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Peter Howitt and R. Preston McAfee*

Unemployment cannot arise in general equilibrium theory because the auctioneer costlessly communicates to all potential buyers and sellers of labor services the willingness of anyone to trade on the other side. The work of Edmund Phelps et al showed how it can arise if you remove the auctioneer and require people to use real resources sending and receiving these signals. Axel Leijonhufvud argued that this was also the way to understand the message of Keynes's General Theory. In such a world large-scale unemployment can arise from communication failures, in which the difficulties that people are having selling in one market discourage them from transmitting their willingness to buy in other markets, thus in turn creating sales difficulties for suppliers in those other markets, etc., in a self-reinforcing manner.

Contrary to Leijonhufvud's arguments, subsequent work along these lines has led to the idea of a unique, optimal natural rate of unemployment that would perpetually be maintained except for serially uncorrelated errors in forecasting some nominal aggregate. Modern Keynesian explanations of unemployment, such as those of Stanley Fischer, and Phelps and John Taylor, make no reference to the theory of search and communication, and rely instead upon unexplained temporary wage and price rigidities.

Two recent papers by Peter Diamond have shown how search theory might indeed be used to generate the kind of result envisaged by Leijonhufvud.

The key to Diamond's analysis is an externality related to the common idea that trading is more costly the thinner the market. Specifically, the more activity there is on one side of the market the lower the costs faced by those on the other side wanting to contact a trading partner. Thus the expectation of an abnormally low level of economic activity can lead to the expectation of thin markets and high contacting costs, and thus discourage people from engaging in trading activities, thereby fulfilling the original expectation. People might all be willing to trade at higher levels, but the fact that no one else is doing so dissuades each trader from communicating a willingness to do so himself.

While Diamond's contributions are promising, they are made in terms of quite special models that are hard to relate to everyday labor-market phenomena.¹ The present paper proposes a more conventional model of the labor market. Workers find it equally costly to search whether they are currently employed or unemployed. Wage offers are set by profit-maximizing firms facing a linear technology, and are accepted or rejected by searching workers. Both workers and firms actively engage in contacting activities; namely search and recruiting. In equilibrium each labor market match persists over time.

Under appropriate conditions there exists a continuum of stationary perfect foresight equilibria with different wages and rate of unemployment. Unique equilibria almost never occur. There are often equilibria that are inefficient in the sense that they are Pareto dominated by other equilibria. All equilibria are inefficient in the sense that a Pareto-improvement would result if workers could be induced to search more intensively.

Our non-uniqueness occurs for two reasons, neither of which captures the Keynesian idea of sales difficulties in one market spilling over into others, but both of which involve a related communication failure. The first reason is that, as in Diamond's models, there are external economies of scale in the communication process. Job search is more profitable when firms are recruiting more actively, and recruiting is more profitable when workers are searching more actively. But there is also an external diseconomy. Specifically, the competition from more active recruiting by some firms raises the cost of recruiting to other firms. Because of the interaction of these externalities there will be equilibria with active search and recruiting, and others with less active search and recruiting.

The second reason for multiple equilibria has to do with the reason proposed by Joseph Stiglitz for a kinked oligopolistic demand curve, except that in this case it is a supply-of-labor curve faced by a wage-setting firm. A firm that offers less than the going wage will find its workers searching and eventually quitting. But a firm that offers more cannot reduce its quit rate below zero; nor will it attract potential recruits at any faster rate. This asymmetric reaction of the supply of workers to the firm imparts a stickiness to its wage offers, which leads to the existence of not just multiple equilibria but a continuum thereof, as in the analyses of Frank Hahn, Takashi Negishi, and Geoffrey Woglom.

1. Setup of the Model

The economy has only two tradeable objects: goods and labor-services, neither of which is storeable. There are two kinds of agents; a large number, n , of firms, located uniformly throughout space, and a continuum of workers. All trade consists of bilateral contracts according to which a firm agrees to trade goods for labor services at a specified constant wage rate for as long as the worker remains attached to the firm. The firm agrees not to fire the

worker (an option which he would not choose to exercise in equilibrium in any event), so that all such attachments remain until the worker either quits voluntarily or dies. Death of each worker occurs according to a Poisson distribution with death rate $\delta > 0$.

In order for such a trade to occur, the worker and firm must seek each other out, and make a successful "contact". This is their only means of communicating their respective dispositions to trade. Search in our model consists not of sampling from a distribution of wage-offers but attempting to contact a potential employer. Thus we emphasize the kind of activity highlighted by Jeff Frank and John Seater, according to whom much of the activity of job-search consists of trying to locate a vacancy rather than trying to find a higher wage. However, we do not assume that firms impose any limit upon the number of workers they are willing to hire, as in most models of vacancies. Instead, the cost of finding a vacancy in our model represents the difficulty of communicating information in the labour market.

New workers enter the labor market at the constant (but endogenous) rate L , and search through space in a randomly varying direction at the speed α . As in the analysis of Ken Burdett and Dale Mortenson, searchers can vary the intensity of their search, α , according to a cost function $c(\alpha)$, where:

$$(1) \quad 0 = c(0) = c'(0), \text{ and } c''(\alpha) \cong \underline{c} > 0, \quad c'''(\alpha) \cong 0 \quad \forall \alpha \cong 0.$$

The importance of assuming $c'(0) = 0$ will be examined in Section 6 below. The assumption on c''' is used for technical simplicity (esp. to establish the concavity of the quit-function with respect to the wage gap) and could be dropped without substantive cost. Other than this the restrictions in (1) are standard.

Each worker starts life at a random location in space. The faster he searches, the greater the probability that he will encounter a firm over any given short interval of time. In fact, if he searches twice as fast he will cover twice the distance over that interval, and average time to contact ought to halve, given the randomness of his initial location. Thus from the point of view of the searching worker, contacts "arrive" according to a Poisson process at a rate proportional to α . By similar reasoning this arrival rate is proportional to $n\alpha$.

At each instant, each firm has two kinds of choice to make. The first is what wage to offer to the workers that arrive at that instant. This offer is made on a take-it-or-leave-it basis to the contacted worker, who can accept or reject but not negotiate.

The second choice is how many resources to devote to recruiting. This is the firm's contribution to the communication process. In order to make contact a searching worker must pass close enough to notice a firm. Just how close this is depends upon how heavily the firm advertises its location by putting up signs and sending out recruiters. Through such activities the firm casts a "recruiting net" about its position. According to our previous discussion the rate at which searchers pass into any given net will be proportional to αU , where U is the number of searching workers. Thus the rate at which a firm will be able to hire new workers will be $\theta\alpha U$, where θ measures the intensity of the firm's recruiting efforts. An increase in θ "catches" more workers by increasing the size of the recruiting net and/or increasing the fraction caught of searchers passing into the net.² Recruiting intensity can be varied subject to a cost function $G(\theta)$, satisfying:

$$(2) \quad G(0) = 0, \quad G'(0) \geq 0, \quad G''(\theta) \geq \underline{G} > 0 \quad \forall \theta \geq 0.$$

An important assumption embodied in (2) is that the firm that devotes no resources to recruiting will not be able to hire. Another is that the cost does not depend upon the recruiting efforts of other firms, as it would, for example, if firms were large enough that their recruiting nets overlapped.

The frequency of contacts is thus $\alpha n \theta U$. This implies that from a searching worker's point of view the marginal effect of his own search intensity upon his chance of making a contact is directly proportional to the size of recruiting effort made by the typical firm, whereas from the firm's point of view the marginal effect of its recruiting effort upon the rate of arrival of job candidates is directly proportional to the intensity of search by the typical worker. This reciprocal externality in the communication process is crucial for what follows.³

In stationary equilibrium all wage offers and recruiting intensities are identical, and workers know what they are. Thus all contacts will result in an acceptance. For to leave the labor force rather than accept the wage would mean it was not optimal to have spent resources searching for the contact in knowledge of the wage offer. The only other alternative would be to continue searching for the same wage, which would likewise waste search costs. Thus all contacts last for life, even though the worker always has the option of searching elsewhere throughout the duration of the job, and all searchers will be new entrants to the labor force who have yet to make their first contact. (Exogenous turnover could obviously be added to the model with no substantive cost.)

2. The Worker's Choice Problem

Each worker has an instantaneous utility function: $v(y, \alpha, l) = y - c(\alpha) + \gamma(l)$, where y denotes the rate of consumption of goods, equal to the wage rate if working and zero otherwise, c is the cost of search, described by (1), l is a dichotomous leisure variable equal to 0 if the worker is either working or

searching (or both) and 1 otherwise, and $\gamma(1) - \gamma(0) = \gamma > 0$. The function thus changes discontinuously when the worker switches from leisure to searching at an arbitrarily small speed. This discontinuity imposes a fixed cost of labor force participation whose importance is discussed below in Section 6. The one sense in which workers of the same type differ is that they have different values of γ , the value of leisure, which will cause them to make different participation decisions.

Consider the choice problem facing an unemployed worker who has decided to participate. Suppose all firms are offering the same wage w and recruiting at the same intensity θ . Then the expected present value of finding a job at date t beyond the initiation of search is $\frac{we^{-rt}}{r+\delta}$ where $r (> 0)$ is the exogenous rate of time discount. The expected cost of termination at date t is $\frac{c}{r}(1 - e^{-rt})$. The probability of finding a job at t is the product of $e^{-\delta t}$ (the probability of surviving until t) times $\alpha n \theta e^{-\alpha n \theta t}$ (the probability of the first offer "arriving" at t). The probability of terminating search at date t is $(\alpha n \theta + \delta)e^{-(\alpha n \theta + \delta)t}$. Thus the expected present value of search is:

$$(3) \quad EPV = \int_0^{\infty} \left[\left(\frac{w}{r+\delta} e^{-rt} \right) e^{-\delta t} \alpha n \theta e^{-\alpha n \theta t} - \frac{c(\alpha)}{r} (1 - e^{-rt}) (\alpha n \theta + \delta) e^{-(\alpha n \theta + \delta)t} \right] dt$$

$$= \frac{\frac{w \alpha n \theta}{r+\delta} - c(\alpha)}{r+\delta + \alpha n \theta}$$

This formula has a straightforward capital-theoretic interpretation. The value of a "ticket to search" is the expected instantaneous net return, consisting of the return $\frac{w}{r+\delta}$ times the probability of finding a job in the next instant, $\alpha n \theta$, minus the sure cost $c(\alpha)$, all discounted at a rate equal to the pure interest rate r plus the rate $(\delta + \alpha n \theta)$ at which the ticket can be expected to depreciate because of termination through death or contact.

