# Second-order odd-harmonic repetitive control and its application to active filter control

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# Second-order odd-harmonic repetitive control and its application to active filter control

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*Abstract*— **High order repetitive control has been introduced to overcome performance decay of repetitive control systems under varying frequency of the signals to be tracked/rejected or improving the interhamonic behavior. However, most high order repetitive internal models used to improve frequency uncertainty are unstable, as a consequence practical implementations are more difficult. In this work a stable, second order odd-harmonic repetitive control system is presented and studied.**

**The proposed internal model has been implemented and validated in a shunt active filter current controller. This high order controller allows dealing with the grid frequency variations without using adaptive schemes.**

## I. INTRODUCTION

Repetitive Control [1], [2] is a well established, Internal Model Principle [3] based technique which allows tracking/rejecting periodic signals of known frequency. Unfortunately, its performance decays dramatically when the signal frequency varies [4]. In order to overcome this problem two major approaches have been proposed. The first one is based on the adaptation of the sampling period in accordance with the signal frequency variation. Although this approach offers good results [5], [6], [7] it implies introducing frequency observers and moving the stability analysis from a Linear Time Invariant framework into a Linear Time Varying one. The second approach is based on the introduction of higher order internal models [8], [4]. The parameters of these high order controllers can be tuned according to different criteria: most of them are related with making the internal model robust in front frequency variations [9], [4], or with a trade-off between this robustness issue and amplification of non-harmonic frequencies [9], [10], [11], [12]. However, most of the reported higher order internal models are unstable and, although this does not make the closed-loop unstable, it clearly limits its performance and yields well-know linear control limitations play an important role in the controller implementation and design [13], [14]. In summary, the instability complicates the implementation and restricts the use of this control scheme in practical situations.

There exist many applications, like the ones related with power electronics, that do not usually deal with generic periodic signals but with odd-harmonic periodic signals. In order to take advantage of this fact, an odd-harmonic digital repetitive internal model was introduced in [15] and later extended to continuous-time implementations in [16]. Similarly, other specific internal models like the ones for $6l \pm 1$ harmonic signals have also been reported [17]. This paper propounds a second-order odd-harmonic internal model which is proven to be stable and improves robustness to frequency variations. Its experimental validation is carried out in an Active Filter (AF).

AFs are power electronics devices intended to overcome the power quality problems caused by nonlinear loads. In this context, the control objective is to achieve a power factor close to 1, as well as load current harmonics and reactive power compensation [18], [19]. Most AF controllers are based on two hierarchical control loops, an inner one in charge of assuring the desired current and an outer one in charge of determining the required shape as well as the appropriate power balance. Repetitive control has proved to be an efficient control technique for the inner controller [20]; however, the frequency variations undergone by most distribution grids can degrade its performance, as mentioned above. In order to avoid this problems the proposed second-order odd-harmonic internal model has been introduced in the inner control loop, this yielding very good performance and robustness under network frequency variations.

## II. ODD-HARMONIC REPETITIVE CONTROL

Repetitive control bases its performance on the introduction of a generator of periodic signal to be tracked/rejected inside the controller. Figure 1 shows the scheme of these generators, which are usually constructed by the feedback connection (either positive or negative, i.e. $\sigma = 1$ or $\sigma = -1$, respectively), of a time delay $W(z)$, in series with a low-pass filter $H(z)$ that reduces the gain at high frequency and improves closed-loop robustness, this yielding the generic internal model

$$I(z) = \frac{\sigma W(z)H(z)}{1-\sigma W(z)H(z)}. \tag{1}$$

It is worth mentioning that the original internal model was constructed using $W(z) = z^{-N}$, $N$ being the discrete time period of the signal to be tracked/rejected, $H(z) = 1$ and $\sigma = 1$.

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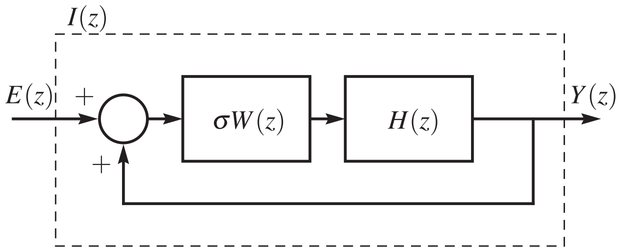


Fig. 1. Generic repetitive control internal model scheme, where $W(z)$ is a delay function, $H(z)$ a null-phase low pass filter, and $\sigma \in \{-1,1\}$.

