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Abstract

This paper qualifies and quantifies what is meant by higher price level and dispersion in an oligopoly market with imperfectly informed consumers for both Fixed Sample Search and Sequential Search. The objective is to identify the conditions under which prices become lower and price dispersion reduces as a function of consumers' information. Surprisingly, the mean price is an increasing function of search intensity and price dispersion is an inverse U-shaped function of the proportion of informed consumers.

JEL classification: D11; D43; D83; L11

Keywords: Search; Information; Competition; Price dispersion

1. Introduction

It has been conjectured that traditional markets might exhibit higher prices and price dispersion than electronic markets.[1] One main reason is that a market with more and better informed consumers should reduce prices and price dispersion for a homogeneous good. The aim of this paper is to discuss this presumption.

We consider an oligopolistic market that consists of a finite number of identical firms producing a homogeneous good at a constant marginal cost and consumers using a Fixed Sample Search (FSS in the following) or a Sequential Search (SS) technology. It is an extension in some directions

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[1] For an analysis, see Brynjolfsson and Smith, 2000

of Stahl (1989) and Varian's (1980) papers. Both consider the effect of information on the price distribution. Varian derives the equilibrium price distribution for an exogenous FSS technology with informed and uninformed consumers. Stahl computes the equilibrium price distribution in the case of SS with two types of consumers, one of whom has a zero search cost. He derives some asymptotic results on the price distribution notably when the number of firms increases. Other authors have considered this effect on the price distribution (Diamond, 1971; Rosenthal, 1980; Stiglitz, 1987). Diamond shows that the monopoly price is the sole equilibrium for an infinitesimal consumers' search cost and an infinite number of firms. Stahl obtains the same result when the number of firms tends to infinity. Janssen and Moraga-Gonzalez (2004) show that an increase in the number of firms does not necessarily lead to an increase in the expected price. For that, they consider a market with two types of FSS consumers. Some are informed, meaning that they search prices costlessly. Others are high cost consumers, meaning that they pay a positive search cost. They prove the existence of three types of equilibria characterized by low, middle or high search intensity. Moreover, they show that when the number of firms tends to infinity, the expected price increases unless the search is of the low type intensity. Janssen et al. (2005) extend Stahl's results to the case of costly sequential search.[2]

Our paper addresses the same issues. Its contribution is to quantify the level of prices and price dispersion as a function of consumers' information and search intensity both in the case of FSS and SS. For this sake, we use two statistics, namely the expected price and variance of the equilibrium price distribution. We show how these two statistics change with consumers' information both for FSS and SS. Surprisingly, the mean price is an increasing function of search intensity (i.e. the sample size of informed consumers) and price dispersion is an inverse U-shaped function of the proportion of informed consumers.

Section 2 presents the general framework. Section 3 analyses the effect on the expected price and on the variance of the price distribution of a change in either the proportion of informed consumers or in the search intensity of these consumers which corresponds to the sample size. Section 4 deals with the same issue in the SS case. Finally, Section 5 concludes.

2. General framework

There are n identical firms selling a homogeneous product at a marginal cost c which we assume w.l.g. to be zero. Firms do not price discriminate between consumers. Each firm is assumed to maximize its expected profit. Consumers have an identical unitary demand function for the product given by $d(p) = 1$ if $0 \leq p \leq v$ and zero elsewhere. v is the maximal willingness to pay of the consumers. Consumers are imperfectly informed about prices. Specifically, the population A of consumers is distributed according to a vector $\overrightarrow{a_k} = (a_1, a_k)$ where a_j with $j = 1, k$ is the proportion of consumers who visit j firms randomly. Moreover, since $a_1 + a_k = 1$, let $a_k = a$ and $a_1 = 1 - a$. Thus, we call informed consumers or shoppers, those who select a random sample of k among n firms with $1 < k \leq n$. They are in proportion a and to avoid trivial cases, we suppose that $0 < a < 1$. Uninformed consumers are assumed to be evenly distributed among firms. In each period, a new cohort of consumers enters the market and firms simultaneously choose a new price which remains fixed during the period. We consider only symmetric equilibria. Let $F(p; a, k)$

[2] Baye and Morgan (2001) consider a related issue: a profit seeking gatekeeper collects prices from firms and provides information to consumers who access the list of advertised prices. They show a conflict of interest between lower price dispersion and the maximizing behavior of the gatekeeper.

be the cumulative equilibrium distribution of prices depending on the parameters a and k. The following Lemma is proved by Varian.

