A second order phase transition in CDT

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About this talk

Quantum Gravity is perturbatively nonrenormalizable.

One way to deal with it is to treat QG as an effective field theory. Alternatively QG may have a UV fixed point (Asymptotic Safety). There is growing evidence that the latter scenario might indeed be realized.

So far all evidence comes from functional renormalization group techniques. In this talk I describe our attempts to address the question of the existence of a UV fixed point from an orthogonal point of view, using Causal Dynamical Triangulations.

Content of this talk

- Motivation and review of the CDT framework
- Detailed investigation of the phase transitions
- Moving towards the continuum limit
- Outlook

Based on:

J. Ambjorn, S. Jordan, J. Jurkiewicz, and R. Loll, "A Second-order phase transition in CDT", arXiv:1108.3932

The relativistic particle

Standard action for the relativistic particle:

$$S[x(\xi)] = m \int_{\text{Path}} dl = m \int_0^1 d\xi \sqrt{\left(\frac{dx_\mu}{d\xi}\right)^2}$$

A classically equivalent alternative action:

$$S[x(\xi), g(\xi)] = \int_0^1 d\xi \sqrt{-g} \left(g^{-1} \frac{\partial x^\mu}{\partial \xi} \frac{\partial x_\mu}{\partial \xi} + m^2 \right)$$

g is an internal metric on the one-dimensional manifold given by the parametrization ξ. It is varied together with x to get the equations of motion.

Quantizing the relativistic particle

Using periodic boundary conditions in time direction, the covariant quantization leads to the path integral

$$Z = \int \frac{\mathcal{D}g}{\mathrm{Vol(diff)}} \int \mathcal{D}x \exp(-S[x[\xi], g[\xi]]).$$

To work with this expression we need to do the following steps:

- Discretize the particle paths by introducing a minimal length scale a.
- Calculate observables of interest, taking care of renormalization wherever necessary.
- Take the continuum limit $a \to 0$.

A path integral of gravity

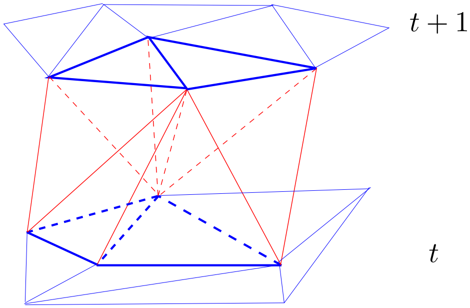
In analogy to the relativistic particle, a pure gravity path integral can be defined as follows:

$$Z = \int_{\text{geometries}} \mathcal{D}[g] \exp(iS_{EH}[[g]])$$

- Discretization can be done in a coordinate free way by introducing a simplicial complex. The sum is then effectively over equivalence classes of metrics.
- S_{EH} is the Einstein-Hilbert action.
- Which geometries should be summed over?

Causal Dynamical Triangulations (CDT)

CDT proposes to sum over causal triangulations, which are Lorentzian triangulations with a foliated structure. Each leaf of the foliation is labeled by a time parameter.



Blue links are spacelike

Red links are timelike

The Regge action

The discretized and Wick-rotated version of the Einstein-Hilbert action is called the Euclidean Regge action. In 3+1 dimensions it has the following form:

$$S^{\mathrm{eucl}}_{\mathrm{Regge}} = -(\kappa_0 + 6\Delta)N_0 + \kappa_4 N_4 + \Delta N_4^{(4,1)}$$

- κ_0: linearly related to the bare inverse Newton constant
- κ_4: linearly related to the bare cosmological constant
- Δ: defines the space/time anisotropy
- $N_0, N_4, N_4^{(4,1)}$: number of simplices of various types

The CDT path integral

The CDT prescription allows us to convert the formal expression for the continuum path integral into a form which is suitable to be explored using Monte Carlo simulations:

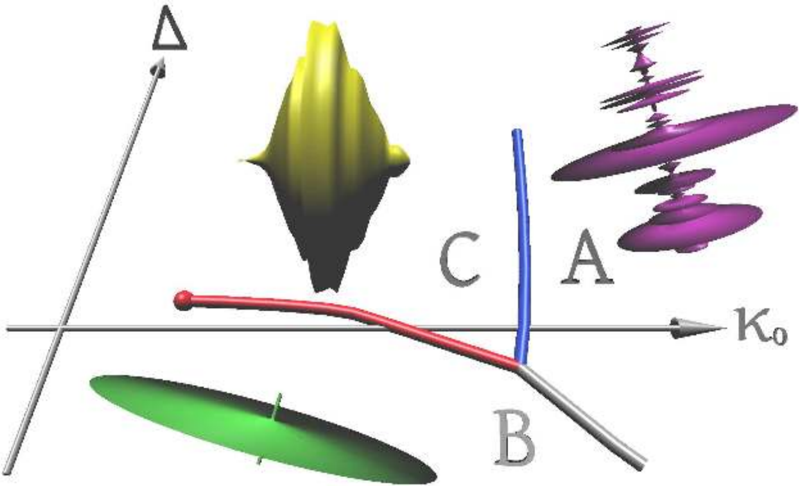
$$Z(G,\Lambda) = \int_{\text{geometries}} \mathcal{D}[g] \exp(iS_{EH}[[g]])$$

$$\Downarrow$$

$$Z(\kappa_0,\kappa_4,\Delta) = \sum_{T\in\mathcal{T}} \frac{1}{C(T)} \exp(-S^{\text{eucl}}_{\text{Regge}}(T))$$

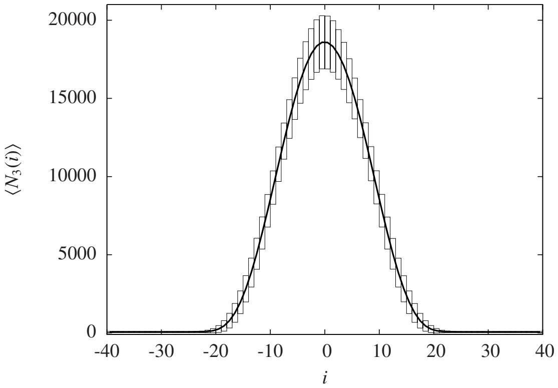
$1/C(T)$ is the measure on the space of triangulations, with $C(T)$ being the order of the automorphism group of the triangulation T.

The CDT phase diagram



Emergent de Sitter spacetime

The emergent geometries in Phase C have been successfully matched with an Euclidean de Sitter spacetime!



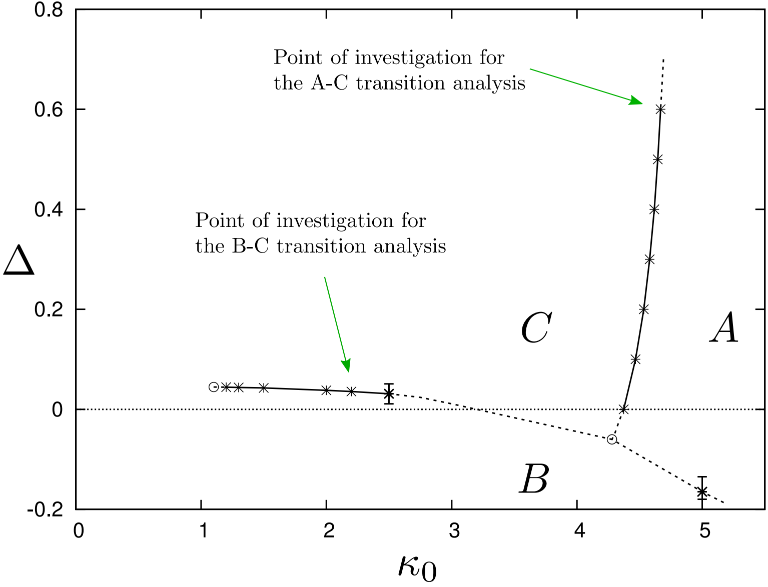
Does a continuum limit exist?

CDT has had remarkable successes in its infrared sector, by making connection to deSitter minisuperspace models.

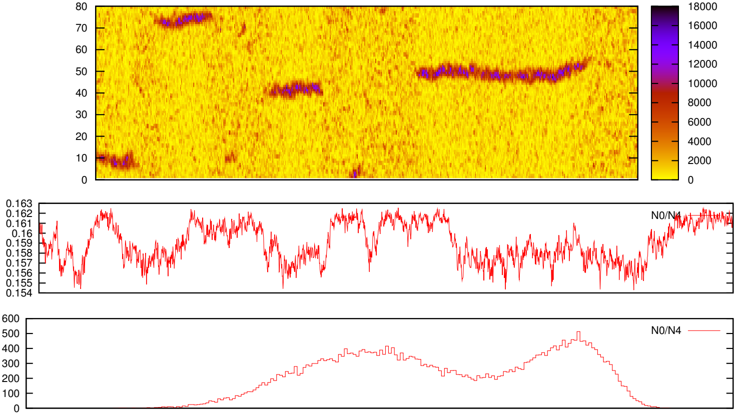
But is CDT really a regularization of a continuum theory of Quantum Gravity? In other words: can we, in analogy to the relativistic particle, take a continuum limit by sending the lattice spacing to zero?

