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ABSTRACT

Quah's [1993a] transition matrix analysis of world income distribution based on annual data suggests an ergodic distribution with twin peaks at the rich and poor end of the distribution. Since the ergodic distribution is a highly non-linear function of the underlying transition matrix, it is estimated extremely noisily. Estimates over the foreseeable future are more precise. The Markovian assumptions underlying the analysis are much better satisfied with an analysis based on five-year transitions than one-year transitions. Such an analysis yields an ergodic distribution with 72% of mass in the top income category, but a prolonged transition, during which some inequality measures increase. The rosy ergodic forecast and prolonged transition arise because countries' relative incomes move both up and down at moderate levels, but once countries reach the highest income category, they rarely leave it. This is consistent with a model in which countries search among policies until they reach an income level at which further experimentation is too costly. If countries can learn from each other's experience, the future may be much brighter than would be predicted based on projecting forward the historical transition matrix.

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1. Introduction

This paper revisits the transition matrix analysis of world-income distribution dynamics, and argues that the data are consistent with a model in which countries search among policies until they reach an income level at which further experimentation is too costly.

Quah (1993a) classifies countries into groups by relative income, and estimates a transition matrix giving the probability that countries move between groups. He finds that rich countries typically stay rich, and that poor countries typically stay poor, but that middle income countries are likely to transit to wealth or poverty. The estimated ergodic income distribution associated with these transition probabilities has twin peaks, with many rich countries, many poor countries, and relatively few middle income countries. This twin peaks result has motivated theoretical work on growth models with multiple steady states, in which countries above a cutoff level of income converge to a high income level, while those below the cutoff fall into a poverty trap.

We start by updating Quah's analysis to include more recent data and then testing hypotheses about the ergodic distribution using techniques developed in Onatski (2000). We find that the point estimate of the ergodic distribution has twin peaks, although the rich peak is much larger than the poor peak. However, the ergodic distribution is estimated very imprecisely. When we follow Quah in using annual data to estimate the transition matrix, we cannot reject the hypotheses that the ergodic distribution has a single peak at the rich end of the income range or is equal to the distribution as of the end of the sample. The ergodic distribution is estimated sufficiently imprecisely that beliefs about the long-run distribution of world income must be heavily influenced by priors.

Nonetheless, a slightly modified Markovian analysis can yield more precise estimates of the short- and medium-run evolution of the world relative income distribution. More important, it can shed light on a possible mechanism generating the data. The assumption of a first-order Markov process is much better satisfied with five-year data than with annual data, and we therefore modify Quah's analysis by estimating transition probabilities over five-year intervals rather than annual intervals. The implied ergodic distribution has a much larger rich than poor peak, with 72% of countries in the richest income category.

However, transition to this steady state is very slow. Though estimates of the ergodic distribution are extremely noisy, the distribution over the next hundred years can be estimated more precisely, because uncertainty in estimates of the transition matrix taken to a relatively small power is much smaller than uncertainty in estimates of the transition matrix taken to an infinite power. We find that under the maintained Markovian assumptions, the coefficient of polarization and the standard deviation of log income will rise for hundreds of years.

The driving force behind the rosy long-run forecast and the prolonged transition is that countries' incomes move both up and down at moderate income levels, but there are very few transitions downwards once countries have reached the richest income category. Prosperity is almost an absorbing state.

The data are consistent with a simple model in which countries search for policies which enhance their long-run income, and optimally stop searching once their income reaches a certain level. The model encompasses two possible cases. In the first case, the effect of policies depends heavily on countries' particular characteristics, so attempts to imitate prosperous countries' policies are likely to be unsuccessful. In the second, "End of History" case, capitalism and democracy are a universal recipe for prosperity, but this has only recently become evident to policy makers with the fall of the Soviet Union and the introduction of market-oriented reforms around the world. In this case, the future may be much brighter than suggested by projecting forward a Markovian matrix based on the 1960-2000 period.

The rest of the paper is organized as follows. In Section 2 we describe the transition matrix approach and our data. Section 3 argues that the ergodic distribution is estimated extremely imprecisely. Section 4 argues that Markovian assumptions are better satisfied by estimating the transition matrix using five-year data, and that the associated ergodic distribution has most of its mass in the top income category. Section 5 shows that transition to the ergodic distribution is very slow, and is more precisely estimated. Section 6 argues that a model in which countries search over policies can explain the data.

2. Transition matrix framework

The transition matrix approach to analyzing growth, pioneered by Quah (1993a), allows for a more flexible relationship between the level of income and the growth rate of income than the standard convergence approach in which countries' growth rates are assumed to be a linear (or sometimes quadratic) function of their (log) income levels.

We follow Quah [1993a] in assuming that each country's relative income follows a first-order Markov process with time-invariant transition probabilities. That is, a country's (uncertain) income tomorrow depends only on its income today. In the discrete version of this approach, one assumes that all countries could be divided into several relative income groups. Quah (1993a) divides countries into five groups: those with less than 1/4 of the world average per capita income; those between 1/4 and 1/2 of world average income; those between 1/2 world average income and world average income; those between 1 and 2 times world average income, and those with income greater than twice the world average.[2]

Quah (1993a) estimated a transition matrix using annual data on GDP per capita for 118 countries from 1962 to 1984, summarized in Table 1. All probabilities on the main diagonal of the transition matrix are higher than 0.9. The only nonzero transition probabilities are those on the three main diagonals of the matrix. The estimated ergodic distribution has two approximately equal peaks at the two ends of the income range. Taken together, the peaks constitute about half of the mass of the ergodic distribution.

In this paper we use Kraay's [1999] data on real GDP per capita computed using a chain index as described in Summers and Heston (1991) for 140 countries from 1960 to 1996 extended from version 5.6 of the Penn World Tables (1991). We have 22 more countries than Quah did, but our results are fairly similar when we restrict ourselves to the 118 countries chosen by Quah.[3]

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[2]Note that Quah's procedure can potentially generate what we will term a "Lake Wobegon long-run distribution" of world income, in which all countries have above average income. Classifying countries' income relative to that of the leading countries rather than relative to world income eliminates this anomaly.

[3]We use the series for real GDP per capita in constant dollars using a chain index instead of that for real GDP per capita (Laspeyres Index) that was used by Quah.

To extend the Penn World Tables chain index of GDP to additional years, Kraay uses growth rates of expenditure components reported by the World Bank. The expenditure components were weighted by their constant-price local currency shares. Summers and Heston weight the expenditures components by their PPP-adjusted shares. However, the difference matters only if PPP and non-PPP shares are different and growth rates of expenditure components differ. Although PPP-adjusted investment shares are typically lower than the unadjusted ones for developing countries, there is not much difference in growth rates of the expenditure components.

Kraay also extends the Penn World Tables to cover new countries, but we use only the time extension described above, because data for the countries in the original Penn World Tables sample is much better.

We exclude countries from the sample if extraction of oil or other non-renewable natural resources accounts for more than 15% of GDP, leaving 128 countries in the sample.[4] Revenues from extraction of non-renewable resources are treated as income in national accounts, but should be considered as asset sales and not

[4] We use data on mining and quarrying from the United Nation's National Accounts Statistics: Main Aggregates and Detailed Tables (1999). The countries thus excluded are Angola, Bahrain, Botswana, Congo, Gabon, Indonesia, Iran, Iraq, Kuwait, Namibia, Nigeria, Oman, Qatar, Saudi Arabia, Syria, Trinidad and Tobago, Venezuela, United Arab Emirates, and Zambia. We do not have figures on mining and quarrying for Algeria. However, this country is classified by World Bank as an "oil" country, so we exclude it from our sample too.

counted as income. Moreover, it seems likely that income dynamics for countries extracting non-renewable resources are governed by somewhat different dynamics than those for other countries. For example, GDP in oil countries moves around a lot with the price of oil. Under the plausible hypothesis that income dynamics are different for resource extracting countries and other countries, including resource extracting countries in the analysis will yield a biased estimate of income dynamics for countries without these resources. On the other hand, under the hypothesis that resource extracting countries are governed by the same dynamics as other countries, excluding countries with substantial natural resources will not bias estimation of the transition matrix, but will merely reduce precision of the estimates.

