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Second order analysis of dynamic pressure profiles, using measured horizontal wave flow velocity component

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Abstract

There is an increasing need for pressure field knowledge under wave flows due to its importance in coastal structure design. Several studies have been conducted in such a way as to accurately learn the behavior of those pressure fields in trying to define reliable formulae for design purposes, describing with a better degree of detail the maximum values achieved as well as the stochastic aspects of the wave action.

This article intends to present the evolution of pressure fields under the action of incident regular waves, as a function of its phase and relative depth, comparing them with a linear and a second order wave theory. These wave fields were obtained in an indirect way, through the horizontal wave flow velocity component.

The measurements were carried out in the unidirectional wave tank of the Faculty of Engineering of the University of Porto. The wave tank was 4,8 m wide, 24,5 m long and has a maximum water depth of 0,40 m near the test section.

Some profiles of the dynamic pressure are represented for each relative water depth (z/d) and for different wave phases. The results obtained with the horizontal velocity component were compared with the results given by the theories and with the related values through the measured mean water elevations.

The agreement between the different values is reasonable, especially for some wave phases. The differences could be explained by secondary or higher order effects.

1 Introduction

Coastal structures are submitted to different kinds of actions, which can be determined through a wave theory analysis. These actions are frequently the cause of damage in maritime structures, affecting the global stability or the integrity of the blocks. Acting forces on the structures result from pressure diagrams under the wave action, which means that its calculation has great importance, having significance in coastal structures design.

The evolution of the dynamic pressure profiles due to regular incident waves, as a function of its phase and relative depth will be presented. These results were obtained on an indirect way, through the measured horizontal velocity component and through the water elevation measured in the laboratory and the linear and 2nd Order Stokes wave theory values.

2 Pressure due to wave flow

2.1 Introduction

Some important studies can be referred, showing the importance of this area of investigation. Most of the studies, like Wist *et al.* [1], Moritz [2], Zhang [3], Choi *et al.* [4] and Briggs *et al.* [5] were based on comparisons between values obtained with second order methods and the linear theory and measured data. The main conclusions point out for the importance of the consideration of non-linear effects in statistical methods for wave kinematics prediction.

As seen before, the determination of pressure profiles under wave action, acting in a coastal structure has a relevant importance in its design, allowing the achievement of the resultant force. Some works related with the determination of these wave-induced forces can also be referred, such as Fuhrboter [6], Burchart [7], Allsop [8], Franco [9], Howarth [10], Martin [11], Allsop *et al.* [12], McConnel *et al.* [13], Bélorgey *et al.* [14], Bullock *et al.* [15], Luís [16], Muttray [17], Walkden *et al.* [18], Martinelli [19], Mason [20], Tickell [21] and Tzang [22]. They mainly studied scale effects and compared results obtained in field measurements with those measured in the laboratory and the predicted by theory ones.

The estimation of wave-induced dynamic pressures is, therefore, essential as it permits the calculation of the wave forces and moments. Wave theories are used to predict wave surface profiles and water motion and since the coastal design depends on its predictions, care must be taken in the selection of the best wave theory.

2.2 Pressure considering the linear wave theory

This theory considers waves as bi-dimensional, of small amplitude and progressive.

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Waves are described in a sinusoidal way, where crest and trough heights are equal. The nonlinear terms in the boundary conditions are ignored, which is only possible when velocities are small (when waves have very small amplitude). It provides, however, a good first estimative of wave parameters.

According to this theory, the water surface displacement is expressed by,

$$\eta = \frac{H}{2}\cos(kx - \sigma t) \tag{1}$$

where k represents the wave number, equal to $2\pi/L$, where L means the wavelength and σ the angular frequency, equal to $2\pi/T$, where T represents the wave period.

Hydrostatic pressure, p, in a point at the depth (-z) in repose, when no wave effect is considered, is equal to

$$p = -\rho g z = -\gamma z \tag{2}$$

where ρ is the fluid mass density, g the gravitational acceleration and γ the weight density.

If the wave flow is considered, an additional factor directly related with the water elevation has to be taken in account and total pressure will be the result of the addiction of two contributions, the hydrostatic pressure and the dynamic pressure due to acceleration, reaching the highest value under the crest and the lowest value under the trough, when the elevation reaches respectively the maximum and the minimum value.

Since only the linear terms will be considered, the velocity potential is expressed by,

$$\phi(x, z, t) = -\frac{Hg}{2\sigma}\frac{\cosh[k(d+z)]}{\cosh(kd)}\operatorname{sen}(kx - \sigma t) \tag{3}$$

The pressure field associated with a progressive wave is determined from the unsteady Bernoulli equation, developed for irrotational flows, that can be expressed by,

$$\frac{p}{\rho} + gz + \frac{1}{2}\left(u^2 + v^2\right) - \frac{\partial\phi}{\partial t} = C(t) \tag{4}$$

where u and v represent the horizontal and vertical components of velocity, respectively .

