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Second-order Phase Transitions, Inflationary Universe, and Formation of Galaxies

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At the critical point of a second-order phase transition, statistical fluctuations are correlated and enhanced in amplitude. We explore this phenomenon in the early universe as a possible mechanism for the formation of galaxies. In an inflationary universe, such dynamical effects on galactic scales are consistent with the constraints imposed by the horizon. Spontaneous breakdown of lepton number provides a model where these ideas are realized. The two-point correlation function for density fluctuations is calculated and agrees with the observed correlation for galaxies. An estimate of the density contrast is shown to be of the required magnitude.

I. INTRODUCTION

The formation of galaxies has been a longstanding puzzle in cosmology.[1,2] How did galaxies or localized inhomogeneities of matter originate in a universe that is known to have been exceedingly homogeneous and isotropic from the very early times? Several hypotheses have been advanced over the years and most of the processes involved are well understood by now.[2] However, there are some aspects of this problem which remain unclear.

The universe to a high degree of accuracy can be described by the isotropic and homogeneous Friedmann-Robertson-Walker (FRW) metric[3]:

$$ds^2=dt^2-R^2(t)\left[\frac{dr^2}{1-kr^2}+r^2d\theta^2+r^2\sin^2\theta\, d\phi^2\right] . \tag{1}$$

There are arguments to suggest that the evidence for homogeneity and isotropy goes back to at least the grand-unified-theory (GUT) phase transition. The dissipation of any anisotropy surviving this phase transition would have produced $\sim 10^{40}$ photons per baryon as against an observed value of $10^{9\pm1}$ photons per baryon.[4] Further, observations of the cosmic microwave background show that fluctuations are less than 1 part in 10 000 in magnitude.[5] On the other hand, the material inhomogeneity of the universe as measured by the density contrast

$$\delta=\frac{\rho-\bar{\rho}}{\bar{\rho}} , \tag{2}$$

where ρ is the density of matter and $\bar{\rho}$ is the average density, is in the range 1–10, the value depending on whether one evaluates it at the scale of galaxies or galactic clusters. If we trace the evolution via gravitational forces of δ back, this present-day value shows that we should have a contrast of $\sim 10^{-4}$ at the epoch when radiation and matter decouple.[2] A typical galaxy has about 10^{68} baryons and statistical fluctuations on this scale have a contrast of only $\sim 1/\sqrt{N} \sim 10^{-34}$. Thus generating fluctuations of the required contrast is a problem. Of course, one could assume, and it is generally true, that δ starts growing not from the time of decoupling but from an earlier epoch. The problem then is a lack of a natural choice of initial time. An early start like the Planck time (10^{-43} sec after the big bang) produces too much growth and may lead to a present-day world of black holes only while some natural later epochs like the Compton time of baryons ($\sim 10^{-23}$ sec) give insufficient time for growth.[1]

The intergalactic correlation function, regarding galaxies as density perturbations, has been empirically estimated to have a simple power-law spectrum[2,6]:

$$\langle n(\vec{x})n(\vec{y})\rangle \simeq \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\xi(k) ,$$
$$\xi(k)=\frac{1}{k^\alpha},\quad \alpha=1.2\pm0.1 . \tag{3}$$

This equation is reminiscent of second-order phase transitions where it is well known that at the critical point long-range power-law-type correlations can develop. Further, the amplitude of statistical fluctuations is enhanced at a critical point and this could give us the required increase in the initial contrast. Thus we look for a second-order phase transition as a possible solution to the problem of generating fluctuations of high contrast.

In a gauge theory, the absorption and emission of gauge bosons in a thermal bath modifies the Higgs potential so as to develop potential barriers between local minima, leading in general to a first-order phase transition.[7] The GUT phase transition is expected to be strongly first order in character while the breakdown of the weak group

$$G_W = \mathrm{SU}(2)\times \mathrm{U}(1) \rightarrow \mathrm{U}(1)_{\mathrm{EM}}$$

is most likely a weakly first-order phase transition.[8]

