*^2}{18r_*^2x^4}, \quad (5.43)$$

$$\Delta_\Psi \simeq -\frac{\beta_4\phi_0^4\Phi_*(5\Phi_*^2\sqrt{\varepsilon}-27\sqrt{x})}{54r_*^2x^{7/2}} + \frac{\phi_0^2\Phi_*^2}{18r_*^2x^4}, \quad (5.44)$$

where we used the approximation (5.5). The integrated solutions to gravitational potentials are given by

$$\Phi(r) \simeq \frac{\Phi_* r_*}{6r}\left[1-\frac{4\beta_4\phi_0^4x^{3/2}\sqrt{\varepsilon}}{M_{\rm pl}^2} + \frac{\phi_0^2\Phi_*}{6M_{\rm pl}^2x}\right], \quad (5.45)$$

$$\Psi(r) \simeq -\frac{\Phi_* r_*}{6r}\left[1+\frac{\beta_4\phi_0^4(16x^{3/2}\sqrt{\varepsilon}+3)}{2M_{\rm pl}^2} + \frac{\phi_0^2\Phi_*}{6M_{\rm pl}^2x}\right]. \quad (5.46)$$

As long as the corrections to $\Phi_{\rm GR}$ and $\Psi_{\rm GR}$ remain small, the post-Newtonian parameter can be estimated as

$$\gamma \simeq 1-\frac{3\beta_4\phi_0^4(8x^{3/2}\sqrt{\varepsilon}+1)}{2M_{\rm pl}^2}. \quad (5.47)$$

At $r = r_t$ this reduces to $\gamma - 1 \simeq -15\beta_4\phi_0^4/(2M_{\rm pl}^2)$, so the bound $|\gamma-1| < 2.3\times10^{-5}$ translates to $|\beta_4|\phi_0^4 < 3\times10^{-6}$. Taking into account Eq. (5.42), local gravity constraints can be satisfied for

$$|\beta_4|\phi_0^4 \lesssim 10^{-6}M_{\rm pl}^2. \quad (5.48)$$

At $r = r_v$ it follows that

$$\gamma \simeq 1-3\frac{\phi_0^2}{M_{\rm pl}^2}. \quad (5.49)$$

Hence the resulting experimental bound is the same as Eq. (4.40), i.e.,

$$\phi_0 \lesssim 3\times10^{-3}M_{\rm pl}. \quad (5.50)$$

If r_v is far outside the solar-system scale, we do not need to impose the condition (5.50).

In summary, under the conditions (5.48) and (5.50) with β_3 in the range (5.16), the deviation from GR is sufficiently small such that the model is consistent with local gravity experiments. When $\beta_3 \to 0$, it follows that r_t goes to infinity and that χ' vanishes for both $r < r_*$ and $r > r_*$. In the limit $\beta_3 \to 0$ (i.e., $\varepsilon \to 0$), Eq. (5.39) reduces to $\gamma \simeq 1 - 2\beta_4\phi_0^4/M_{\rm pl}^2$, so the local gravity bound is satisfied for $|\beta_4|\phi_0^4 \lesssim 10^{-5}M_{\rm pl}^2$. In this case, the deviation of γ from 1 is directly related with the existence of the $\beta_4 X^2$ term in G_4.

VI. CONCLUSIONS

In this paper, we have studied the screening mechanism of the fifth force in a generalized class of Proca theories. The breaking of $U(1)$ gauge invariance for an Abelian vector field gives rise to nontrivial derivative self-interactions described by the Lagrangians (2.3)–(2.5), in addition to the Lagrangian $\mathcal{L}_2$ associated with the mass term. The equations of motion in these generalized Proca theories are of second order without Ostrogradski instabilities, while the number of propagating DOF remains three (two transverse and one longitudinal) as in the original Proca theory.

In the presence of a matter source, we derived the equations of motion up to the Lagrangian $\mathcal{L}_4$ for a general curved space-time and then applied them to a spherically symmetric and static background described by the line element (3.1). First, we showed that the transverse components of the spatial vector field A^i vanish identically to satisfy the compatibility with the spherically symmetric background and the regularity of solutions at the origin. Thus, we focused on the propagation of the longitudinal scalar component of A^i with A^μ of the form (3.7).