The searching worker thus chooses α so as to maximize EPV. The first order condition for this problem is:

$$(4) \quad n\theta(c(\alpha) - \alpha c'(\alpha)) = (r + \delta)c'(\alpha) - n\theta w$$

which can be rewritten as:

$$(5) \quad -(n\theta) \int_0^{\alpha} xc''(x)dx - (r + \delta)c'(\alpha) + n\theta w = 0$$

Restrictions (1) imply that any solution to (5) is a unique, global maximum to the worker's problem, which we denote by $\hat{\alpha}(w, \theta)$. They also guarantee the existence of such a solution for any $(w, \theta) \geq 0$, for as α increases from 0 to ∞ , the LHS decreases monotonically from $n\theta w$ to $-\infty$. From (1) and (5),

$$(6) \quad \alpha \leq 2\sqrt{w/c} \quad \forall (w, \theta) \geq 0, \text{ and } \lim_{w \rightarrow \infty} \hat{\alpha}(w, \theta) = \infty \quad \forall \theta > 0$$

Finally, from (5) and (3) the optimized value of EPV is

$$(7) \quad EPV^* = \frac{w}{r + \delta} - \frac{c'(\alpha)}{n\theta} = \left(\frac{1}{r + \delta}\right) \int_0^{\alpha} xc''(x)dx$$

The above-mentioned externality in the communication process shows up in the fact that the worker's optimal search-intensity, α , and hence his expected utility, EPV^* , depends not only upon the real wage but also upon the intensity, θ , with which firms are recruiting. This dependency of household decisions upon endogenous market variables other than relative prices is similar not only to the idea of vacancies but also to Robert Clower's notion of effective demand, according to which household demand functions depend upon the realized sales of labor services as well as the real wage rate, a notion which was central to Leijonhufvud's analysis. In the present case the dependency does not result from any stickiness in wage-setting, or from any failure of agents to realize perceived gains from trade.⁴

It follows from (5) that

$$(8) \quad \hat{\alpha}(w, \theta) = 0 \text{ if and only if } w = 0 \text{ or } \theta = 0.$$

For any $(\theta, w) > 0$; (5) implies that:

$$(9) \quad \frac{\partial \hat{\alpha}}{\partial w} = \frac{1}{\left(\frac{r+\delta}{n\theta} + \alpha\right) c''(\alpha)} > 0$$

$$(10) \quad \frac{\partial \hat{\alpha}}{\partial \theta} = \frac{c'(\alpha) \frac{r+\delta}{\theta}}{(\alpha n \theta + r + \delta) c''(\alpha)} > 0$$

Although no quitting occurs in equilibrium, nevertheless each firm must be able to conjecture what would happen if it were to alter wages in such a way as to induce quitting. Suppose all other firms were hiring with intensity $\bar{\theta}$ at wage \bar{w} . If a single firm paid its new workers the wage $w < \bar{w}$, these new workers would begin searching at a rate that was optimal for the incremental wage $\bar{w} - w$, $\hat{\alpha}(\bar{w} - w, \bar{\theta})$. Since it is no more costly to search while employed than while unemployed they would not quit in order to search. But they would quit at a rate equal to the rate of successful contacts:

$$(11) \quad q(\bar{w} - w, \bar{\theta}) = \begin{cases} 0 & \text{if } w \geq \bar{w} \\ n\bar{\theta}\hat{\alpha}(\bar{w} - w, \bar{\theta}) & \text{if } w \leq \bar{w} \end{cases}$$

From our earlier results:

$$(12) \quad q(\bar{w} - w, \bar{\theta}) = 0 \text{ if and only if } w \geq \bar{w} \text{ or } \bar{\theta} = 0, \text{ and } \lim_{\bar{\theta} \rightarrow \infty} q(\bar{w} - w, \bar{\theta}) = \infty \forall \bar{w} - w > 0.$$

And, for all $\bar{\theta} > 0$, $\bar{w} > w$;

$$(13) \quad \frac{\partial q}{\partial (\bar{w} - w)} = \frac{n\bar{\theta}}{\left(\frac{r+\delta}{n\bar{\theta}} + \alpha\right) c''(\alpha)} \Bigg|_{\alpha = \hat{\alpha}(\bar{w} - w, \bar{\theta})} > 0, \quad \frac{\partial q^+(0, \bar{\theta})}{\partial (\bar{w} - w)} = \frac{(n\bar{\theta})^2}{(r+\delta) c''(0)} > 0$$

$$(14) \quad \frac{\partial q}{\partial \bar{\theta}} = n \hat{\alpha}(\bar{w}-w, \bar{\theta}) + \frac{nc'(\alpha)(r+\delta)}{(\alpha n \bar{\theta} + r + \delta)c''(\alpha)} \Big|_{\alpha = \hat{\alpha}(\bar{w}-w, \bar{\theta})} > 0$$

Equations (12) and (13) show a crucial kink in the quit function. Specifically, as the firm's wage rate is increased from 0 to ∞ with the wage and recruiting intensities of other firms held constant at $(\bar{w}, \bar{\theta})$, the quit rate at first falls steadily until the wage reaches \bar{w} , at which point it is still falling at a non-zero rate, according to the second part of (13). But at that point the quit rate abruptly stops falling, and remains at zero for all further increases in the wage. Also, note that the quit function is concave in the wage differential (when positive):

$$(15) \quad \frac{\partial^2 q}{\partial (\bar{w}-w)^2} = - \frac{n\bar{\theta}}{[\frac{r+\delta}{n\bar{\theta}} + \alpha]c''(\alpha)^2} \{c'' + (\frac{r+\delta}{n\bar{\theta}} + \alpha)c'''\} < 0 \text{ if } \bar{\theta} > 0 \text{ and } \bar{w}-w > 0.$$

To analyze the participation decision suppose that the value of leisure, γ , is distributed across all potential entrants at any instant according to a continuously differentiable cumulative distribution function Ψ . Let $\bar{L} > 0$ be the exogenous rate of birth of potential entrants at each instant. The fraction of these who actually enter will be the fraction for whom EPV* exceeds the expected present value of leisure, $\frac{\gamma}{r+\delta}$. Define the integral function $I(\alpha) = \int_0^{\alpha} xc''(x)dx$, with $(I, I') > 0$ iff $\alpha > 0$. Then according to (7), the flow of actual entrants will be:

$$(16) \quad L = \bar{L} \Psi(I(\hat{\alpha}(w, \theta))) \equiv \hat{L}(w, \theta).$$

Assume that there is a positive greatest lower bound, $\underline{\gamma}$, to the value of leisure for all workers. Then

$$(17) \quad \hat{L}(w, \theta) > 0 \text{ if and only if } \hat{\alpha}(w, \theta) > I^{-1}(\gamma) > 0.$$

To sharpen this condition note that, from (6) and (7), $I(\hat{\alpha}(w, \theta)) < w$ and $\lim_{\theta \rightarrow \infty} I(\hat{\alpha}(w, \theta)) = w$. Therefore:

$$(18) \quad \begin{cases} \hat{\alpha}(w, \theta) > I^{-1}(\gamma) \text{ only if } w > \gamma, \text{ and for any } w > \gamma \\ \text{there is some } \hat{\theta} \text{ such that } \hat{\alpha}(w, \theta) > I^{-1}(\gamma) \text{ if } \theta > \hat{\theta}. \end{cases}$$

In a stationary state the number of unemployed workers, U , will consist of those new entrants who have yet to find a job. The flow into unemployment, L , will equal the flow out of unemployment due to death and contact, $(\delta + \alpha n \theta)U$. Thus:

$$(19) \quad U = \frac{1}{\delta + \alpha n \theta} \hat{L}(w, \theta).$$

3. The Firm's Decision Problem

When all other firms offer \bar{w} and recruit with intensity $\bar{\theta}$, any given firm that recruits at the rate θ will face a flow of new workers equal to $\hat{\alpha}(\bar{w}, \bar{\theta})U\theta$. Thus, from (19) the arrival rate of workers to the firm, per unit of θ , is:

$$(20) \quad \sigma(\bar{w}, \bar{\theta}) = \frac{\hat{\alpha}(\bar{w}, \bar{\theta})}{\delta + \hat{\alpha}(\bar{w}, \bar{\theta})n\bar{\theta}} \hat{L}(\bar{w}, \bar{\theta}).$$

This is independent of the firm's own choice variables w and θ because the firm conjectures that it is too small to influence the workers' search behavior.

It follows from (16), (17) and (19) that:

$$(21) \quad 0 \leq \sigma(w, \theta) \cdot \theta \leq \bar{L}/n \quad \forall (w, \theta) \geq 0, \text{ and}$$

$$(22) \quad \sigma(w, \theta) > 0 \text{ if and only if } \hat{\alpha}(w, \theta) > I^{-1}(\gamma).$$

Also, from (9), (10), (16), (17) and (20):

$$(23) \quad \frac{\partial \sigma}{\partial w} = \left(\frac{\alpha}{\delta + \alpha n \theta} \right) \frac{\partial \hat{L}}{\partial w} + \frac{\hat{L} \delta}{(\delta + \alpha n \theta)^2} \frac{\partial \hat{\alpha}}{\partial w} > 0 \text{ if and only if } \alpha > I^{-1}(\gamma), \text{ and}$$

$$(24) \quad \frac{\partial(\theta \sigma)}{\partial \theta} = \left(\frac{\alpha}{\delta + \alpha n \theta} \right) \frac{\partial \hat{L}}{\partial \theta} + \frac{\hat{L} \delta}{(\delta + \alpha n \theta)^2} \frac{\partial(\theta \hat{\alpha})}{\partial \theta} > 0 \text{ if and only if } \alpha > I^{-1}(\gamma).$$

Each firm produces subject to a constant returns production function $Q = f \cdot z$, where z is its labor force, and discounts profits at the rate r . To allow at least the possibility of gains from trade, assume that $f > \gamma$. The contribution of a worker hired today to the expected value of the firm is $\int_0^{\infty} e^{-(r+\delta+q)t} (f-w) dt = \frac{f-w}{r+\delta+q}$, since $e^{-(\delta+q)t}$ is the probability that the worker will still be with the firm at date t . Since the rate of hiring is $\theta \sigma$, the contribution of its current hiring to the expected value of the firm is $\pi(w, \theta) = \theta \sigma(\bar{w}, \bar{\theta}) \left(\frac{f-w}{r+\delta+q(\bar{w}-w, \bar{\theta})} \right) - G(\theta)$. The firm will choose w and θ , both non-negative, so as to maximize this function for given values of \bar{w} and $\bar{\theta}$.