Odd-harmonic repetitive control uses an internal model which introduces infinite gain only at a certain frequency and its odd harmonics [15]. This internal model is constructed using $W(z) = z^{-N/2}$ and $\sigma = -1$ in (1):

$$I_o(z) = \frac{-H(z)}{z^{\frac{N}{2}} + H(z)}. \tag{2}$$

With $H(z) = 1$, (2) has the poles in $z = \exp(2(2k-1)\pi j/N)$, which are uniformly distributed over the unit circle[1]. As a consequence (2) provides infinite gain at the odd-harmonic frequencies $\omega_k = 2(2k-1)\pi/N$, with $k = 1,2,\ldots,N/2$.

Besides the internal model, which assures steady state performance, repetitive controllers include a stabilizing controller, $G_x(z)$, which assures closed-loop stability. Traditionally, repetitive controllers are implemented in a "plug-in" fashion, i.e. the repetitive compensator is used to augment an existing nominal controller $G_c(z)$, as depicted in Figure 2. This nominal compensator is designed to stabilize the plant, $G_p(z)$, and provides disturbance attenuation across a broad frequency spectrum.

*Theorem 1:* [15] The closed-loop system of Figure 2 is stable if the following sufficient conditions are fulfilled:

1) The closed loop system without the repetitive controller is stable, i.e.

$$T_o(z) = \frac{G_c(z)\,G_p(z)}{1 + G_c(z)\,G_p(z)}$$

is stable.

2) $\| W(z) H(z) (1 - T_o(z) G_x(z)) \|_\infty < 1$, where $H(z)$ and $G_x(z)$ must be selected to meet this condition.

*Remark 1:* It is advisable to design the controller $G_c(z)$ with a high enough robustness margin. Moreover, $H(z)$ is usually selected according to the desired bandwidth and the required robustness, while $G_x(z)$ is designed using phase cancellation techniques [21], which for minimum-phase systems results in[2]: $G_x(z) = k_r (T_o(z))^{-1}$. Finally, as argued in [22], $k_r$ must be designed looking for a trade-off between robustness and transient response.

For the generic internal model (1), the sensitivity transfer function of the control loop shown in Figure 2 can be written

[1]Note that, as there is no pole in $z = 1$, there is no infinite gain in dc-frequency, i.e. no integrator

[2]There is no problem with the improperness of $G_x(z)$ because the internal model provides the repetitive controller with a high positive relative degree.

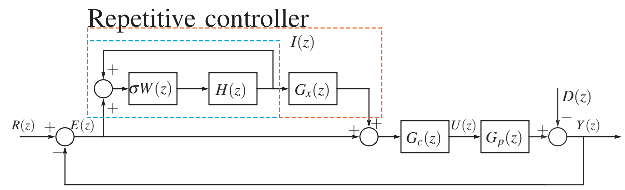


Fig. 2. Block-diagram of the repetitive controller plug-in approach.

as

$$S(z) = \frac{E(z)}{R(z)} = S_o(z) S_{Mod}(z) \tag{3}$$

where

$$S_o(z) = \frac{1}{1 + G_c(z)\,G_p(z)} \tag{4}$$

stands for the sensitivity function of the system without repetitive controller and $S_{Mod}(z)$ is the modifying sensitivity function

$$S_{Mod}(z) = \frac{1 - \sigma W(z) H(z)}{1 - \sigma W(z) H(z) (1 - G_x(z) T_o(z))}. \tag{5}$$

The poles of the closed-loop system are given by the poles of $S_o$, i.e the poles of the closed-loop without repetitive controller, and the ones of $S_{Mod}$. For the case in which $G_x(z) = k_r (T_o(z))^{-1}$, $\sigma = -1$, $W(z) = z^{-\frac{N}{2}}$ and $H(z) = 1$, the poles of $S_{Mod}$ are

$$z = \sqrt[N/2]{|1-k_r|}\, e^{\frac{2(2k-1)\pi j}{N}},\ k = 1,\ldots,\frac{N}{2}. \tag{6}$$

These poles are uniformly distributed over a circle of radius $\sqrt[N/2]{|1-k_r|}$ which, for stability requirements, should be within the unit circle, i.e. $k_r \in (0,2)$.