The maximum likelihood estimates of the transition probabilities using annual data and the corresponding ergodic distribution are reported in Table 2. The point estimate of the ergodic distribution has twin peaks at the polar ends of the income distribution, in accordance with Quah's (1993a) results. However, our estimates have a much greater peak at the rich end of the income range and a much smaller one at the poor end of the income range. This is not because we included more countries than Quah did, and not because our sample is longer. When we estimate the transition matrix using Quah's choice of countries and time period, we get similar results to those we get with the full sample and longer time period. One possible reason for the discrepancy is that the data in Penn World Tables release 5.6 may differ from the data in the version of the tables circulating at 1993.[5]

As one can see, the only non-zero estimated transition probabilities are those between adjacent groups. We will assume therefore that the true transition probabilities satisfy this condition, which we will call the triple diagonal condition. This assumption is in accord with logic, as per capita income does not halve or double in a single year. As discussed below, this assumption considerably simplifies the analysis.

Assuming this triple diagonal condition is satisfied, the ergodic probabilities, denoted π, bear a simple relation to the probability of transition between groups

[5] As discussed below, the ergodic distribution is extremely sensitive to the underlying transition matrix. Therefore, even small revisions in the data may change the shape of the estimated ergodic distribution substantially. Thus, for example, the data from release 4 of the tables implies that there is a large peak at the poor end of the ergodic distribution and a small peak at the rich end which is exactly opposite to what the data from the current tables (release 5.6) suggest.

One factor that causes the difference is as follows. Both old and new versions of the tables show that Saudi Arabia's income relative to the world average started around the threshold between groups 4 and 5, then rose, and then fell again to near the threshold. However, according to the older version, Saudi Arabia started above the threshold and ended below it, whereas according to the newer version, Saudia Arabia started below and ended above the threshold. This difference has a large effect on the size of the rich peak because very few countries transited to and from group 5.

i and j, denoted p_{ij}:

$$\frac{\pi_1}{\pi_2} = \frac{p_{21}}{p_{12}}, \quad \frac{\pi_2}{\pi_3} = \frac{p_{32}}{p_{23}}, \quad \frac{\pi_3}{\pi_4} = \frac{p_{43}}{p_{34}}, \quad \frac{\pi_4}{\pi_5} = \frac{p_{54}}{p_{45}}. \tag{2.1}$$

To see this, note that after one transition, the probability of being in the first group equals the probability of initially being in the first group and remaining there, plus the probability of initially being in group 2 and transiting to group 1. Thus, in the ergodic distribution, $\pi_1 = \pi_1(1 - p_{12}) + \pi_2 p_{21}$. Simplifying yields $\frac{\pi_1}{\pi_2} = \frac{p_{21}}{p_{12}}$, and the remaining equalities in (1) follow by induction.

Formula (2.1) radically simplifies the calculation of the ergodic probabilities given the transition matrix. More important, it simplifies hypothesis tests on the shape of the ergodic distribution, to which we turn in the next section.

3. Tests on the shape of the ergodic distribution: annual data

The estimated ergodic probabilities are very sensitive to changes in estimated transition probabilities. These in turn are sensitive to small counterfactual changes in the data. One illustration of this is provided by the fact that there were 40 transitions from group 1 to group 2, and 56 transitions from group 2 to group 1, whereas the number of states 1 and 2 ever observed were approximately equal (916 vs. 976). Had there been only seven more transitions from group 1 to group 2 and seven fewer transitions from group 2 to group 1, the estimated $\hat{p}_{12}$ would have been $\frac{47}{916}$, and thus larger than the estimated $\hat{p}_{21}$ of $\frac{49}{976}$, so the estimated ergodic distribution would not have had a peak at the lowest end of the distribution.

A simple calculation shows that the above counterfactual changes are indeed small in the following sense. If the number of countries is large then the joint distribution of the set $\sqrt{N_i}(\hat{p}_{ij} - p_{ij})$ is approximately normal with means 0, variances $p_{ij}(1-p_{ij})$, and covariances $-\delta_{ig}p_{ij}p_{gh}$, where N_i is the number of states i ever observed and δ_{ig} equals one if $i = g$ and zero otherwise (Anderson and Goodman, 1957). Therefore the standard deviations of $\hat{p}_{12}$ and $\hat{p}_{21}$ given N_1 and N_2 are approximately 0.007. Thus if the Markov model of transitions were true it would be not unusual to get 7 more or less transitions out of a state observed 1000 times in total (recall that we have 916 observations of state 1 and 976 observations of state 2).

In this section we push the above analysis of sensitivity further and formally test shape restrictions on the ergodic distribution. With annual data, we cannot reject either the hypothesis that the ergodic distribution is equal to the distribution as of 1996, the last year of our sample; or the hypothesis that the ergodic distribution has a single peak at the rich end of the income range.

Let π denote the vector of the ergodic probabilities. A quite general form of a shape restriction hypothesis is as follows.

$$H : f_1(\pi) = 0, f_2(\pi) \geq 0, \tag{3.1}$$

where f_1 and f_2 are some, possibly nonlinear, vector functions. The following examples illustrate the usefulness of the type of hypothesis above.

One might want to know if the ergodic distribution is identical to some particular distribution π^*. The π^* distribution might be the distribution at the end of the sample period, or flat, or degenerate. Obviously, such a hypothesis would correspond to $f_1(\pi) \equiv \pi - \pi^*$ and $f_2(\pi) \equiv 0$. Another interesting hypothesis about the ergodic distribution is that on the number and importance of the peaks. If one wants to test the hypothesis that the ergodic distribution has a single peak at, say, group 3, one could consider $f_1(\pi) \equiv 0$ and $f_2(\pi) \equiv (\pi_2 - \pi_1, \pi_3 - \pi_2, \pi_3 - \pi_4, \pi_4 - \pi_5)'$. Of course, such a hypothesis does not distinguish between a steep peak in the distribution, with π_3 much greater than π_2 or π_4, and a "bump" in the distribution with π_3 only slightly greater than π_2 or π_4. However, the somewhat more complicated hypothesis $f_1(\pi) \equiv 0$

and $f_2(\pi) \equiv (\pi_2 - \delta\pi_1, \pi_3 - \delta\pi_2, \pi_3 - \delta\pi_4, \pi_4 - \delta\pi_5)'$ where δ is a positive number greater than 1, does distinguish between peaks and bumps.

Below we formulate and test several hypothesis about the ergodic distribution having form (3.1) with linear f_1 and f_2. Given the triple diagonal assumption, linear restrictions on the ergodic probabilities, π can be reformulated as linear restrictions on the transition probabilities. A theory of likelihood ratio tests for the linear inequality restrictions on the transition probabilities was developed in Onatski (2000). The equality restrictions could be tested using fairly standard likelihood ratio tests. (Note that because the distribution of estimates of the transition probabilities can be well approximated by the normal distribution, one expects good finite sample properties of the asymptotic likelihood ratio test of the linear restrictions on the transition probabilities. Without the triple diagonal assumption, simple linear restrictions on π would be equivalent to complex non-linear restrictions on the transition probabilities, so that one would expect very bad finite sample properties of the asymptotic test.)