Developing this equation (Dean [23]) and neglecting the small velocity square terms, for a point at depth z, eqn. (4) reduces to,

$$\frac{p}{\rho} = -gz + \frac{\partial\phi}{\partial t} \tag{5}$$

Considering a progressive wave, the pressure will then be equal to,

$$p = -\rho g z + \rho g \frac{H}{2}\frac{\cosh[k(d+z)]}{\cosh(kd)}\cos(kx - \sigma t) \tag{6}$$

or, more simply,

$$p = -\rho g z + \rho g \eta K_p(z) \tag{7}$$

where K_p represents the *pressure response factor*, equal to,

$$K_p = \frac{\cosh[k(d+z)]}{\cosh(kd)} \tag{8}$$

Eqn. (6) can be calculated for the various relative depths, obtaining the following equations for the subsurface pressure under a wave, presented in Table 1.

Table 1: Variation of total pressure with water depth

	Relative depth	Total pressure
Deep water	$d>L_0/2$	$p = \rho g(-z+\eta)$
Transitional water	$L/25<d<L_0/2$	$p = -\rho g z + \rho g \frac{H}{2}\frac{\cosh[k(d+z)]}{\cosh(kd)}\cos(kx-\sigma t)$
Shallow water	$d<L/25$	$p = -\rho g z + \rho g \eta e^{kz}$

2.3 Dynamic pressure and the Stokes 2nd Order wave theory

Linear wave theory gives a good estimation of wave properties, very useful for a first approximation, but the ocean waves often are not small in amplitude and, generally, the most interesting ones (in the engineering point of view) are the bigger ones (that conduce to greater forces and bigger sediment transport). When leading with waves with greater amplitude while traveling toward shore into shallow water or when the wave steepness is bigger, it is suggested to consider an high order theory, with more complicated mathematical formulaes, Demirbilek [24].

According to the 2nd Order Stokes theory, waves have not a purelly sinusoidal shape (since the nonlinear effects are considered, it cannot be represented by a curve like a sin or a cos), being the crests heights more peaked and the troughs flatter. This theory introduces some corrections in the linear wave theory and is more complete in the mathematical point of view, becoming the most popular when considering shallow water, giving predictions of wave characteristics close to the real ones.

Particles displacement trajectories are not closed, like in the linear theory, existing a mass transport in the direction of the wave propagation.

The velocity potential is expressed by,

$$\begin{aligned}\phi(x,z,t) &= -\frac{Hg}{2\sigma}\frac{\cosh[k(d+z)]}{\operatorname{senh}(kd)}\operatorname{sen}(kx-\sigma t) \\ &+ \frac{3\pi H^2}{16T}\frac{\cosh(2k(d+z))}{\operatorname{senh}^4(kd)}\operatorname{sen}(2(kx-\sigma t))\end{aligned} \tag{9}$$

Once the velocity potential is known, one can obtain the water surface elevation deriving the function above in order to time, obtaining,

$$\eta = \frac{H}{2}\cos(kx-\sigma t) + \frac{\pi H^2}{8L}\left(\frac{2+\cosh(2kd)}{\operatorname{senh}^3(kd)}\right)\cosh(kd)\cos(2(kx-\sigma t)) \tag{10}$$

Dynamic pressure at any distance below the fluid surface can be obtained (Demirbilek [24]) by,

$$p_d = \rho g \frac{H}{2} \frac{\cosh[k(d+z)]}{\cosh(kd)} \cos(kx - \sigma t)$$
$$+\frac{3}{8}\rho g \frac{\pi H^2}{L} \frac{\tanh(kd)}{\mathrm{senh}^3(kd)}\left(\frac{\cosh(2k(d+z))}{\mathrm{senh}^3(kd)} - \frac{1}{3}\right)\cos(2(kx - \sigma t)) \tag{11}$$
$$-\frac{1}{8}\rho g \frac{\pi H^2}{L} \frac{\tanh(kd)}{\mathrm{senh}^2(kd)}\left(\cosh(2k(d+z)) - 1\right)$$

3 Indirect analysis of dynamic pressure

3.1 Introduction

The main objective of this work was to assesss profiles of dynamic pressure p_d. Considering experimental data from measurements taken in the Hydraulics Laboratory of the Faculty of Engineering of Porto and two diferent theoretical approaches, dynamic pressure due to wave flows through the horizontal component of the velocity at different points were achieved.