Lemma 1. *Whenever $0 < a < 1$, if $F(p; a, k)$ is a Nash equilibrium conditional on the distribution of consumer search $\overrightarrow{a_k}$, then it is atomless. The support $[b(a, k), v]$ of the price distribution is connected.*

The probability that a firm, say i, randomly chosen among k firms sets the lowest price is $(1 - F(p_i; a, k))^{k-1}$. There are C_k^n ways of choosing k firms among n. The probability that an informed consumer will buy the good from firm i is then $(C_{k-1}^{n-1}/C_k^n)(1 - F(p_i; a, k)^{k-1}) = (k/n)(1 - F(p_i; a, k)^{k-1})$.

Lemma 2. *The expected demand of a firm fixing price p, given that rivals play the mixed strategy $F(p; a, k)$, is*

$$D(p; F(p; a, k)) = \frac{A}{n}[(1 - a) + ka(1 - F(p; a, k))^{k-1}].$$

Definition 3. A distribution $F(p; a, k)$, conditional on the distribution of consumers' search behavior (a, k), is a symmetric Nash-equilibrium, if the expected profit $E\pi(p, F(p; a, k))$ is equal to a constant (say π) for all p in the support of $F(p; a, k)$ and not greater than π for any other p.

By Lemma 1, v is fixed in equilibrium and by Definition 3,

$$\pi = E\pi(p, F(p; a, k)) = \frac{A}{n}(1 - a)v. \tag{1}$$

Resolving Eq. (1) leads to the equilibrium price distribution $F(p; a, k) = 1 - ((1 - a)(v - p)/kap)^{1/(k-1)}$. The lower bound of the distribution, conditional on (a, k), is obtained by resolving $F(b; a, k) = 0$, which leads to $b(a, k) = (1 - a)v/((k - 1)a + 1)$. Note that the equilibrium price distribution depends neither on n nor on A but only on (a, k). Ideally, our model can sketch an internet market where some consumers are aware of a shopbot and others not (Kephart and Greenwald, 1999).

3. Fixed sample search and prices

3.1. Fixed sample search and expected price

Let us define $E(p; a, k) = \int_{b(a,k)}^{v} pf(p; a, k)\,\mathrm{d}p$ to be the expected price of the market equilibrium with $f(p; a, k)$ the equilibrium density function for parameters (a, k). Let $k > 1$ be fixed and $0 < a < 1$.

Lemma 4. $\partial F(p; a, k)/\partial a > 0$ *for all p such that $b(a, k) < p < v$. As a corollary, the expected price of the market decreases from the monopoly price v to the marginal cost c when the proportion a of informed consumers increases from 0 to 1.*

The proof is by simple derivation. The result is equivalent to the property of first order stochastic dominance of the price distribution. Moreover, Stahl shows that the price distribution converges to the degenerate price distribution with unit mass at v when a tends to 0 and converges weakly to the degenerate price distribution with unit mass at marginal cost of 0 when a tends to 1. Now, let a be fixed and $2 \leq k \leq n$.

Table 1
Expected price paid by informed (italic values) and uninformed consumers (roman values) ($v = 1$)

a	k											
	2		3		4		5		10		15	
0.1	*0.87*	0.90	*0.84*	0.91	*0.81*	0.91	*0.78*	0.91	*0.66*	0.93	*0.58*	0.94
0.2	*0.76*	0.81	*0.70*	0.82	*0.66*	0.84	*0.62*	0.85	*0.48*	0.88	*0.40*	0.90
0.3	*0.65*	0.72	*0.59*	0.75	*0.54*	0.77	*0.50*	0.79	*0.37*	0.84	*0.30*	0.87
0.4	*0.55*	0.64	*0.49*	0.68	*0.44*	0.71	*0.40*	0.73	*0.29*	0.81	*0.23*	0.85
0.5	*0.45*	0.55	*0.40*	0.60	*0.35*	0.65	*0.32*	0.68	*0.22*	0.78	*0.17*	0.83
0.6	*0.36*	0.46	*0.31*	0.53	*0.28*	0.59	*0.25*	0.63	*0.17*	0.75	*0.13*	0.80
0.7	*0.27*	0.37	*0.23*	0.46	*0.20*	0.52	*0.18*	0.57	*0.12*	0.71	*0.09*	0.78
0.8	*0.18*	0.27	*0.16*	0.37	*0.14*	0.45	*0.12*	0.51	*0.08*	0.67	*0.06*	0.75
0.9	*0.09*	0.16	*0.08*	0.27	*0.07*	0.35	*0.06*	0.42	*0.04*	0.62	*0.03*	0.71