We start with the hypothesis that the ergodic distribution is identical to the income distribution in 1996, the last year of our sample. In 1996, 28% of countries were in the poorest income category, 22% of countries were in Group 2, and 18%, 10%, and 22% of countries were in Groups 3, 4, and 5 respectively. Therefore, in terms of the transition probabilities, the hypothesis has the following form

$$\frac{p_{12}}{p_{21}} = \frac{22}{28}, \ \frac{p_{23}}{p_{32}} = \frac{18}{22}, \ \frac{p_{34}}{p_{43}} = \frac{10}{18}, \text{ and } \frac{p_{45}}{p_{54}} = \frac{22}{10}.$$

As was mentioned before, the distribution of $\sqrt{N_i}(\hat{p}_{ij} - p_{ij})$ can be approximated by a normal distribution with means 0, variances $p_{ij}(1 - p_{ij})$, and covariances $-\delta_{ig}p_{ij}p_{gh}$. Hence, the distribution of $\hat{p}_{ij}$ is approximately the same as if p_{ij} were probabilities from the multinomial distribution and we observed N_i trials for each $i = 1, ..., 5$. To test the above hypothesis we therefore can apply the asymptotic likelihood ratio test as we would apply it to test the restrictions on the multinomial transition probabilities. The likelihood ratio statistic is 7.54, smaller than the 95% critical value of the chi-squared distribution with 8-4=4 degrees of freedom, 9.49.[6]

Another interesting hypothesis that we test is that of a single peak at the richest end of the income range. It can be formulated in the following form.

$$p_{k,k+1} \geq \delta p_{k+1,k}, \ \ k = 1, ..., 4.$$

[6] We also compare the steady-state distribution with the distribution as of 1989, because in 1996 we have 12 missing observations and in 1989 there are only 2 such observations. The result does not change much. The likelihood ratio statistics in this case is 7.97.

The importance of the peak can be regulated by appropriate choice of δ: the greater δ, the more pronounced the peak. If δ is equal to, say, 1.2, then the ergodic size of the largest group is at least as large as about 2 times the size of the smallest group.

As shown in Onatski (2000), the asymptotic distribution of the likelihood ratio statistic for this test is

$$\frac{1}{2^4}\sum_{i=0}^{4}\binom{4}{i}\chi_i^2,$$

where $\binom{4}{i}$ are binomial coefficients, χ_i^2 are the cumulative distribution functions of chi-squared distributions with i degrees of freedom, and χ_0^2 is the cumulative distribution function of zero constant. Hence, one can find (numerically) the exact asymptotic critical value for the likelihood ratio statistic. It happens to be equal to 6.50.

The value of the likelihood ratio test statistics for the single peak test, the corresponding p-values, and critical value of the test for different δ are shown at Figure 1. We cannot reject a single peak at the rich end of the income range for peaks with δ up to about 1.2, which is large. The p-values of the test for deltas close to one are as large as about 0.36.

Figure 2 compares simulated finite sample and theoretical distribution of LR statistics for the rich peak hypothesis. The finite sample distribution is simulated for the true transition probabilities, $p^0_{ij} = 0.01$ for $|i - j| = 1$. The restriction set is the null set of the rich peak hypothesis. The importance of the peak, δ, is taken to be equal to one. Three thousand Markov chains with initial distribution coinciding with the initial distribution in the data are drawn. The figure demonstrates that the finite sample distribution of the likelihood ratio statistics for the true transition probabilities chosen is very close to the theoretical one.

Even the procedure used in this paper may substantially underestimate uncertainty in the ergodic distribution of world income. Transitions for single countries may be correlated with transitions of neighbors. Belgium and the Netherlands, or Liberia and Sierra Leone, may be subject to correlated shocks. Treating each country as a separate data point may lead to underestimation of the standard errors.

As Quah (1993b) notes, the transition matrix approach might be misspecified due to the arbitrary division of countries into income groups. Results on the number and positions of peaks might not be robust to the choice of the income groups. For example, Chad Jones (1997) groups countries by output per worker in a somewhat different manner and finds that the ergodic distribution has a single peak in the middle income state.

This problem has led Quah (1997) to suggest the continuous stochastic kernel approach. We use the transition matrix approach as the first step to analyze the statistical significance of inferences about the shape of the steady-state distribution, but extending the present analysis to the stochastic kernel approach is an interesting topic for future research. We expect that the ergodic distribution will be no more precisely estimated with this technique, given that kernel estimation is generally noisy with small samples.

The non-informativeness of the data suggest that people's beliefs about the long-run world income distribution are likely to be influenced primarily by their prior beliefs, rather than updated based on the data. However, before abandoning empirical work along these lines, it is worth taking another look at the data.

4. Five-year Transition Period

It seems natural to consider transition periods longer than 1 year. The assumption of a one-period Markov process is likely to be violated. A group 3 country that experiences a recession in a particular year and falls just over the borderline into group 2 is less likely than other group 2 countries to fall into group 1 the next year, and more likely than most group 2 countries to transit to group 3 in the following year. Considering transition periods longer than one year reduces the impact on the estimated transition matrix of high frequency fluctuations in income of countries that happened to be close to the threshold between different groups at the beginning of the period.

One way to see why moving to longer periods is desirable is to note that the predicted 5-year mobility based on the one-year transition matrix is much greater than the actual 5-year mobility in the data.[7] The one-year transition matrix implies that the probability of transiting from group 5 to group 4 in a single year is 0.004. Neglecting some second order terms, this implies that the probability of transiting over a 5 year period should be approximately 5*0.004, or 0.02. In fact, the actual 5-year transition probability is only 0.007, or less than half of that implied by the one-year data. (See Table 3.) This suggests that using 5-year data may provide a more accurate picture of long-run dynamics than using annual data. Note that the 10-year mobility implied by the 5-year Markov transition

[7] This result is similar to what others find in social mobility studies. For a review of these studies, see Bartholomew (1973). Two reasons might lie behind this result. First, the countries might have different transition probabilities. If, similar to a popular stayer-mover model setting, half of the countries transit between groups extremely slowly, and half of the countries transit fast, then the estimated one-year transition matrix is in fact a weighted sum of two different transition matrices. Under reasonable conditions, stated for example in Shorrocks (1976), this would imply the result we observed. Second, some mobility in one-year transition matrix is due to short-run fluctuation of countries around a threshold separating two adjusent groups. For example, for 1-year data, there were four $1 \to 2$ transitions and five $2 \to 1$ transitions corresponding to The Gambia. These transitions reflect The Gambia's balancing on the boundary between the poorest and the second poorest groups. When we consider 5-year transitions, only one $1 \to 2$ transitions and two $2 \to 1$ transitions remain.

assumption is only in fact a little less than actual 10-year mobility. (See Table 4.)

The estimated ergodic distribution with five-year data has more than half its mass at the top group. (See Table 3.) As discussed below, this is because countries rarely exit group 5. Spain accounts for the sole transition out of the top group, and it is worth noting that Spain rejoined the top group shortly thereafter.[8]

Note that had we calculated the transition matrix using 10-year data (or 35-year data), the associated long-run distribution would be degenerate, with all its mass in the top income category. A working paper version of Quah [1993a] estimated the ergodic distribution for a 23 year transition period. He found 57% of countries in the rich peak and only 16% of countries in the poor peak, which is somewhat similar to our estimates.

The scarcity of transitions from Group 5 to Group 4 makes the estimated ergodic distribution extremely sensitive to counterfactuals. Had Spain never tran-

[8] Our sample does not include the formely communist countries of Czechoslovakia, East Germany, USSR, and Yugoslavia. However, this does not influence the estimated transition rate from Group 5 to Group 4. All these countries, with the exception of East Germany belonged to Groups 3 and 4. East Germany moved up once from group 4 to group 5.

The exclusion of oil countries does affect p_{54}. If the oil countries are included in the sample, Gabon, Iraq, Oman, Saudi Arabia, Trinidad and Tobago, and Venezuela would have transited from 5 to 4. Moreover, Argentina and Puerto Rico would have been included in Group 5 but would have transited to Group 4 at some point.

In the Quah sample, the transitions from 5 to 4 are due to Gabon, Iraq, Spain, and Venezuela. Gabon, Iraq, and Venezuela are excluded from our sample.

sited from Group 5 to Group 4, the estimated ergodic distribution would have been degenerate, with all the mass in the highest income category. We would have an estimated steady state in which all countries are above average.