One possibility for a second-order phase transition in the early universe is spontaneous breakdown of lepton number (acting as a global symmetry on the Lagrangian). This has been suggested as a possible mechanism for generating neutrino masses.[9,10] There have been diverse experimental indications that neutrinos might have a mass of a few eV.[11] Theoretical considerations have enhanced the attractiveness of the concept for a number of reasons. Neutrino masses emerge quite naturally in some of the grand unified theories.[12] Further, if the neutrinos have a mass of about 10 eV, they may be the dominant component in the mass density of the universe and may condense into galactic halos.[13] This would give a consistent picture of the dark matter in the universe and gravitational stability of galactic clusters (in the sense of the virial theorem). The kinematic and gravitational effects of neutrino masses also affect theories of galaxy formation. The increased mass density helps to amplify density perturbations.[14] A potential contradiction between theories of galaxy formation based on adiabatic perturbations[1,2] and observations might also be resolved by massive neutrinos. The predictions of these models for the fluctuations of the 2.7-K background radiation seem to be on the edge of contradiction with the experimental limits.[5] In theories with massive neutrinos one can start out with a smaller contrast and avoid this problem.[15]

Spontaneous breakdown of lepton number, in addition to these effects, has the dynamical effect that at the phase-transition point long-wavelength statistical fluctuations tend to get correlated and enhanced. An estimate of the enhancement shows that it is of adequate magnitude. The fluctuations can be gravitationally transmitted to the baryon-photon medium to form protogalaxies which grow by gravitational instability (or the Jeans-Lifshitz mechanism) to the present-day structures. In Sec. II, we discuss models with spontaneous breakdown of lepton number and compute the two-point correlation function of the leptonic current. The result is very close to the empirical estimate in Eq. (3). Section III gives an estimate of the contrast. This requires a brief discussion of the inflationary universe. Since the length scale of a galaxy at the transition point exceeds the horizon scales of the standard FRW universe at that time, the correlations, which are a dynamical effect, can be generated at these scales only in an inflationary-universe scenario. Section IV gives a summary of our results and an assorted set of remarks on various aspects of the problem.

This paper is an expanded version of a previously circulated report.[16]

II. LEPTON-NUMBER VIOLATION AND THE CORRELATION FUNCTION

Two models of generating neutrino masses by spontaneous breakdown of lepton number have been proposed. We shall refer to these as the "singlet model"[9] and the "triplet model."[10] A combination of the two models is also possible.[17] In the singlet model, one introduces fermions (of right chirality) which are singlets under the standard gauge group

$$\mathrm{SU}(3)_C \times G_W = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \ .$$

These singlet fermions, denoted ν_R, will be referred to as right-handed neutrinos or heavy neutrinos since the physical heavy neutrinos which will emerge are predominantly composed of these. In addition to the usual Higgs field, which is a weak doublet, there is a singlet Higgs field ϕ which carries lepton number -2. This allows invariant neutrino couplings of the form

$$\mathscr{L}_I = g(\nu_R^T \sigma_2 \nu_R \phi + \nu_R^\dagger \sigma_2 \nu_R^* \phi^*) \ . \tag{4}$$

The singlet field develops a large vacuum expectation value $\langle \phi \rangle$ which is much larger than the weak breaking scale (~ 250 GeV) but less than GUT scales. This breaks the lepton-number symmetry of the Lagrangian and gives a Majorana-type mass to the ν_R's. The breakdown of the weak group G_W to electromagnetism $[\mathrm{U}(1)_{\mathrm{EM}}]$ gives a Dirac mass term connecting ν_R's to the light, left-chirality neutrinos ν_L. The physical states are mixtures of these of the form

$$N_R = \nu_R + \frac{m}{M}\nu_L, \quad N_L = \nu_L - \frac{m}{M}\nu_R \ , \tag{5}$$

where m is the Dirac mass, M is the Majorana mass, $M = g\langle \phi \rangle$, and $m \ll M$. N_R are heavy neutrinos of mass $\simeq M$ and N_L are light neutrinos of mass $\simeq m^2/M$. Since lepton number is initially a good global symmetry, its spontaneous breakdown produces a physical Goldstone boson called the Majoron. The Majoron and the heavy neutrinos couple to ordinary matter only through the mixing terms. This has two consequences: The long-range force between ordinary leptons (say electrons) due to exchange of the massless Majoron is suppressed sufficiently so as to meet the experimental bounds on long-range forces.[18] Secondly, in the hot early phase of the universe, the heavy neutrinos remain more or less stable (i.e., except for higher-order processes and many-body final-state decays) even after lepton number breaks down. Since their predominant mode of decay is into light neutrinos by Majoron emission, the decay process is possible only after the weak group breaks down.