A continuum limit is typically expected to be found at a critical point in the phase diagram. We need to look for a second-order phase transition in the CDT phase diagram.

Choosing the points of investigation

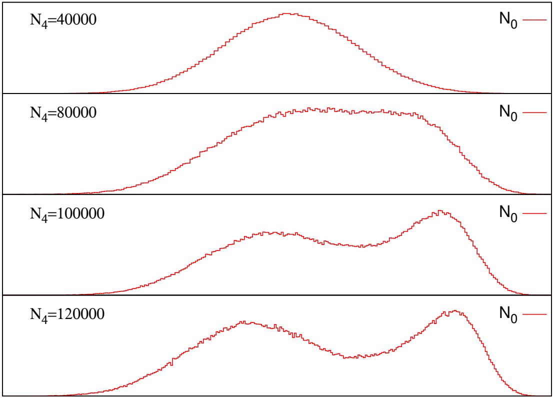


These plots show the characteristic signal of a phase transition inside the Monte Carlo simulation, namely the flipping between both phases.



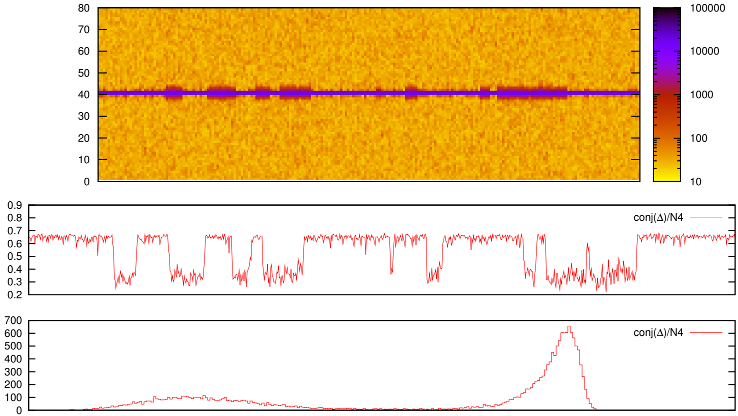
Histogram analysis at the A-C transition

We have measured the number of vertices at the A-C transition point, for various system sizes. The histograms develop a double peak signature with increasing 4-volume, thus signaling a first-order transition.



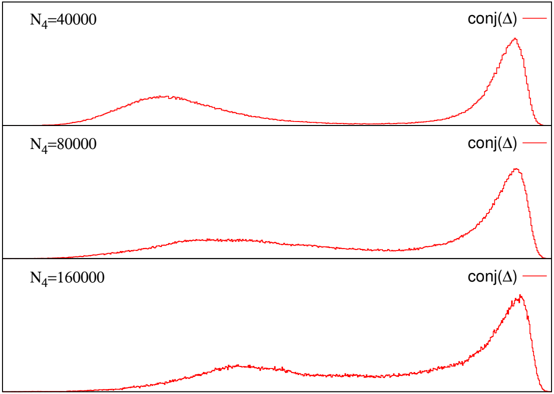
Monte Carlo evolution at the B-C transition

At the B-C transition we also observe the flipping of phases. The time extension is small even in phase C, which is the physically interesting phase.



Histogram analysis at the B-C transition

The histograms for the observable $\mathrm{conj}(\Delta) = N_4^{(4,1)} - 6N_0$ at the B-C transition show a double peak signature, but its strength decreases with increasing 4-volume. No definite conclusions can be made based on this plot.



Methods to determine the transition order

The histogram analysis left the order of the B-C transition open. Therefore we use two additional methods to clarify this issue:

- Measurement of the critical exponent which governs the shift of the phase transition with 4-volume.
- Analysis of the minima of Binder cumulants.

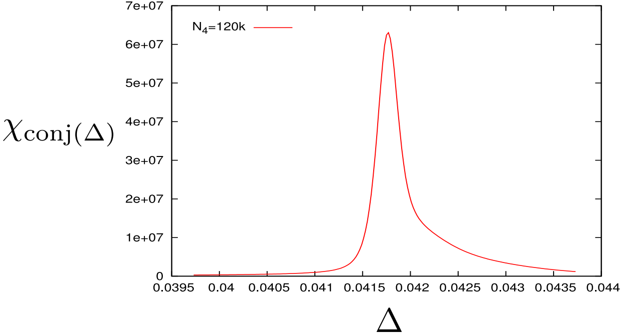
We will explain both methods and the associated results in detail on the following slides.