Of course, this "Lake Wobegon" distribution in which most countries have more than twice world average income is impossible.[9] However, it is possible to redefine the groups so as to avoid this paradoxical result.[10] Jones [1997] estimates a Markov transition matrix in which countries' income is measured relative to the income of the leading country. In the rest of this paper, we measure income relative to the average income of the five leading countries.[11]

Table 5 shows the transition matrix and associated ergodic distribution when countries are classified by their income relative to the average population-weighted income of the five richest countries. The groups consist of those with less than one-sixteenth of this income, between one-sixteenth and one-eighth, between one-eighth and one-fourth, between one-fourth and one-half, and more than one-half

[9] Lake Wobegon is a fictional community in which all the children are above average.

[10] In independent work, Pearlman [2000] also notes the possibility that a transition matrix estimated with Quah's income groupings can lead to a logically impossible ergodic distribution of income. His approach to resolving this problem involves categorizing countries relative to the geometric mean of income among countries of the world. Note that when Pearlman moves to this type of analysis, he finds a large peak of the ergodic distribution at the bottom of the distribution. This is exactly the opposite of our results. The difference is presumably due to our focus on five-year data and to the exclusion of non-renewable natural resource producers from our analysis.

[11] Measuring income relative to the average income of the five leading countries is somewhat less sensitive to the behavior of a single country than Jones' procedure.

the average population-weighted income of the five richest countries. In the estimated ergodic distribution, 75% of countries are in the richest income category. As before, Spain accounts for the sole transition out of the top group, and it rejoins the top group shortly thereafter.[12]

The estimated ergodic distribution is fairly noisy. Formal hypothesis tests using the likelihood ratio test suggest that we cannot reject the hypotheses that the steady state has as much as 95%, or as little as 34% of the mass in the rich peak. We estimated the critical value of the test from below as follows. First, we found those transition probabilities that maximize the likelihood under the restriction that $\pi_5 \geq 0.95$ or $\pi_5 \leq 0.34$. We took these transition probabilities as a set of pseudo-true transition probabilities and simulated 1000 corresponding Markov chains starting from the distribution of countries between the groups actually observed in 1960. The length of simulated chains was chosen to be equal to the length of our data interval, (1995-1960)/5=7. For each simulation we computed the likelihood ratio statistic, thus obtaining an empirical distribution of the likelihood ratio statistic corresponding to the pseudo-true transition probabilities chosen. The 95% quantile of the empirical distribution is an estimate of

[12] Spain slipped out of the top group in 1980, when its per capita GDP fell to 1.99 times the world average. By 1985 it had fallen to only 1.93 times the world average, but by 1987 it had returned to the top group.

the critical value of the global test of the restriction $\pi_5 \geq 0.95$ or $\pi_5 \leq 0.34$. The estimate is from below so that we are rejecting the null too often, which makes non-rejection safe.

5. Transition path analysis

Note that the structure of the implied transition matrix suggests that it may take quite a while to get close to the steady state. Although countries tend to remain in Group 5 once they get there, they bounce around a lot on their way. There are currently many countries in Group 1, and it is likely to take them a long time to reach Group 5, under current trends.

A useful criterion of speed of convergence to the ergodic distribution is the asymptotic half-life of convergence, h. It indicates how many periods it takes for the norm of the difference between the current distribution and the ergodic distribution to decrease by half. The formula for the half life is as follows:

$$h = -\frac{\log 2}{\log |\lambda_2|},$$

where λ_2 is the second largest eigenvalue (after 1) of the transition probability matrix. For the five-year transition matrix, h is equal to 58.9. That is, it would take 58.9*5≈295 years to reduce the distance between the ergodic and current distribution by half.

The measure h is an asymptotic measure, so the influence of the initial distribution on convergence is not taken into account. Initial convergence to the ergodic distribution might be faster if the initial distribution turns out to be favorable. We simulated evolution of the countries' income distribution as of 1989, according to our estimated 5-year transition matrix. We found that the square root quadratic difference between the 1996 distribution and the ergodic distribution was 0.54. After 57 periods (285 years) it becomes 0.27, and after 115 periods (575 years) it becomes 0.14. This is in accordance to the theoretical half life calculations.

We simulated the evolution of the Gini coefficient[13] of expected countries' income distribution for the next 2500 years. The estimates and corresponding 95% confidence bands are given in Figure 3. One can see that it is likely that the Gini coefficient will decrease begin decreasing immediately.

In contrast, the standard deviation of log income and the coefficient of polarization are likely to rise for hundreds of years, even though they may currently be greater than their values in the ergodic distribution. Figure 4 shows the transition

[13] Each country was assumed to have the average relative income of its relative income category in 1989.

path for standard deviation of log income, and Figure 5 shows the polarization coefficient[14] of Esteban and Ray [1994].

Note that the standard error bands are much tighter over the transition than in the ergodic distribution. This is because the uncertainty in the estimated transition matrix is blown up much more in the ergodic distribution than in the distribution 100 years from now.

Thus, even though the ergodic distribution of income is very imprecisely estimated, and point estimates suggest that most of the mass will be at the top of the distribution in the ergodic distribution, if recent trends in international income mobility continue, some measures of inequality and polarization will worsen for hundreds of years, creating long-lived twin peaks.

[14] The polarization coefficient is as defined in Esteban and Ray [1994]. That is

$$P = c\sum_{i=1}^{5}\sum_{j=1}^{5}\pi_i^{1+\alpha}\pi_j|y_i - y_j|,$$

where we take $\alpha = 1$, and $y_i = i$. The constant c is defined so that maximum polarization is equal to 1. We define the polarization coefficient with respect to the group number, not actual income. Thus countries in group 1 are treated as having income 1, countries in group 2 have income 2, etc.

6. Explaining the Results: A Robust Fact and a Potential Model

In this section, we argue that the high proportion of countries in the top-income group in the estimated ergodic distribution and the prolonged transition both arise because countries often transit to lower income groups at moderate income levels, but rarely transit down once in the top group. This section argues that the scarcity of exits from the top group is reasonably robust empirically, and is consistent with a simple model of search among alternative policies.

The high proportion of countries in the top income group in the ergodic distribution is due to the scarcity of exits from Group 5. Recall that $\frac{\pi_5}{\pi_4} = \frac{p_{54}}{p_{45}}$. With only one transition from Group 5 to Group 4, this ratio is more than ten.

The scarcity of transitions out of Group 5 and hence the large peak at the rich end of the ergodic distribution seem reasonably robust to alternative model speculations, including varying the income cutoffs, weighting by population, or examining longer time periods. By playing around with the boundary between the second-highest group and the highest group, it is possible to attain point estimates of the steady state with somewhat less mass in the top group. However, this tends to be due to regions or countries such as Puerto Rico or Israel which bounce a bit around the threshold.[15] The most legitimate case of a country falling out of the top group is Argentina, which does not count as rich with the cutoff of 50% of GDP of the five largest countries, but would show up with a 45% cutoff. We conjecture that as long as a reasonable kernel is used, the continuous stochastic kernel approach would also suggest that transitions out of wealth are rare.

In Table 6, transition probabilities are weighted by the population of the country. Under this specification, 83% of the mass in the estimated ergodic distribution is in the highest income category, and the low peak of the distribution disappears. This reflects the fact that many of the countries which move from the second-lowest category to the lowest category are small African countries. Note, however, that there is still a peak in the second lowest category.

If anything, transitions out of the group of rich countries seem even rarer when transition matrices are constructed using longer periods. The ten year transition matrix and the 35-year transition matrix show no transitions out of the richest group.

The infrequency of transitions also seems to hold up over even longer periods. DeLong [1988] identifies 23 countries which were rich in 1870. The only countries

[15] It is unclear whether Puerto Rico should be in the data set, since its economy is so intertwined with that of the United States.

on this list which were not rich 130 years later are Argentina and Chile, both of which are currently in the second highest income group.