Externalities in the communication process show up here as well as in the worker's choice problem. The firm's profits will depend not only upon the real wage it offers but also upon the arrival rate σ which it takes as given, and which is functionally dependent upon the amount of resources that others have chosen to devote to the communication process. Specifically, (a) if workers were to increase their search intensity, α , independently of \bar{w} and $\bar{\theta}$, then, according to (20), the arrival rate and hence the firm's profits would be increased, whereas (b) if other firms were to recruit more intensively without this affecting the intensity of search, the arrival rate would decrease.

These two externalities are merged when the functional dependency of search intensity upon other firms' recruiting intensity is taken into account, as described by (21) and (22) and as shown in Figure 1. As long as the wage being offered by all firms is greater than the value of leisure for at least some firms, ($\bar{w} > \gamma$), then according to (18) and (22) the arrival rate will be

positive whenever other firms are recruiting with at least some minimal intensity. But holding wages constant, (8) and (22) ensure that the arrival rate will fall to zero when that intensity gets small enough (before $\bar{\theta}$ reaches zero). Also, (21) ensures that the arrival rate will fall asymptotically to zero when the intensity gets indefinitely large. What's happening is that if other firms are not recruiting intensively enough no worker will find it worthwhile to search, whereas if other firms are recruiting very intensively the workers who enter will find jobs with them so quickly that the number of workers at any point in time who still haven't found a job will fall to zero. In either case the rate of arrival of searchers into a net of given size will fall to zero. Thus the externality is at first an economy, as $\bar{\theta}$ raises σ , and eventually a diseconomy as $\bar{\theta}$ lowers σ .

The fact that a firm takes σ as given also reflects its inability to alter unemployed workers' search behavior by unilateral increases in its wage-offer. We are assuming that workers cannot be expected to search more intensively as a result of a single firm offering to pay above the market wage, because the chance of stumbling upon that particular firm during the random search process is too close to zero to matter. This extreme assumption rules out any form of advertising which might allow workers to direct their search toward a high-wage firm. The consequences of relaxing this extreme assumption are explored briefly in Section 6 below.

Because of this assumption the decision by a firm to change its wage will have only two effects. It will change the wage-bill associated with any given amount of employment, and it will affect its employees' quit-rates, but it will not affect the flow-supply of new workers. Furthermore, the above-mentioned kink in the quit-function implies that the firm will not find it optimal to offer above the market wage, \bar{w} . For to do so would have no effect other than to raise the firm's wage-bill.

Thus the range over which the firm selects its wage can be restricted to the interval from 0 to \bar{w} . But within that interval, (15) guarantees that the firm's objective function is strictly convex in its wage, so that the optimal wage must occur at either 0 or \bar{w} . The situation is depicted in Figure 2. (The dashed line may be ignored for now.)

4. Equilibrium

An equilibrium is a situation in which each firm chooses the same (w, θ) , each worker chooses the same α , the stocks of employed and unemployed workers are constant, and each agent makes his choice optimally with full knowledge of the values of all market variables.⁵ Formally it is a pair⁶ $(w, \theta) > 0$ which solves the firm's decision problem given $(\bar{w}, \bar{\theta})$, with $(w, \theta) = (\bar{w}, \bar{\theta})$. The set of equilibria can be characterized as follows:

Proposition 1: The pair $(w, \theta) > 0$ is an equilibrium if and only if:

$$(a) \quad F(w, \theta) \equiv \sigma(w, \theta) \left(\frac{f-w}{r+\delta} \right) - G'(\theta) = 0, \text{ and}$$

$$(b) \quad H(w, \theta) \equiv \frac{f-w}{r+\delta} - \frac{f}{r+\delta+q(w, \theta)} \geq 0.$$

Proof: (i) only if: The necessary first-order condition for the firm's choice of θ is (a). From (a) and (ii), $\sigma > 0$. Therefore

$$\pi(w, \theta) \geq \pi(0, \theta) \text{ implies (b). (ii) if: Take any } (\tilde{w}, \tilde{\theta}) \geq 0. \text{ Then}$$

$$\pi(w, \theta) - \pi(\tilde{w}, \tilde{\theta}) = \left[\left(\frac{f-w}{r+\delta} \right) \sigma \theta - G(\theta) - \left(\frac{f-\tilde{w}}{r+\delta} \right) \sigma \tilde{\theta} + G(\tilde{\theta}) \right] + \left[\frac{f-w}{r+\delta} - \frac{f-\tilde{w}}{r+\delta+q(w-\tilde{w}, \theta)} \right] \sigma \tilde{\theta}.$$

The first term is non-negative, from (a) and the convexity of G in (2). The second term is non-negative, from (b) and the discussion surrounding Figure 2. ||

Condition (b) of Proposition 1, governing the value of w , is an inequality rather than an equality because, as shown in Figure 2, it need only

guarantee that the firm cannot profit from reducing its wage below \bar{w} . Thus the wage-condition defines not a one-dimensional locus but a two-dimensional subset of $w - \theta$ space, as in Figure 3. Formally:

Proposition 2: There is a continuous, positive-valued function $\tilde{\theta}(w)$ on $[\underline{y}, f)$, with $\lim_{w \rightarrow f} \tilde{\theta}(w) = \infty$, such that $H(w, \theta) \geq 0$ if and only if $\theta \geq \tilde{\theta}(w)$.

Proof: From (12) and (14), $H(w, \theta)$ increases monotonically from $\frac{-w}{r+\delta} < 0$ to $\frac{f-w}{r+\delta} > 0$ as θ goes from 0 to ∞ , so $\tilde{\theta}(w)$ is well-defined by the equation $H = 0$. By the definition of H , $q(w, \tilde{\theta}(w)) = \frac{w(r+\delta)}{f-w}$.

Therefore $\lim_{w \rightarrow f} q(w, \tilde{\theta}(w)) = \infty$. Since q is continuous on $[\underline{y}, f)$,

$$\lim_{w \rightarrow f} \tilde{\theta}(w) = \infty. \parallel$$

The other factor lending indeterminacy to our results is the above-mentioned externality, which shows up in condition (a) of Proposition 1 governing the equilibrium value of recruiting intensity. The function F is the marginal (private) profit from recruiting. For any given wage the firm will take the marginal benefit $\sigma\left(\frac{f-w}{r+\delta}\right)$ as given, while the marginal cost $G'(\theta)$ is strictly increasing with recruiting effort; thus any θ that sets $F = 0$ will be uniquely optimal. But for all firms taken together this marginal benefit will depend upon the recruiting intensity, as shown in Figure 1. As shown in Figure 4, marginal profit is negative for small (but positive) recruiting intensities, because it takes at least some minimal intensity before any workers will begin searching, and until then there is no point even trying to recruit. But marginal profit is also negative for very large recruiting intensities because marginal cost keeps rising whereas the reduction in the pool of unemployed discussed earlier makes the marginal benefit fall toward zero. So if there is any recruiting intensity for which the marginal profit is strictly positive (θ_0 in Figure 4) there must be at least two intensities at which it is zero, one larger and one smaller than θ_0 .

Likewise, as Figure 5 illustrates, there will, generically, be an even number of wages for which a given recruiting intensity satisfies the recruiting condition. For the marginal profit of recruiting obviously becomes negative for any wage higher than the marginal product. It also becomes negative for any wage below the minimum value of leisure, at which point there will be no searchers to catch no matter how big the net.

These results are formalized by the following proposition, which is illustrated in Figure 6:

Proposition 3: There exist θ' and θ'' , with $0 < \theta' < \theta'' < \infty$, such that $F < 0$ for all $(w, \theta) > 0$ outside the open box $S \equiv (\underline{y}, f) \times (\theta', \theta'')$.
If $F(w_0, \theta_0) > 0$ then there is a closed loop of solutions to $F = 0$ surrounding (w_0, θ_0) .

Proof: The second sentence follows from the first. For any continuous arc starting at $(w_0, \theta_0) \in S$, where $F > 0$, and reaching the boundary of S where $F < 0$ must pass through a point where $F = 0$, since F is continuous. The first sentence is proved in three stages. First, that $F < 0$ when $w \notin (\underline{y}, f)$ and $\theta > 0$ follows immediately from (2), (18) and (22). Next, define θ' by the equation $\hat{\alpha}(f, \theta') = I^{-1}(\underline{y})$. By (8), (10) and (18), θ' is well-defined and positive. By (2), (9), (10) and (22), if $(w, \theta) \in (\underline{y}, f) \times (0, \theta']$ then $F = -G'(\theta) < 0$. Finally, define θ'' by the equation $\frac{\bar{L}f}{n\theta(r+\delta)} - G'(\theta) = 0$. Conditions (2) guarantee that θ'' is well defined and $\theta'' > 0$. (If $\theta'' \leq \theta'$ then redefine θ'' as $2\theta'$.) By (21), $F \equiv \frac{\bar{L}(f-w)}{n\theta(r+\delta)} - G'(\theta) < \frac{\bar{L}f}{n\theta(r+\delta)} - G'(\theta) \leq 0$ if $(w, \theta) \in (\underline{y}, f) \times [\theta'', \infty)$. ||

This allows us to state a general existence result, illustrated by

Figure 7:

Proposition 4: If there is some (w_0, θ_0) such that $F(w_0, \theta_0) > 0$ and $\theta_0 \cong \tilde{\theta}(w_0)$, then there is a continuum of equilibria.

Proof: By Proposition 3, a horizontal line starting at (w_0, θ_0) must cross the closed loop of solutions to $F=0$ to the right of (w_0, θ_0) . At this point $H \cong 0$, by Proposition 2. Similarly, all points on the closed loop in some neighborhood must have $\theta - \tilde{\theta}(w) > 0$, and thus satisfy $H \cong 0$. These points constitute the desired continuum. ||

Holding all other parameters constant there is some $\underline{f} \cong 0$ such that the premise of Proposition 4 is satisfied for all $f \cong \underline{f}$. To see this, pick any $\theta_0 > 0$. From (6) and (22) there is some $w_0 > 0$ such that $\sigma(w_0, \theta_0) > 0$. Therefore there is some $\underline{f}_1 > 0$ such that $F(w_0, \theta_0) > 0$ if $f \cong \underline{f}_1$. Likewise, since $q(w_0, \theta_0) > 0$ (by (12)), there is some $\underline{f}_2 > 0$ such that $H(w_0, \theta_0) \cong 0$ if $f \cong \underline{f}_2$. By Proposition 2, (w_0, θ_0) satisfies the premise of Proposition 4 if $f \cong \underline{f} \equiv \max(\underline{f}_1, \underline{f}_2)$.

The premise of Proposition 4 is satisfied also if the number of potential entrants, \bar{L} , is large enough. To see this, take any $w \in (\underline{y}, f)$. By (12) and (18), θ_0 can be chosen so that $\sigma(w_0, \theta_0) > 0$ and $H(w_0, \theta_0) \cong 0$. That $F(w_0, \theta_0) > 0$ if \bar{L} is large enough follows from (16) and (20).