Theorem 5. *Let $0 < a < 1$ and k^1, k^2 be two integers with $2 \leq k^1 < k^2 \leq n$; then $k^1 < k^2 \Rightarrow E(p; a, k^1) < E(p; a, k^2)$. Moreover, $E(p; a, 2) > c = 0$ and $\lim_{k\to+\infty} E(p; a, k) = v$.* [3]

Note that the result of Theorem 5 contradicts the presumption that prices should fall when consumers search more actively. There are two effects from a change in k. First, each store contemplates a linear increase in the number of visits made by shoppers. This positive effect encourages low pricing in order to gain an enlarged common market. Second, the probability of gaining the common market decreases exponentially, which encourages high pricing. On average, the second effect dominates the first.

Let us consider the expected price paid by shoppers. Let $(p_1, \ldots, p_k)$ be a random sample from the cumulative distribution $F(p; a, k)$. Define $p_{\min} = \min[p_1, \ldots, p_k]$ to be the minimum price out of a sample of k prices. The associated cumulative distribution function $G(p; a, k)$ is:

$$G(p; a, k) = 1 - (1 - F(p; a, k))^k = 1 - \left(\frac{(1-a)(v-p)}{kap}\right)^{k/(k-1)}.$$

Let $E(p_{\min}; a, k)$ be the expected value of $p_{\min}$ given parameters (a, k). Let k^1 and k^2 be integers.[4]

Theorem 6. *Let $0 < a < 1$, then $1 < k_1 < k_2 \Rightarrow E(p_{\min}; a, k^1) > E(p_{\min}; a, k^2)$. In addition, $\lim_{k\to+\infty} E(p_{\min}; a, k) = 0$. Let $0 < a^1 < a^2 < 1$ and $k > 1$ then $G(p; a^1, k) < G(p; a^2, k)$.*

Table 1 shows the expected price paid by uninformed consumers and by informed consumers.

Informed consumers benefit from an increase in k, while at the same time the situation of uninformed consumers gets worse. When k tends to infinity, shoppers expect to pay the marginal cost and uninformed consumers expect the monopoly price. In this case, the total expected consumer surplus depends only on the proportion of informed consumers, but increasing the proportion of informed consumers will benefit all consumers.

3.2. Fixed sample search and price dispersion

We study the effect of a and k on price dispersion measured by the variance of market prices. Let $k > 1$.

[3] Note that $k \to +\infty \Rightarrow n \to +\infty$. Proofs are reported in an appendix available on the JEBO website.

[4] More generally, results of theorem 5 and 6 hold for k real.

Table 2
Standard deviation of the market price as a function of a and k ($v = 1$)

a	k									
	2	3	4	5	8	9	10	15	24	30
0.1	0.05	0.08	0.09	0.10	0.12	0.13	0.13	0.14	*0.15*	0.14
0.2	0.10	0.14	0.16	0.17	0.20	0.20	0.20	*0.20*	0.20	0.19
0.3	0.13	0.18	0.21	0.23	0.24	0.25	*0.25*	0.24	0.23	0.22
0.4	0.16	0.22	0.25	0.27	0.28	*0.28*	0.28	0.27	0.25	0.24
0.5	0.18	0.25	0.28	0.30	*0.31*	0.31	0.31	0.30	0.27	0.26
0.6	0.19	0.27	0.31	0.32	*0.34*	0.34	0.34	0.32	0.29	0.28
0.7	**0.20**	0.29	0.33	0.35	*0.36*	0.36	0.36	0.34	0.31	0.30
0.8	0.19	**0.29**	0.34	0.37	*0.39*	0.39	0.38	0.37	0.33	0.31
0.9	0.16	0.28	**0.35**	0.38	0.41	*0.41*	0.41	0.39	0.36	0.34

Theorem 7. *Price dispersion is an inverse U-shaped function of a. Moreover* $\lim_{a\to 0} var(p; a, k) = 0$ *and* $\lim_{a\to 1} var(p; a, k) = 0$.