The transition matrix estimated with five-year data in the post-1960 period seems a fairly good guide to behavior of income over the 1870-2000 period. Using the estimated five-year matrix to project the year 2000 incomes of 23 rich countries that were rich in 1870 suggests that in expectation, 20.87 of the countries would be in the richest group at the end of the period, 1.11 would be in group 4, and 1.12 would be in lower income groups. If anything, mobility out of the top group is slightly lower than would have been predicted based on 5-year data since 1960, since none of the 23 rich countries in 1870 fell below the second-highest group by 2000, and since Chile's classification as rich in 1870 is suspect.[16]

The example of Argentina shows that countries can exit the top income group. But Argentina is an anomaly. To say that countries rarely exit out of the richest group is not to say such exits are impossible.

What model can explain the tendency of countries to move both up and down at lower income levels, but to stay rich once they become rich? Chari, Kehoe,

[16] DeLong [1988, p. 1149] explains that the Argentine 1870 data "should not under any circumstances be cited for any purposes dealing with Argentinean development alone. The estimate is sufficiently shaky to be unacceptable for such purposes, although it is barely acceptable as an estimate for a comparative project like this one." He then goes on to explain that the Chilean 1870 estimate is "perhaps the shakiest of all, and places Chile close to the cutoff for inclusion in the sample."

and McGrattan [1996] consider a model in which countries change policies, and these policies determine the countries' quasi-steady-state incomes. (We use the qualifier "quasi" because countries are subject to further changes in policy.) They note that there seems to be less mobility in the tails of the distribution than in the center, but they do not model why this is the case. Ideally, a model of endogenous policy determination would generate this effect.

Most of the political economy literature has, perhaps appropriately, focused on models in which the pursuit of self-interest by individuals or groups within the political system leads to sub-optimal outcomes for society as a whole. However, as emphasized by Piketty [1995], differences of belief about appropriate policy given a common objective function may also play a role. Nyerere may have saddled Tanzania with African Socialism to preserve his political power, but it is also possible that he made an honest mistake.

A theory in which bad policies are chosen because of bargaining failure among rent-extracting interest groups and individuals does not seem to predict that downward income movements should be frequent in the middle income groups but rare in the top groups. Politicians, lobbyists, and unions are presumably equally grasping in India, Costa Rica, France, and the U.S.

In contrast, a theory in which politicians search for good policies but do not know what policies are best suggests a reason why countries would cease experimenting with policy changes once they become rich enough.

Suppose that each country's quasi-steady-state income is a function of its policies, and countries search over policies until they find policies which make them rich. Countries may need to search either because the same policies work for all countries, but political leaders do not know which policies work, or because the effect of policies is extremely sensitive to a country's historic, geographic, and cultural circumstances, so it is difficult to learn from other countries' experience. For example, some have interpreted the disastrous output performance in transition economies as the effect of attempting to impose western institutions in an inappropriate environment.

Suppose that countries can periodically draw new policies, and the associated quasi-steady-state relative incomes, from an urn. Technological progress increases the absolute income of all countries over time, but does not affect their relative income. Countries' convergence towards their quasi-steady-state relative incomes can be approximated by the standard neo-classical growth model. (Actually, capital accumulation would be affected by the prospect that quasi-steady-state relative income will evetually change, but we assume those changes are rare.) Income may also be subject to measurement error, and short-run fluctuations from business cycles. Countries endowed with non-renewable natural resources will have incomes greater than those associated with their policies. All countries draw a quasi-steady-state relative income from the same urn. Since countries only rarely have a chance to change policies, to a first approximation, their payoff is the payoff of the destination state associated with their new policies (plus any rents associated with natural resources).

Suppose that the distribution of quasi-steady-state relative income drawn from the urn is $f(x)$, and the welfare level associated with income x is $U(x)$. It will be optimal to experiment until income is above some cutoff level, and then cease to experiment. At the cutoff level of income, x^*, nations will be indifferent whether to search for better policies. The value of staying with these policies will be $U(x^*)/(1-\delta)$, where δ is the discount rate. The value of searching will be $V = E(U(x)) + \delta \int_{x^*}^{\infty} f(x) \ \frac{U(x)}{1-\delta} dx + \delta \mathrm{p}(x < x^*)V$. Hence x^* is implicitly defined by $\frac{U(x^*)}{1-\delta} = E(x) + \delta \int_{x^*}^{\infty} f(x) \frac{U(x)}{1-\delta} dx + \delta \mathrm{p}(x < x^*) \frac{U(x^*)}{1-\delta^*}$.

In the model as sketched above, no country ever exits the wealthy state. In fact, of course, there are occasional exits, such as Argentina in this century, Spain a few centuries ago, and China a few centuries before that. (The Netherlands and the U.K. lost their political power and their positions as the leading economic power, but they did not exit the group of rich countries.) However, while these examples are salient, there is a lot of history out there, and the exit rate per century is low. It would be straightforward to extend the model to allow for political economy factors or exogenous changes in the appropriate set of policies that lead to occasional exits from the top income category.

The idea that countries stop searching once they are rich enough seems reasonable. Countries seem more willing to take risks when they have little left to lose. Hugo Chavez was overwhelmingly elected president of Venezuela [Barrionuevo, 1998], whereas Ross Perot got only 19% of the vote in the United States.

Radical policy changes are often, albeit not always, associated with extra-legal changes of government, and such changes of government are very rare in rich countries. Based on Banks' [1997] data over the 1960-1996 period, there were no coups out of the 764 country-year observations in countries with more than twice world average income. Of the 508 country-year observations in countries with incomes between 1 and 2 times world average income, there were 10 coups (5 of which took place in Argentina). Poorer countries had 128 coups out of approximately 2600 country-year observations. This provides some additional support for the view that rich countries are unlikely to risk radical changes in policy.

A model in which poor countries search for policies to make them more wealthy may sound odd at this moment in history when there is broad consensus on the policies that lead to wealth. However, historically, it seems plausible that when India adopted socialist planning, China adopted communism, and much of Latin America adopted import-substituting industrialization, they did so in the belief that these policies would more rapidly make them rich and powerful.

If countries could quickly recognize and correct policy mistakes, they might be able to rapidly converge on a set of policies that would lead to prosperity. In practice, however, opportunities to correct policy mistakes may be rare. Given that countries are subject to many shocks, and that some policies may create good long-run outcomes but bad short-run outcomes, it may be difficult to identify the effects of policies quickly. Moreover, once adopted, policies create their own constituents and their own ideological adherents. Nehru may not have intended to enrich corrupt bureaucrats when he adopted licensing requirements, but subsequent governments wishing to liberalize have to reckon with the political influence of these bureaucrats. Once governments have publicly adopted a policy, and educated party activists and the population to believe in it, it may be hard to abandon.

A search model should also fit some other facts. It should allow for convergence among rich countries, as found, for example, by Mankiw, Romer, and Weil

(1992) among OECD countries. Under a model in which countries draw their quasi-steady-state relative income from a distribution, there will be a threshold above which countries cease experimenting, and thus their quasi-steady-state relative income stays constant. This is nonetheless consistent with convergence in relative incomes, since some of the countries observed with income just over the threshold will be in the process of transiting to their steady-state relative income. All those countries will be transiting from below, and the further they are below their steady-state relative income, the faster they will grow, so each country will grow more quickly as it passes the threshold level of income than as it approaches its steady-state income.

A search model should also be consistent with the finding that growth rates among low income countries are no higher than among middle income countries, and that, as Quah argues, there may even be a peak at the bottom of the distribution. This fact can be matched if few policies lead to good outcomes and many lead to bad outcomes. All happy countries are alike, but there are many ways to be unhappy.

It is plausible that $f(x)$ has a lot of mass at low income levels if good policies are complements. Privatizing electricity may do limited good unless there are adequate steps to ensure that the new privatized firm will not simply be subsidized by the state; there are strong enough banking regulations to ensure that commercial banks will not be politically pressured into bailing out the privatized firm in the expectation that they in turn will be bailed out by the central bank; there is adequate corporate governance to ensure that the managers and controlling shareholders of the privatized firm do not devote all their energies to stealing from, rather than managing the firm; and there is adequate regulation to ensure that the firm has appropriate incentives to invest and does not charge monopoly prices. Getting all these policies right is not easy.

