



Seasonal Autoregressive Integrated Moving Average (SARIMA) Model for the Analysis of Frequency of Monthly Rainfall in Osun State, Nigeria

Samuel Olorunfemi Adams^{1*}, Bello Mustapha¹ and Auta Irinew Alumbugu²

¹*Department of Statistics, University of Abuja, Nigeria.*

²*Department of Mathematics and Statistics, Federal Polytechnic Nasarawa, Nigeria.*

Authors' contributions

This work was carried out in collaboration among all authors. Author SOA designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author BM managed the analyses of the study. Author AIA managed the literature searches. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/PSIJ/2019/v22i430139

Editor(s):

- (1) Dr. Manuel Malaver de la Fuente, Universidad Marítima del Caribe, Av. El Ejército, Catia La Mar, Estado Vargas, Venezuela.
(2) Dr. Christian Brosseau Distinguished, Professor, Department of Physics, Université de Bretagne Occidentale, France.

Reviewers:

- (1) Olumide Adesina, Olabisi Onabanjo University, Nigeria.
(2) Janilson P. Assisi, Federal Rural University of the Semi-Arid (UFERSA), Brazil.
(3) Ramesh M. Mirajkar, Dr. Babasaheb Ambedkar College, India.
Complete Peer review History: <http://www.sdiarticle3.com/review-history/50701>

Original Research Article

Received 26 May 2019
Accepted 06 August 2019
Published 10 August 2019

ABSTRACT

The Seasonal Autoregressive Integrated Moving Average (SARIMA) model is proposed for Osun State monthly rainfall data and the analysis was based on probability time series modeling approach. The Plot of the original data shows that the time series is stationary and the Augmented Dickey-Fuller test did not suggest otherwise. The graph further displays evidence of seasonality and it was removed by seasonal differencing. The plots of the ACF and PACF show spikes at seasonal lags respectively, suggesting SARIMA (1, 0, 1) (2, 1, 1). Though the diagnostic check on the model favoured the fitted model, the Auto Regressive parameter was found to be statistically insignificant and this led to a reduced SARIMA (1, 0, 1) (1, 1, 1) model that best fit the data and was used to make forecast.

*Corresponding author: E-mail: samuel.adams@uniabuja.edu.ng, adamssamuel22003@yahoo.com;

Keywords: SARIMA model; time series; Dickey-Fuller test; autoregressive parameter; rainfall data.

1. INTRODUCTION

The highly variable nature of rainfall as compared with the relatively stable nature of the temperature appears to have imbued more relevance to the former as the major component in the study of climate in a particular region. There is need to understand the dynamical processes that determine changes that occur in climate system, though this has been very difficult and challenging to climate scientists till today [1]. The change has significantly contributed to the increase of global disasters caused by weather, climate and water related hazards as both developed and developing countries of the world are bearing the burden of repeated floods, temperature extremes and storms in which Nigeria is not left out [2]. Water resources are essential renewable resources that are the basis for existence and development of a society. Proper utilization of these resources requires assessment and management of the quantity and quality of the water resources both partially and temporally. Water crises caused by shortages, floods and diminishing water quality, among others are increasing in all parts of the world. The growth in population demands for increased domestic water supplies and at the same time results with a higher consumption of water due to expansion in agriculture and industry [2]. Mismanagement and lack of knowledge about existing water resources and the changing climatic conditions have consequences of an imbalance of supply and demand of water. A few literature exist in the time series analysis of monthly rainfall in some states in Nigeria, they include; Akwa-Ibom State, Uyo [3], Ile-Ife, Osun State [4], Umuahia, Abia State [5], Ilorin, Kwara State [6], Portharcourt, Rivers State [7], Akure, Ondo State [8–10], Ikeja Lagos [11], Ota, Ogun State [12] and Ogbomosho, Oyo State [13]. This study therefore attempts to identify and construct the best SARIMA model that best fits, explains the underlying generating process and satisfactorily forecast into the future of the monthly frequency of rainfall in Osun State.

2. SARIMA MODELING

Rainfall data are time structured and time series analyses are often employed in the analysis of the data. The data were subjected to seasonal autoregressive integrated moving average (SARIMA). Modeling. An ARIMA model is an algebraic statement that describes how a time

series is statistically related to its own past. The seasonal ARIMA model incorporates both non-seasonal and seasonal factors in a multiplicative model given as;

$$ARIMA(p, d, q) \times (P, D, Q)_s \quad (2.1)$$

Where; p = non-seasonal AR order, d = non-seasonal differencing, q = non-seasonal MA order, P = seasonal AR order, D = seasonal differencing, Q = seasonal MA order, and S = time span of repeating seasonal pattern.