Measurements were obtained, for regular incident waves with different periods, heights e water depths, totalizing six test conditions. For each regular wave, 50 phases, corresponding to an equal number of dynamic pressure profiles were considered. According to that, 300 distinct profiles were produced and some of them will be presented.

3.2 Dynamic pressure and the linear wave theory

Isolating the term that corresponds to the dynamic pressure, p_d, from equation (7), it is obtained,

$$p_d = \rho g \eta K_p(z) \tag{12}$$

equal to,

$$p_d = \rho g \frac{H}{2} \frac{\cosh[k(d+z)]}{\cosh(kd)} \cos(kx - \sigma t) \tag{13}$$

In linear wave theory, the horizontal component of the velocity is expressed by,

$$u = \frac{gHk}{2\sigma} \frac{\cosh[k(d+z)]}{\cosh(kd)} \cos(kx - \sigma t), \tag{14}$$

equal to,

$$u = \frac{k}{\rho\sigma} \rho g \frac{H}{2} \frac{\cosh[k(d+z)]}{\cosh(kd)} \cos(kx - \sigma t) \tag{15}$$

Eqn. (13), can then be retyped as a function of ρ, C and u,

$$p_d = \frac{\rho\sigma}{k}u = \frac{\rho\frac{2\pi}{T}}{\frac{2\pi}{L}}u = \rho\frac{L}{T}u = \rho C u \tag{16}$$

where C represents the wave celerity, equal to L/T.

We can conclude that, according to the linear theory, knowing the values of ρ, C and u, one can assesss to the dynamic pressure p_d.

3.3 Dynamic pressure and the 2nd Order Stokes wave theory

The horizontal component of the velocity of a fluid particle can be calculated derivating the velocity potential expression in order to x, obtaining for the 2nd Order Stokes theory,

$$\begin{aligned} u = {} & \frac{gHk}{2\sigma}\frac{\cosh[k(d+z)]}{\cosh(kd)}\cos(kx-\sigma t) \\ & + \frac{3}{4}\left(\frac{\pi H}{T}\right)\left(\frac{\pi H}{L}\right)\frac{\cosh[2k(d+z)]}{\operatorname{senh}^4(kd)}\operatorname{sen}[2(kx-\sigma t)] \end{aligned} \tag{17}$$

equivalent to,

$$\begin{aligned} & u - \frac{3}{4}\left(\frac{\pi H}{T}\right)\left(\frac{\pi H}{L}\right)\frac{\cosh[2k(d+z)]}{\operatorname{senh}^4(kd)}\operatorname{sen}[2(kx-\sigma t)] \\ & = \frac{gHk}{2\sigma}\frac{\cosh[k(d+z)]}{\cosh(kd)}\cos(kx-\sigma t) \end{aligned} \tag{18}$$

Multiplying both terms by $\rho\,\sigma/k$, we have,

$$\begin{aligned} & \rho\frac{\sigma}{k}\left(u - \frac{3}{4}\left(\frac{\pi H}{T}\right)\left(\frac{\pi H}{L}\right)\frac{\cosh[2k(d+z)]}{\operatorname{senh}^4(kd)}\operatorname{sen}[2(kx-\sigma t)]\right) \\ & = \rho\frac{\sigma}{k}\left(\frac{gHk}{2\sigma}\frac{\cosh[k(d+z)]}{\cosh(kd)}\cos(kx-\sigma t)\right) \end{aligned} \tag{19}$$

equal to,

$$\begin{aligned} & \rho C\left(u - \frac{3}{4}\left(\frac{\pi H}{T}\right)\left(\frac{\pi H}{L}\right)\frac{\cosh[2k(d+z)]}{\operatorname{senh}^4(kd)}\operatorname{sen}[2(kx-\sigma t)]\right) \\ & = \rho g\frac{H}{2}\frac{\cosh[k(d+z)]}{\cosh(kd)}\cos(kx-\sigma t) \end{aligned} \tag{20}$$

The second term of this expression is equal to the first term of the dynamic pressure expression, which substituting in eqn. (11) conduces to,

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$$p_d = \rho C\left(u - \frac{3}{4}\left(\frac{\pi H}{T}\right)\left(\frac{\pi H}{L}\right)\frac{\cosh[2k(d+z)]}{\mathrm{senh}^4(kd)}\mathrm{sen}[2(kx-\sigma t)]\right)$$
$$+\frac{3}{8}\rho g\frac{\pi H^2}{L}\frac{\tanh(kd)}{\mathrm{senh}^3(kd)}\left(\frac{\cosh[2k(d+z)]}{\mathrm{senh}^3(kd)}-\frac{1}{3}\right)\cos[2(kx-\sigma t)] \qquad (21)$$
$$-\frac{1}{8}\rho g\frac{\pi H^2}{L}\frac{\tanh(kd)}{\mathrm{senh}^2(kd)}\left(\cosh[2k(d+z)]-1\right)$$

