

Secrecy Optimization for Diffusion-Based Molecular Timing Channels

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Abstract—Security in the context of molecular communication systems is an important design aspect that has not attracted much attention till date. This paper analyzes the information-theoretic secrecy of diffusive molecular timing channels when the distance of the eavesdropper is assumed to be random and uniformly distributed. Using an existing upper bound on the timing channel capacity, we calculate the optimal secrecy rate and optimal transmission rate for Bob which would help in achieving an improved secrecy throughput performance. Based on this optimal rate, we calculate the maximum achievable throughput. We then use this formulation to minimize the generalized secrecy outage probability (GSOP) by simultaneously maximizing the average fractional equivocation and minimizing the average information leakage rate. The numerical results show that while choosing the system parameters, there is always a trade-off between different performance metrics like GSOP, average fractional equivocation, and average information leakage rate. The proposed secrecy optimization provides a robust understanding of the physical layer secrecy at the molecular level, enabling the design of secure molecular communication systems.

Index Terms—Average fractional equivocation, average information leakage rate, generalized secrecy outage probability, information-theoretic secrecy, molecular timing channel.

I. INTRODUCTION

Traditional communication engineering research has mostly focused on the transmission of information from the transmitter to the receiver using electromagnetic (EM) waves travelling over wired, wireless, or optical media. However, with the advancements in the field of nanotechnology, it is now possible to develop and deploy nano and microscale devices that need to transmit and receive information at the microscopic level. At such small scales, the conventional means of communication using EM waves fail to deliver promising results. For such microscale environments, molecular communication (MC) where information is exchanged chemically by exchanging information molecules has emerged as a promising communication option [1], [2], [3]. Though MC research from an engineering perspective is relatively new, this kind of communication involving molecules such as pollen, pheromones, hormones,

etc., is very prominent in natural and biological systems [4], [5].

Transmission of the information is achieved by modulating certain characteristics of these information molecules. Some of these characteristics include time of release [3], concentration [2], number [6], position [7], or type [8]. Furthermore, the transportation of the information molecules can be achieved via pure diffusion, flow assisted diffusion, molecular motors, and engineered bacteria [2]. In a diffusion-based MC channel, the channel is usually some kind of aqueous medium connecting the transmitter to the receiver. An information molecule released in such a medium propagates through the underlying process of Brownian motion, which results from the random collisions with the molecules of the surrounding fluid [9]. Though most of the mathematical reasoning and theoretical framework of conventional communication can be used for characterizing MC systems, there are, however, certain aspects of MC are fundamentally distinct from the EM communication.

In timing-based MC systems, the information to be transmitted is encoded in the time of release of information molecules [10]. Once released in the fluid medium, an information molecule takes a random path to the receiver. In diffusion-based molecular timing (DB-MT) channels, the random propagation delay associated with this random path acts as additive noise. The presence or absence of any drift in the fluid media significantly affects the nature of the additive noise term. In flow assisted environments where a positive drift is present in the fluid medium, this additive noise term is characterized by inverse Gaussian (IG) [11]. For drift free environments, this noise term follows a Lévy distribution [12]. Unlike the IG distribution with exponentially decaying tails, the Lévy distribution is α -stable having algebraic tails [13]. The stability of a Lévy distributed random variable results in the non-existence of finite moments, making it difficult to characterize and analyze the drift-free diffusive MC channels. To overcome this problem, the use of exponentially truncated Lévy statistics for obtaining the capacity bounds in diffusive molecular timing channel was first discussed in [14].

The challenges of physical layer security at the nanoscale level were first highlighted by [15]. In particular, the authors discussed the concept of bio-chemical cryptography, wherein biological macro-molecule structure and configuration could be employed to keep up the information integrity. The potential research directions for using bio-chemical cryptography using observations from nature was discussed in [16]. The expressions for secrecy capacity in terms of thermodynamic

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transmitter power, the distance of eavesdropper, and the radius of the receiver was first calculated in [17]. The authors first considered the capacity expressions of [18] and then introduced the concept of eavesdropping in it. An Energy-Saving algorithm was implemented in [19], wherein secrecy was obtained using the Diffie-Hellman method. Further, a programmed biological entity modelling was adopted in [20], where two attack scenarios were considered. In [21] authors demonstrated the potential of accurate passive eavesdropper detection and localization in molecular communications. Authors used the attributes of the random-walk channel to detect an eavesdropper and then estimated its position accurately. A viable and low complexity physical layer security algorithm for secure molecular communications was highlighted in [22].