In the triplet model, there are no singlet neutrinos. A Higgs field H, which is a triplet under the weak group and which carries lepton number -2, is introduced. The ordinary left-handed neutrinos directly acquire a Majorana mass through couplings of the form

$$\mathscr{L}_I = -\alpha(\bar{l}_L^c)Hl_L + \text{H.c.} \; , \tag{6}$$

where l_L is the standard lepton doublet and

$$H = \begin{bmatrix} \frac{h^+}{\sqrt{2}} & h^{++} \\ h^0 & -\frac{h^+}{\sqrt{2}} \end{bmatrix} . \tag{7}$$

The neutral component of H, viz., h^0, acquires a vacuum expectation value. Again the long-range Majoron force can be kept below experimental limits. However, Majoron emission can be a significant mode of energy loss from stars. As red giant stars are not seen to die too quickly by this mechanism, there is the astrophysical constraint $\langle h^0 \rangle \lesssim 100$ keV.[19] We shall see later that this constraint is too severe; we are not able to generate enough contrast for the density perturbations with this value of $\langle h^0 \rangle$. Thus although much of the discussion in this section would apply to both models, we shall concentrate more on the "singlet" scheme.

A combination of these two models, extension to a multigenerational scheme, and embedding in a grand unified theory, are possible.[17,20]

We shall now consider the density perturbations at the transition point. The terms in the Lagrangian relevant to this discussion can be written as

$$\mathscr{L} = -\bar{N}[\gamma\cdot\partial + \mu\gamma_0\gamma_5 - g(\phi+\phi^*) - g\gamma_5(\phi-\phi^*)]N$$

$$- |\partial\phi|^2 - V(\phi) \; , \tag{8a}$$

$$V(\phi) = \lambda(\phi^*\phi - v^2)^2 \; , \tag{8b}$$

where N is a Majorana spinor[21]

$$N = \frac{1}{\sqrt{2}} \begin{bmatrix} \nu_R \\ \sigma_2\nu_R^* \end{bmatrix} . \tag{9}$$

Since lepton number is a good symmetry at temperatures above the transition point T_c, we have to introduce a chemical potential μ corresponding to nonzero lepton-number density. We shall work in Euclidean space with periodicity (or antiperiodicity) of the fields along imaginary time of period $\beta = 1/T$.[7,8,22]

As remarked in the Introduction, the phase transition connecting the vacuums with $\langle\phi\rangle = 0$ and $\langle\phi\rangle \neq 0$ will be of the second order since lepton number is not gauged. This follows from the fact that the theory can be modeled near the critical point by a pure scalar Higgs theory in three dimensions.[23] The major contribution to the thermal corrections to $V(\phi)$ is from Higgs self-couplings since the fermion-Higgs coupling g can be chosen small enough compared to λ so as not to affect $V(\phi)$ significantly. The phase transition occurs at a critical temperature $T_c \simeq \sqrt{6}v$. Further at T_c, the Higgs field ϕ will be massless. The theory contains no mass parameters and long-range correlations are possible.

The Noether variation of $\mathscr{L}$ under the phase transformation $\nu \to \nu e^{i\theta}$ gives the contribution of the fermions N to the leptonic current as

$$J_\mu(x) = i\bar{N}\gamma_\mu\gamma_5 N \; . \tag{10}$$

$J_0(x)$ can be taken as a measure of the number density of the fermions. This interpretation is good for $T \geq T_c$ since lepton number is a good symmetry and approximately good even for a short range of temperatures below T_c. This follows from the fact that as the system cools below T_c, although lepton-number-violating processes begin, they are initially suppressed by mass over energy (m/E) factors.[21] At temperatures corresponding to $m/E \simeq 1$ (i.e., when the fermions become nonrelativistic), our interpretation would be invalidated. [Analogously, since lepton number is not conserved we have $\mu = 0$ at $T \ll T_c$, but $\mu \neq 0$ for $T \geq T_c$. μ starts decreasing as T drops below T_c, being reduced to zero by temperatures at which $m/E \sim 1$. Upon heating the system, the reverse occurs but $\mu(T)$ could in general follow a different path giving a hysteresis behavior.] Thus to study the density fluctuations we have to compute the correlation function $\langle J_0(\vec{x})J_0(\vec{y})\rangle$ at the critical point T_c.