## III. Odd-harmonic high order repetitive control

High-Order Repetitive Control (HORC) was mainly introduced either to improve robustness against performance reduction under uncertainty/variation of the reference/disturbance frequency or to reduce the amplification of interharmonic frequencies. HORC uses an internal model equivalent to the one in Figure 1, but replacing the delay by a weighted addition of several delays, namely:

$$W(z) = \sum_{k=1}^{M} w_k z^{-kN}. \tag{7}$$

Similarly, the controller architecture uses the plug-in approach of Figure 2, which is stable if the previously introduced conditions are fulfilled.

According to the desired performance, several criteria have been introduced to select the weights $w_k$ in (7). In order to assure high gain at harmonic frequencies, the following constraint is usually demanded:

$$\sum_{k=1}^{M} w_k = 1 \tag{8}$$

In [8], HORC was introduced to improve the interharmonic amplification, and this was done through $\min_{w_k} \|S_{Mod}(z)\|_2$.

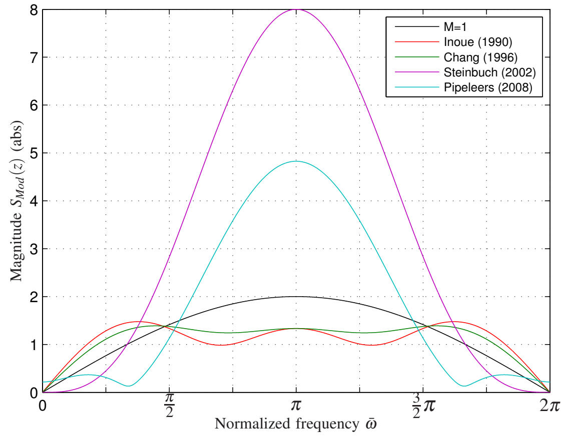


Fig. 3. Magnitude response of $S_{Mod}(z)$: comparison of [8], [9], [4] and [11] for $G_x = 1/(T_o(z))^{-1}$, $H(z) = 1$ and $M = 3$.

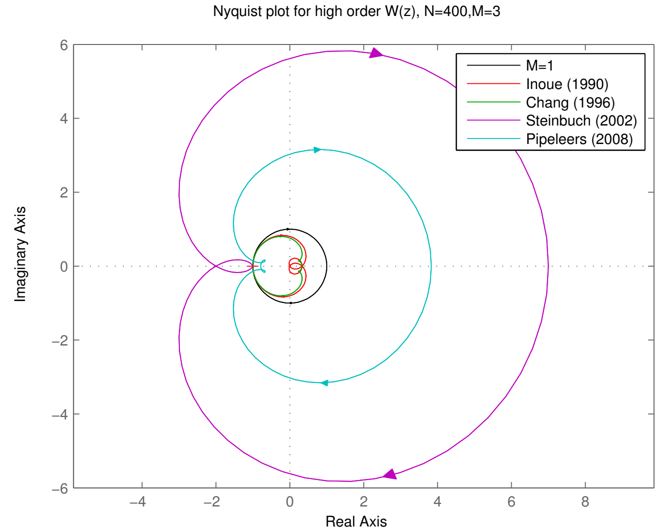


Fig. 4. Nyquist plot of $-W(z)$ for traditional repetitive control ($M = 1$) and the high order repetitive control ($M = 3$) tuned according to [8], [9], [4] and [11].

The analytical solution of this problem is $w_k = 1/M$, $\forall k$. In [9], a trade-off between the harmonic and the interhamonic behavior was formulated via $\min_{w_k} \|S_{Mod}(z)\|_\infty$. In [4], in order to minimize sensitivity against frequency variations, $W(z)$ is selected maximally flat at harmonic frequencies, and an analytical solution is obtained. In [10], the results in [9] are generalized by solving $\min_{w_k} \|G(z)S_{Mod}(z)\|_\infty$, $G(z)$ being a weight function which defines the frequency interval in which $S_{Mod}(z)$ will be minimized; results in [4] and [9] are particular cases of this generic formulation.

In [11], the constraint (8) is eliminated. This reduces the gain obtained at harmonic frequencies, i.e. it yields a performance reduction, but allows a better control of the interharmonic behavior.