4 Experimental Set-Up

The measurements were carried out in the unidirectional wave tank of the Faculty of Engineering of the Porto University, schematised in Figure 1. The wave tank is 4.8 m wide, 24.5 m long and has a maximum water depth of 0.60 and 0.40 m, near the piston type wave generator and in the measuring section, respectively. To avoid tri-dimensional effects and diffraction, during the tests with regular waves, it was used a thin diving wall, that isolated the measuring section from the rest of the tank. A small steepness beach was also constructed, diminishing wave reflection near the bottom by causing the dissipation of wave energy.

Wave probes, placed in the section where the velocities were measured, registered the instantaneous water surface elevation.

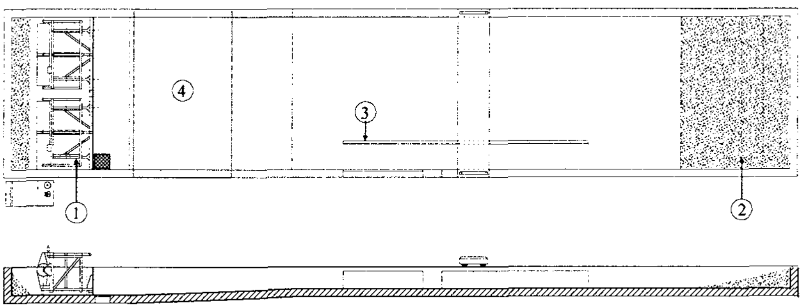

Figure 1: Layout of the Hydraulics Laboratory, FEUP (1–Wavemaker; 2–Dissipating beach; 3–Thin dividing wall; 4–Wave tank)

The measurement of the velocities was made using a Laser Doppler Anemometry optical system, together with the simultaneous recordings of the corresponding water levels. A detailed description of the experimental set-up can be found in Taveira-Pinto [25].

5 Results and discussion

As mentioned above, six different test conditions were used, which are described in table 2,

Table 2: Measured wave properties (m)

Measurement	d (m)	$\overline{H}_m$ (m)	$\overline{T}_m$ (s)	$\overline{L}_m$ (m)	$\overline{C}_m$ (m/s)
1	0,18	0,02734	0,9053	1,0035	1,115
2	0,20	0,03273	0,9053	1,0400	1,155
3	0,22	0,02809	0,9090	1,0665	1,190
4	0,18	0,03559	1,2029	1,4700	1,225
5	0,20	0,03415	1,2047	1,5225	1,270
6	0,22	0,03651	1,2062	1,5535	1,295

where d represents the water depth, $\overline{H}_m$ the mean measured wave height, $\overline{T}_m$ the mean measured wave period, $\overline{L}_m$ the mean measured wave length and $\overline{C}_m$ the mean measured wave celerity.

Considering the available data, the dynamic pressure can be calculated considering different approaches:

- through the mean measured horizontal component of the velocity, represented by pd (eqn. 16);
- through the water elevation obtained by the linear theory, represented by pd1 (eqn. 1 and 12);
- through the measured water elevation, represented by pd2 (eqn. 12);
- through the mean measured horizontal component of the velocity, represented by pds (eqn. 16 and 17);
- through the water elevation obtained by the 2nd Order Stokes theory (considering only the non linear terms), represented by pd1s (eqn. 10 and 12);
- through the 2nd Order Stokes theory, represented by pd2s (eqn.11).

The following figure 2 shows the isolines of the dynamic pressure p_d in the prototype (in kN/m^2), obtained through the horizontal component of the mean velocity, for the six test conditions. In yy axis is represented the relative water depth, z/d, and in xx axis, the wave phase in degrees. Figure 3 and 4 represent the isolines of the dynamic pressure pd1, obtained through the water elevation calculated by the linear theory.