The transmission of information securely from the authorized transmitter (Alice) to the authorized receiver (Bob) has always been an essential consideration in any communication scenario [23]. In case of MC, the secure transmission of information becomes even more daunting given the limited computational capabilities of the nanodevices and the fact that the instantaneous channel state information (CSI) of eavesdropper (Eve) is not known to Alice for most practical scenarios. Since the information molecules in case of MC do not have the inherent capability to distinguish the receivers (both Bob and Eve), it becomes imperative to focus on secrecy whenever the predator receiver tries to retrieve sensitive information. In the case of MC, the computational and transmission capabilities of the devices involved is minimal, making the security introduced at the physical layer a handy, easy to implement tool to combat the menace of Eve as the random motion of the information molecules increases the probability of a molecule getting absorbed by Eve rather than by Bob.

In this work, we consider a purely diffusive molecular timing channel and derive the optimal design parameters (optimal secrecy and Bob's transmission rates) which would be useful for minimizing the generalized secrecy outage probability (GSOP), maximizing average fractional equivocation, and finally minimizing average information leakage rate. Although the secrecy performance metrics employed in this case are motivated from [24], where the performance of secrecy in quasi-static fading channels was discussed, the fundamental problem formulation and noise models considered in this work are different. The main contributions of this paper are as follows:

- Unlike [17], where the authors assume knowledge of the instantaneous CSI of Eve, in our case, the uniformly distributed distance of the eavesdropper means that there is no prerequisite knowledge about the CSI of Eve. As such, different secrecy performance metrics have been employed in this work.
- We first use the upper bound on the capacity given in [14] to obtain an approximate expression for the upper bound on the channel capacity. Based on this approximate expression, we calculate the transmission probability when eavesdropper distance is uniform distributed, which would be useful for calculating maximum achievable throughput, which is defined as probability of successful transmission of particles times the achievable secrecy

rate.

- Using the throughput analysis, we calculate the optimal value of the secrecy rate and Bob's transmission rate.
- Based on the optimal design parameters values, we minimize GSOP, maximize average fractional equivocation and minimize the average information leakage rate, respectively.

The rest of the manuscript is organized as follows. Section II highlights the system model with a schematic diagram depicting an eavesdropping scenario. Section III illustrates various secrecy performance metrics and their implications on the system design. Section IV provides the numerical results which validate the system model, and finally, Section V concludes the paper.

Notation: The following notations would be used throughout the script: $\mathcal{U}(a, b)$ is a Uniformly distributed random variable (RV) over the interval $[a, b]$ and $\ln(\cdot)$, represents the natural logarithm. Random variables are represented by upper case letters like R, T_n , while their realization is represented using the respective lower case letters. $\frac{\partial}{\partial x}$ represents the partial differentiation of a function with respect to x . $h(X/Y)$ is equivocation which represents the differential entropy of X conditioned to the observed signal Y at the destination. We use $f_{T_n}(t_n)$ to represent the probability density function (PDF) of a continuous RV T_n . $\mathbb{P}(\cdot)$ denotes the probability. Expectation operator is denoted by $\mathbb{E}(\cdot)$. The Modified Bessel's function and incomplete Bessel or leaky aquifer function are represented as $K_v(z)$ and $K_v(x, y)$, respectively. Note that in this work, we use the terms information molecule and information particle interchangeably.

II. SYSTEM MODEL

We consider the transmission of information from the authorized transmitter, Alice to a legitimate receiver, Bob over a diffusion-based molecular timing (DB-MT) channel, where the information to be transmitted is encoded in the time of release of the information molecule, in the presence of an eavesdropper, Eve. Fig.1 represents this scenario of eavesdropping. For the sake of simplicity, we have considered a single particle system where Alice is a point source located at the origin ($x = 0$). Meanwhile, Bob and Eve are assumed to be absorbing receivers. Furthermore, the transmission occurs over a time-slotted channel with τ_m being the length of each time slot. In this work, the distance (d_E) of Eve from Alice is assumed to be uniform distributed. Let T_t be the time of release of an information molecule. After being released, each information molecule follows an independent and identically distributed (iid) propagation path and arrives at the receiver (either Bob or Eve) at time T_a . This arrival time is sum of T_t and random propagation delay T_n . Mathematically, it can be written as

$$T_a = T_t + T_n, \quad (1)$$

where $T_n = T_{n_B}$ if the information molecule arrives at Bob and $T_n = T_{n_E}$ if the information molecule arrives at Eve, with T_{n_B} and T_{n_E} being the random propagation time taken by an information particle to reach Bob and Eve, respectively. This propagation delay (T_n) for the drift free channel can