The leading-order gravitational interaction in the vector-field equations should come from the gravitational potentials $\Psi_{\rm GR}$ and $\Phi_{\rm GR}$, whose interior and exterior solutions around a compact body ($\rho_m \simeq \rho_0$ for $r < r_*$ and $\rho_m \simeq 0$ for $r > r_*$) are given, respectively, by Eqs. (3.17) and (3.18). After substituting these solutions into the vector equations of motion under the weak-gravity approximation ($\Phi_* = \rho_0 r_*^2/M_{\rm pl}^2 \ll 1$), it is possible to derive analytic solutions of the vector field A^μ (the temporal component ϕ and the transverse mode χ') for a given Lagrangian.

In Sec. IV we obtained analytic vector field profiles and corrections to the leading-order gravitational potentials $\Phi_{\rm GR}$ and $\Psi_{\rm GR}$ in the presence of the vector Galileon Lagrangian $\mathcal{L}_3 = \beta_3 X \nabla_\mu A^\mu$ by assuming that the temporal component ϕ is of the form (4.9). Provided that the parameter $s_{\beta_3} = 3(\beta_3\phi_0 M_{\rm pl})^2/(4\rho_0)$ is larger than the order of 1, derivative self-interactions lead to the suppression of the longitudinal mode $\chi'(r)$. The fifth force can be screened in such a way that the model is compatible with solar-system constraints of gravity. For $s_{\beta_3} \ll 1$ the screening occurs partially at the distance larger than r_V given by Eq. (4.26), in which case the solar-system experiments lead to the bound $\phi_0 \lesssim 3 \times 10^{-3} M_{\rm pl}$. As shown in Fig. 1, we have numerically confirmed that our analytic solutions of the vector field are sufficiently trustable even for the continuous density profile like Eq. (4.28).

In Sec. V we studied the vector Galileon theories up to the Lagrangian $\mathcal{L}_4$ which contains a derivative self-coupling term $\beta_4 X^2$ in the function G_4. When $\beta_3 = 0$, we showed the existence of the solution where $\chi'(r)$ vanishes everywhere. If the Lagrangian $\mathcal{L}_3$ is present in addition to $\mathcal{L}_4$ and the former is subdominant to the latter, we obtained the solution $\chi'(r) = -\beta_3 r/(8\beta_4)$ for $r \lesssim r_t = r_*/(4\varepsilon)^{1/3}$, where ε is given by Eq. (5.16). For $r > r_t$ the effect of the coupling β_3 manifests itself in the longitudinal mode, such that its amplitude decreases as $|\chi'(r)| \propto r^{-1/2}$ for $r_t < r < r_v = r_t/(4\beta_4^2\phi_0^4)^{1/3}$ and $|\chi'(r)| \propto r^{-2}$ for $r > r_v$ (see Fig. 2). The solar-system constraint at $r = r_t$ provides a mild bound $|\beta_4|\phi_0^4 \lesssim 10^{-6} M_{\rm pl}^2$. If r_v is within the solar-system scale, we also obtain the bound $\phi_0 \lesssim 3 \times 10^{-3} M_{\rm pl}$ from the estimation (5.49) of the post-Newtonian parameter.

We have thus shown that the screening mechanism of the longitudinal scalar for the vector field is at work in the presence of cubic and quartic derivative self-interactions. It will be of interest to study whether the similar mechanism holds or not with the Lagrangian $\mathcal{L}_5$. We leave this analysis for a future work.

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APPENDIX EXPRESSIONS FOR THE COEFFICIENTS $\mathcal{C}_i$

The coefficients of the gravitational equations (3.8)–(3.10) are given by

$$\mathcal{C}_1 = 4X_\phi[1 - 2c_2(G_{4,X} + 2X_\phi G_{4,XX})],$$

$$\mathcal{C}_2 = 4\chi' X_\phi G_{3,X} + 2e^{2\Psi}\phi\phi'[1 - 2c_2(G_{4,X} + 2X_\phi G_{4,XX})],$$

$$\mathcal{C}_3 = -32 X_\phi X_\chi G_{4,XX},$$

$$\mathcal{C}_4 = -2\chi'(X_\phi + X_\chi)G_{3,X},$$

$$\mathcal{C}_5 = -4[G_4 - 2(X_\phi + 2X_\chi)G_{4,X} - 4X_\chi(X_\phi + X_\chi)G_{4,XX}],$$