Figure 3 compares the results obtained for $G_x = 1/(T_o(z))^{-1}$, $H(z) = 1$ and $M = 3$ when using the different tuning techniques previously introduced. It is important to state that the results from [4] obtain perfect tracking, i.e. zero gain, at the harmonic frequencies while their neighborhoods are flat, this meaning robustness against small variation in the signal frequency. Unfortunately, interharmonic frequencies are notoriously amplified. On the contrary, results from [8] and [9] do not amplify much interharmonic frequencies, but they do not improve robustness against frequency variations. Finally, [11] offers an interesting trade-off between both issues: it has no perfect tracking (no zero gain) at harmonic frequencies, but the gain is maintained small in a 20% of the frequency region while the interharmonic amplification in kept bounded, below the values obtained from [4].

*Proposition 2:* The weights obtained in [4] yield

$$W(z) = 1 - (1 - z^{-N})^M$$

and, as a consequence, the internal model resulting from (1) with $\sigma = 1$ and $H(z) = 1$ is

$$I(z) = \frac{1 - (1 - z^{-N})^M}{(1 - z^{-N})^M}, \tag{9}$$

its poles being $z = \sqrt[N]{1}$ with multiplicity $M$.

*Proof:* By straightforward calculation. ■

From Proposition 2 it is immediate that the poles coincide with those of the traditional repetitive controller ($M = 1$). The pole multiplicity increase improves robustness against frequency variations [4] (i.e. with $M > 1$) but implies internal models which are not BIBO stable.

Figure 4 shows the Nyquist plot of $-\sigma W(z)H(z)$ with $\sigma = 1$, $H(z) = 1$ and $N = 400$ for the options previously analyzed[3]. The Nyquist plot of the standard repetitive controller, i.e. with $M = 1$, is over the unit circle and, therefore, it is marginally stable. As a consequence, the tuning obtained with those methods which do not improve robustness under frequency uncertainty [8], [9] generate a Nyquist plot which is contained inside the unit circle, except at tangential points corresponding to the harmonic frequencies poles. Differently, those methods which improve robustness [4] encircle the -1 point many times. Although, as shown in Proposition 2, the internal model for $H(z) = 1$ does not contain poles outside the unit circle, with the introduction of a low pass $H(z)$ inside the internal model the Nyquist plot will vary slightly. This variation will change the number of the encirclements of $z = -1$ and, consequently, poles outside the unit circle may appear in most cases.

The optimization based tuning [11] generates internal models with poles outside the unit circle depending on the selected filter $H(z)$ and weight function. It is important to note that these internal models do not contain integrators or poles at the harmonic frequencies for the specific tuning shown in Figure 3, which generates an internal model without poles outside the unit circle.

From the theoretical point of view, the existence of poles outside the unit circle in the internal model does not com-

[3]Note that, as the internal model is composed of a positive feedback, the Nyquist criterium has to be applied to $-W(z)H(z)$.

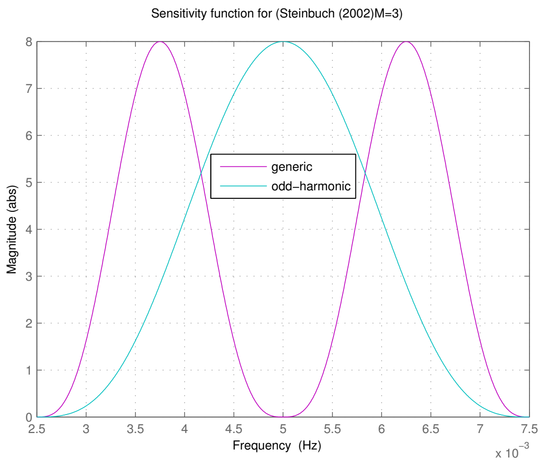


Fig. 5. Sensitivity functions of generic and odd-harmonic high-order repetitive controllers with $M=3$, $H(z)=1$ and $N=400$.

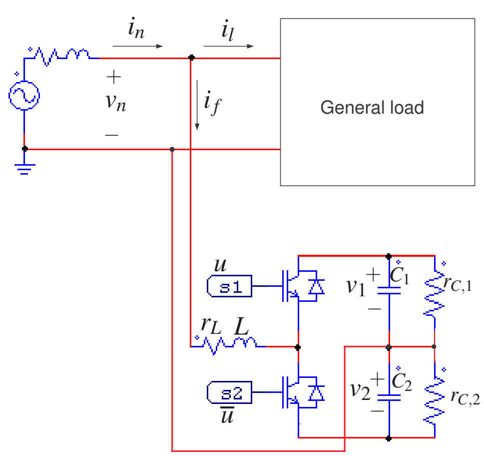


Fig. 6. Single-phase shunt active filter connected to the network-load system.

promise the closed-loop stability, but it complicates the implementation of this internal models in practical applications. As an example, sophisticated anti-windup schemes must be included in order to avoid problems with saturated control actions.