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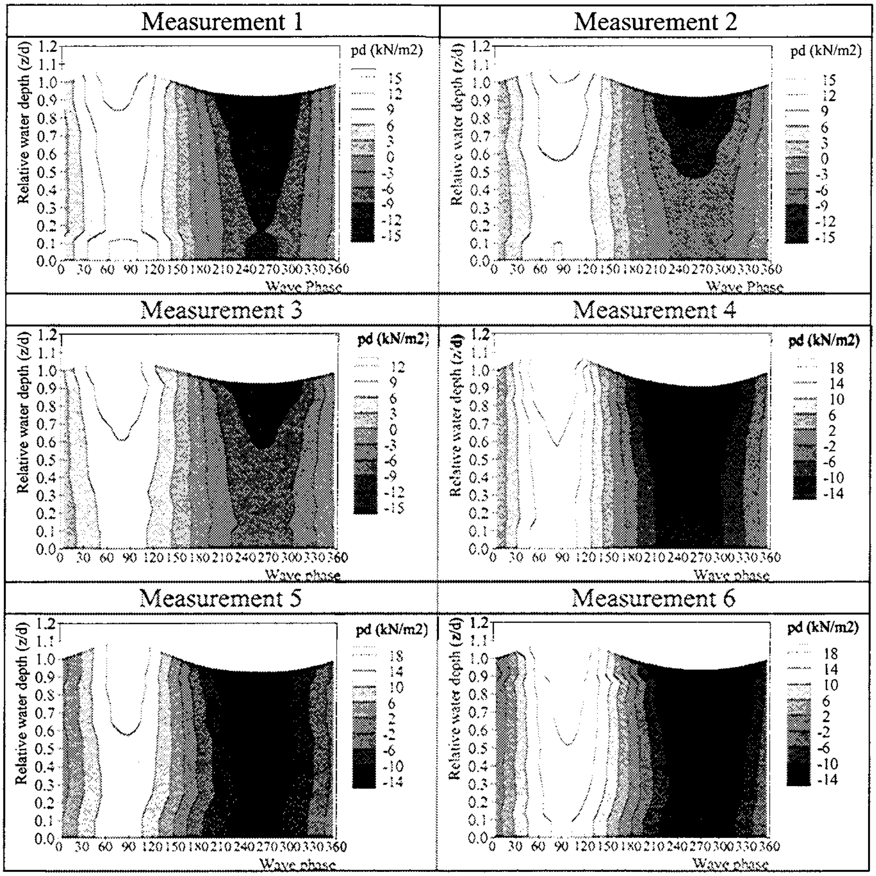

Figure 2: Dynamic pressure isolines (pd)

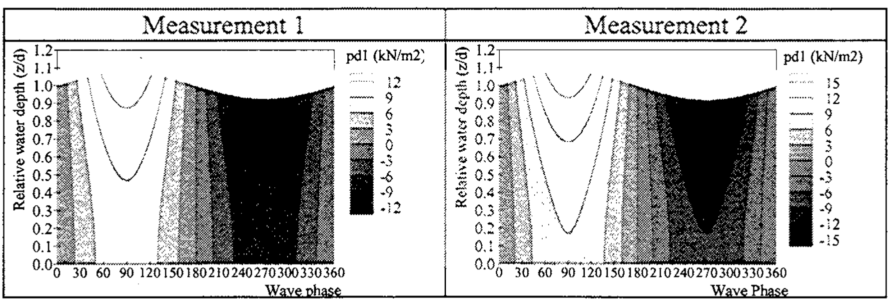

Figure 3: Dynamic pressure isolines (pd1)

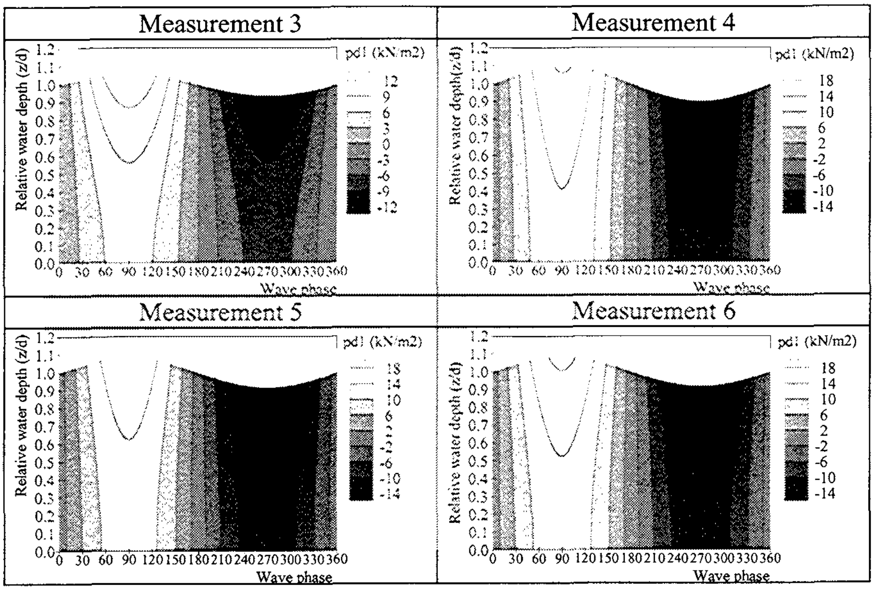

Figure 4: Dynamic pressure isolines (pd1)

Mean water elevation measured in the six tests and the equivalent water elevation obtained by the theory (linear and Stokes second order) is represented in figures 5 and 6. yy axis represents the relative water depth (z/d) and the xx axis, the wave phase in degrees. These values were used in the determination of the dynamic pressure, pd1, pd2 and pd1s.