Transition points for finite 4-volume

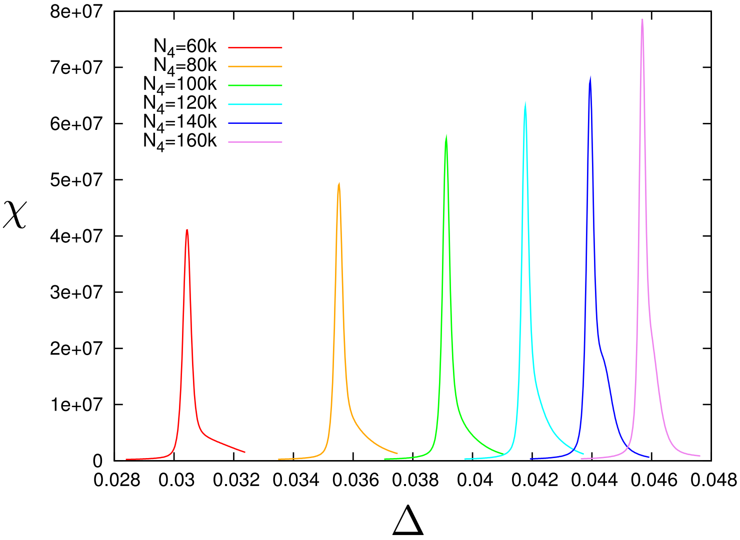
The notion of a transition point is ambiguous for finite lattices. We consider the susceptibility

$$\chi_{\mathrm{conj}(\Delta)} = \langle \mathrm{conj}(\Delta)^2 \rangle - \langle \mathrm{conj}(\Delta) \rangle^2$$

as a function of Δ and use the location of the maximum to define the transition point for that system size.



Susceptibility curves for various 4-volumes



The shift exponent

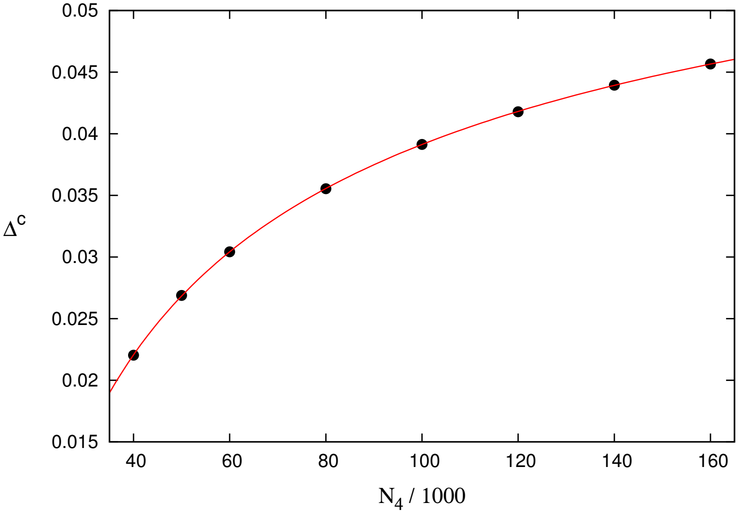
For sufficiently large system sizes, the location of the phase transition (denoted by Δ^c) as a function of system size is governed by a power-law:

$$\Delta^c(N_4) = \Delta^c(\infty) - C N_4^{-1/\tilde{\nu}}$$

The shift exponent $\tilde{\nu}$ tells us about the order of the phase transition:

$$\begin{aligned} \tilde{\nu} = 1 : &\quad \text{first order transition} \\ \tilde{\nu} \neq 1 : &\quad \text{second order transition} \end{aligned}$$

Measuring the power-law function



The shift exponent for the B-C transition

A careful analysis shows that the two data points with the smallest 4-volume lie outside the scaling region. A fit through the remaining data points yields:

$$\tilde{\nu} = 2.51 \pm 0.03$$

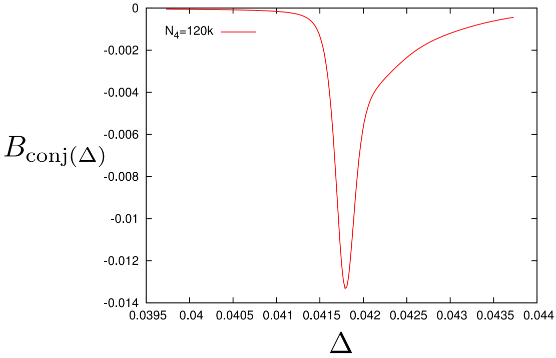
This result makes a strong case for a second order transition, since the prediction $\tilde{\nu} = 1$ for a first order transition is clearly violated.