All previous works have been formulated for generic internal models. However, they can be transformed into odd-harmonic internal models changing $\sigma=1$ by $\sigma=-1$, $N$ by $N/2$ and reformulating $w_k$.

From now on, assume that $\sigma=-1$ and set $M=2$. Then, the internal model is:

$$I_{hodd}(z)=-\frac{\left(2z^{-\frac{N}{2}}+z^{-N}\right)H(z)}{1+\left(2z^{-\frac{N}{2}}+z^{-N}\right)H(z)}. \tag{10}$$

*Proposition 3:* The second-order, odd-harmonic internal model (10), $H(z)$ being a null-phase filter with $\|H(z)\|_\infty<1$, is stable.

## IV. The Active Filter

### A. The boost converter

The system architecture is depicted in Figure 6. A load is connected to the power source, while an active filter is applied in parallel in order to fulfill the desired behavior, i.e. to guarantee unity power factor at the network side. A boost converter with the ac neutral wire connected directly to the midpoint of the dc bus is used as active filter. The averaged (at the switching frequency) model of the boost converter is given by

$$L\frac{di_f}{dt} = -r_L i_f - v_1\frac{d+1}{2} - v_2\frac{d-1}{2} + v_n \tag{11}$$

$$C_1\frac{dv_1}{dt} = -\frac{v_1}{r_{C,1}} + i_f\frac{d+1}{2} \tag{12}$$

$$C_2\frac{dv_2}{dt} = -\frac{v_2}{r_{C,2}} + i_f\frac{d-1}{2} \tag{13}$$

where $d$ is the duty ratio, $i_f$ is the inductor current and $v_1$, $v_2$ are the dc capacitor voltages; $v_n=V_n\sqrt{2}\sin(\omega_n t)$ is the voltage source,[4] $L$ is the converter inductor, $r_L$ is the inductor parasitic resistance, $C_1, C_2$ are the converter capacitors and $r_{C,1}$, $r_{C,2}$ are the parasitic resistances of the capacitors. The control variable, $d$, takes its value in the closed real interval $[-1,1]$ and represents the averaged value of the Pulse-Width Modulation (PWM) control signal injected to the actual system.

Due to the nature of the voltage source, the steady-state load current is usually a periodic signal with only odd-harmonics in its Fourier series expansion, so it can be written as $i_l=\sum_{k=0}^{\infty} a_k\sin(\omega_n(2k+1)t)+b_k\cos(\omega_n(2k+1)t)$.

### B. Control objectives and control architecture

The active filter goal is to assure that the load is seen as a resistive one. This can be stated as $i_n^*=I_d^*\sin(\omega_n t)$, i.e. the source current must have a sinusoidal shape in phase with the network voltage[5]. Another collateral goal, necessary for a correct operation of the converter, is to assure constant average value of the dc bus voltage[6], i.e. $<v_1+v_2>_0^*=v_d$, where $v_d$ must fulfill the boost condition ($v_d>2\sqrt{2}v_n$). It is also desirable for this voltage to be almost equally distributed among both capacitors ($v_1\approx v_2$).

This paper uses the control architecture presented in [20], changing the regular odd-harmonic internal model by the here proposed second-order odd-harmonic model.

The controller is designed using a two level approach, as portrayed in Figure 7: first, an inner current controller forces the sine wave shape for the network current and, second, an outer control loop yields the appropriate active power balance for the whole system. The output of this loop is the amplitude of the sinusoidal reference for the current control loop. The active power balance is achieved if the energy stored in the active filter capacitors, $E_C=v_1^2+v_2^2$, is equal to a reference value, $E_C^d$.

[4] $\omega_n=2\pi/T_p$ rad/s is the network frequency.
[5] $x^*$ represents the steady-state value of signal $x(t)$.
[6] $<x>_0$ means the dc value, or mean value, of the signal $x(t)$.