The perturbative expansion of this function is diagrammatically represented in Fig. 1. The integrals lead to power-law infrared divergences, the situation worsening as more and more ϕ propagators are included. In fact, if we have j ϕ propagators, the divergence is as η^{1-j} where η is the momentum scale. The reason for these divergences is very simple. In a Fourier expansion of ϕ as

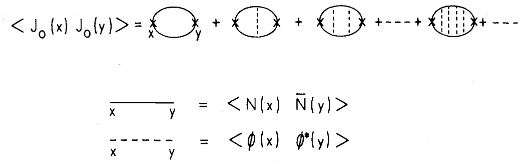

FIG. 1. Perturbative expansion for the two-point correlation function for lepton-number density.

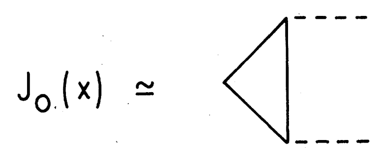

FIG. 2. Diagram expressing lepton-number density in terms of Higgs fields.

$$\phi(x)=\sum \phi_n(\vec{x})e^{-i\omega_n \tau}, \tag{11}$$

where $\omega_n=2\pi nT$, the mode $n=0$ has no natural infrared cutoff since the effective temperature-dependent mass term for ϕ vanishes at T_c. In other words, the ϕ propagator $\langle \phi(\vec{x})\phi(\vec{y})\rangle$ scales as η instead of the canonical η^2 behavior. The prescription to deal with these divergences is to "integrate out" all fields except the $n=0$ mode of ϕ. This gives an effective field theory in $3=4-\epsilon$ dimensions. Calculations can then be done in a perturbative ϵ expansion.[8,23] This technique which has been used for scalar and gauge theories requires only a slight extension to include composite operators of fermions. We have to express every operator of interest in terms of the $n=0$ mode of ϕ. This is essentially identical to the Zimmermann-Lowenstein normal-product definition of composite operators with the trivial difference that we have a temperature-dependent field theory.[24] Thus we look for a representation of $J_0(\vec{x})$ in terms of the $n=0$ mode of ϕ. To lowest order in g, this is given by the triangle diagram of Fig. 2, whose low-momentum behavior is

$$J_0(\vec{x})\simeq i\frac{8g^2}{T}\phi_0(\vec{x})^*\phi_0(\vec{x}) \times \int \frac{d^3k}{(2\pi)^3}\sum_n \frac{(\omega_n'-i\mu)^3}{[(\omega_n'-i\mu)^2+\omega_k{}^2]^3}, \tag{12}$$

where $\omega_k{}^2=|\vec{k}|^2$. The summation over n refers to the different fermionic modes circulating around the triangle and ω_n' are the fermionic Matsubara frequencies $\omega_n'=2\pi(n+\frac{1}{2})T$. The sum can be performed by the standard trick of transforming it to a complex integral,[22]

$$J_0(\vec{x})\simeq \frac{3g^2}{2T}\phi_0(\vec{x})^*\phi_0(\vec{x}) \times \int \frac{d^3k}{(2\pi)^3}\frac{1}{\omega_k}[n_k(1-n_k)-\bar{n}_k(1-\bar{n}_k)], \tag{13a}$$

$$n_k=\frac{1}{e^{(\omega_k-\mu)/T}+1}, \quad \bar{n}_k=\frac{1}{e^{(\omega_k+\mu)/T}+1}. \tag{13b}$$

n_k and $\bar{n}_k$ are the occupation numbers for the fermions and antifermions, respectively. At the high temperatures we shall be concerned with, it is reasonable to neglect $n_k{}^2$ and approximate Eq. (13a) as

$$J_0(\vec{x})\simeq \frac{3g^2}{2T}\phi_0(\vec{x})^*\phi_0(\vec{x}) \times \int \frac{d^3k}{(2\pi)^3}\frac{1}{\omega_k}(n_k-\bar{n}_k) \tag{14a}$$

$$\equiv \frac{3g^2}{2T}\phi_0(\vec{x})^*\phi_0(\vec{x})n\left\langle\frac{1}{\omega_k}\right\rangle \Delta L, \tag{14b}$$

where $n=N/V$ is the mean number density of fermions; it is a constant over space. $\Delta L=(n-\bar{n})/n$ is the lepton asymmetry, and $\langle 1/\omega_k\rangle$ is defined by the equality of (14a) and (14b). The propagators for the $n=0$ mode of ϕ is given by

$$\langle \phi_0(\vec{x})\phi_0(\vec{y})^*\rangle = \int \frac{d^3k}{(2\pi)^3}e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\left[\frac{T}{\omega_k{}^2}\right]. \tag{15}$$