Proposition 8. *The variance decreases to zero as k tends to infinity.*

Claim 9. *The variance is a inverse U-shaped function of k.*[5]

Table 2 shows the standard deviation of the equilibrium price distribution for different values of a and k and $v = 1$. The bold values indicate the maximum value of the standard deviation as a function of a. The italic values indicate the maximum value of the standard deviation as a function of k. By Theorem 7, the standard deviation is an inverse U-shaped function of a. However, Table 2 shows that this function is left skewed. The variance decreases when there are at least 70% of informed consumers (with $k = 2$). This critical proportion increases steadily with k . For $k = 3$, we have $a = 80\%$; for $k = 4$, we have $a = 90\%$; for $k = 8$, at least 99% of consumers must be informed for the price variance to start decreasing. Thus, price dispersion increases for a wide range of parameter a. Proposition 8 shows that when k increases sufficiently price dispersion will decrease to zero. However, since prices converge to the monopoly price, reduction in dispersion will be around the monopoly price.

Moreover, for a given a, the standard deviation is a steadily increasing function of k for low values of k. The standard deviation increases with k while k is lower than $k = 8$. For value of k for which the variance decreases, it will be at a low pace. Consequently, Stigler's presumption (1961, 214) that "*Price dispersion, is ..., the measure of ignorance in the market*" is not generally true for FSS. In general a higher proportion of informed consumers or more active informed consumers will lead to substantial price dispersion.

Consider the changes in expected price and variance induced by a change in v or in the marginal production cost c. Given a and $k > 1$, and with a change in notation, let $E\,(p; c, v)$ be the expected market price with marginal production cost c and willingness to pay v, and $E(p; 0, 1)$ be the case where $c = 0$ and $v = 1$. The following lemma shows that the expected market price is a linear function of v, and the variance, a quadratic function of v.

Lemma 10. $E(p; c, v) = (v - c)E(p; 0, 1) + c$; $E(p_{\min}; c, v) = (v - c)E(p_{\min}; 0, 1) + c$; $var(p; c, v) = (v - c)^2 var(p; 0, 1)$.

[5] This last result has only been shown by numerical simulations for all $0.05 \le a \le 0.95$, a varying by steps of 0.05.

Lemma 10 implies that the qualitative results obtained thus far with a change in parameters (a, k) are independent on the particular value of v or marginal cost c and are thus analogous to the one of $E(p; 0, 1)$ and $var(p; 0, 1)$ given by Tables 1 and 2. Lemma 10 will be used in Section 4 for the study of prices and price dispersion in the case of endogenous SS.

4. Sequential search equilibria

Condition 11. Search by consumers is sequential. A proportion a of consumers, with $0 < a < 1$, have a zero search cost. A proportion $1 - a$ of consumers supports a search cost $t < v$, **except on the first visit which costs zero**. If all firms have been sampled, then with no additional cost, consumers buy the good at the lowest price p among the n firms whenever $p \leq v$.

A search strategy from a known distribution has a reservation price property (Kohn and Shavell, 1974). The reservation price $r(t)$ is such that it is optimal for the searcher with search cost t to accept any price less than or equal to $r(t)$, while he will search further at a price higher than $r(t)$. Three conditions must be met in a rational search equilibrium: first, firms choose the price distribution so as to maximize profits given the search strategy of consumers; second, given their beliefs on the price distribution, consumers use an optimal search strategy defined by the stopping rule $r(t)$; finally, consumers' beliefs are correct and correspond to the equilibrium price distribution used by firms. Suppose that $F(p; r^*)$ is the actual price distribution where r^* is the maximal price of the support of the equilibrium distribution. In the case of SS, the expected gain for a consumer from one additional visit if the current best price offer is z is

$$\mathrm{EG}(z) = \int_{b(r^*)}^{z} (z - p)\,\mathrm{d}F(p; r^*) = \int_{b(r^*)}^{z} F(p; r^*)\,\mathrm{d}p \tag{2}$$

where $b(r^*)$ is the lower bound in the support of the price distribution (i.e. such that $F(b(r^*); r^*) = 0$). The stopping rule is such that if z is the current best offer, then if $\mathrm{EG}(z) > t$, then continue to search, and if $\mathrm{EG}(z) \leq t$, stop and buy at price z.[6] Thus, we define $r(t)$ by $\mathrm{EG}(r(t)) = t$. Lemma 12 and Propositions 13 and 14 are shown in Stahl's paper.