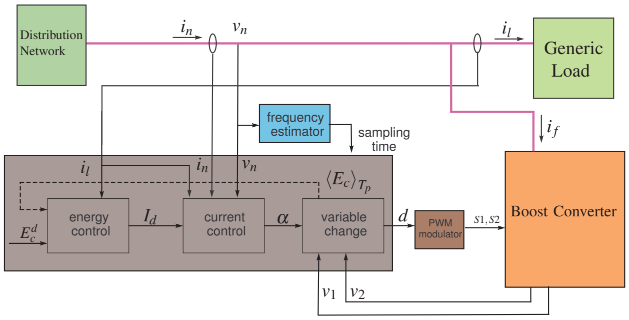


Fig. 7. Global architecture of the control system.

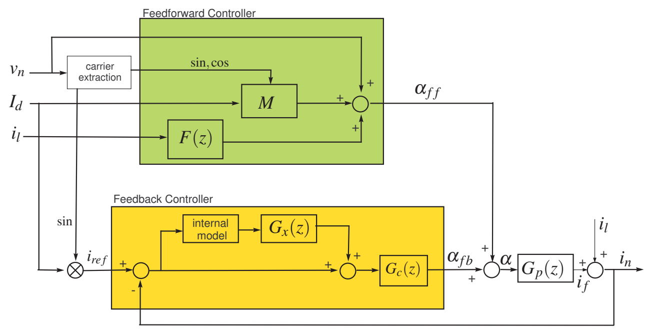


Fig. 8. Current control block diagram.

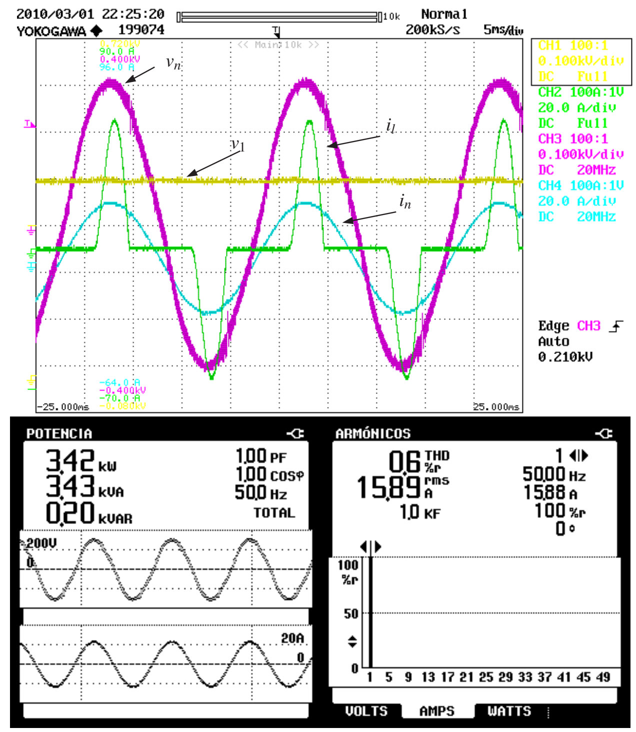


Fig. 9. Nonlinear load and the active filter connected to source (50Hz). (top) $v_n$, $i_n$, $i_l$ and $v_1$ *vs* time; (bottom) PF, $\cos\phi$ and THD for $i_n$.

## C. The current-loop controller

A linear controller is designed to force a sinusoidal shape in $i_n$. This controller consists of two parts, as pictured in Figure 8:

- A feedforward control action corresponding to the nominal control action that may keep the system tracking the desired trajectory [20].
- A feedback controller which compensates uncertainties and assures closed-loop stability. This controller uses second-order odd-harmonic repetitive control under a plug-in scheme. As the nominal period of the signal to be tracked/rejected is $T_p = 1/50$ s and the sampling period is selected to be $T_s = 5\cdot 10^{-5}$s (the PWM switching period), then $N = T_p/T_s = 400$.

  The plant discrete-time model of (11), once filtered by an anti-aliasing device with time constant $\tau$, answers to:

$$G_p(z) = \mathscr{Z}\left[\frac{-1}{Ls+r_L}\cdot\frac{1}{\tau s+1}\cdot\frac{1-e^{-T_s}}{s}\right]_{T_s} \qquad (14)$$

  which gives a minimum-phase system. The inner loop uses the lag controller

$$G_c(z) = -\frac{0.6305z-0.629}{z-0.9985},$$

  which provides a phase margin of $140^o$. Also,

$$H(z) = \frac{1}{4}z+\frac{1}{2}+\frac{1}{4}z^{-1},$$

  while $k_r = 1$ has been selected. This value allows to reach the steady state in the fastest manner. It is very important that the inner-loop is fast so it can be time-decoupled from the outer one.