Binder cumulants

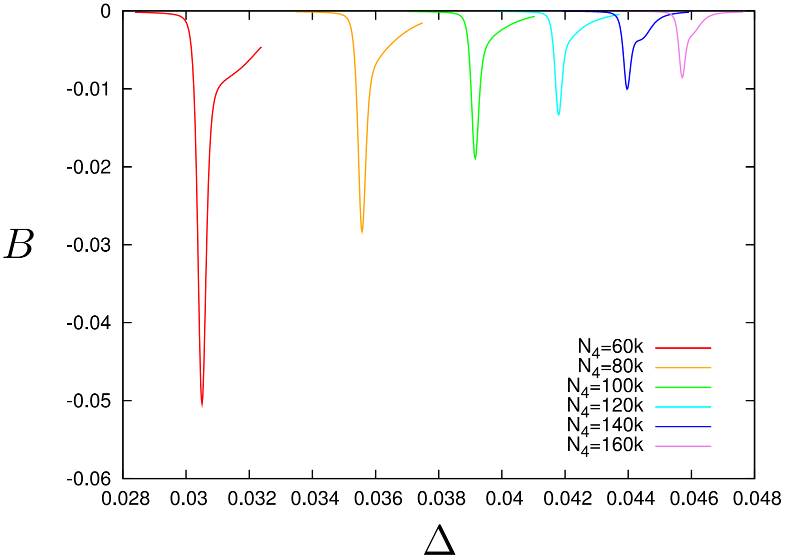
Now we consider the Binder cumulant

$$B_O = \frac{1}{3}\left(1 - \frac{\langle O^4 \rangle}{\langle O^2 \rangle^2}\right)$$

as function of Δ, which has a minimum at the transition.



Binder cumulants for various 4-volumes



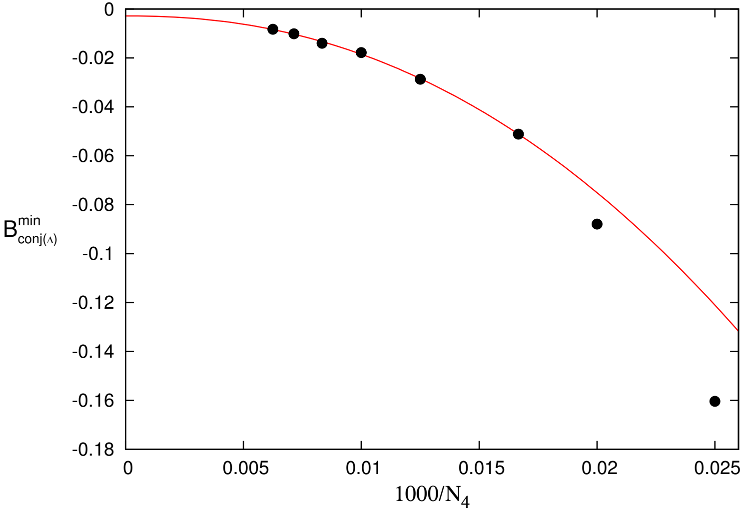
Convergence of Binder cumulant minima

The behaviour of the Binder cumulant minima in the infinite volume limit decides about the order of the phase transition:

- If the transition is first order, the BC minima either converge to a nonzero value or they don't converge at all.
- If the transition is second order, the BC minima converge to zero.

Thus we need to plot the Binder cumulant minima versus system size and extrapolate the curve (which for sufficiently large system sizes follows a power-law) to $N_4 \to \infty$.

Measuring the Binder cumulant minima



Extrapolation of the Binder cumulant minima

Performing the extrapolation of the Binder cumulant minima for various observables, we find very good agreement with a value of zero, supporting the second order nature of the transition.