Using this we can calculate $\langle J_0(\vec{x})J_0(\vec{y})\rangle$ [we use expression (14a) for one J_0 and expression (14b) for the other; bringing out one factor of n facilitates integration over $\vec{x}$ and $\vec{y}$ which we shall do later]:

$$\langle J_0(\vec{x})J_0(\vec{y})\rangle \simeq \left[\frac{3g^2}{8\pi}\right]^2 n(\Delta L)^2 \times \int \frac{d^3k}{(2\pi)^3}e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\xi(k), \tag{16}$$

where

$$\xi(k)=\frac{T_c}{|\vec{k}|}. \tag{17}$$

The enhancement of the long-wavelength fluctuations is now manifest. $\xi(k)$ shows typical critical-point behavior $1/k^\alpha$ with an index $\alpha=1$.

The correlation function for density fluctuations, viz.,

$$\langle [n(\vec{x})-n][n(\vec{y})-n]\rangle$$

is given by the connected part of $\langle J_0(\vec{x})J_0(\vec{y})\rangle$. In Eq. (16) we have written down only the connected part. There are, however, more terms in the connected part itself, the most significant of which is the ideal-gas result $n\delta(\vec{x}-\vec{y})$. (This corresponds to the fermion loop with no ϕ propagators in Fig. 1.) These contributions are negligible compared to what Eq. (16) gives.

The density fluctuations are passed onto the baryon-photon component as we discuss below. Since the contrast $\delta \ll 1$ all through the times we are interested in, linear perturbation theory gives an adequate description of the growth of δ. Each mode grows by itself and we expect the spectral behavior of Eqs. (16) and (17) to be preserved to the present epoch. Comparing with the correlation function of Eq. (3), we see that our lowest-order result $\alpha=1$ is in good agreement with the experimental value $\alpha=1.2\pm0.1$. From the theory of the ϵ expansion,[25] it is obvious that corrections of higher order in g and ϵ will modify our lowest-order result slightly.

III. ESTIMATE OF CONTRAST AND INFLATIONARY UNIVERSE

We carry out the Fourier transform in Eq. (16) to get

$$\langle J_0(\vec{x})J_0(\vec{y})\rangle \simeq \left[\frac{3g^2}{8\pi}\right]^2 \frac{n(\Delta L)^2}{2\pi^2}\frac{T_c}{|\vec{x}-\vec{y}|^2} . \tag{18}$$

(As in the case of the Coulomb potential the integral involves a limiting procedure.) Integrating $\vec{x}$ and $\vec{y}$ over a volume $4\pi l^3/3$ and taking the square root we get the contrast for a fluctuation of linear size l:

$$\delta=\frac{\Delta N}{N}\simeq\frac{3g^2}{4\pi}\left[\frac{T_c l}{2\pi}\right]^{1/2}\frac{|\Delta L|}{\sqrt{N_l}} . \tag{19}$$

N_l is the number of particles in the volume. There is an extra ideal-gas contribution to δ which is equal to $1/\sqrt{N_l}$. The enhancement of δ comes from the factor $(T_c l)^{1/2}$.

We now turn to numerical estimates. The coupling constant g^2 is to be evaluated at T_c by use of the renormalization-group equations in the ϵ expansion.[23] Since the critical point is a fixed point of the "flow" generated by the renormalization group, $g^2(T_c)$ is numerically fixed instead of being related to its value at another temperature. For estimation purposes, we shall assume that $g^2/4\pi$ is at least $\sim 10^{-2}$.