$$\mathcal{C}_6 = -e^{2\Phi}(G_2 - 2X_\phi G_{2,X}) + [e^{2\Psi}\phi\phi'\chi' + 2\chi''(X_\phi + X_\chi)]G_{3,X} + \frac{1}{2}e^{2\Psi}\phi'^2[1 - 2c_2(G_{4,X} + 2X_\phi G_{4,XX})],$$

$$\mathcal{C}_7 = 4\chi' X_\phi G_{3,X} + 4e^{-2\Phi}\chi'\chi'' G_{4,X} + 8[e^{-2\Phi}\chi'\chi''(X_\phi + X_\chi) - e^{2\Psi}\phi\phi' X_\chi]G_{4,XX},$$

$$
\begin{aligned}
\mathcal{C}_8 &= 2(1-e^{2\Phi})G_4 - 4[X_\chi + (1-e^{2\Phi})X_\phi]G_{4,X} - 8X_\phi X_\chi G_{4,XX}, \\
\mathcal{C}_9 &= 4X_\phi[1-2c_2(G_{4,X}+2X_\chi G_{4,XX})], \\
\mathcal{C}_{10} &= 2\chi'(X_\chi - X_\phi)G_{3,X} + 2e^{2\Psi}\phi\phi'[1-2c_2(G_{4,X}+2X_\chi G_{4,XX})], \\
\mathcal{C}_{11} &= 4[G_4 + 2(X_\phi - 2X_\chi)G_{4,X} + 4X_\chi(X_\phi - X_\chi)G_{4,XX}], \\
\mathcal{C}_{12} &= -e^{2\Phi}(G_2 - 2X_\chi G_{2,X}) - e^{2\Psi}\phi\phi'\chi' G_{3,X} + \frac{1}{2}e^{2\Psi}\phi'^2[1-2c_2(G_{4,X}+2X_\chi G_{4,XX})], \\
\mathcal{C}_{13} &= 4\chi' X_\chi G_{3,X} + 4e^{2\Psi}\phi\phi'(G_{4,X}+2X_\chi G_{4,XX}), \\
\mathcal{C}_{14} &= 2(1-e^{2\Phi})G_4 - 4X_\chi(2-e^{2\Phi})G_{4,X} - 8X_\chi^2 G_{4,XX}, \\
\mathcal{C}_{15} &= 2[G_4 + 2(X_\phi - X_\chi)G_{4,X}], \\
\mathcal{C}_{16} &= 2[G_4 + 2\{2(c_2+2)X_\phi - X_\chi\}G_{4,X} + 4X_\phi(X_\phi - X_\chi)G_{4,XX} - 2X_\phi], \\
\mathcal{C}_{17} &= -2[G_4 + 2(X_\phi - 2X_\chi)G_{4,X} + 4X_\chi(X_\phi - X_\chi)G_{4,XX}], \\
\mathcal{C}_{18} &= 2\chi' X_\phi G_{3,X} - 2e^{2\Psi}\phi\phi'[1-2(c_2+3)G_{4,X} \\
&\quad + 2(X_\chi - 2X_\phi)G_{4,XX}] + 2e^{-2\Phi}\chi'\chi''[G_{4,X} + 2(X_\chi - X_\phi)G_{4,XX}], \\
\mathcal{C}_{19} &= -2[G_4 - 4X_\chi(G_{4,X} + X_\chi G_{4,XX})], \\
\mathcal{C}_{20} &= -e^{2\Phi}G_2 + (2\chi'' X_\chi + e^{2\Psi}\phi\phi'\chi')G_{3,X} - \frac{1}{2}e^{2\Psi}\phi'^2[1-2(c_2+2)G_{4,X} - 8X_\phi G_{4,XX}] \\
&\quad + 2e^{2\Psi}\phi\phi'' G_{4,X} - 2e^{2\Psi-2\Phi}\phi\phi'\chi'\chi'' G_{4,XX}, \\
\mathcal{C}_{21} &= 2e^{2\Psi}\phi\phi'(G_{4,X} - 2X_\chi G_{4,XX}) + 2e^{-2\Phi}\chi'\chi''(G_{4,X} + 2X_\chi G_{4,XX}).
\end{aligned} \tag{A1}
$$

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