Lemma 12. *Consumers, with zero search costs, visit $k = n$ firms with probability 1 whenever the price distribution $F(p; r^*)$ is a continuous distribution.*

This is a direct consequence of the fact that the event $p = b(r^*)$ has a zero probability. Now, the price distribution is indeed continuous.

Proposition 13. *Under* Condition 11, *the only equilibrium is a price dispersed equilibrium such that consumers with search cost t are uninformed (i.e. they sample only one firm) and consumers with zero search cost are informed. Moreover, the equilibrium price $F(p; r^*)$ is an atomless distribution.*

We link FSS with SS. The function $\mathrm{EG}(z)$ defined in Eq. (2) clearly increases in z up to infinity and $\mathrm{EG}(b(r^*)) = 0$. A unique solution exists such that $\mathrm{EG}(r(t)) = t$. The decision rule of the consumer is to accept any price $z \leq \min(r(t), v)$ and continue to search as long as $z > \min(r(t), v)$. In the event that for all n visits of a consumer, prices exceed $r(t)$, he will shop at no additional

[6] Considering the equality is irrelevant in the decision to stop for density functions.

Table 3
Reservation price (% of v) with $\tau = 1\%$

a	k							
	2	3	4	5	10	15	24	30
0.1	10.3	10.8	11.2	11.6	13.7	15.6	18.9	20.8
0.2	5.3	5.7	6.1	6.5	8.3	9.9	12.7	14.3
0.3	3.6	4.0	4.3	4.7	6.3	7.8	10.2	11.8
0.4	2.7	3.1	3.4	3.7	5.2	6.6	8.8	10.2
0.5	2.2	2.5	2.8	3.1	4.5	5.7	7.8	9.1
0.6	1.9	2.1	2.4	2.7	4.0	5.1	7.0	8.2
0.7	1.6	1.8	2.1	2.3	3.5	4.5	6.3	7.4
0.8	1.4	1.6	1.8	2.0	3.1	4.0	5.6	6.6
0.9	1.2	1.4	1.5	1.7	2.6	3.4	4.9	5.7

cost at the store with the lowest price among the n sellers conditionally on that price being lower than or equal to v. This implies that $r^* \leq r(t)$ since otherwise some consumers with search cost t would search more than once, which would contradict Proposition 13. Moreover, since at a price $p > v$ no consumer will buy, we have $r^* = \min(r(t), v)$. Hence, the equilibrium price distribution in the case of SS differs from that of FSS by the fact that $k = n$ and r^* is endogenous, so that

Proposition 14. *The equilibrium price distribution is given by* $F(p; r^*) = 1 - ((1-a)(r^* - p)/nap)^{1/(n-1)}$ *with support* $[b(r^*), r^*]$ *where* $r^* = \min(r(t), v)$ *and* $b(r^*) = (1-a)r^*/((n-1)a+1)$.

In the case where $\min(r(t), v) = v$, there is no difference between $F(p; r^*)$ and $F(p; a, n)$. Now, let $E(p; r^*)$ be the expected market price with maximal willingness to pay r^*. The following lemma shows the dependence of r^* on t.

Lemma 15. *Define* $t = \tau v$ *with* $0 < \tau < 1$. $r^*(t)$ *is such that*:

$$r^*(t) = \begin{cases} r(t) = \frac{t}{[1-E(p;1)]} & \text{if } \tau < [1 - E(p;1)] \\ v & \text{if } \tau \geq [1 - E(p;1)] \end{cases} \tag{3}$$

According to Lemma 15, the knowledge of the expected market price $E(p; 1)$ and search cost t is sufficient to determine $r^*(t)$. It has the advantage that $E(p; 1)$ is completely specified as a function of (a, k) (Table 1; Section 3) with $k = n$ for SS. Moreover, $r(t)$ is linear in t and v. For example, if the high search cost is $\tau = 5\%$, then in the case $a = 0.1$ and $n = 30$, r^* is equal to v since $r(t) = 1.04 > v$ corresponding to 5 times the figure in Table 3.