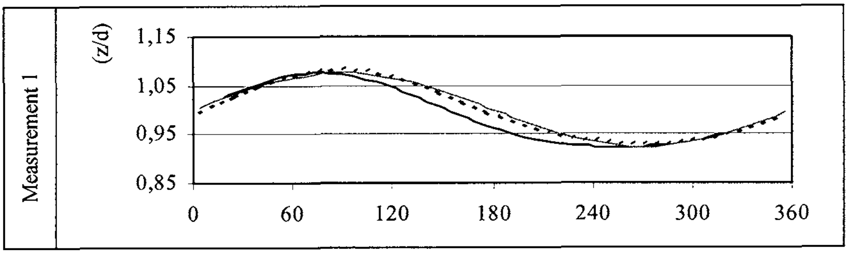

Figure 5: Measured and obtained by linear and 2nd Order Stokes theory mean water surface elevation

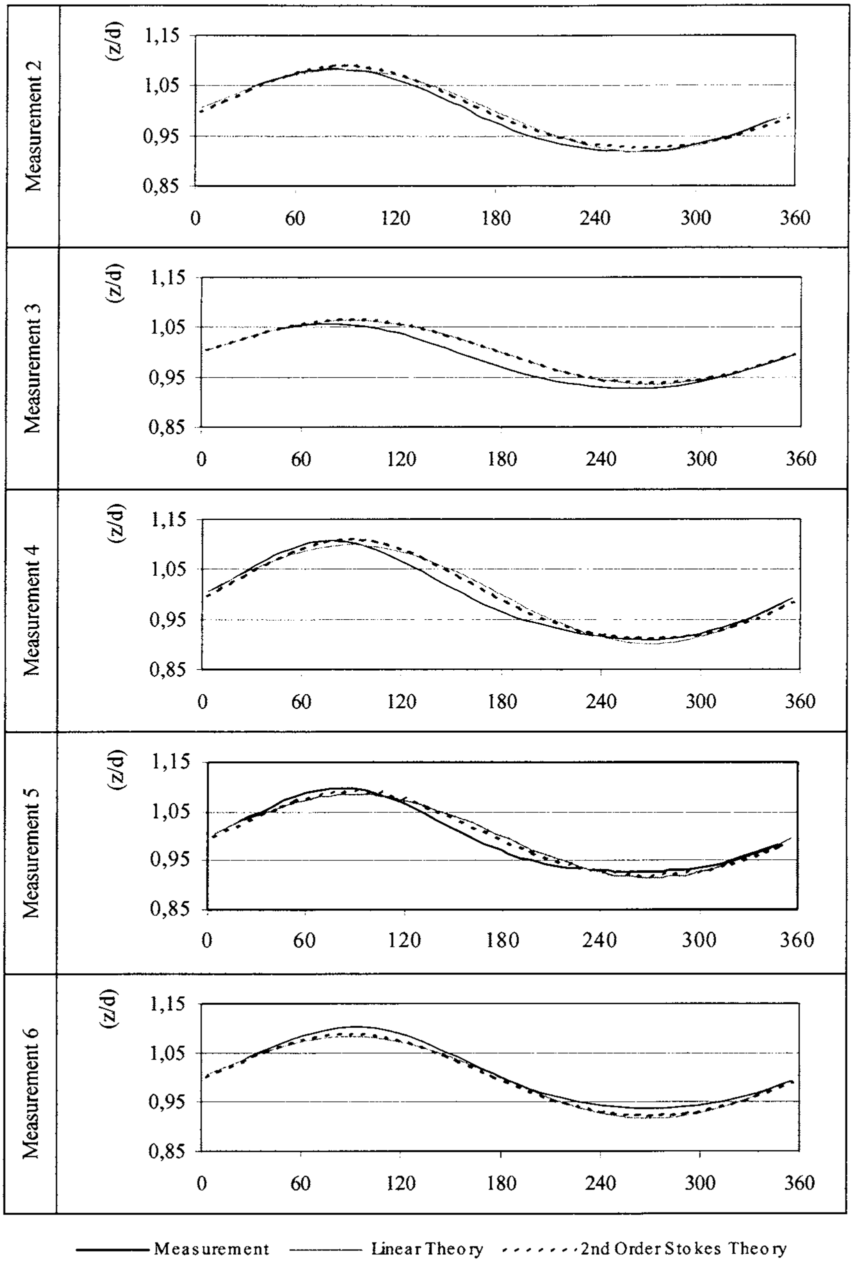

Figure 6: Measured and obtained by linear and 2nd Order Stokes theory mean water surface elevation