Without differencing operations, the model could be written more formally as;

$$\phi(B^s)\varphi(B)(X_t - \mu) = \theta(B^s)\theta(B)W_t \quad (2.2)$$

The non-seasonal components are:

$$AR: \varphi(B) = 1 - \phi_1 B - \dots - \phi_p B^p \quad (2.3)$$

$$MA: \theta(B) = 1 + \theta_1 B + \dots + \theta_q B^q \quad (2.4)$$

The seasonal components are:

$$Seasonal AR: \phi(B^s) = 1 - \phi_1 B^s - \dots - \phi_p B^{ps} \quad (2.5)$$

$$Seasonal MA: \theta(B^s) = 1 + \theta_1 B^s + \dots + \theta_q B^{qs} \quad (2.6)$$

Note that on the left side of equation (2.2) the seasonal and non-seasonal AR components multiply each other, and on the right side of equation (2.2) the seasonal and non-seasonal MA components multiply each other (14).

The SARIMA modeling approach is concerned with finding a parsimonious seasonal ARIMA model that describes the underlying generating process of the observed time series. Box and Jenkins (15) established a three step modeling procedure: Identification, estimation and diagnostic checking steps. The identification step is to tentatively choose one or more ARIMA/SARIMA model(s) using the estimated ACF and PACF plots. The ACF plot of the AR (Auto Regressive)/ SAR (Seasonal Auto Regressive) process shows an exponential decay while its PACF plot truncates at lag p /seasonal lag p and diminishes to zero afterwards. The ACF plot of the MA process truncates to zero after lag q while its PACF decays exponentially to zero. The two processes:

AR (p)/SAR(P) and MA (q)/SMA(Q), could be combined to form the ARMA (p, q)/SARMA (P, Q) process which has ACF and PACF that decays exponentially to zero. The maximum likelihood estimation method could be used to estimate the parameters of the identified model(s) in the identification stage. The last diagnostic checking stage involves assessing the adequacy of the identified and fitted models through possible statistically significant test on the residuals to verify its consistency with the white noise process e.g. the Ljung-Box test [16]. Finally, the best fitting model would be selected among other satisfactory, competing models e.g. the information criteria statistics on the basis of the AIC or BIC rule of thumb, the Models with the smallest information criterion is the best and forecast is made with the model of best fit.

3. APPLICATION OF SARIMA MODEL

The data used in this study which is the frequency of the monthly rainfall in Osun State Nigeria, was collected from the National Bureau of Statistics, Nigeria from the year 1981-2015. The behavior of the data was observed and the estimation of the expected models was carried out using the method of likelihood with the plots of the Autocorrelation function (ACF) and Partial Autocorrelation Function (PACF) of the difference and non-difference series. After several iterations, some models were suggested; SARIMA(1,0,1)(1,1,1)₁₂, (1,0,2)(1,1,1)₁₂, (1,0,1)(2,1,1)₁₂, (102)(1,1,2)₁₂, (2,0,1)(2,1,1) (1,2,1)₁₂, (1,0,1)(1,1,2)₁₂ as presented in the Table 1;

Comparing the SARIMA(1,0,1)(1,1,1)₁₂, (1,0,2)(1,1,1)₁₂, (1,0,1)(2,1,1)₁₂, (102)(1,1,2)₁₂, (2,0,1)(2,1,1)(1,2,1)₁₂, (1,0,1)(1,1,2)₁₂ the suggested models were compared based on the criteria; AIC, Standard error, log likelihood, square sigma estimated and coefficient respectively, clearly, SARIMA (1,0,1)×(1,1,1)₁₂ proved to be the appropriate model with minimum Akaike information criterion (AIC) of 4721.14. The selected model was then used to describe and forecast the time series observations.

From the plots in Fig.1 it could be seen that the time series observation displays a wave like pattern an evidence of seasonality and no trend is observed which implies that the time series might be stationary. The sinusoidal or periodic pattern in the ACF plot is again suggesting that the series has a strong seasonal effect also, the PACF plot is neither suggesting otherwise. In order to verify the stationarity claim of the visual displays, the Augmented Dickey-Fuller [17] test was performed.

Table 2 depicts the Augmented Dickey-Fuller Test, the hypothesis.

H₀: The series is unit root non stationary vs H₁: the series is unit root stationary P-value < 0.05, indicating a strong evidence against the null hypothesis at 5% level of significance. In order to eliminate the seasonal effect from the time series observation, it was subjected to a seasonal differencing and the data is re-examined visually as seen in Fig. 4.