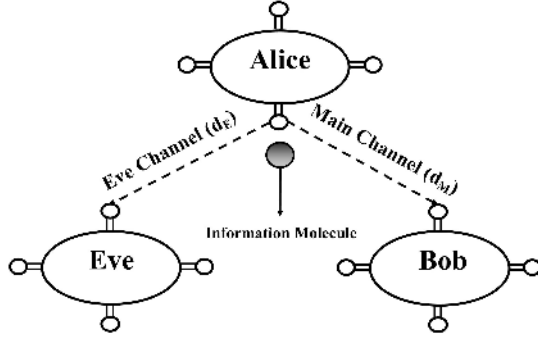


Fig. 1. Scenario of eavesdropping in Diffusion-based molecular communication.

be modeled as an α -stable Lévy distributed random variable (Lévy(μ, c)) [13]. The PDF of Lévy distributed RV R can be represented as

$$f_R(r; \mu, c) = \sqrt{\frac{c}{2\pi(r-\mu)^3}} \exp\left(-\frac{c}{2(r-\mu)}\right), \quad (2)$$

where μ is location parameter and c is scale parameter. In the case of a purely diffusive MC channel, the distance d between the transmitter and the receiver and the diffusion coefficient D of the information molecule in the given fluid media parametrize the scale parameter c (also called the Lévy noise parameter) which is given as

$$c = \frac{d^2}{2D}. \quad (3)$$

The additive noise term T_n can thus be written as $T_n \sim \text{Lévy}(0, d^2/(2D))$, i.e.,

$$f_{T_n}(t_n) = \frac{d}{\sqrt{4\pi D t_n^3}} \exp\left(-\frac{d^2}{4D t_n}\right). \quad (4)$$

Once released in the fluid media, it is justified to assume that the information molecule undergoes degradation because of certain environmental factors. We adopt an exponential degradation model for the lifetime of the information molecules which can be modeled mathematically as [12]

$$h(\tau) = \alpha e^{-\alpha\tau}, \quad \tau > 0, \quad (5)$$

where α is the *degradation parameter* and $h(\tau)$ is the exponentially decaying lifetime. Using this exponentially decay model, the truncated version of the first arrival time distribution i.e., the truncated Lévy distribution is expressed as [14]

$$f_{t_d} = \begin{cases} 0, & \text{for } t_d \leq 0 \\ k' \sqrt{\frac{d^2}{4\pi D t_d^3}} e^{-\frac{d^2}{4D t_d}} e^{-\alpha t_d} & \text{for } t_d > 0, \end{cases} \quad (6)$$

where $k' = \exp(p)$ is the normalizing factor and p is a scaled version of the noise parameter given by $p = \sqrt{2\alpha c}$.

Note that, this exponentially degrading lifetime for the information molecules allows us to consider an inter-symbol interference (ISI) free channel for our analysis. Taking an approach similar to [12], we assume that the timing channel is divided into time slots of duration τ_m . In order to avoid any

ISI τ_m is taken to be *sufficiently large*. A *sufficiently large* τ_m satisfies [12, eq.(7)]

$$\tau_m \gg \tau_x + \mathbf{E}(T_n), \quad (7)$$

where τ_x is the symbol interval within which a transmission can occur. For a given value of the parameter c , let L be a Bernoulli distributed RV with $L = 1$ for the case where the molecule arrives at the receiver within a time slot, such that $\Pr(L = 1) = p_\tau$ and $\Pr(L = 0) = 1 - p_\tau$, where $p_\tau = \Pr(T_n < \tau_m)$. In general, p_τ represents the hitting probability of the molecule. Using this formulation, the existing upper bound on the capacity of molecular timing channels as given by [14, eq.(38)] is taken. Compared to the most existing literature on molecular timing channels without drift, only the authors in [14] adopt a highly realistic exponential degradation model for the lifetime of the information molecules, which well models the natural decay of the molecules [25]. This leads to a totally new and complex mathematical analysis for the molecular timing channel which is, to the best of our knowledge, not reported elsewhere in the literature. This has motivated us to consider the exponentially truncated Lévy distribution as in [14]. Mathematically the capacity upper bound is expressed as

$$C_{ub} = \max_{\tau_x} p_\tau (\ln(\tau_x + \tau_n) - h(T_n|L = 1)). \quad (8)$$