The dynamic pressures in the prototype, pd, pd1, pd2, pds, pd1s and pd2 profiles, for the first test condition (Measurement No. 1) are shown in figures 7 and 8, as a function of the relative water depth (z/d) and for different wave phases. The dynamic pressure is expressed in kN/m^2 . Considering a geometric scale of 1/100, the correspondent model values are one hundred times smaller than the prototype ones (represented in the figures).

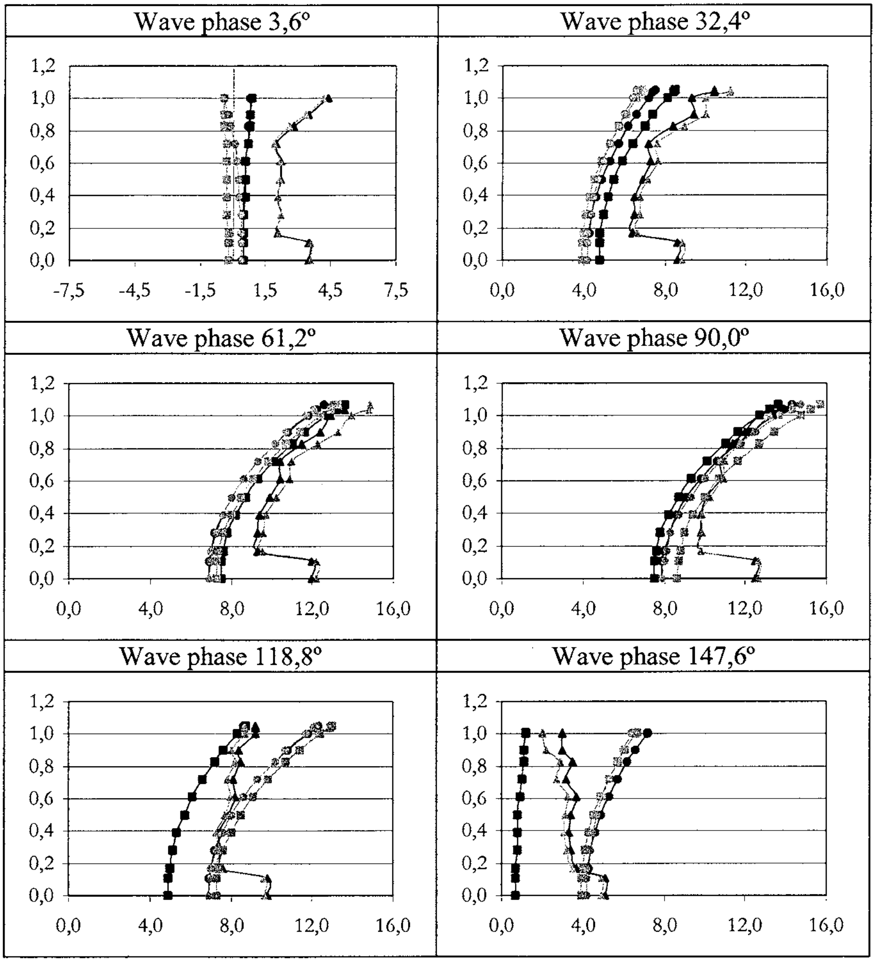

Figure 7: Dynamic pressure profiles (kN/m^2) as a function of the relative water depth (z/d) and for different wave phases (Measurement No. 1)