Similarly it can be shown that holding all other parameters constant, the premises of Proposition 2 will be satisfied if (a) $c(\alpha)$ is of the form $k\Gamma(\alpha)$ for $k > 0$ and a fixed $\Gamma(\cdot)$ satisfying (1), and k is small enough, or (b) $G(\theta)$ is of the form $h \cdot \xi(\theta)$ for $h > 0$ and a fixed $\xi(\cdot)$ satisfying (2), and h is small enough.

Unique equilibria will almost never occur because that would require 0 to be the maximum value of $F(w, \theta)$ on the set defined by $H \cong 0$, attained at a unique point in that set. Indeed, a countable number of equilibria will also be rare, for similar reasons.

The existence of a whole continuum of equilibria is due mainly to the kink that makes the wage-setting condition an inequality. But the externality that yields a closed loop of solutions to the recruiting condition also plays a role in generating multiple equilibria, as suggested in Figures 4 and 5. Specifically, it is easy to derive conditions under which there are multiple equilibria with the same wage but different recruiting intensities. For example, take the case studied above where $F(w_0, \theta_0) > 0$ for $f \cong \underline{f} > 0$. Define $\theta'_0 > 0$ by the equation $\hat{\alpha}(w_0, \theta'_0) = I^{-1}(\underline{y})$. As with θ' , defined in Proposition 3, the closed loop must pass entirely to the right of (w_0, θ'_0) . By (12) there is some $\underline{f} \cong \underline{f}$ such that $H(w_0, \theta'_0) > 0$ when $f \cong \underline{f}$. Therefore if f is large enough then $\tilde{\theta}(w_0) < \theta'_0$ and all points on the closed loop with $w = w_0$ are equilibria. The argument is illustrated in Figure 8.

By the same token, for large enough f all the points on the closed loop with wages no greater than w_0 will be equilibria, in which case there will exist an upward-sloping curve of equilibria. The significance of such an upward-sloping segment of the loop will be brought out in the next section.

5. Unemployment, Dynamics, and Efficiency

Let z be the number of workers employed in a stationary equilibrium. The flow into employment is $\alpha n \theta U$. The outflow is δz . Thus stationarity, (19), and (20) require:

$$(25) \quad z = \alpha n \theta U / \delta = n \theta \sigma(w, \theta) / \delta.$$

The size of the labor force will be:

$$(26) \quad U + z = \hat{L}(w, \theta) / \delta$$

The rate of unemployment will be:

$$(27) \quad \frac{U}{U+z} = \frac{\delta}{\delta + \hat{\alpha}(w, \theta)n\theta}$$

The participation rate will be:

$$(28) \quad \frac{L}{\bar{L}} = \Psi(I(\hat{\alpha}(w, \theta)))$$

The average duration of unemployment will be:

$$(29) \quad d = \frac{1}{\hat{\alpha}(w, \theta)n\theta}$$

Comparative-statics predictions of the usual sort are hard to make with all the indeterminacy of the present model. However, we may regard (w, θ) as exogenous variables for purposes of generating empirical predictions, holding constant all parameters and allowing (w, θ) to vary from one equilibrium to another. Doing this and using (25)~(29) together with (9), (10), (16), (23), and (24), yields the prediction that an increase in either the wage or recruiting intensity, *ceteris paribus*, will cause an increase in employment, an increase or no change in the labor force and the participation rate, and a decrease in the rate and average duration of unemployment. The effects on the number of unemployed will be ambiguous, since both exogenous changes will increase the flow of entrants into unemployment but also will cause people to find jobs more quickly, with an indeterminate net effect on the stationary size of the pool of unemployed.

In general there is no guarantee that *ceteris paribus* increases in either w or θ can be observed. But the arguments at the end of the previous section guarantee that they can be if, for example, the marginal product of labor is large enough, in which case there will exist equilibria with the same wage but different recruiting intensities. Likewise if there exists an upward

sloping curve of equilibria, which there will if the marginal product of labor is large enough, the predictions of the previous paragraph apply to any movement along that curve, as w and θ increase together.

To examine the effects upon these labor market variables of the exogenous variables f , δ , G' and c' (the latter two in the sense described in the third paragraph following Proposition 4) we need to make some assumption about which equilibrium will be selected following a shock. We have not been able to work out a complete analysis of the system's dynamics. But consider the case where for any w there is at most one equilibrium with $\frac{\partial F}{\partial \theta} < 0$, as for example, in the case where equilibria consist entirely of a single closed loop with exactly two equilibrium θ 's for each w , where the equilibria with $\frac{\partial F}{\partial \theta} < 0$ are those on the right-hand side of the loop. We call these "high-level" equilibria. Suppose that each high-level equilibrium satisfies:

$$(30) \quad \hat{\alpha}(w, \theta) > I^{-1}(\bar{\gamma}) \text{ , and}$$

$$(31) \quad \frac{\alpha c''(\alpha)}{c'(\alpha)} \cdot \frac{\theta G''(\theta)}{G'(\theta)} > 1. \quad (\alpha = \hat{\alpha}(w, \theta))$$

Condition (30) guarantees that the rate of entry into the labor force is fixed at its upper limit \bar{L} in the neighborhood of every high-level equilibrium.

Given this, condition (31) rules out unbounded economies of scale, in the sense that it is a second-order condition for an interior solution to maximizing the Hamiltonian of the social planning problem:

$$\text{Max}_{\{\alpha, \theta, z, U\}} \int_0^{\infty} e^{-rt} (fz(t) - nG(\theta(t)) - Uc(\alpha(t))) dt$$

subject to

$$\dot{z} = \alpha n \theta U - \delta z$$

$$\dot{U} = L - (\delta + \alpha n \theta) U .$$

Under these conditions, the following analysis shows that starting in the neighborhood of any high-level equilibrium there is a perfect foresight equilibrium trajectory with a fixed real wage and with the recruiting intensity converging upon its high-level equilibrium value for that wage.

Along any such trajectory the searching workers choose a search plan $\{\alpha(t)\}_{t=0}^{\infty}$ so as to maximize

$$\int_0^{\infty} e^{-(r+\delta)t} x \left(\frac{w\alpha n\theta}{r+\delta} - c(\alpha) \right) dt$$

subject to

$$\dot{x} = -x\alpha n\theta$$

$$x = e^{-\int_0^t \alpha n\theta d\tau}$$

where $x(t) = e^{-\int_0^t \alpha n\theta d\tau}$ is the probability that the searcher will still be without a job, conditional upon still being alive, at date t . Maximizing the Hamiltonian of this problem requires

$$(32) \quad c'(\alpha) = \left(\frac{w}{r+\delta} - \lambda \right) n\theta$$

where λ is the costate variable associated with x . The Euler equation together with (32) implies:

$$(33) \quad \dot{\lambda} = (r+\delta)\lambda + c(\alpha) - \alpha c'(\alpha)$$

The number unemployed will vary according to:

$$(34) \quad \dot{U} = L - \delta U - \alpha n\theta U$$

The firms will continue to choose their recruiting intensely according to the condition $F = 0$, with $\sigma = \alpha U$:

$$(35) \quad G'(\theta) = \left(\frac{f-w}{r+\delta} \right) \alpha U$$

From (32) and (35):

$$(36) \quad c' \left(\frac{G'(\theta)}{\beta U} \right) - n\theta \left(\frac{w}{r+\delta} - \lambda \right) = 0$$

where $\beta = \frac{f-w}{r+\delta} > 0$. Let $(\bar{w}, \bar{\theta})$ be any high-level equilibrium. Then $\bar{U} = \frac{L}{\delta + \hat{\alpha}(\bar{w}, \bar{\theta}) n \bar{\theta}}$ and $\bar{\lambda} = \frac{1}{r+\delta} I(\hat{\alpha}(\bar{w}, \bar{\theta}))$ satisfy (32) ~ (35) with $\dot{\lambda} = \dot{U} = 0$.

Furthermore, (31) ensures that (36) defines an implicit function $\hat{\theta}(\lambda, U)$ in the neighborhood of $(\bar{\lambda}, \bar{U})$, with

$$(37) \quad \frac{\partial \hat{\theta}}{\partial \lambda} < 0, \quad \frac{\partial \hat{\theta}}{\partial U} > 0.$$

Replacing α with $G'(\theta)/\beta U$ and θ with $\hat{\theta}(\lambda, U)$ in (33) and (34) defines a dynamical system in (λ, U) which characterizes a perfect foresight path along which firms are choosing their recruiting intensity optimally, households are searching at the optimal speed, and real wages are constant. The trajectories exist in the neighborhood of $(\bar{\lambda}, \bar{U})$. To show that one of them converges upon $(\bar{\lambda}, \bar{U})$ we need only show that $(\bar{\lambda}, \bar{U})$ is a saddle-point; i.e., that the Jacobian of the system:

$$\begin{bmatrix} r + \delta - \alpha c''(\alpha) \frac{G''}{\beta U} \frac{d\hat{\theta}}{d\lambda}, & -\alpha c''(\alpha) \left[\frac{G''}{\beta U} \frac{d\hat{\theta}}{dU} - \frac{G'}{\beta U^2} \right] \\ -\frac{n}{\beta} [\theta G'' + G'] \frac{d\hat{\theta}}{d\lambda}, & -\delta - \frac{n}{\beta} [\theta G'' + G'] \frac{d\hat{\theta}}{dU} \end{bmatrix}$$

has a negative determinant. This follows from (1), (2), (32) and (37).

This means that we can legitimately assume in this case that following a displacement the real wage will remain unchanged and the recruiting intensity will converge upon the new value of the high-level equilibrium θ for that wage. Thus we can conduct comparative-statics experiments by examining the recruiting condition (a) of Proposition 1, assuming $\frac{\partial F}{\partial \theta} < 0$, holding w constant, and calculating:

$$(38) \quad \frac{\partial \theta}{\partial f} > 0, \quad \frac{\partial \theta}{\partial \delta} < 0, \quad \frac{\partial \theta}{\partial c'} < 0, \quad \frac{\partial \theta}{\partial G'} < 0.$$

These results, together with the definitions (25), (27) and (29) produce the comparative-statics predictions tabulated in Table 1:

Exogenous Variable \ Endogenous Variable	Marginal Product of Labor	Death Rate	Marginal Cost of Search	Marginal Cost of Recruiting
Total Employment	+	-	-	-
Rate of Unemployment	-	+	+	+
Duration of Unemployment	-	+	+	+

TABLE 1

Because of assumption (30), there is no effect upon the labor force or the participation rate.