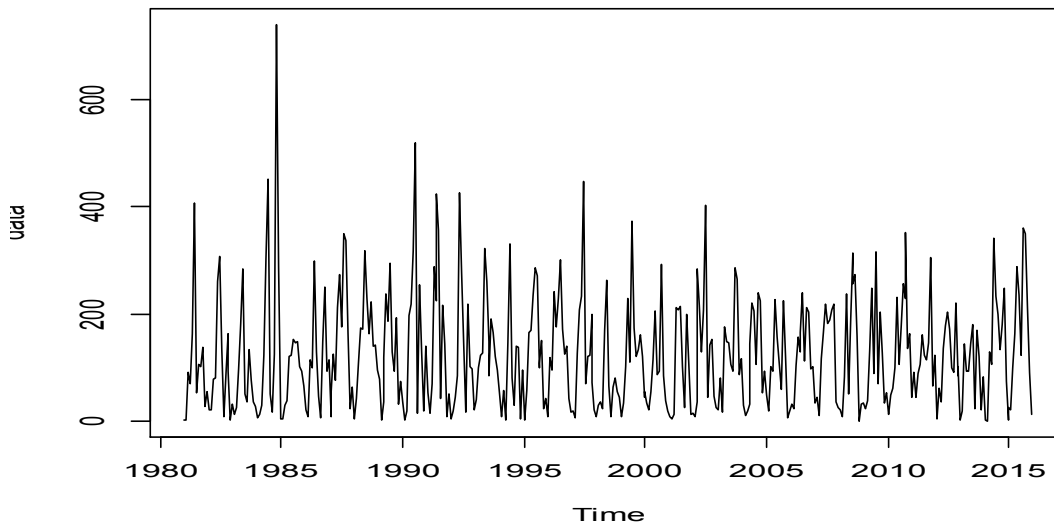


Fig. 1. Frequency of monthly rainfall in Osun State

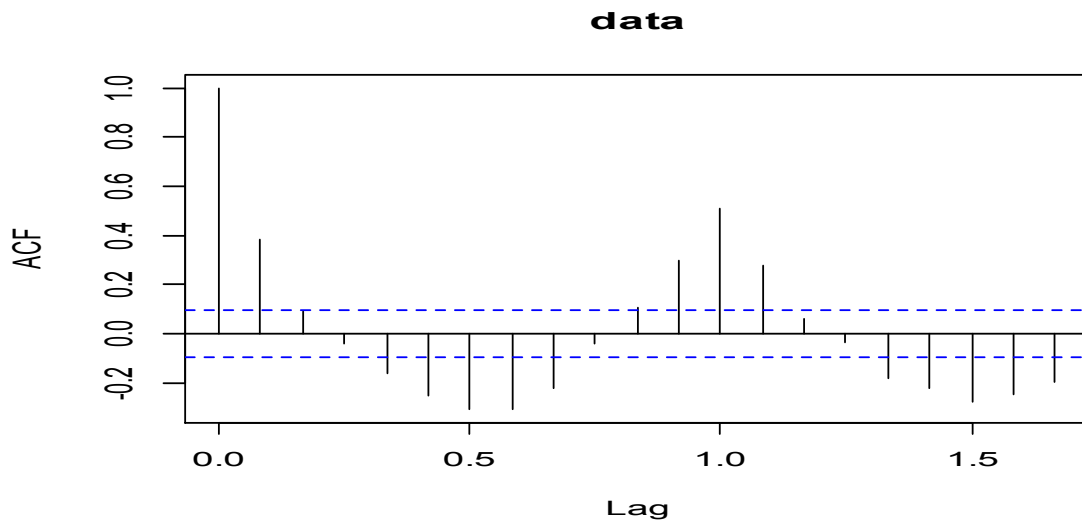


Fig. 2. ACF plot of the frequency of monthly rainfall in Osun State

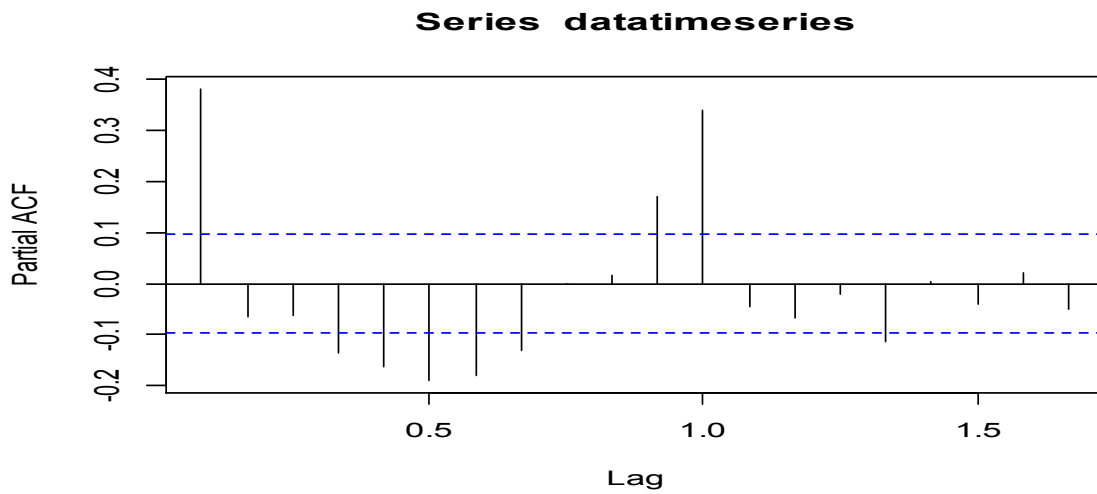


Fig. 3. PACF plot of the frequency of monthly rainfall in Osun State

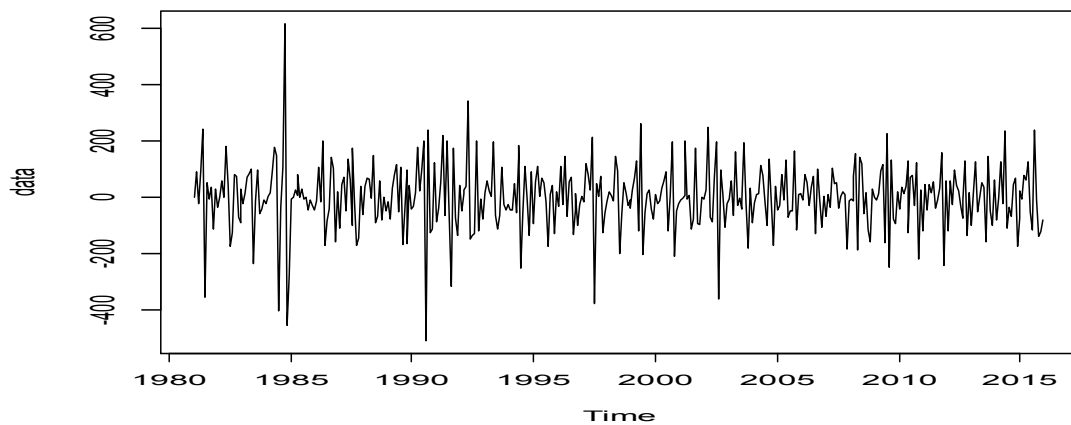


Fig. 4. Plot of diff (1) of monthly rainfall in Osun State

Table 1. Summary of some estimated candidate SARIMA models

| Candidate model | Coefficient | S.E | Sigma ² | Log likelihood | AIC |
|-----------------|-------------|--------|--------------------|----------------|---------|
| AR1 | 1.1418 | 0.0200 | | | |
| AR2 | -0.1553 | 0.0213 | | | |
| MA1 | -1.0000 | 0.0131 | 5697 | -2354.59 | 4723.17 |
| SAR1 | 0.1062 | 0.0233 | | | |
| SAR2 | 0.0640 | 0.0231 | | | |
| SMA1 | -0.9292 | 0.0322 | | | |
| AR1 | 0.1897 | 0.3071 | | | |
| MA1 | -0.0374 | 0.3127 | | | |
| SAR1 | 0.0893 | 0.0593 | 5775 | -2355.57 | 4721.14 |
| SMA1 | -0.9121 | 0.0388 | | | |
| AR1 | -0.3062 | 0.8815 | | | |
| MA1 | 0.4603 | 0.8803 | | | |
| MA2 | 0.0794 | 0.1361 | 5771 | -2355.5 | 4723.01 |
| SAR1 | 0.0910 | 0.0594 | | | |
| SMA1 | -0.9135 | 0.0387 | | | |
| AR1 | 0.0645 | 0.3733 | | | |
| AR2 | 0.0176 | 0.3882 | | | |
| MA1 | 0.0868 | 0.3992 | 5738 | -2355.1 | 4724.20 |
| SAR1 | 0.5672 | 0.3728 | | | |
| SMA1 | -1.3955 | 0.3971 | | | |
| SMA2 | 0.4279 | 0.3571 | | | |
| AR1 | 0.2082 | 0.3220 | | | |
| MA1 | -0.0586 | 0.3293 | | | |
| SAR1 | 0.1020 | 0.0591 | 5742 | -2354.99 | 4721.97 |
| SAR2 | 0.0618 | 0.0571 | | | |