The following lemma states the properties of the reservation price $r(t)$ as a function of a and $k = n$.

Lemma 16. *For* $0 < a < 1$, $(\mathrm{d}r(t)/\mathrm{d}a) < 0$; $(\mathrm{d}r(t)/\mathrm{d}n) > 0$ *and* $(\mathrm{d}r(t)/\mathrm{d}t) > 0$. *Moreover*, $\lim_{a\to 0} r(t) = +\infty$; $\lim_{a\to 1} r(t) = t$; $\lim_{n\to +\infty} r(t) = +\infty$.

Thus for a small enough a or a large n, $r^*(t) = v$ (i.e. $r(t) \geq v$). We call this case the FSS type equilibrium. For a large enough a, $r^*(t) = r(t) < v$ which we call the SS type equilibrium.

4.1. Sequential search and expected market price

We turn to the properties of $E(p; r^*(t))$ and $E(p_{\min}; r^*(t))$ as a function of a, n and t.

Table 4
Expected price paid by informed (italic values) and uninformed consumers (roman values) with endogenous search ($\tau = 0.01$; $v = 100$)

a	$k = n$											
	2		3		4		5		10		15	
0.1	*9.00*	9.31	*9.00*	9.74	*9.00*	10.18	*9.00*	10.61	*9.00*	12.67	*9.00*	14.58
0.2	*4.00*	4.29	*4.00*	4.69	*4.00*	5.08	*4.00*	5.47	*4.00*	7.29	*4.00*	8.93
0.3	*2.33*	2.60	*2.33*	2.97	*2.33*	3.33	*2.33*	3.69	*2.33*	5.32	*2.33*	6.80
0.4	*1.50*	1.74	*1.50*	2.08	*1.50*	2.42	*1.50*	2.74	*1.50*	4.24	*1.50*	5.59
0.5	*1.00*	1.22	*1.00*	1.53	*1.00*	1.84	*1.00*	2.14	*1.00*	3.51	*1.00*	4.75
0.6	*0.67*	0.86	*0.67*	1.14	*0.67*	1.42	*0.67*	1.70	*0.67*	2.96	*0.67*	4.10
0.7	*0.43*	0.59	*0.43*	0.84	*0.43*	1.10	*0.43*	1.34	*0.43*	2.50	*0.43*	3.55
0.8	*0.25*	0.38	*0.25*	0.59	*0.25*	0.82	*0.25*	1.04	*0.25*	2.07	*0.25*	3.02
0.9	*0.11*	0.20	*0.11*	0.36	*0.11*	0.54	*0.11*	0.73	*0.11*	1.62	*0.11*	2.45

Theorem 17. *An increase in the proportion a decreases the expected price paid by uninformed consumers and the expected price paid by shoppers. Moreover,* $\lim_{a \to 0} E(p; r^*(t)) = v$; $\lim_{a \to 1} E(p; r^*(t)) = 0$.

For $n \geq 2$, an increase from n to $n + 1$ firms increases the expected market price. Moreover, $\lim_{n \to +\infty} E(p; r^*(t)) = v$.[7] *The effect of n on the expected price paid by shoppers is ambiguous. However,* $\lim_{n \to +\infty} E(p_{\min}; r^*(t)) = 0$.

An increase in t increases the expected price paid by uninformed and informed consumers whenever $\tau < [1 - E(p; 1)]$ (i.e. $r^*(t) < v$). For $\tau \geq [1 - E(p; 1)]$, the expected price does not depend on t (FSS type equilibrium).