## D. The energy shaping controller

The outer controller assures the mean value of the energy stored in the capacitors, $\langle E_c(t)\rangle_{T_p}$, to be close to the desired reference value, $E_c^d$, and is made up of two parts:

- A feedforward term which makes $I_d^{ff} = a_0$. This assures the energy balance in the ideal case ($r_L = 0$ and $r_C = 0$) and takes into account $i_l$ characteristics and changes instantaneously.
- A feedback term which compensates dissipative effects and system uncertainties.

  The dynamics of the plant can be modelled by an integrator and the losses in the inductor and capacitors parasitic resistances can be considered as an additive disturbance [20]. So, the PI controller

$$I_d^{fb} = k_i\frac{T_s(z+1)}{2(z-1)}\Delta E + k_p\Delta E, \qquad (15)$$

  where $\Delta E = E_c^d - \langle E_c(t)\rangle_{T_p}$, will regulate $\langle E_c(t)\rangle_{T_p}$ to the desired value $E_c^d$ with null steady-state error.

# V. Experimental setup and results

## A. Experimental setup

The experimental setup is composed of a full-bridge diode rectifier (nonlinear load), the previously described single-phase active filter and ac power source (*PACIFIC Smartsource, 140-AMX-UPC12*) that acts as a variable frequency

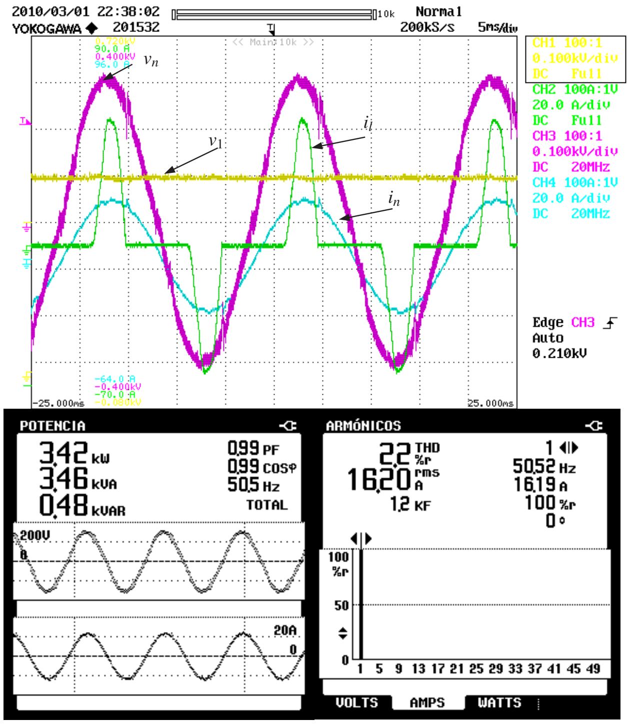


Fig. 10. Nonlinear load and the active filter connected to source (50.5Hz). (top) $v_n$, $i_n$, $i_l$ and $v_1$ *vs* time; (bottom) PF, $\cos\phi$ and THD for $i_n$.

ac source. The active filter is connected in a shunt manner with the rectifier to compensate its distorted current.

The active filter controller has been implemented on a DSP based hardware, i.e. within a digital framework, with a nominal sampling frequency equal to the switching frequency of 20 kHz.

### B. Experimental results

In the first experiment a rectifier is connected to the 50Hz ac source, $v_s$. The rectifier current, $i_l$, has a total harmonic distortion(THD) of 62.6% and an RMS value of 19.56A. As Figure 9 shows, when the active filter is connected in parallel with the rectifier the shape of the current at the source port is nearly sinusoidal, $i_n$, with a THD of 0.6%, while the power factor (PF) and $\cos\phi$ at the port are unitary. The figure shows that the mean value of $v_1$ is maintained almost constant[7].

In the next experiment the same nonlinear load is connected to a 50.5Hz ac source, $v_s$. As it can be shown in Figure 10 the PF and the $\cos\phi$ return to unitary values and the THD for $i_n$ is 2.2%.

## VI. Conclusions

A new stable second order internal model for repetitive controllers has been proposed and studied. This controller has been validated experimentally in the current loop of a shunt active filter. This repetitive controller allows the active filter to operate in a wide frequency range without reducing its performance and without the need of a frequency observer.

[7] $v_2$ is not show due the limited number of channels in the instrumentation.

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