| SMA1 | -0.9317 | 0.0420 | | | |
| AR1 | 0.1698 | 0.3141 | | | |
| MA1 | -0.0181 | 0.3184 | | | |
| SAR1 | 0.5540 | 0.3822 | 5739 | -2355.1 | 4722.2 |
| SMA1 | -1.3815 | 0.4055 | | | |
| SMA2 | 0.4152 | 0.3643 | | | |

Table 2. Unit root and stationarity tests of Osun state monthly rainfall

| Test | Test statistics | Lag order | p-value |
|---------------|-----------------|-----------|---------|
| Dickey-Fuller | -13.626 | 0 | 0.01 |

Table 3 depicts the Augmented Dickey-Fuller Test, the hypothesis;

H_0 : The series is unit root non stationary vs H_1 : the series is unit root stationary P-value < 0.05, indicating a strong evidence against the null hypothesis at 5% level of significance.

Table 3. Unit root and stationarity tests of Osun state monthly rainfall

| Test | Test Statistics | Lag Order | p-value |
|---------------|-----------------|-----------|---------|
| Dickey-Fuller | -12.085 | 1 | 0.032 |

4. FORECASTING WITH THE FITTED MODEL

One of the objectives of fitting SARIMA model to data is to be able to forecast its future values. The model that best fits the data is SARIMA (1,0,1)×(1,1,1)₁₂. The fitted model is therefore used to forecast for four [4] years i.e. January 2016 – December 2020.

The model validation is concerned with checking the residual of the model to determine if the model contains any systematic pattern which can be removed to improve on the selected model may appear to be the best among a number of models considered it become necessary to do diagnostic checking to verify that the model is adequate.