Sketch of the proof. By Lemma 10, $E(p; r^*(t)) = r^*(t)E(p; 1)$ and $E(p_{\min}; r^*(t)) = r^*(t)E(p_{\min}; 1)$. The results are derived through the combined effect of each parameter a or n on $E(p; 1)$ or $E(p_{\min}; 1)$ and $r(t)$. □

An increase in n has an ambiguous effect on $E(p_{\min}; r^*(t))$. On the one hand, by Theorem 6, $E(p_{\min}; 1)$ decreases due to an increase in the sample size, but on the other hand, the reservation price $r(t)$ increases which increases the shoppers' expected price. Surprisingly, figures in Table 4 shows that the expected price paid by informed consumers remains constant with $k = n$ in the case where $r^*(t) = r(t) < v$, which is a striking difference with FSS search. The increase in $r(t)$ compensates for the decrease in prices. However, since $E(p_{\min}; 1) \to 0$ and $r^*(t) \to v$ when $n \to +\infty$, we have that $E(p_{\min}; r^*(t)) \to 0$ for $n \to \infty$.

Note the linearity in v of figures in Table 4, by the linearity of $E(p; r^*(t))$ and $E(p_{\min}; r^*(t))$ in $r^*(t)$ and $r^*(t)$ in v.

4.2. Sequential search and price dispersion

We turn to the properties of $var(p; r^*(t))$ as a function of a, n and t.

Lemma 18. $var(p; r^*(t)) = r(t)^2 var(p; 1) = (\tau v / [1 - E(p, 1)])^2 var(p; 1)$, *if* $\tau < [1 - E(p, 1)]$ *and* $var(p; r^*(t)) = v^2 var(p; 1)$ *if* $\tau \geq [1 - E(p, 1)]$

[7] Stahl's paper contains the result for a and the limit result for n.

Table 5
Standard deviation with endogenous search $\tau = 0.01$, $v = 100$

a	k							
	2	3	4	5	10	15	24	30
0.1	0.54	0.83	1.05	1.22	1.81	2.21	2.74	3.02
0.2	0.50	0.77	0.97	1.13	1.66	2.01	2.50	2.74
0.3	0.47	0.72	0.91	1.06	1.56	1.90	2.35	2.60
0.4	0.43	0.68	0.85	0.99	1.47	1.80	2.23	2.48
0.5	0.39	0.63	0.80	0.93	1.40	1.72	2.15	2.38
0.6	0.35	0.58	0.74	0.88	1.33	1.64	2.05	2.28
0.7	0.31	0.53	0.69	0.81	1.26	1.56	1.97	2.19
0.8	0.26	0.47	0.62	0.75	1.18	1.47	1.87	2.08
0.9	0.19	0.39	0.53	0.66	1.08	1.36	1.75	1.95

Table 6
Standard deviation with endogenous search $\tau = 0.1$, $v = 100$

a	k							
	2	3	4	5	10	15	24	30
0.1	*5.23*	*7.73*	*9.33*	*10.47*	*13.21*	*14.14*	*14.54*	*14.49*
0.2	5.04	7.74	9.73	11.31	16.56	20.13	*19.75*	*19.20*
0.3	4.67	7.24	9.08	10.57	15.58	19.00	*22.99*	*22.11*
0.4	4.31	6.77	8.50	9.93	14.72	17.98	22.31	*24.30*

Proposition 19. *An increase in the search cost t increases price dispersion whenever $r(t) < v$, that is $\tau < [1 - E(p; 1)]$ (SS type). For $\tau \geq [1 - E(p; 1)]$, price dispersion does not depend on t (FSS type).*

Theorem 20. *The variance is an inverse U-shaped function of a. For a given n, let* $a^*(t) = \arg\max var(p; r^*(t))$, *then* $\mathrm{d}a^*(t)/\mathrm{d}t > 0$ *for the SS type and* $\mathrm{d}a^*(t)/\mathrm{d}t = 0$ *for the FSS type.*

By Lemma 16, the variance is of the FSS type (i.e. $r(t) > v$) for $a \to 0$ and of the SS type for $a \to 1$. Moreover, there exists a unique a^* such that $var(p; r^*(t))$ is an increasing function of a as long as $a < a^*$ and is a decreasing function of a for $a > a^*$. Although the variance is an inverse U-shaped function in a as shown by Theorem 20, Table 5 shows that for $n \leq 30$ and $\tau = 0.01$, we have $a^* \leq 0.1$. In the SS case, there is a benefic effect in reduction of price dispersion due to $\mathrm{d}r(t)/\mathrm{d}a < 0$. Note also the linearity of standard deviation in v (by Lemmas 15 and 18). In Table 6, the standard deviation is of the FSS type (i.e. $r(t) \geq v$) for the italic values. The other figures are of the SS type, and since standard deviation is linear in t for SS (by Lemma 18), these figures are proportional to those in Table 5.