The poor performance of countries at the bottom of the income distribution relative to middle income countries is consistent with the hypothesis that there are a great many potential policies which lead to quasi-steady-state relative income of less than 1/16 the average income of the five richest countries of the world; and that the odds of choosing policies associated with moderate incomes are not too great relative to the odds of choosing policies associated with high incomes.

Suppose that all countries initially start with bad policies. Suppose that the threshold below which countries search coincides with the boundary between the richest income category and the second richest income category. Most countries with relative incomes in the bottom relative income group will have quasi-steady-state relative incomes in this group. In contrast, a greater proportion of the countries observed in the second highest income category will not be in a quasi-steady-state associated with this income, but instead will be transiting through this state on the way to a quasi-steady-state income in the top category. Hence, growth need not be lower among countries observed in the second highest income category than in the lowest income category, even though regression to the mean in i.i.d draws of quasi-steady-state relative income implies that growth of quasi-steady-state relative income will be greater at low levels of quasi-steady-state relative income than at high levels.

It is worth noting that two key aspects of our empirical strategy are appropriate under the search model. Excluding producers of natural resources makes sense in estimating the threshold under a search model, since countries with natural resources may have incomes above the quasi-steady-state relative income associated with their policies. Countries with a quasi-steady-state relative income below the threshold will optimally search for new policies, even if their incomes including natural resources, are above the threshold. This may help explain why Argentina fell out of the top group, since much of its wealth was based on natural resources. It also explains why other natural resource producers moved from Group 5 to Group 4.[17]

[17] Moreover, if each economy behaves as a closed neo-classical economy, producers of non-

The fact that the only countries to exit the top income group are producers of natural resources is thus consistent with the model. Many multiple equilibria models, in contrast, would suggest that a lucky discovery of oil could permanently move a country into a better equilibrium.

The search model implies that once incomes of non-resource producers cross a threshold, they do not fall back. This generates a strong non-linearity in income dynamics, justifying the use of a discrete Markovian transition matrix analysis.

An obvious question that arises with a search model is why countries do not simply imitate other successful countries. Why didn't Tanzania simply adopt U.S. or British institutions? There are at least two possibilities. First, it is possible that the institutions which work in some settings will not do so in other settings. If the appropriate policies are very sensitive to the existing institutions, culture, and economic and political conditions in a country, imitation would not be successful.

Alternatively, the same basic policies may be appropriate everywhere, but

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renewable natural resources will be particularly subject to downward transitions, because they will optimally accumulate more than the quasi-steady-state level of capital given their policies and then run down this stock later. If, as seems to be the case empirically, natural resource prices do not systematically increase over time, it will be optimal to extract natural resources and sell them, investing in physical and human capital beyond the point at which the rate of return on these assets declines to the discount rate. Later, after the natural resources are exhausted (or the flow diminishes), it will be optimal to draw down this capital stock, and growth will be negative.

policy makers may not have understood this, or may not have recognized what characteristics of prosperous countries were best to imitate. It is not clear on *a priori* grounds that imitating success is the best strategy. Communism promised faster transition than capitalism, along with more equal distribution, so the experience of looking at capitalist countries was not enough to convince people to adopt capitalism. Dependency theory implies that those countries lucky enough to industrialize first can use their initial advantages to exploit developing countries that participate in the world system, and that the best strategy for latecomers may be separation. Many of those who argue for the East Asian model disagree over its content, making imitation difficult.

Francis Fukuyama [1992] has suggested that the collapse of the Soviet Union heralded the end of history: everyone now accepts that liberal democracy, combined with a market-oriented economy, is the best form of organizing society. If Fukuyama is correct, both about the superiority of market economics combined with liberal democracy, and about the worldwide consensus on this superiority, then in the future there may be many more upward transitions and many fewer downward transitions. In fact, there is evidence that many countries are adopting much more market-oriented policies than in the past (see Easterly, 2000), although there is much less evidence that growth has accelerated in poor countries. We are either at the end of history or at the start of a new fad.

7. Conclusion

This paper first argues that beliefs about the very long-run evolution of the world income distribution must rely heavily on our priors, since empirical estimates of the ergodic distribution are noisy. With annual data, we cannot reject the hypotheses that the ergodic distribution is equal to the distribution as of 1996, the last year of our sample, or that the ergodic distribution has a single peak at the rich end of the income range.

Nonetheless, when the Markovian analysis is conducted using five-year data, it comes closer to satisfying the maintained assumptions and yields more precise estimates of what would happen to the world income distribution over the foreseeable future if previous trends continue. With five-year data, the estimated transition matrix yields an ergodic distribution in which most countries are in the richest income category. However, the transition path to the steady state is extremely long. According to our estimates, the half life of convergence is likely to be 303 years. The Gini coefficient may decrease immediately, but the standard deviation of countries' log income and the coefficient of polarization are likely to increase for hundreds of years. Estimates over this period involve raising the transition matrix to lower powers, and hence are much less noisy.

More important, the transition matrix analysis can shed light on possible processes generating the data. In particular, the rosy ergodic distribution and prolonged transition arise because countries frequently transit down from middle income states, but rarely transit out of wealth. This is consistent with a model in which countries search among policies and cease experimenting once income exceeds a certain cutoff. If countries learn about optimal policy from each others' experience, the transition matrix may become more favorable in the future.

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Table 1. Quah's estimates of transition matrix and ergodic distribution, 1962 to 1984, 1-year transitions.

	Upper endpoint:				
(Number)	0.25	0.5	1	2	Inf.
456	0.97	0.03			
643	0.05	0.92	0.04		
639		0.04	0.92	0.04	
468			0.04	0.94	0.02
508				0.01	0.99
Ergodic	0.24	0.18	0.16	0.16	0.27

Table 2. Our estimates of transition matrix and ergodic distribution, 1960 to 1996, 1-year transitions.

	Upper endpoint:				
(Number)	0.25	0.5	1	2	Inf.
916	0.956	0.044			
976	0.057	0.902	0.041		
1024		0.042	0.929	0.029	
582			0.031	0.945	0.024
803				0.004	0.996
Ergodic	0.13	0.10	0.10	0.09	0.59

Table 3. Estimates of transition matrix and ergodic distribution, 1960 to 1996, 5-year transitions

	Upper endpoint:				
(Number)	0.25	0.5	1	2	Inf.
178	0.942	0.058			
201	0.145	0.757	0.098		
194		0.094	0.792	0.114	
108			0.097	0.823	0.080
148				0.007	0.993
Ergodic	0.12	0.05	0.05	0.06	0.72

Table 4. Squared 5-year transition matrix vs. estimated 10-year transition matrix

Squared 5-year transition matrix				
0.25	0.5	1	2	Inf.
0.90	0.10	0.01		
0.25	0.59	0.15	0.01	
0.01	0.15	0.65	0.18	0.01
	0.01	0.16	0.69	0.14
		0.00	0.01	0.99

Estimated 10-year transition matrix				
0.25	0.5	1	2	Inf.
0.89	0.11			
0.25	0.61	0.15		
	0.13	0.66	0.20	
		0.14	0.71	0.14
				1.00

Table 5. Estimates of transition matrix and ergodic distribution: 1960 to 1996; 5-year transitions
(Division into groups relative to 5 richest countries)

	Upper endpoint:				
(Number)	1/16	1/4	1/2	1	Inf.
178	0.933	0.067			
201	0.144	0.756	0.100		
194		0.088	0.799	0.113	
108			0.093	0.824	0.083
148				0.007	0.993
Ergodic	0.09	0.04	0.05	0.06	0.75

Table 6. Estimates of transition matrix and ergodic distribution: 1960 to 1996; 5-year transitions
(Division into groups relative to 5 richest countries. Population weighted transitions.)