The plots Fig. 7 comprise of the time plot of the residuals, ACF and the PACF plot of the residuals respectively. The plot clearly shows

Table 4. Forecast of monthly rainfall in Osun state from 2016 to 2020

| Month/years | Forecast | Lo 80 | Hi 80 | Lo 95 | Hi 95 |
|-------------|-----------|-----------|----------|-------------|----------|
| Jan 2016 | 15.36211 | -82.25182 | 112.9768 | -133.925716 | 164.6507 |
| Feb 2016 | 30.07109 | -68.75338 | 128.8956 | -121.067900 | 181.2101 |
| Mar 2016 | 87.03243 | -11.83149 | 185.8963 | -64.166884 | 238.2317 |
| Apr 2016 | 135.47231 | 36.60574 | 234.3389 | -15.731056 | 286.6757 |
| May 2016 | 217.30619 | 118.43930 | 316.1731 | 66.102330 | 368.5101 |
| Jun 2016 | 221.34064 | 122.47370 | 320.2076 | 70.136708 | 372.5446 |
| Jul 2016 | 180.92120 | 82.05425 | 279.7881 | 29.717249 | 332.1251 |
| Aug 2016 | 156.42678 | 57.55984 | 255.2937 | 5.222836 | 307.6307 |
| Sep 2016 | 215.29227 | 116.42532 | 314.1592 | 64.088317 | 366.4962 |
| Oct 2016 | 202.80709 | 103.94012 | 301.6741 | 51.603108 | 354.0111 |
| Nov 2016 | 68.80167 | -30.06542 | 167.6688 | -82.402493 | 220.0058 |
| Dec 2016 | 53.45305 | -45.41474 | 152.3208 | -97.752177 | 204.6583 |
| Jan 2017 | 22.87171 | -77.59039 | 123.3338 | -130.771816 | 176.5152 |
| Feb 2017 | 33.48983 | -67.02443 | 134.0041 | -120.233470 | 187.2131 |
| Mar 2017 | 84.42676 | -16.09105 | 184.9446 | -69.301961 | 238.1555 |
| Apr 2017 | 136.37262 | 35.85437 | 236.8909 | -17.356773 | 290.1020 |
| May 2017 | 207.18579 | 106.66748 | 307.7041 | 53.456301 | 360.9153 |
| Jun 2017 | 220.21331 | 119.69499 | 320.7316 | 66.483810 | 373.9428 |
| Jul 2017 | 182.39962 | 81.88129 | 282.9179 | 28.670109 | 336.1291 |
| Aug 2017 | 149.68948 | 49.17116 | 250.2078 | -4.040030 | 303.4190 |
| Sep 2017 | 212.07930 | 111.56098 | 312.5976 | 58.349791 | 365.8088 |
| Oct 2017 | 199.83608 | 99.31773 | 300.3544 | 46.106535 | 353.5656 |
| Nov 2017 | 67.77333 | -32.74517 | 168.2918 | -85.956453 | 221.5031 |
| Dec 2017 | 58.44459 | -42.07480 | 158.9640 | -95.286561 | 212.1758 |
| Jan 2018 | 23.24208 | -78.49374 | 124.9779 | -132.349433 | 178.8336 |
| Feb 2018 | 34.57036 | -67.20853 | 136.3493 | -121.087027 | 190.2277 |
| Mar 2018 | 83.61879 | -18.16338 | 185.4010 | -72.043613 | 239.2812 |
| Apr 2018 | 135.06850 | 33.28591 | 236.8511 | -20.594546 | 290.7316 |
| May 2018 | 202.12505 | 100.34239 | 303.9077 | 46.461902 | 357.7882 |
| Jun 2018 | 219.42295 | 117.64028 | 321.2056 | 63.759786 | 375.0861 |
| Jul 2018 | 186.30029 | 84.51762 | 288.0830 | 30.637129 | 341.9635 |
| Aug 2018 | 136.99492 | 35.21224 | 238.7776 | -18.668248 | 292.6581 |
| Sep 2018 | 203.84346 | 102.06079 | 305.6261 | 48.180291 | 359.5066 |
| Oct 2018 | 199.05576 | 97.27305 | 300.8385 | 43.392541 | 354.7190 |
| Nov 2018 | 66.53447 | -35.24841 | 168.3174 | -89.129019 | 222.1980 |
| Dec 2018 | 61.65131 | -40.13266 | 163.4353 | -94.013840 | 217.3165 |
| Jan 2019 | 23.94533 | -78.40262 | 126.2933 | -132.582346 | 180.4730 |
| Feb 2019 | 35.09976 | -67.27187 | 137.4714 | -121.464135 | 191.6637 |
| Mar 2019 | 83.59608 | -18.77772 | 185.9699 | -72.971136 | 240.1633 |
| Apr 2019 | 135.20616 | 32.83206 | 237.5803 | -21.361512 | 291.7738 |
| May 2019 | 201.22227 | 98.84812 | 303.5964 | 44.654520 | 357.7900 |
| Jun 2019 | 219.49075 | 117.11659 | 321.8649 | 62.922989 | 376.0585 |
| Jul 2019 | 186.99776 | 84.62360 | 289.3719 | 30.430001 | 343.5655 |
| Aug 2019 | 135.52501 | 33.15085 | 237.8992 | -21.042751 | 292.0928 |
| Sep 2019 | 203.03491 | 100.66076 | 305.4091 | 46.467149 | 359.6027 |
| Oct 2019 | 199.01391 | 96.63972 | 301.3881 | 42.446099 | 355.5817 |
| Nov 2019 | 66.56296 | -35.81142 | 168.9374 | -90.005152 | 223.1311 |
| Dec 2019 | 62.48961 | -39.88596 | 164.8652 | -94.080322 | 219.0595 |
| Jan 2020 | 24.25371 | -78.62394 | 127.1314 | -133.084081 | 181.5915 |
| Feb 2020 | 35.43322 | -67.46663 | 138.3331 | -121.938523 | 192.8050 |
| Mar 2020 | 83.76050 | -19.14149 | 186.6625 | -73.614511 | 241.1355 |
| Apr 2020 | 135.35698 | 32.45470 | 238.2593 | -22.018483 | 292.7324 |
| May 2020 | 201.04265 | 98.14032 | 303.9450 | 43.667115 | 358.4182 |
| Jun 2020 | 219.66535 | 116.76301 | 322.5677 | 62.289804 | 377.0409 |