Our discussion of the effects of a ceteris paribus change in w or θ suggest that, whenever such comparisons are possible, the equilibria to the northeast are superior, since they involve higher activity levels and lower rates and average durations of unemployment. To explore this suggestion in terms of expected utilities let us take as our two measures of welfare the expected utility of the unborn worker, m , and expected profits, e . Then m is the expected maximum of EPV^* and the present value of leisure. According to (7):

$$(39) \quad m = \frac{1}{r+\delta} [\Psi(I(\alpha))I(\alpha) + \int_{I(\alpha)}^{\infty} \gamma d\Psi(\gamma)]$$

The expected present value of profits for any firm is:

$$(40) \quad e = \frac{1}{r} ((f-w)\frac{z}{n} - G(\theta)) = \frac{1}{r} ((f-w)\sigma\theta/\delta - G(\theta))$$

But, from this and (a) of Proposition 1, if (w, θ) is a stationary equilibrium, then expected profits will equal:

$$(41) \quad e(\theta) = \frac{1}{r} \left[\left(\frac{r+\delta}{\delta} \right) \theta G'(\theta) - G(\theta) \right]$$

with

$$(42) \quad e'(\theta) = \frac{1}{r\delta} [(r+\delta)\theta G''(\theta) + rG'(\theta)] > 0$$

Also, from (9), (10) and (39), expected utility is increasing in both w and θ . Thus any equilibrium to the northeast of another is Pareto-superior.

All equilibria are Pareto-inefficient in the following weaker sense. Suppose that all searching workers could be induced (costlessly) to search more intensively, with no change in the participation rate. Keep the wage constant and allow firms to vary their recruiting intensity in response to the induced change in the arrival rate $\left(\frac{d\sigma}{d\alpha} = \frac{d}{d\alpha} \left(\frac{\alpha}{\delta + \alpha n \theta} \hat{L} \right) = \frac{\delta \hat{L}}{(\delta + \alpha n \theta)^2} > 0 \right)$ so as to keep the marginal profit from recruiting equal to zero. From (a) of Proposition 1 and the definition of π , $\frac{d\theta}{d\alpha} = \frac{d\sigma}{d\alpha} \cdot \frac{(f-w)}{(r+\delta)G'(\theta)} > 0$ and $\frac{d\pi}{d\alpha} = \frac{\partial \pi}{\partial \sigma} \cdot \frac{d\sigma}{d\alpha} + \frac{\partial \pi}{\partial \theta} \cdot \frac{d\theta}{d\alpha} = \frac{\partial \pi}{\partial \sigma} \cdot \frac{d\sigma}{d\alpha} = \theta \frac{(f-w)}{r+\delta} \frac{d\sigma}{d\alpha} > 0$. From (3), $\frac{dEPV}{d\alpha} = \frac{\partial EPV}{\partial \theta} \cdot \frac{d\theta}{d\alpha} + \frac{\partial EPV}{\partial \alpha} = \frac{\partial EPV}{\partial \theta} \cdot \frac{d\theta}{d\alpha} = \frac{w\alpha n + c(\alpha)}{(r+\delta + \alpha n \theta)^2} \cdot \frac{d\theta}{d\alpha} > 0$. Thus both expected profits and expected utility of participating workers could be increased, with no loss to non-participating workers. This inefficiency is an obvious consequence of the externality in communication.⁷

6. Varying the Assumptions

The assumption that the marginal cost of search falls continuously to zero when the speed of search becomes small is crucial for the existence of equilibrium. For suppose on the contrary that $c'(0) > 0$. Then it would take at least some positive wage-discrepancy (specifically, $\frac{r+\delta}{n\theta} c'(0)$) before an employee

would begin searching on the job for a higher wage. Thus the firm would always want to offer less than the going market wage, since a small reduction below the market wage would reduce the wage bill without in any way affecting the supply of workers. The kink in the firm's objective function shown in Figure 2 would occur at $\bar{w} - \frac{r+\delta}{n\theta} c'(0)$ rather than at \bar{w} .

The attempt by firms to exploit the workers' positive marginal search cost by offering strictly less than the going wage would drive the wage down to zero, for reasons analogous to those in Diamond's (1971) model, where the output price was driven up to the monopoly price. In this case, however, the equilibrium would be degenerate, since with $w=0$ and $\gamma > 0$ no worker would ever search. (Similar results have been found by Stiglitz.)

In the same connection it is important that we have required all wage offers to consist of a constant wage for life. If a firm were free to offer a non-constant wage profile, then the same reason would probably prevent the existence of any non-degenerate equilibrium, as the following heuristic reasoning suggests. Such an equilibrium would require all firms to pay some equilibrium profile $w^*(t)$ with a positive expected present value to the worker. But a firm would always prefer to offer a little below the going wage-profile, in the form of a slightly deferred profile equal to zero until time Δ and $w^*(t-\Delta)$ after that. The saving to the firm in the form of a reduced present-value of the expected wage-bill per recruit would be of order Δ . But the cost would only be of order Δ^2 . This is because a small deferral would induce only a small amount of extra on-the-job search, lasting for only a small time. Somewhat more formally, if all other firms were offering $w^*(\cdot)$ in a non-degenerate equilibrium, the expected present value to the firm of each new recruit to a firm deferring by Δ would be:

$$V(\Delta) = \int_0^{\Delta} \exp\{-(\delta+r)t - \int_0^t \tilde{q}(\tau, \Delta) d\tau\} dt + \exp\{-(\delta+r)\Delta - \int_0^{\Delta} \tilde{q}(\tau, \Delta) d\tau\} V(0)$$

where $\tilde{q}(\tau, \Delta)$ is the rate at which workers quit at time τ when their wages are deferred by Δ . It is readily seen that $V'(0) = f - (r + \delta + \tilde{q}(0, 0))V(0)$. But $\tilde{q}(0, 0) = 0$, since it doesn't pay to search for what you've just found. Also, $q \geq 0$, and $\int_0^{\infty} e^{-(\delta+r)t} w^*(t) dt > 0$ if the equilibrium is to be non-degenerate. Thus $V(0) = \int_0^{\infty} \exp\{-(r+\delta)t + \int_0^t \tilde{q}(\tau, 0) d\tau\} (f - w^*(t)) dt < \frac{f}{r+\delta}$, implying $V'(0) > 0$. The firm would want to set $\Delta > 0$ and the equilibrium could not be sustained.

By restricting the firm to pay the same wage for the lifetime of a contract we guarantee that any undercutting of rivals will result in workers searching on the job for the duration of the contract, making the costs of undercutting take on the same order of magnitude as the benefits.

While this restriction upon wage-setting is somewhat arbitrary, it should be remembered that once contact has been made the situation in which a wage is agreed upon is one of bilateral monopoly. This is because whatever the equilibrium wage the presence of positive search costs implies that the worker's reservation wage is less than that, while positive recruiting costs imply that the firm's reservation wage is above that. Since there is no unique solution to the problem of bilateral monopoly, any imposed solution must be to some extent arbitrary. Allowing firms to make offers on a take-it-or-leave-it basis implicitly gives them most of the bargaining power. To prevent this bargaining power from driving wages to zero we must ensure that there is enough of a threat of quitting, which is the point of assuming that the marginal cost of searching falls continuously to zero and of requiring wage offers to have constant wages.⁸

The advantage of the take-it-or-leave-it assumption that it seems to mimic the actual behavior of many labor markets. But it also understates the bargaining power of workers in many situations. A more symmetrical solution is the one proposed by Diamond (1982a) according to which

the wage is not chosen by either side but instead results from an unspecified bargaining process that always results in the gains from trade being split in a fixed proportion. This assumption would replace our inequality governing wage-determination by an equality, thus destroying the basis for a continuum of equilibria. However, it would not destroy the basis of multiplicity, which would still arise because of the externality.

More specifically, the gain from trading at the wage w to a worker who has already encountered a firm, and whose best alternative is to recommence searching for the same wage, is $\frac{w}{r+\delta} - EPV^* = \frac{c'(\hat{\alpha}(w,\theta))}{n\theta}$, by (7). The gain to the firm, whose best alternative is to lose the contact and continue as before is $\frac{f-w}{r+\delta}$. Let β be the fraction of the total gain accruing to the firm. Then in any equilibrium,

$$(43) \quad J(w,\theta) \equiv \frac{f-w}{r+\delta} - \beta \frac{c'(\hat{\alpha}(w,\theta))}{n\theta} = 0, \quad \beta = \frac{\beta}{1-\beta} > 0.$$

Reasoning as before we see that a stationary equilibrium with this bargaining solution to wage determination consists of all $\{w,\theta\} > 0$ satisfying (43) and the condition $F=0$ governing the recruiting intensity.

It is easily verified that the locus of points satisfying (43) is an upward-sloping curve starting at $(w,\theta) = ((1-\beta)f,0)$, and projecting onto the entire horizontal axis, as in Figure 9. Equilibria consist of points on this curve such that $F=0$. There will still generally be multiple equilibria (if any exist) because $F < 0$ at both ends of the curve, to the left of θ' and to the right of θ'' . Note that in this case the welfare comparisons and comparative-statics propositions discussed in Section 5 in which w and θ change in the same direction, ceteris paribus, are indeed operational since all equilibria can be ordered in a northeasterly direction.

One of our extreme assumptions mentioned earlier is that workers cannot be induced in any way to direct their search toward a deviant high-wage firm. This was what ensured that no firm would contemplate offering above the market wage. Relaxing this assumption in a coherent fashion is largely an unsolved problem. But some insight can be gained from the following attempt. Suppose that the signs and recruiting agents used by the firm are more effective the higher the wage they offer. We might suppose, for example, that a sign offering to pay above the going wage will cause extra commotion on the part of searchers who notice it, which then attracts the attention of others who might otherwise have missed the sign. Formally, suppose that the cost of recruiting is $G(\theta, w - \bar{w})$, with G convex, $G_1 > 0$ and $G_2 < 0$ when $\theta > 0$, and $G_1(0, w - \bar{w}) = 0$.