The hitting probability of the molecule p_τ is analytically derived in [14, eq.(32)] as

$$p_\tau = k' \sqrt{\frac{c}{2\pi}} \left(2\sqrt{\frac{p}{c}} K_{1/2}(p) - \sqrt{\frac{1}{\tau_n}} K_{1/2}\left(\alpha\tau_n, \frac{c}{2\tau_n}\right) \right). \quad (9)$$

From [14, eq.(38)] it is evident that for lower values of levy noise parameter the effect of logarithmic term in the upper bound on capacity is more prominent compared to other term. The expression of the capacity can be written as

$$C_{ub} \approx \frac{\ln(\tau_x + \tau_n)}{b} \approx \frac{\ln(\tau_m)}{b}, \quad (10)$$

where b is the scaling constant. Now by substituting the expression of τ_m from [12, eq.(7)] and putting $\tau_x = 1$ we have,

$$\begin{aligned} \ln(\tau_m) &\approx \ln\left(\tau_x + e^{-d\sqrt{\frac{\alpha}{D}}} \sqrt{\frac{d^2}{4\alpha D}}\right) \\ &\approx \ln\left(e^{-d\sqrt{\frac{\alpha}{D}}} \sqrt{\frac{d^2}{4\alpha D}}\right). \end{aligned} \quad (11)$$

The approximation is valid when $\left(e^{-d\sqrt{\frac{\alpha}{D}}} \sqrt{\frac{d^2}{4\alpha D}}\right)$ is small, i.e., $\ln(1+x) \approx \ln(x)$ for small x . Thus, using properties of the logarithmic function and curve fitting techniques, the approximate solution for the upper bound on capacity is obtained as

$$C \approx \frac{1 + \ln(d) - \ln(\sqrt{4D\alpha})}{b}. \quad (12)$$

As noted in [14, eq.(38)], the noise parameter $c = d^2/2D$ should be typically small for a purely diffusive MC channel so that the receiver does not need to wait for too long for the

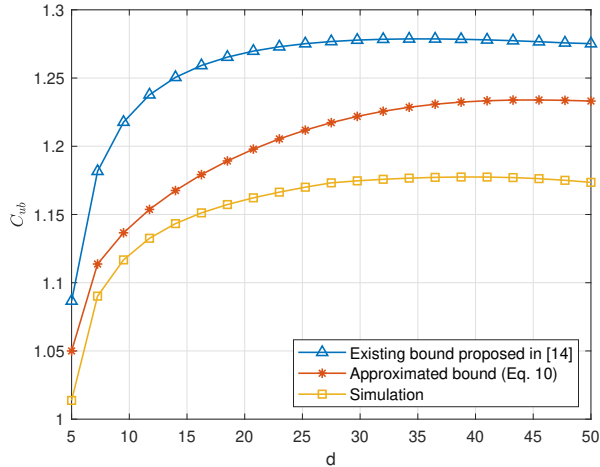


Fig. 2. Existing [14] and proposed approximate of upper bounds on the channel capacity, along with the simulation result for parameter values of $D = 500\mu\text{m}^2/\text{s}$ and $\alpha = 0.01\text{s}^{-1}$. For all the simulation results in this paper, we have used particle based simulations, where the results are averaged over 30,000 independent realizations of the system.

information molecule to arrive. Since D is a property of the system, a small value of c can be obtained by keeping the separation d between the transmitter and the receiver small. In this work too, we have considered small values for d . As can be observed from the capacity bound plots obtained in [14], for small values of the degradation parameter α , the capacity bound initially increases slightly with d , then decreases exponentially. Note that, our analysis holds for small values of d (which should be the case for any practical purely diffusive MC channel) for which the upper bound on capacity increases slightly with d . The validity of the approximation made in this work is confirmed using analytical as well as simulation plots obtained for small values of d as can be seen from Fig.2 which shows the proposed approximation along with the existing expression for capacity upper bound [14, eq.(38)]. A close match of the proposed approximation and existing expression is seen from the figure.