| Month/years | Forecast | Lo 80 | Hi 80 | Lo 95 | Hi 95 |
|-------------|-----------|-----------|----------|------------|----------|
| Jul 2020 | 187.51743 | 84.61510 | 290.4198 | 30.141889 | 344.8930 |
| Aug 2020 | 134.83001 | 31.92767 | 237.7323 | -22.545539 | 292.2056 |
| Sep 2020 | 202.67424 | 99.77190 | 305.5766 | 45.298687 | 360.0498 |
| Oct 2020 | 199.17805 | 96.27568 | 302.0804 | 41.802451 | 356.5537 |
| Nov 2020 | 66.70668 | -36.19591 | 169.6093 | -90.669246 | 224.0826 |
| Dec 2020 | 62.98184 | -39.92203 | 165.8857 | -94.396044 | 220.3597 |

that the residuals appear to be randomly scattered, no evidence exists that the error terms are correlated with one another as well as no evidence of existence of an outlier. The residuals or errors are therefore conceived as an independently and identically distributed (i.i.d) sequence with a constant variance and a zero

mean. The ACF and the PACF plot of the residuals show no evidence of a significant spike indicating that the residuals seem to be uncorrelated. Therefore, the SARIMA (1,0,1) (1,1,1)₁₂ model appears to fit well and can be used to predict the frequency of rainfall in Osun State Nigeria.