To generate a nonzero contrast, we should have a lepton excess. Any GUT scenario of baryogenesis which preserves $B-L$ naturally generates a lepton excess also. This would be equipartitioned among particle species carrying lepton number and the neutrinos get their share.[26] We consider the model in the context of a grand unified theory. However, for our purpose any history of the universe which generates a lepton excess prior to T_c is sufficient. One could also rely on lepton excess as an observational fact and use an appropriate value for ΔL. Thus in any case we can take $\Delta L=\Delta B\simeq 10^{-8}$.

Immediately before the decoupling of matter and radiation (which occurs at a temperature of about 0.3 eV), the dissipative effects of photon diffusion reach a peak. This effect, which is due to the increase of Compton cross section at low energies, tends to wash out all density perturbations on a scale below $\sim 10^{12}M_\odot$ (Refs. 3 and 27); $M_\odot$ is the mass of the Sun. Although slightly below the peak value of the Jeans mass, perturbations above this scale (sometimes called the Silk mass) can survive into matter-dominated era. Thus we have to consider fluctuations of this scale.

Assuming the present density of matter to be about $\frac{1}{10}$ of the critical density, i.e., $\rho_b\simeq 0.1\rho_c$, a mass scale of $10^{12}M_\odot$ implies a length scale of $l_0\simeq 10$ Mpc. l in Eq. (19) is the length scale of the fluctuation at the critical point T_c; it is thus $l_0\simeq 10$ Mpc appropriately scaled down. The adiabatic nature of the expansion of the universe gives $lT=$const, so that

$$T_c l = T_0 l_0 \simeq 10^{26} , \tag{20}$$

where we have taken T_0, the present neutrino temperature, to be ~ 1 K. Notice that $T_c l$ is independent of T_c.

With these numbers, Eq. (19) gives $\delta\simeq 10^3/\sqrt{N_l}$. $10^{12}M_\odot$ corresponds to $\sim 10^{69}$ baryons. In the radiation-dominated era, before particle-antiparticle annihilations, this corresponds to $\sim 10^{77}$ particles of any species which is relativistic ($m \ll T$) at that temperature. Thus $N_l\simeq 10^{77}$ and $\delta\simeq 10^{-36}$. This is at the transition point T_c. We have to study the evolution of δ up to decoupling of matter and radiation.

Since fluctuations on the scale of $10^{12}M_\odot$ cannot be accommodated within the standard cosmology based on the FRW model, we must consider an inflationary-universe scenario. To see how this failure occurs, we integrate the blackbody distribution formula over the volume within the horizon. This gives the number of photons (or any other species relativistic at T) within the horizon in the radiation-dominated era as

$$N_\gamma=\frac{8\zeta(3)}{3\pi^7}\left[\frac{45}{32\pi G T^2}\right]^{3/2}\simeq 10^{-2}\left[\frac{1}{GT^2}\right]^{3/2} . \tag{21}$$

At a temperature of $\sim 10^3$ GeV, $N_\gamma \simeq 10^{45}$. Thus a fluctuation of 10^{77} particles extends far beyond the horizon. Since dynamical effects are confined to within the horizon by requirements of causality, the enhancement of the contrast cannot be produced on the scales we need in the FRW universe.

This is one aspect of the horizon problem. The inflationary universes which were designed to solve the horizon, flatness, and monopole problems can solve this problem as well.[28] For the sake of completeness, we recall some salient features of these models. The key ingredient is a strongly first-order phase transition in the early universe. This is usually identified as the GUT phase transition. Above this transition point $T_{\rm GUT}$ ($\simeq 10^{14}$ GeV), the universe exists in the symmetric Higgs vacuum ($\langle \Sigma \rangle = 0$) of the grand-unification gauge group. Below $T_{\rm GUT}$, $\langle \Sigma \rangle = 0$ remains a local minimum, but the symmetry-breaking minimum ($\langle \Sigma \rangle \neq 0$) is energetically favored (i.e., it is the true vacuum). The transition from $\langle \Sigma \rangle = 0$ to $\langle \Sigma \rangle \neq 0$ is not immediate since it has to proceed by tunneling effects. This gives the supercooling characteristic of first-order phase transitions. If, in addition, the symmetry breaking is entirely radiatively induced, i.e., it is of the Coleman-Weinberg type, the transition to the true vacuum is considerably delayed and the universe persists in a metastable state close to the symmetric vacuum for a long time ($\sim 10^{-30}$ sec). With the vacuum energy normalized to zero in the present universe, the metastable state enjoys a large cosmological constant [$\Lambda \simeq 10^{20}$ (GeV)^2] and the universe undergoes exponential expansion (inflation) by factors as large as $\sim 10^{500}$ over this time.