	Upper endpoint:				
(Number i	1/16	1/8	1/4	0.50	Inf.
5.99	0.722	0.278			
8.92	0.079	0.897	0.025		
2.14		0.073	0.771	0.156	
1.95			0.108	0.760	0.132
4.35				0.008	0.992
Ergodic	0.03	0.10	0.03	0.05	0.79

Fig. 1 Values of the LR statistic. Rich peak hypothesis

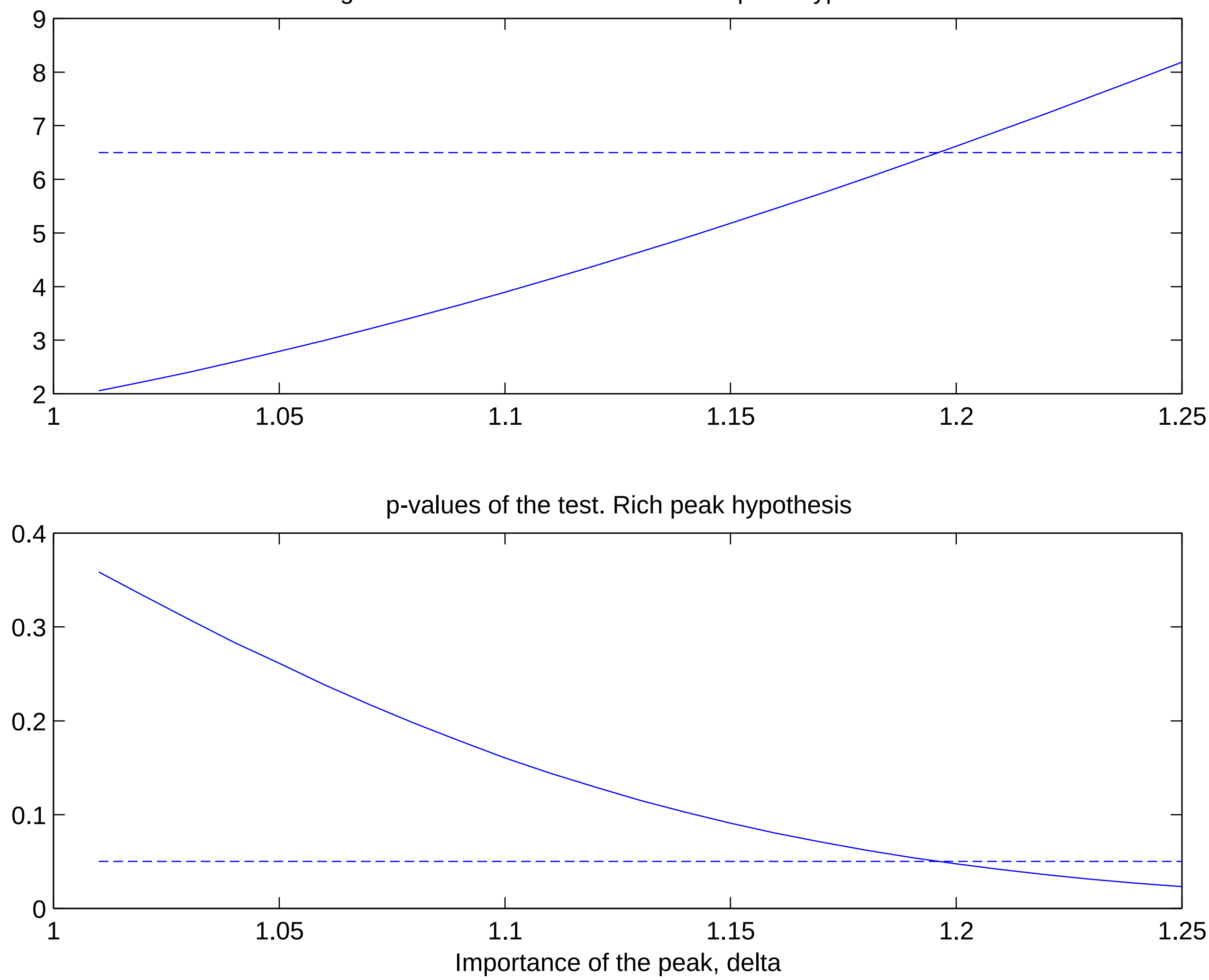
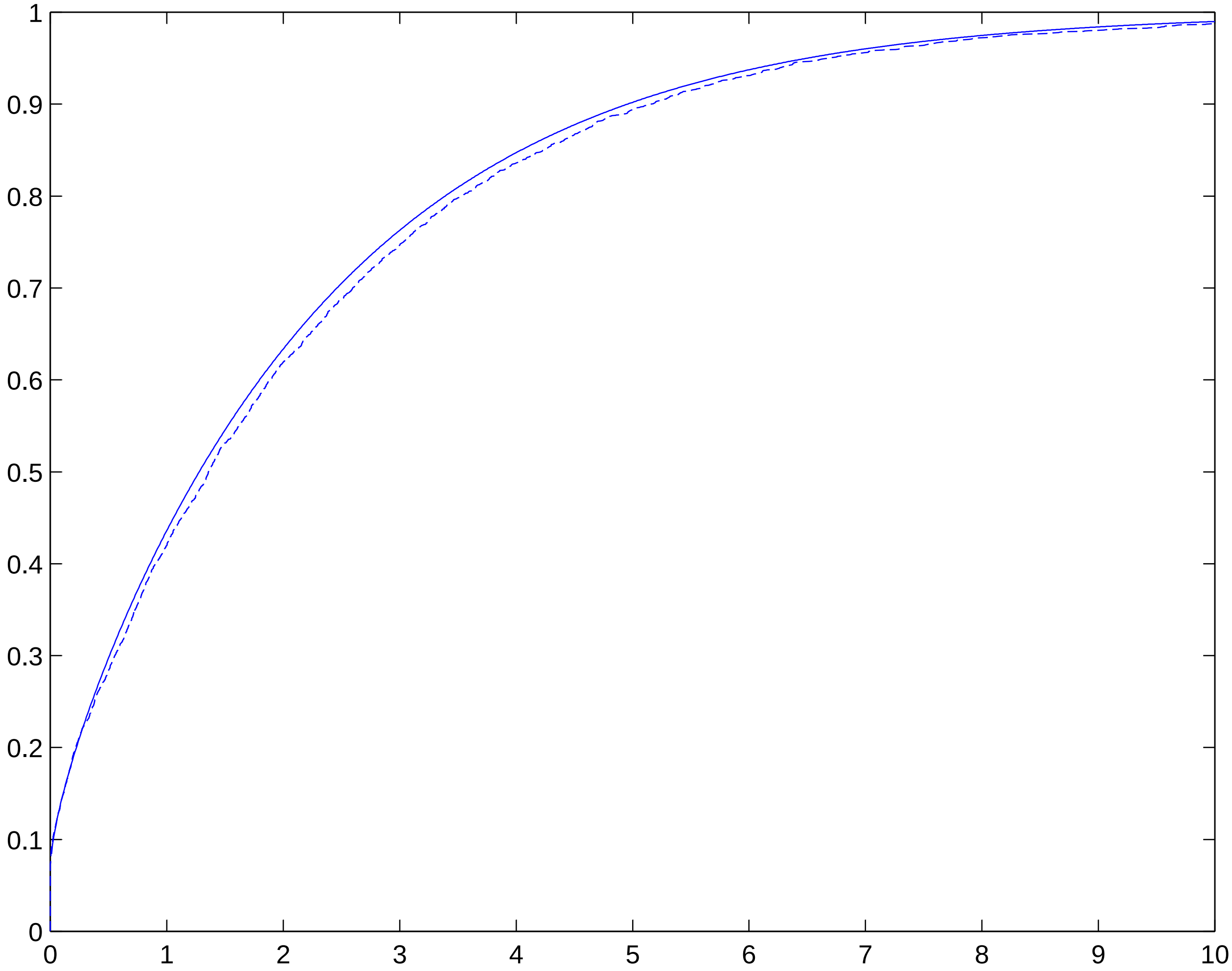