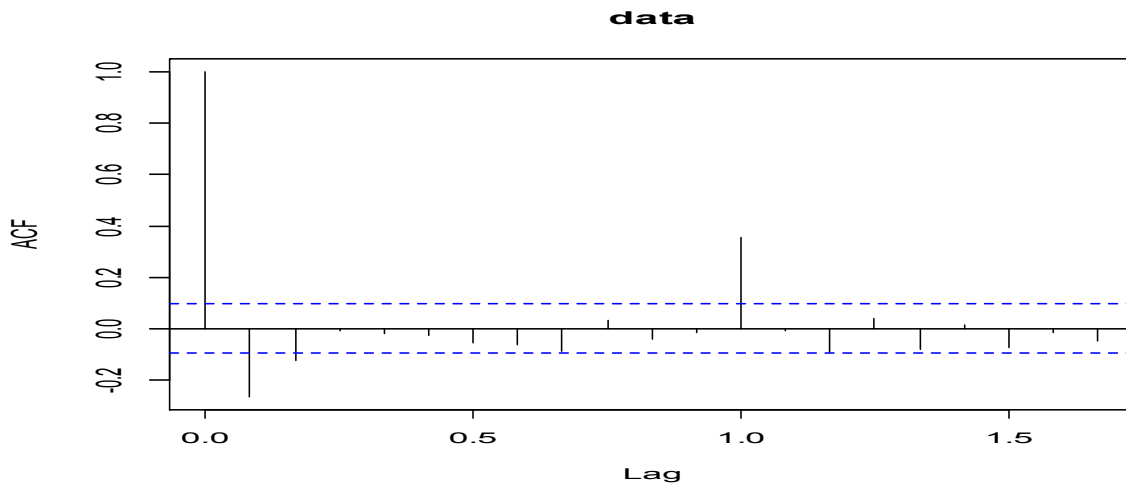


Fig. 5. ACF plot of diff (1) of the frequency of monthly rainfall in Osun State

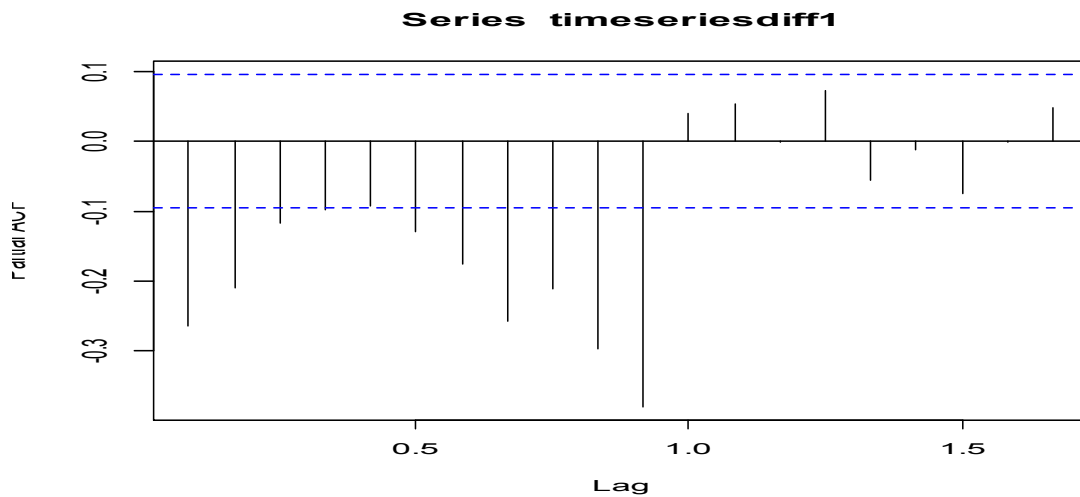


Fig. 6. PACF plot of diff (1) of the frequency of monthly rainfall in Osun State

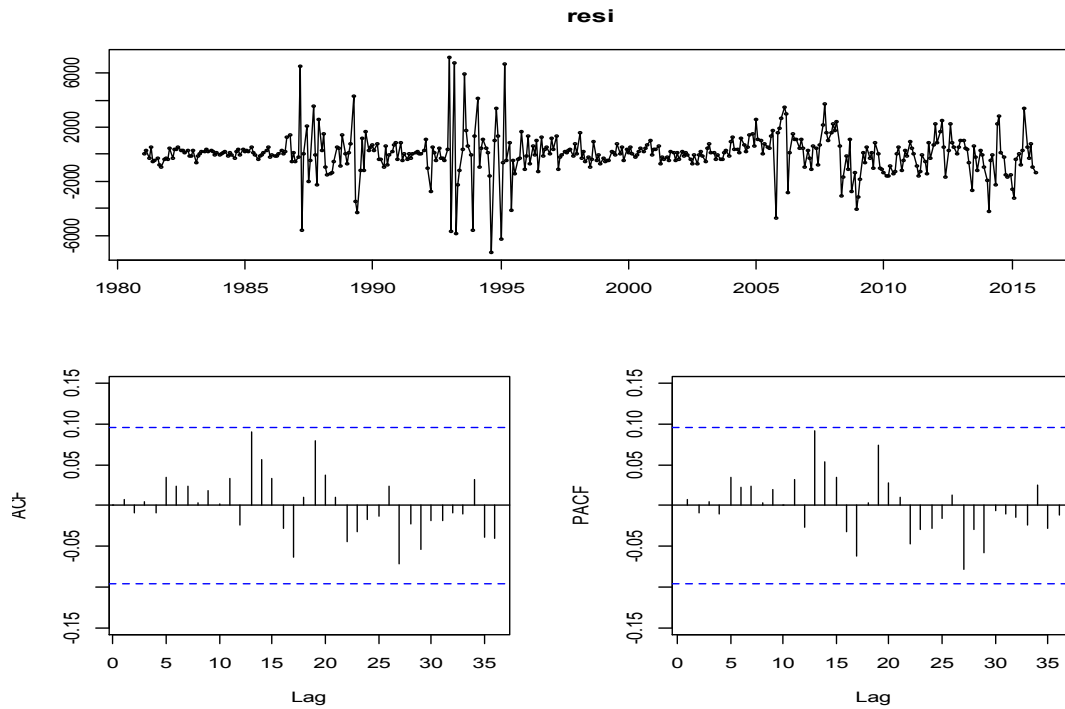


Fig. 7. Model verification plot

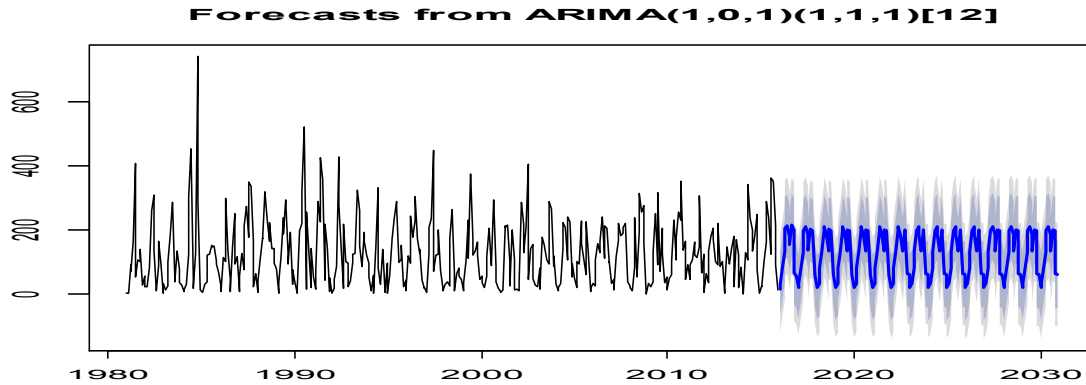


Fig. 8. Forecast plot of monthly rainfall in Osun state from 2016 to 2020

Table 5. Fitted SARIMA (1,0,1)(1,1,1)₁₂ model

| Sigma^2 | Log likelihood | AIC |
|------------------|----------------|---------|
| 5771 | -2355.5 | 4723.01 |

5. CONCLUSION

In this study, the frequency of monthly rainfall from 1981 to 2015 was analysed using seasonal time series modeling approach. The plot of the original data shows that the time series is stationary and has evidence of seasonality, Augmented Dickey Fuller test confirmed the

stationarity claim. Seasonal differencing was done to remove the seasonal effect and SARIMA (1,0,1)(2,1,1)₁₂ model of was obtained, this resulted was found to be statistically insignificant and this consequently led to a new SARIMA (1,0,1)(1,1,1)₁₂ that best fit the data and was used to make forecast. The study has revealed periods of high and low rainfall in Osun State, the rainfall forecast is important for future plans regarding agriculture, and to commodity traders within the stock market, and hence the impact on the economy of Osun state.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Schwarz G. Estimating the dimension of a model. *Annals of Statistics*. 1978;6:461–464.
2. Adejuwon JO, Balogun EE, Adejuwon SA. On the annual and seasonal pattern of rainfall fluctuations in sub-Saharan West Africa. *International Journal of Climatology*. 1990;10(8):39–848.
3. Afahakan IE, Udofia KM, Umoren MA. Analysis of rain rate and rain attenuation for earth-space communication links over Uyo, Akwa-Ibom State. *Nigerian Journal of Technology (NIJOTECH)*. 2016;35(1):137–143.
4. Ajayi GO, Ofoche EBC. Some tropical rainfall rate characteristics at Ile-Ife for microwave and millimeter wave applications. *Journal of Climate and Applied Meteorology*. 1984;23(4):562–567.
5. Akpanta AC, Okorie IE, Okoye NN. SARIMA modelling of the frequency of monthly rainfall in Umuahia, Abia State of Nigeria. *American Journal of Mathematics and Statistics*. 2015;2(2):82–87.