In this case the firm's objective function $\hat{\pi}(w, \theta) = \frac{(f-w)\sigma\theta}{r+\delta+q(\bar{w}-w, \theta)} - G(\theta, w - \bar{w})$ still has a kink in it at $w = \bar{w}$, because of the kink in the quit function. But it is not necessarily downward sloping with respect to a wage greater than \bar{w} . Thus the wage-setting optimality condition becomes a set of two inequalities, requiring profits to fall for $w > \bar{w}$ and $w < \bar{w}$ respectively, instead of the single inequality as before. Assume that $\hat{\pi}(w, \theta)$ is concave for $\theta \geq 0$, $w \geq \bar{w}$. Then Proposition 4 can be replaced by the proposition that $(w, \theta) > 0$ is an equilibrium if

- (i) $\sigma(w, \theta) \left(\frac{f-w}{r+\delta} \right) - G_1(\theta, 0) = 0$
- (ii) $\frac{f-w}{r+\delta} - \frac{f}{r+\delta+q(w, \theta)} \geq 0$
- (iii) $\frac{\sigma(w, \theta)\theta}{r+\delta} + G_2(\theta, 0) \geq 0$

Conditions (i) and (ii) are analogous to (a) and (b) of Proposition 1. Condition (iii) is the extra inequality governing wage-setting. To show sufficiency note that (i) and (iii) are the Kuhn-Tucker conditions for (w, θ) to maximize $\hat{\pi}(\tilde{w}, \tilde{\theta})$ when $(\tilde{w}, \tilde{\theta}) = (w, \theta)$, subject to $(\tilde{w}, \tilde{\theta}) \geq (w, 0)$, over which range π is concave. If $\tilde{w} < w$, then exactly as in Proposition 1, (i) and (ii) imply

$$\hat{\pi}(w, \theta) \geq \sigma \tilde{\theta} \left(\frac{f - \tilde{w}}{r + \delta + q(w - \tilde{w}, \theta)} \right) - G(\tilde{\theta}, 0)$$

But the RHS of this inequality is just $\hat{\pi}(\tilde{w}, \tilde{\theta}) + G(\tilde{\theta}, \tilde{w} - w) - G(\tilde{\theta}, 0)$. Therefore, since $G_2 < 0$ and $\tilde{w} - w < 0$, $\hat{\pi}(w, \theta) > \hat{\pi}(\tilde{w}, \tilde{\theta})$.

Thus all the equilibria existing under the premise of Proposition 4 will exist in this case if they also satisfy (iii). From (23) it follows that for any $\theta > 0$ there is a unique $\tilde{w}(\theta)$ such that (iii) is satisfied if and only if $w \geq \tilde{w}(\theta)$. However, there is no guarantee that $\tilde{w}(\theta)$ is finite for any given θ . Indeed since $\frac{\sigma \theta}{r + \delta}$ is bounded above by $\frac{\bar{L}}{n(r + \delta)}$ it is conceivable that $\tilde{w}(\theta)$ is infinite for all $\theta \leq \theta''$, in which case no equilibria exist even under the premise of Proposition 4. All we can say is that if equilibria exist, then again except in rare cases a continuum of equilibria will exist, and be efficiency and comparative-statics analysis of Section 5 go through as before. Thus while the possibility of advertising high wage offers complicates the analysis it does not necessarily alter our main results.

More specifically this suggests that it is the kink in the firm's objective function at \bar{w} , not the fact that it is always downward sloping to the right of \bar{w} , that produced the continuum of equilibria in the main analysis of the paper. The result is analogous to those of Hahn, Negishi, and Woglom, who assume a similar kink in firms' objective functions with respect to their chosen wages and/or prices.

Another important assumption was that a lumpy value of leisure had to be given up at each date in order to search at even a small intensity. This is what guaranteed that the firm's arrival rate, and hence the marginal profit of recruiting, fell to zero before the recruiting intensity reached zero,

as in Figure 1. As a result it guaranteed that θ' , the left-hand boundary of the set S , beyond which $F < 0$ (see Figure 6) was strictly positive.

While we think it reasonable to suppose that there is at least some fixed leisure cost to searching, our main results are not necessarily destroyed if that cost is zero. To examine that case suppose that the lumpy value of leisure time, γ , is lost not whenever the worker participates, as we have been assuming, but only when he works. Then each worker will participate if and only if $w > \gamma$, irrespective of θ , and will search at the rate $\hat{\alpha}(w-\gamma, \theta)$, since he is, in effect, searching for the incremental wage $w-\gamma$. Workers who contact a firm offering below the market wage will sign a contract only if $w \geq \gamma$. Otherwise they will continue searching. Once they sign a contract they will quit, as before, at the rate $q(\bar{w}-w, \bar{\theta})$, independently of γ , since the incremental wage for which they are searching while on the job will depend upon their existing wage, not the value of leisure.

It is readily seen that the firm's choice problem is the same in this case as before, except that $\sigma(\bar{w}, \bar{\theta})$ must be replaced by $\tilde{\sigma}(\bar{w}, \bar{\theta}; w) \equiv \int_{\gamma}^{\bar{w}} \frac{\hat{\alpha}(\bar{w}-\gamma, \bar{\theta}) \bar{L}}{\delta + \hat{\alpha}(\bar{w}-\gamma, \bar{\theta}) n \bar{\theta}} d\psi(\gamma)$, which depends upon his own wage, since wage-reductions will now lower the arrival rate of willing applicants.

This calls for two major changes. First, conditions (a) and (b) of Proposition 1 are still sufficient to ensure that (w, θ) is an equilibrium (with $\tilde{\sigma}(w, \theta, w)$ replacing $\sigma(w, \theta)$ in (a)). But (b) is no longer necessary. This is because π as a function of w will still be the same as in Figure 2 for $w \geq \bar{w}$, where $\frac{\partial \tilde{\sigma}}{\partial w}(\bar{w}, \bar{\theta}; w) = 0$, but will lie strictly below the curve in Figure 2 for $\gamma < w < \bar{w}$, where $\frac{\partial \tilde{\sigma}}{\partial w} > 0$, reaching $-G'(\theta)$ for $0 \leq w \leq \gamma$. (This is shown by the dashed line in Figure 2.) So it is still never optimal to offer above the

market wage, and if it is non-optimal to offer below the market wage under the assumption of a constant arrival rate, it is a fortiori non-optimal when that offer would lower the arrival rate. In this sense the wage-setting condition for equilibrium becomes easier to satisfy.

The other major change is that, as already discussed, it might not be possible to define a positive θ' as in Figure 6. The marginal profit of recruiting might, for some fixed wages, now behave as in Figure 10, with only one equilibrium. Thus the closed loop of Proposition 3 might have to include part of the vertical axis, which points are not candidates for equilibria. This could destroy the possibility of multiple equilibrium recruiting intensities for a given wage. However, there are assumptions that guarantee against this possibility, the simplest one being that $G'(0) > 0$. In any event the result that equilibria will generally come in continua (if at all) will still hold, as will all our comparative-statics and efficiency results.

7. Conclusion

The preceding analysis obviously stops well short of a fully developed model of the kind of communication failure which Leijonhufvud saw underlying General Theory. But there are similarities, some of which we have observed in passing, which suggest that this way of proceeding holds some promise of leading to such a result. First, there is the externality which, as we have argued, bears some resemblance to the Keynesian idea of effective demand. Second, the unemployment that exists in "low-level" equilibria is involuntary, since unemployed workers face a greater difficulty of finding a job than they do at a higher equilibrium, for reasons not directly related to the wages being offered. Third, the equilibria are all inefficient. Fourth, as Leijonhufvud has stressed

was important for Keynes, in sharp contrast to textbook Keynesianism, the unemployment persists despite the absence of any impediments to wage-flexibility--indeed lower wages are associated with higher rates of unemployment in equilibria with the same or smaller recruiting intensity. (As Section 6 argued the wage-stickiness implied by the kink in the quit function was inessential to our main results.)

Fifth, the argument by which we showed that all equilibria are Pareto-inefficient involved an idea similar to that of the Keynesian multiplier process according to which increases in demand cause further increases in demand. For an exogenous increase in α , the speed of search by workers, which reduces the resource cost to a firm of any given rate of hiring, can also be thought of as increasing the quantity of goods that the firm can sell at a given price, and for a given sales effort, since each contract involves the sale of goods as well as the purchase of labor services. This increase in demand will induce firms to increase their recruiting intensity, making it easier to find jobs. It would thus induce workers to raise α even further, as in the usual multiplier process.

This multiplier-interpretation is somewhat strained because of the absence of money and separate markets for labor and goods, and because goods markets tend to be organized somewhat differently from labor markets. Further research should be directed at incorporating money and these important differences in market organization. Likewise, policy implications and seriously testable empirical restrictions will require an analysis freed from the perfect-foresight stationary equilibrium confines of the present analysis.

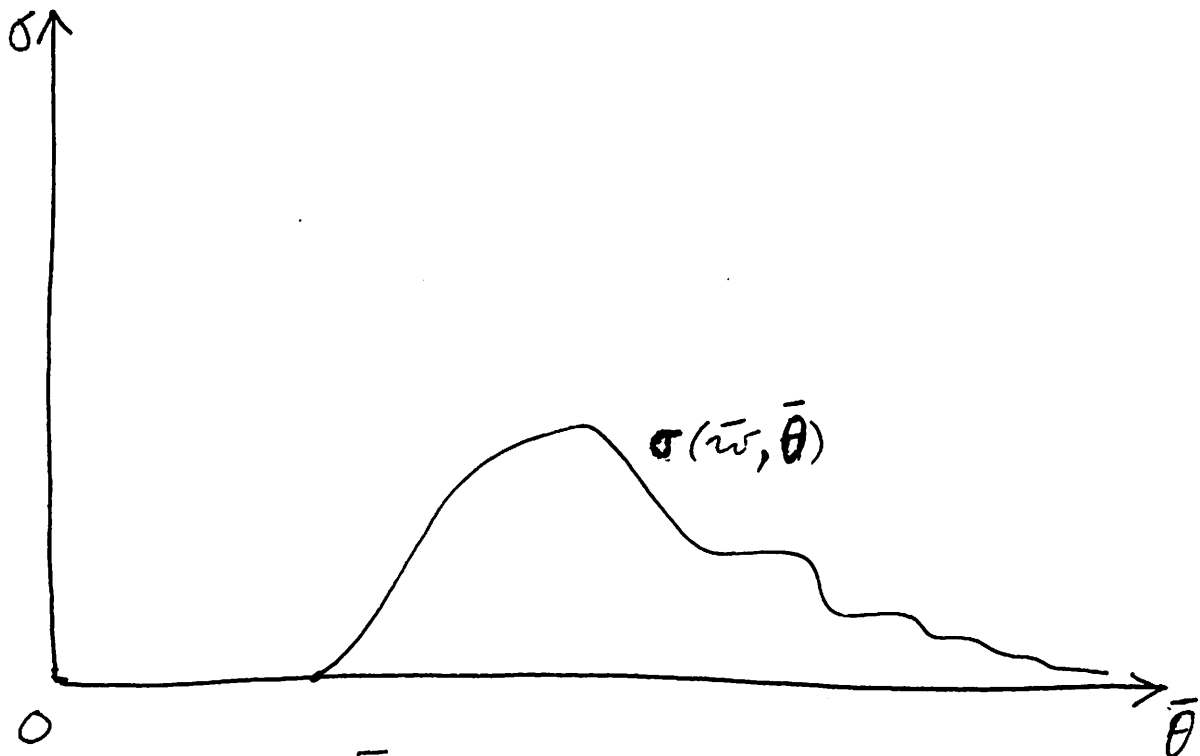


Figure 1. - The dependency of one firm's arrival rate upon other firms' recruiting intensity.