Figure 2. Simulated finite sample and theoretical distribution of the LR statistic



Solid line- theoretical distribution, Dashed line- simulated distribution

Figure 3. Transition Path for Gini Coefficient

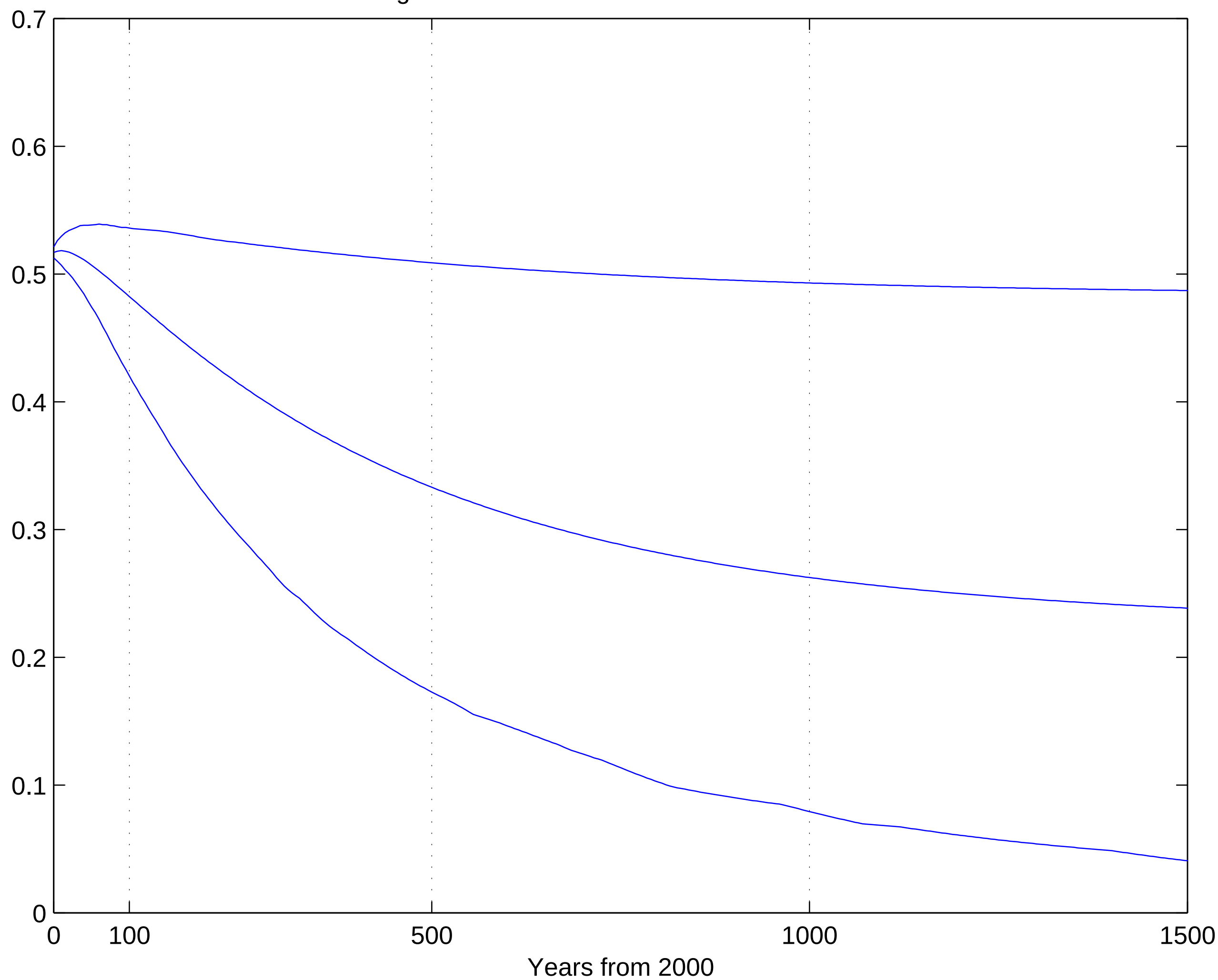


Figure 4. Transition Path for Standard Deviation

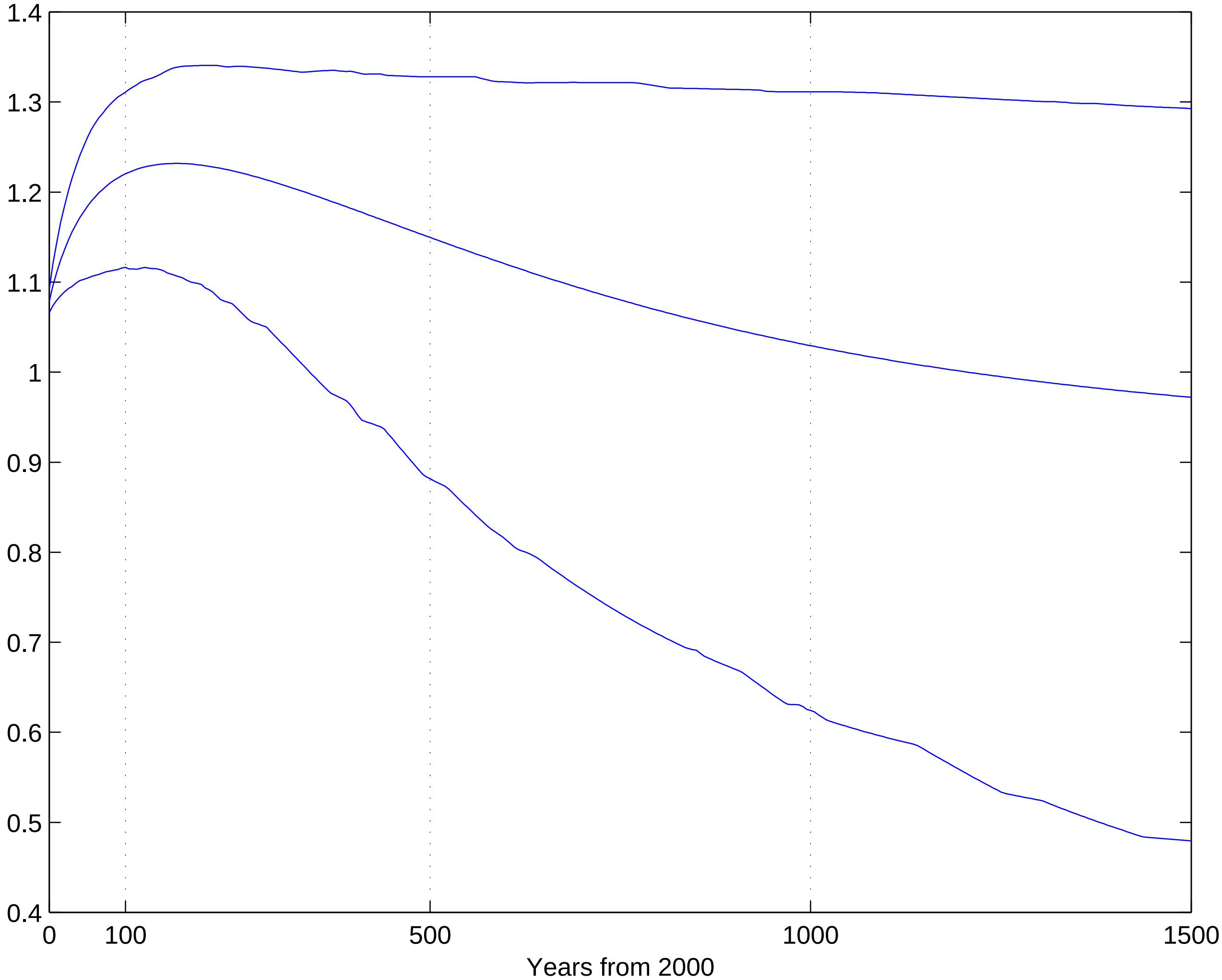


Figure 5. Transition Path for Polarization Coefficient