The universe in this picture originates from a single bubble of true vacuum. The transition is expected to be completed by the time the universe cools to $\sim 10^6$ GeV when the cosmological constant Λ disappears. The radius of the universe ($\simeq$ the horizon distance) would have increased to $\sim 10^{500}$ light years by this time. The release of latent heat reheats the universe to $\sim 10^{14}$ GeV producing a large number of photons and other particles with a local density $\sim T^4$. The universe can be described thereafter by the FRW metric and the subsequent cooling occurs adiabatically as in standard cosmology. We may note that by adiabatically continuing backward this second leg of cooling, one can construct a fictitious FRW history of the universe within the inflationary universe, although with a shifted origin of time. Time-temperature conversions, which we need later, can thus be done with the formalism of standard FRW cosmology.

The lepton-number-violating phase transition occurs during the second leg of cooling. A fluctuation on the scale $10^{12} M_\odot$ which corresponds to a distance scale $\sim 10^6$ cm is well within the horizon.

Questions of detail like the nature of the exit from inflationary expansion, whether GUT transition is radiatively induced or not, etc., remain unclear. However, it seems imperative to have an inflationary universe to solve the horizon and flatness problems. We need only the metrical properties and our considerations are insensitive to these details.

Since the FRW metric is applicable, the first-order (linear) perturbation equations are the standard ones[2,3]:

$$\dot{\delta}_1 - (1+\kappa_1)\frac{\dot{h}}{2} = 0 , \tag{22a}$$

$$\dot{\delta}_2 - (1+\kappa_2)\frac{\dot{h}}{2} = 0 , \tag{22b}$$

$$\ddot{h} + 2\frac{\dot{R}}{R}\dot{h} = 8\pi G[(1+3\kappa_1)\rho_1\delta_1 + (1+3\kappa_2)\rho_2\delta_2] . \tag{23}$$

Here the subscripts 1 and 2 refer to the heavy neutrinos and the baryon-photon medium, respectively. $\kappa = p/\rho$ where p is the pressure. h is the metrical perturbation defined by

$$\sqrt{-g} = R^3(1-h/2) . \tag{24}$$

We have neglected pressure-gradient terms in these equations. In the standard FRW cosmology, this is equivalent to demanding that the scale of the fluctuations be larger than the horizon distance, $l \gg t$. In our case, l is much less than the horizon distance. Nevertheless, pressure gradients can be neglected since $M = 10^{12} M_\odot \gg M_J$, the Jeans mass. We have also set $\vec{\nabla}\cdot\vec{V} = 0$ ($\vec{V}$ is the fluid velocity), since we are interested in the fastest growing mode (cf. Secs. 85 and 86 of Ref. 2).

The growth of δ is the fastest if the expansion is radiation dominated ($\rho \sim T^4$, $R \sim t^{1/2} T \sim t^{-1/2}$). We now discuss whether this is a possibility. The heavy neutrinos, once G_W breaks, decay with a lifetime of $\sim 10^{-10}$ sec.[9] This corresponds to cooling to ~ 130 GeV. If $m_{\nu R} \lesssim 130$ GeV, then $\rho_\nu \sim T^4$ up to ~ 130 GeV. After the decay of ν_R's, we have only relativistic light neutrinos, charge baryons, leptons, photons, Majorons, etc. Thus by choosing $m_{\nu R} \lesssim 130$ GeV, we can assure that the expansion is dominated by radiation up to the epoch of decoupling of matter and radiation. This also gives $\kappa_1 = \kappa_2 = \kappa$.

Equations (22) give, with the initial conditions $\delta_1 \simeq 10^{-36}$, $\delta_2 = 0$,

$$\delta_1 \simeq 10^{-36} + \delta_2 . \tag{25}$$

Thus δ_2 catches up with δ_1 in a short time, differing by less than 1% by the time δ_1 increases by a factor of 100. [As will be clear from Eq. (26) below, this happens by the time T drops by a factor of 10.] Thus we can treat neutrinos, baryons, photons, etc., as a single fluid. The growing mode solution to Eqs. (22) and (23) is then

$$\delta \sim t \sim \frac{1}{T^2} . \tag{26}$$

The contrast of the baryon medium at decoupling can thus be written as

$$\delta \text{ (at decoupling)} \simeq 10^{-36} \left[\frac{T_c}{0.3\text{eV}} \right]^2 . \tag{27}$$

The required contrast of 10^{-4} is easily obtained for $T_c \gtrsim 10^6$ GeV.