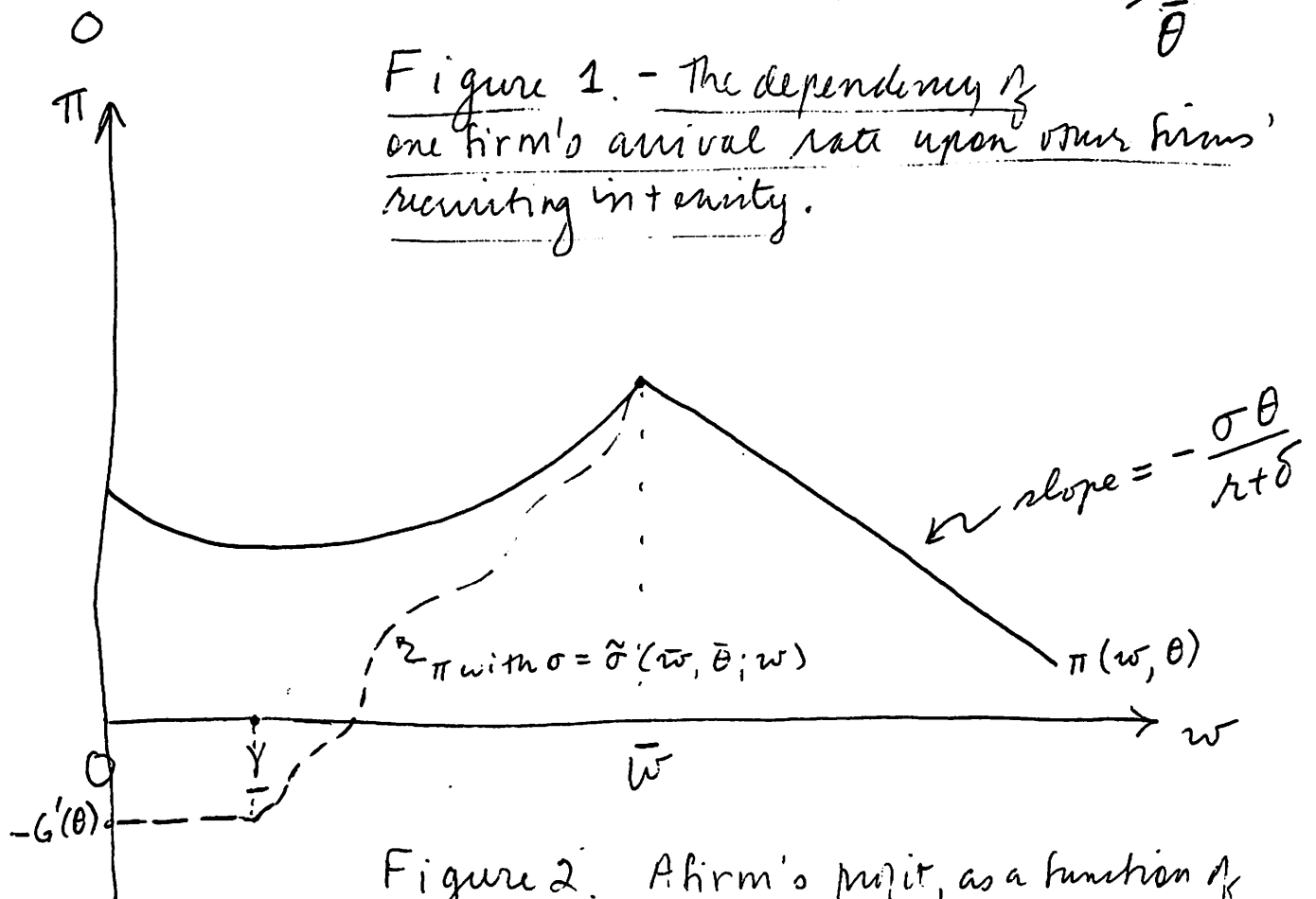


Figure 2. A firm's profit, as a function of its wage-offer, holding constant its recruiting intensity.

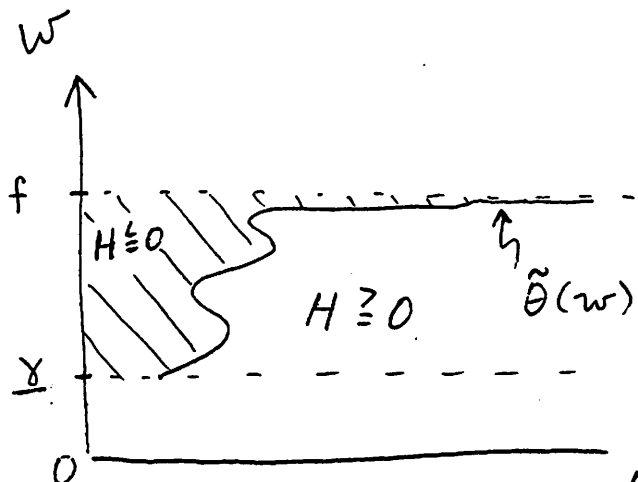


Figure 3. The set of solutions to the inequality governing equilibrium values of the wage: $H \geq 0$.

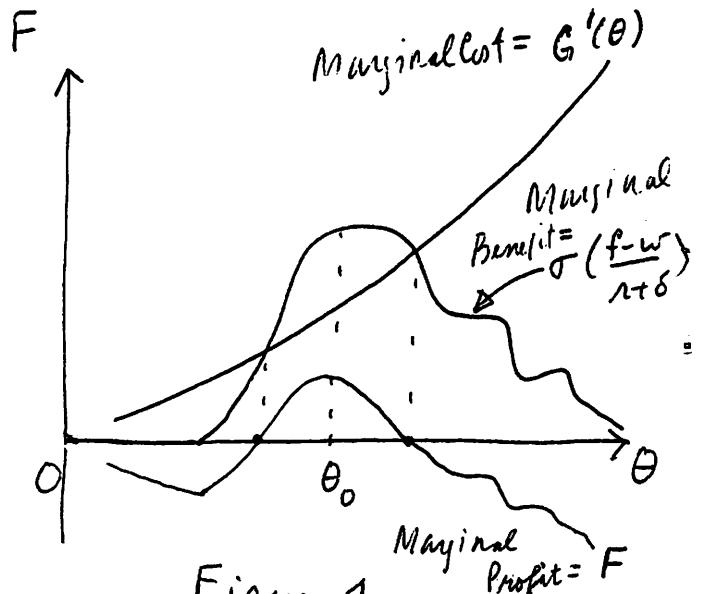


Figure 4. The solutions to the optimal-recruiting equation, for a given wage.

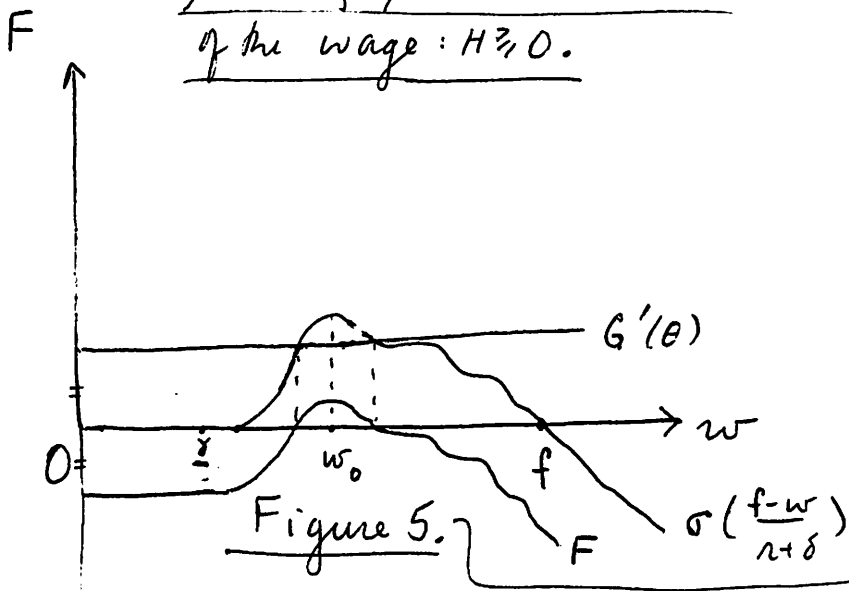


Figure 5.

The ways for which the optimal-recruiting equation is satisfied for given recruiting intensity

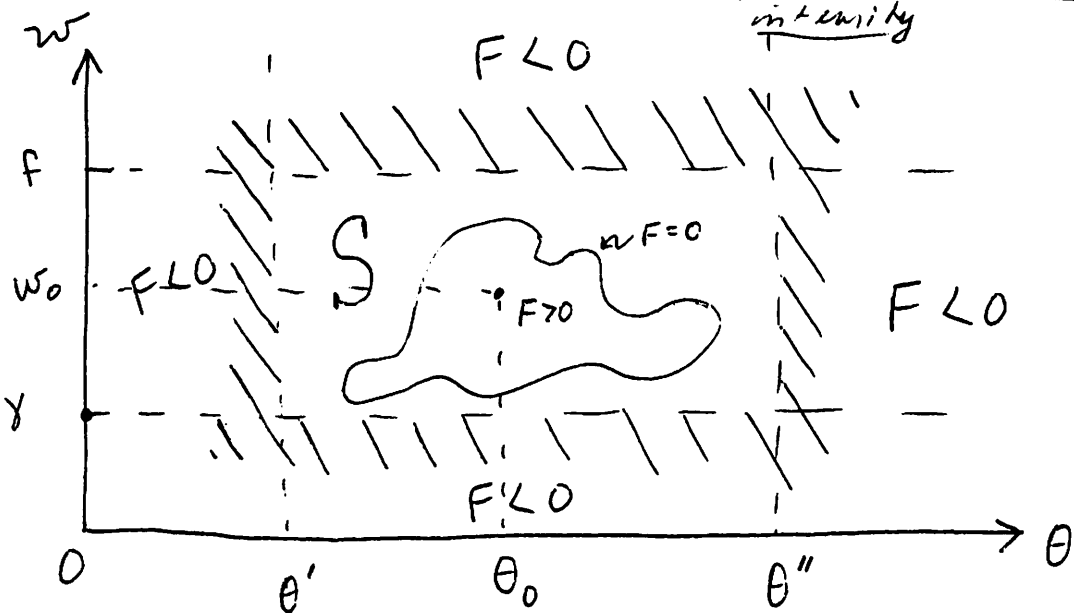


Figure 6: The closed loop of solutions to the optimal-recruiting equation.