Observable O	$B_O^{\min}(N_4 \to \infty)$
$\mathrm{conj}(\Delta)$	$(-3 \pm 4) \cdot 10^{-3}$
$N_4^{(4,1)}$	$(-1 \pm 3) \cdot 10^{-3}$
N_2	$(-1 \pm 3) \cdot 10^{-7}$
N_1	$(-3 \pm 7) \cdot 10^{-6}$
N_0	$(0 \pm 3) \cdot 10^{-4}$

Summary of the numerical results

The analysis of the histograms at the A-C transition showed the emergence of a double-peak signal at large 4-volumes. Thus the A-C transition is clearly first order.

The B-C transition has been analyzed by measuring the shift exponent and by extrapolating Binder cumulant minima. The results strongly support the second order nature of the B-C transition.

Looking for a continuum limit

The second order result motivates us to look for a continuum limit. Thus we would like to send the lattice spacing to zero while holding physical quantities fixed.

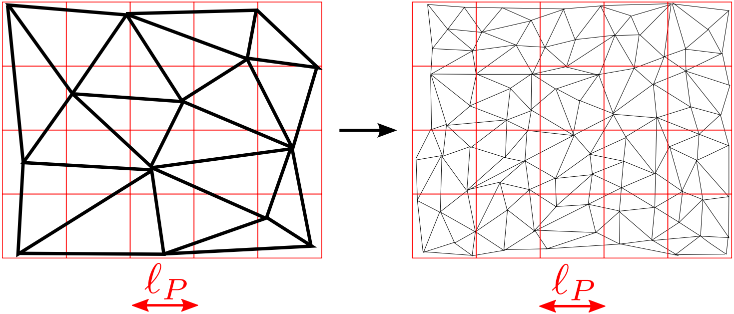
In previous works on CDT a connection was established between the lattice spacing and the Planck length, by analysing the fluctuations of the volume profile:

$$\ell_P^2 \sim \frac{a^2}{k_1(\kappa_0, \Delta, a)}$$

$k_1(\kappa_0, \Delta, a)$ is a quantity measured in the simulations.

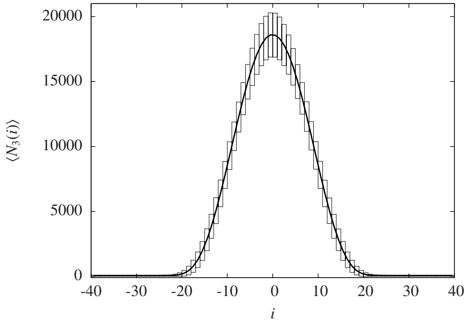
Zooming into the UV

As we move around in the phase diagram, the lattice becomes coarser or finer when we keep the Planck length fixed. Thus we can study RG trajectories of the form $(k_0, \Delta)(a, \ell_P \text{ fixed})$, a being the lattice spacing. To find a continuum limit we need to find such RG trajectories with $a \to 0$.



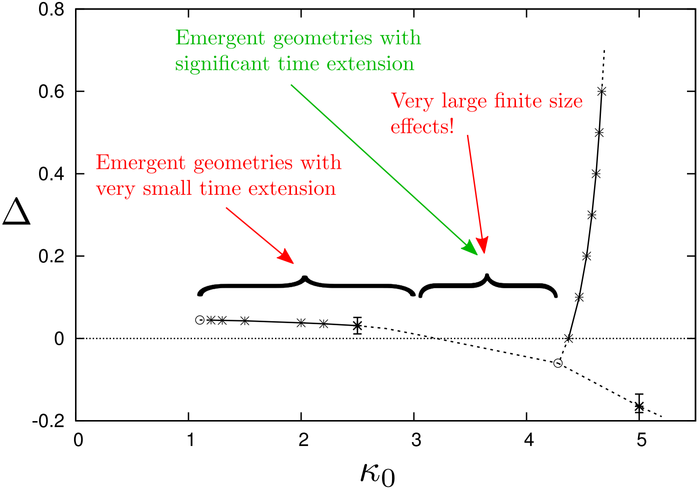
Measuring the Planck length

In order to measure the Planck length as a function of the lattice spacing we need to have emergent geometries with a significant time extension. Far away from the phase transition this is indeed the case:



Where to look for a continuum limit?

Close to the B-C transition the situation is much worse:



Outlook

The following lines of research are currently being pursued:

- Attempt to construct RG flow segments, starting away from the transition to avoid the complications discussed above (A. Kreienbuehl, J. Ambjørn, SJ).
- The enormous finite size effects at the BC transition are maybe due to the large rigidity imposed on the geometry by the foliation constraint. A generalization of CDT is currently being developed, which lacks the foliation constraint, but retains the notion of causality (SJ).

Thank you for your attention!