This is a "natural" value for the singlet model[17]; the triplet model with $T_c \lesssim 100$ keV cannot give us enough contrast.

IV. SUMMARY REMARKS

In summary, we give a chronicle of events. After the big bang, the universe supercools to about 10^6 GeV undergoing exponential expansion. The GUT transition then occurs and the universe is reheated to $\sim 10^{14}$ GeV. Baryogenesis follows and creates a lepton excess as well. During the subsequent cooling, the lepton-number-violating phase transition occurs at $T_c \gtrsim 10^6$ GeV. This gives enhancement of the contrast for long-wavelength statistical fluctuations in the ν_R medium. The contrast is passed onto the baryon-photon medium by gravitation. We get protogalaxies as density perturbations of the neutrino-baryon-photon medium. At ~ 250 GeV, $G_W = SU(2)_L \times U(1)$ breaks, ν_R's mix with ν_L's, and the heavy neutrinos decay away by Majoron emission. The density perturbations in the baryon-photon medium grow into galaxies.

A number of comments are in order.

We would like to emphasize that although most of the discussion was based on spontaneous violation of lepton number, the enhancement on contrast and most of the other results will be true for any second-order phase transition. Since GUT and G_W phase transitions are of the first order, the Majoron model seems to be the only presently available candidate.

Even though GUT's give explicit lepton-number-violating interactions, the superheavy particles mediating these decays would have decoupled by T_c giving us an effective lepton-number-conserving theory. The corrections are relatively insignificant.

The renormalization-group equations in the ϵ expansion are of the form[23]

$$\frac{d\lambda}{d\tau} = \beta_\lambda(\lambda, g, \epsilon), \quad \frac{dg}{d\tau} = \beta_g(\lambda, g, \epsilon) , \tag{28}$$

where $\tau = -\ln |T - T_c|$. Although $g^2(T_c)$ is fixed by the vanishing of the β function, $g^2(T=0)$ is an independent parameter since the integration of these equations requires an arbitrary constant. Thus we have the freedom of choosing $T_c \gtrsim 10^6$ GeV along with $m_{\nu R} \lesssim 130$ GeV.

The model discussed in this paper, it should be emphasized, is only one among several possibilities. The problem of generating fluctuations of adequate contrast has a long history.[1,2] Models range from quantum fluctuations within the first 10^{-43} sec to early anisotropy and white noise fluctuations.[29] There are also models based on primordial black holes, vacuum strings, vacuum walls, and other topological exotics.[30] Also the power-law spectrum of the correlation function in Eq. (3) with $\alpha \simeq 1.2$ can be generated by self-similar clustering of random uncorrelated fluctuations. (This is a scale-invariant solution of the Bogoliubov-Born-Green-Kirkwood-Yvon hierarchy.[2,31])

We have to leave open two minor questions. Our use of the imaginary time formalism obscures the real time development of the phase transition. This aspect is important to determine at what scales our expression for $\langle J_0(x) J_0(y) \rangle$ will apply. If this scale is too small, we will still have fluctuations of high contrast but the correlation function (3) will have to be generated by some other mechanism, e.g., self-similar clustering. One may still have dynamical correlations if there are secondary inflationary phases due to first-order transitions other than the GUT transition, occurring after the second-order transition of interest. In this case, length scales would have to be redefined. This is possible in theories with intermediate mass scales.

Finally, we emphasize that by solving the horizon problem, inflationary universes open up the possibility of generating large-scale fluctuations of high contrast dynamically (by our mechanism or otherwise) instead of postulating inbuilt metrical perturbations as initial conditions.[2]

In our case the key model-independent requirement is the breakdown of a global quantum number with respect to which the fermions have an asymmetry. Since the world can accommodate up to ~ 30 Goldstone bosons associated with breakdown of global symmetries, several models are possible.[32]

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