Claim 21. For a given a, the variance is an inverse U-shaped function of n decreasing to zero as n tends to infinity[8].

Overall, one may say that price dispersion increases steadily with n and remains significant afterwards as shown in Tables 5 or 6. For the SS type, price dispersion increases for a larger range of the parameter value n than in the FSS case as shown in Table 6, due to the added effect of $\mathrm{d}r(t)/\mathrm{d}n > 0$.

[8] The claim that the variance is inverse U-shaped is based on the numerical result for $var(p; 1)$ (Table 2).

Janssen et al. (2005) relax the assumption that high cost consumers' first search is costless and call it truly costly sequential search. With truly costly sequential search, either $r(t) \leq v$, leaving the previous SS analysis unchanged; or $r(t) > v$ and a consumer expects a negative surplus by paying t on a first visit. In that case, Janssen et al. show that the equilibrium is such that only a proportion μ^* of high cost consumers enters, which they call an equilibrium with partial participation.[9] They show that a change in either the proportion of shoppers a or the number of firms leaves the expected price in a partial equilibrium unchanged. In addition, they show that in a partial equilibrium, the price distribution does not vary with a and that an increase in n leads to a mean preserving increase in spread. With truly costly SS search, the FSS type equilibrium (i.e. when $r(t) > v$) is thus replaced by the partial equilibrium concept of Janssen et al.. That is, by Lemma 16, there exists a proportion a^{**} such that $a < a^{**}$, we have a partial equilibrium and for $a > a^{**}$ we have an equilibrium with full participation (i.e. with $r(t) \leq v$). Relying on Janssen et al.'s results, price dispersion is thus constant for $a < a^{**}$ and an inverse U-shaped function of a for $a > a^{**}$ as shown in Theorem 20. However, the italic values in Table 6 shows a decrease in price dispersion in the SS type equilibrium for $a \geq 0.1$. In addition, when the number of firms is large enough, i.e. $n > N(a)$, we have a partial equilibrium. If $n < N(a)$ (i.e. $r(t) < v$) then price dispersion is increasing in n as shown in Table 6 for the italic values. For $n > N(a)$, price dispersion is mean preserving increase in spread with an increase in n. Thus, price dispersion increases with n with truly costly search.

5. Conclusion

The main contribution of this paper is to stress the non-trivial relationship between information, prices and price dispersion. First, the standard presumption, that increasing the proportion of informed consumers decreases the expected price paid by informed or uninformed consumers, is true both for Fixed Sample Search (FSS) and Sequential Search (SS). Second, a higher search intensity of informed consumers (i.e. an increase in the sample size k) increases the expected market price both for the FSS and SS cases. However, a higher search intensity decreases the expected price paid by informed consumers in the FSS case while remaining ambiguous in the SS case, except in the limit when the number of firms tends to infinity. Third, price dispersion is an inverse U-shaped function of the proportion of informed consumers a both for the FSS and SS cases. However for FSS, this function is left skewed leading in general to higher price dispersion with a. For SS, we have shown that as long as the high search cost is relatively low, price dispersion is right skewed. For example, if the high search cost is equal to 1% of v and for a wide range of parameter $k = n$, the maximum price dispersion is attained at around $a^* = 10\%$. For a high search cost equals to 10% of v and for a wide range of parameter k, price dispersion is in general decreasing with a for values larger than approximately $a^* = 20\%$ (Table 6). However increasing the level of the high search cost will increase the value a^* for a given n. Last, for FSS and SS, we have shown that price dispersion increases in k for k not too large.

Waldeck and Darmon (2006) have shown that with adaptive sellers using reinforcement learning, a change in the market structure (a, k) impacts in the same manner on the average price and variance of the price distribution than the Nash predictions of the FSS model do. Results are thus robust with bounded rational players. However, Darmon and Waldeck (in press) show that introducing social learning affects dramatically these results.

[9] It corresponds also to the low search intensity equilibrium as described by Janssen and Moraga-Gonzalez (2004).

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Appendix A. Supplementary Data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.jebo.2006.02.009.

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