6. Edwin A, Martins O. Stochastic characteristics and modelling of monthly rainfall time series of Ilorin, Nigeria. *Open Journal of Modern Hydrology*. 2014;4:67–69.
7. Etuk HE, Moffat UI, Chims EB. Modelling monthly rainfall data of Port Harcourt, Nigeria by seasonal box-Jenkins method. *International Journal of Sciences*. 2013; 7(2):12–19.
8. Ibiyemi TS, et al. Rain rate and rain attenuation prediction with experimental rain attenuation efforts in south-western Nigeria. 20th In Telecommunications Forum (TELFOR); 2012.
9. Obiyemi OO, Ibiyemi TS. Experimental investigation of rainfall effect on digital satellite television reception in Nigeria: Initial results. In *Adaptive Science & Technology (ICAST), IEEE 6th International Conference*; 2014.
10. Obiyemi OO, Ojo JS, Ibiyemi TS. Performance analysis of rain rate models for microwave propagation designs over tropical climate. *Progress In Electromagnetics Research M*. 2014;39:115–122.
11. Okorie IE, Akpanta AC. Threshold excess analysis of Ikeja monthly rainfall in Nigeria. *International Journal of Statistics and Applications*. 2015;1(5):15–20.
12. Omotosho T, et al. One year results of one minute rainfall rate measurement at Covenant University, Southwest Nigeria. In *Space Science and Communication (Icon Space), IEEE International Conference*; 2013.
13. Semire FA, et al. Analysis of cumulative distribution function of 2-year rainfall measurements in Ogbomoso, Nigeria. *International Journal of Applied Science and Engineering*. 2012;10(3):171–179.
14. Akaike HA. New look at the statistical model identification. *Annals of statistics: IEEE Trans. Automat. Contr.* 1974;19:716–723.
15. Box GEP, Jenkins GM. *Time series analysis, forecasting and control*. San Francisco: Holden Day; 1976.
16. Ljung GM, Box GEP. On a measure of lack of fit in time series models. *Biometrika*. 1978;65:297.
17. Dickey DA, Fuller WA. Autoregressive time series with a unit root. *Journal of American Statistical Association*. 1979;74:427–431.

© 2019 Adams et al.; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Peer-review history:

The peer review history for this paper can be accessed here:

<http://www.sdiarticle3.com/review-history/50701>