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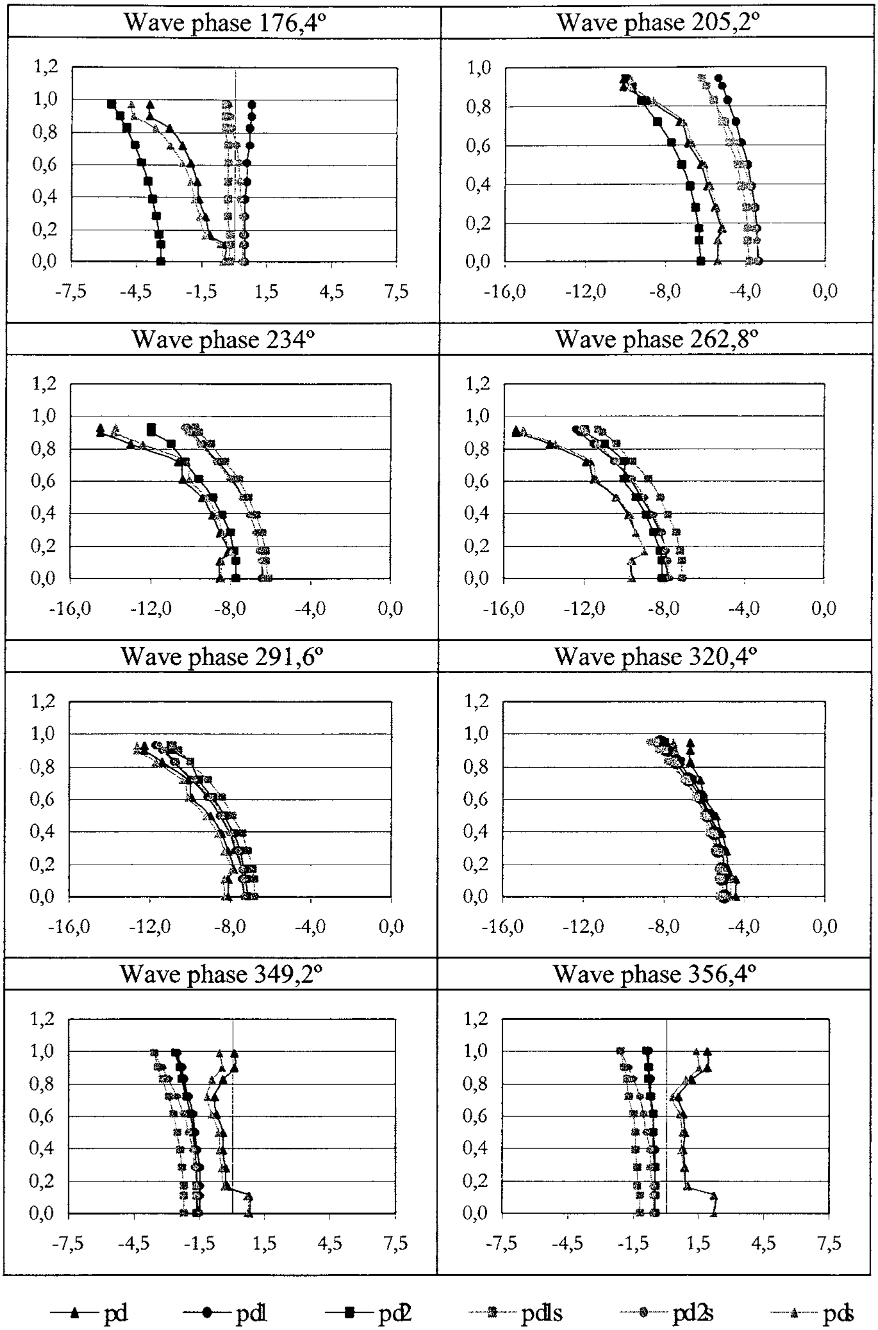

Figure 8: Dynamic pressure profiles (kN/m^2) as a function of the relative water depth (z/d) and for different wave phases (Measurement No. 1)

Attending to the trend shown, the dynamic pressure profiles (pd, pd1, pd2, pd1s and pd2s) are, in general, in relative good agreement. As closer the water elevations are (measured and obtained by theory), closer the calculated profiles are. Differences between profiles can be explained and justified by the different values of the calculated and measured water elevation or by secondary or higher order effects.

Water surface elevation was, in most cases, well predicted by theory (specially in the trough and crest sections) and, unsurprisingly, values obtained with the 2nd Order Stokes theory were closer the measured ones. Same conclusions for the dynamic pressure profiles pd1, pd2 and pd1s can be taken, since they depend on the closeness of water surface elevation profiles.

The dynamic pressure profiles obtained through the horizontal component of the mean velocity measured, were much more irregular, fact that can be mainly explained by the velocity measurement system properties. Generally, these pressures were between pd1 and pd2 (in sections where water elevation was positive) or lower (in sections where water elevation was negative). Possible explanations for these discrepancies can be explained by the presence of non-linear effects, neglected in the using of the linear wave theory (that does not consider the non linear effects in the horizontal component of the velocity).

The dynamic pressure profiles obtained by the 2nd Order Stokes theory were near those calculated by linear theory, being its trend similar to pd1 and pd1s. In general, they were closer to those obtained through the horizontal velocity component (pd) and through the water surface elevation than the calculated by linear theory ones, an expected fact explained by the consideration of the non-linear effects.

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