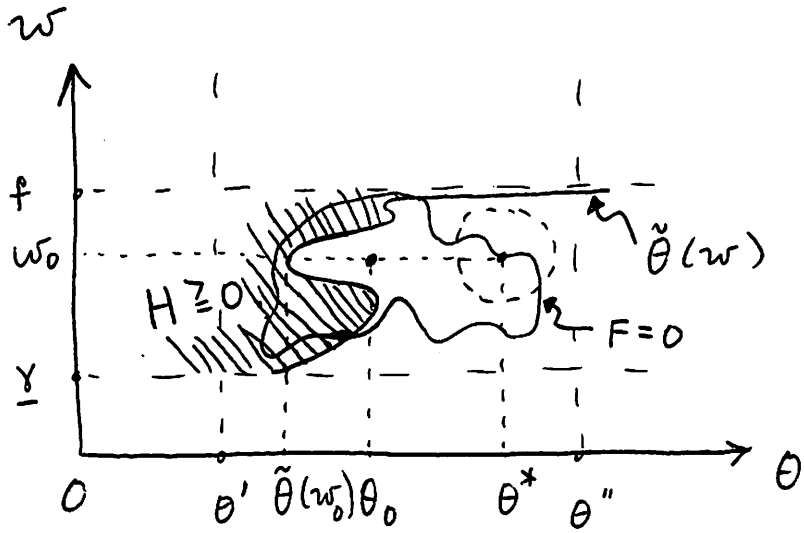


Figure 7. The existence of a continuum of equilibria in a neighbourhood of (w_0, θ^*) .

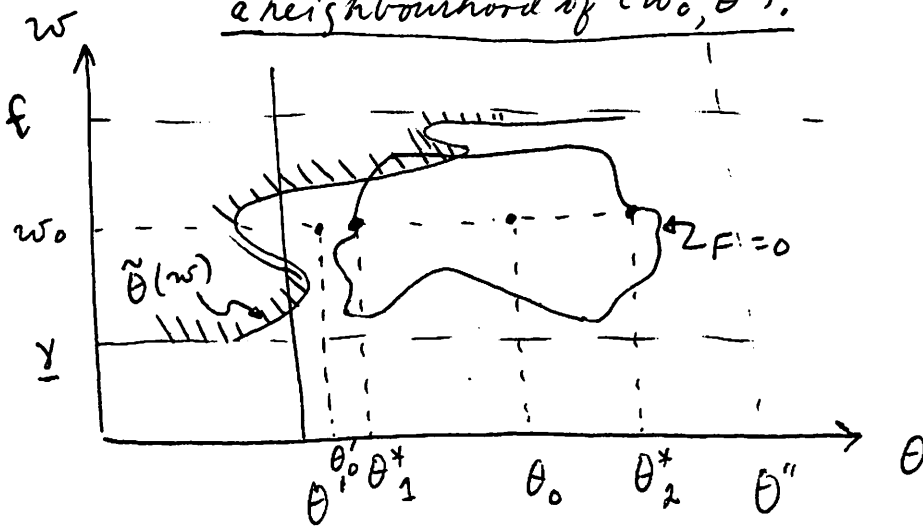


Figure 8. The existence of two equilibria with the same wage (w_0) but different recruiting intensities (θ_1^* and θ_2^*).

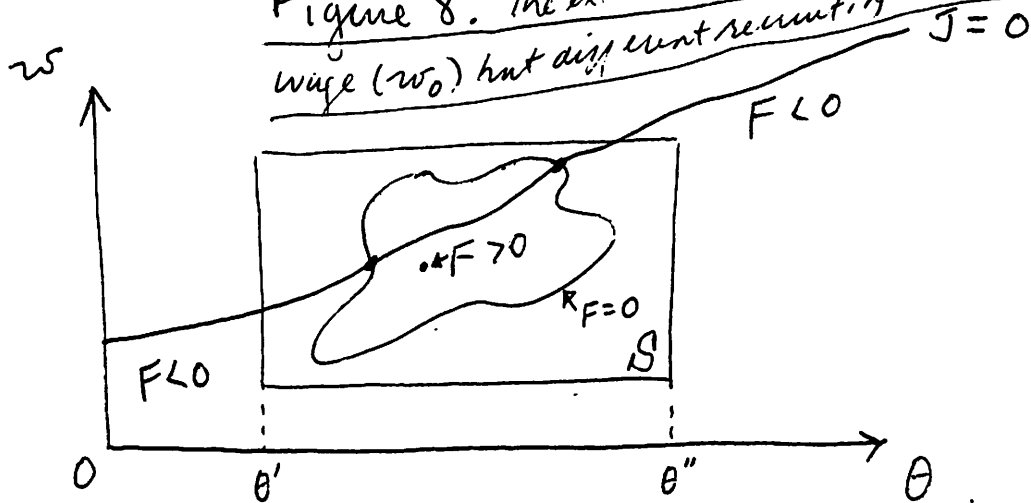


Figure 9. Equilibrium when wages are negotiated instead of being set by firms.

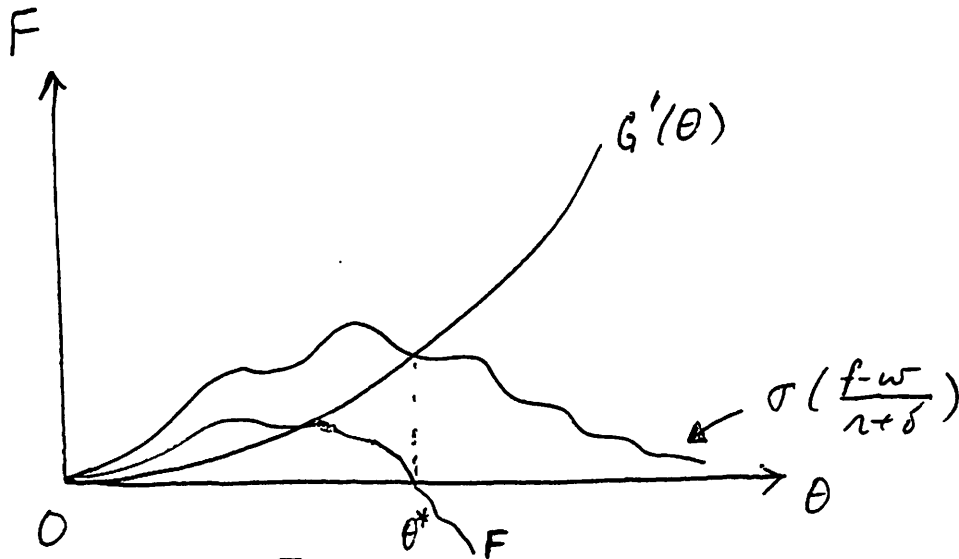


Figure 10 The possibility of a unique ^{optimal} recruiting intensity (θ^*) ~~when~~ for a given wage, when there is no fixed search cost.

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Footnotes

* University of Western Ontario. The authors wish to thank, without implicating, John McMillan and the participants of the Western Ontario theory workshop for useful comments.

¹More specifically: (a) production, instead of using hired inputs through a continuous production function, consists of the random arrival at discrete points in time of opportunities to produce one unit of output, at a random subjective cost to the agent; (b) each agent's ability to store goods is limited to exactly one unit, which requires him to forego any possibility of producing while attempting to sell goods that have already been produced; (c) no agent can attempt to sell goods unless he has already produced them, which, in combination with (b) comes close to an assumption that many have argued discredits search theory as a vehicle for explaining unemployment, namely that work and job-search are mutually exclusive activities; (d) it isn't clear which activity in the model ought to be interpreted as unemployment and which as employment. (Diamond interprets waiting for a production opportunity as unemployment, whereas this kind of waiting is presumably a necessary input into production and ought to be interpreted as employment. The activity of looking for a trading partner seems to us analogous to job search, but he calls it employment.); (e) after each act of production search begins anew with no possibility of recalling the previous trading partner, in contrast to the long-term bilateral relationships typical of real-world labor markets; and (f) prices are negotiated individually each time two potential trading partners meet, instead of being set on a take-it-or-leave-it basis by demanders, as in many labor markets (although clearly many wage contracts do involve substantial negotiations).

²For an informal example, consider workers travelling on a maze of roads. Firms place signs around, informing workers of job opportunities at the firm. If the workers travel twice as fast, they will double the number of signs they see on average. Alternatively, if the firms double the number of signs posted, this will double the number of people seeing their sign. The "recruiting net", θ , is the proportion of the roads from which a sign is visible, while the search rate, α , is the speed searchers travel. Observe that one can increase θ either by erecting more signs, or by placing them up higher.

³This setup is similar to Mortenson's, who assumed that the frequency of contacts would be additive in the two intensities rather than multiplicative. This two-way externality is present in both setups. The model of Christopher Pissarides is also similar, although he takes the number of vacancies, analogous to our recruiting intensity, as given, and hence does not study the full interaction of search and recruiting.

⁴The connection between this kind of externality and Keynesian theories of unemployment built upon Clower's analysis is discussed further by Howitt.

⁵We do not consider asymmetric equilibria with a dispersion of w 's and θ 's.

⁶The notation $(w, \theta) > 0$ indicates $w > 0$ and $\theta > 0$. We are ignoring the degenerate "equilibrium" $(w, \theta) = 0$.

⁷To internalize this externality would require either a collective decision to reorganize the market structure, or a private innovation that resulted in a large-scale "contracting" agency. It is not clear that either could succeed in restoring efficiency since the process of communicating under any but a highly centralized system is likely to involve such externalities,

whereas a highly centralized system is unlikely to be efficient in gathering, collating, and disseminating the highly diverse and particular information of a heterogeneous labor market, where economies of specialization are likely to prevail along with diseconomies of scope.

In particular it is hard to see how Mortenson's compensation scheme or some variant thereof might internalize the externality because (a) the scheme would require traders to be able to identify which party's effort was responsible for the match, whereas within our model no such imputation seems possible, and (b) until a contact is made neither party is able to communicate his willingness to trade according to any scheme, whereas after a contact is made the search and recruiting are bygone so it is too late to agree upon a scheme that would encourage the right amount of search and recruiting.

⁸This suggests an alternative explanation of real-wage stickiness--more precisely, stickiness of real-wage profiles within the terms of a contract--namely that without some such institutional restriction on wage offers the market could not function with firms acting as competitive bidders for labor.