For the sake of clarity, Eve's distance is expressed as d_E and Bob distance is denoted by d_M . Thus the expressions for channel capacities of Bob and Eve in terms of their respective distances from Alice are given by

$$C_B \approx \frac{1 + \ln(d_M) - \ln(\sqrt{4D\alpha})}{b}, \quad (13)$$

$$C_E \approx \frac{1 + \ln(d_E) - \ln(\sqrt{4D\alpha})}{b}. \quad (14)$$

In this work, the distances of neither Bob nor Eve are known at Alice and are assumed to be uniformly distributed. To validate the the performance of our system model, we use the concept of throughput, which basically gives the information about the amount of confidential information propagated throughout the system. Mathematically, throughput is denoted as $\eta = P_{tx}R_S$, where R_S denotes achievable secrecy rate and P_{tx} represents the particle transmission probability from Alice. The transmission probability, also known as the hitting probability, can be

interpreted as quality of service (QoS) measure, which in turn can be written as

$$P_{tx} = \mathbb{P}(R_B \leq C_B), \quad (15)$$

where R_B denotes Bob's transmission rate while C_B represents the maximum achievable channel capacity for Bob. To guarantee whether transmission of particle from Alice is possible, the expression in (15) always holds true. Using (13) and assuming the distance of Bob to be uniformly distributed i.e., $d_m \sim \mathcal{U}(0, \bar{d}_M)$, the expression (15) is modified as

$$P_{tx} = \mathbb{P}\left(R_B \leq \frac{A + \ln(d_M)}{b}\right) = \mathbb{P}(d_M \geq e^{R_B b - A}), \quad (16)$$

where $A = 1 - \ln(\sqrt{4D\alpha})$. Using probability definition the expression (16) can be modified to be written as

$$P_{tx} = \int_{e^{R_B b - A}}^{\bar{d}_M} \frac{1}{\bar{d}_M} dx = 1 - \frac{e^{R_B b - A}}{\bar{d}_M}. \quad (17)$$

III. SECURE TRANSMISSION DESIGN

In this section, we optimize the secrecy performance of the system described in Section II. Since the instantaneous CSI of neither Bob nor Eve is known to Alice, it is very difficult to characterize the system in terms of exact secrecy. One way to characterizing the secrecy performance of the system is to use a performance metric such as the secrecy outage probability (SOP). However, the classical SOP has certain constraints which sometimes are too stringent for practical systems, and a system designer will find it very difficult to adopt the optimal design parameters based on the classical SOP, as the resulting conditions are too stringent for any practically feasible system. Moreover, the classical SOP neither gives information about Eve's decodability nor does it give the rate at which confidential information is leaked to Eve. To overcome these limitations of the classical SOP, we use newer secrecy metrics such as GSOP, average fractional equivocation, and average information leakage rate. These secrecy performance metrics give insights about how information integrity is maintained when the instantaneous CSI of Eve is not known to Alice. In this work, we study the optimal values for the secrecy and Bob's rate, which minimizes the GSOP, maximizes the average fractional equivocation, and minimizes average information leakage rate.

We then examine the significance of the proposed secrecy metrics from the perspective of a system designer. The proposed secrecy metrics lead to different optimal system design parameters as compared to the optimal parameters obtained using the classical SOP. Furthermore, the optimal transmission design based on the classical SOP results in a large secrecy loss, if the actual system requires a low decodability at the eavesdropper or a low information leakage rate. It is interesting to note that by adopting the optimal design based on the classical SOP would lead to a large secrecy loss when the secrecy performance is measured in terms of the secrecy metrics used in this paper.

In a practical scenario where partial secrecy is experienced, the maximum achievable fractional equivocation is given mathematically as [24, eq.(17)]

$$\Delta = \begin{cases} 1, & \text{for } C_E \leq C_B - R_S \\ \frac{C_B - C_E}{R_S}, & \text{for } C_B - R_S < C_E < C_B \\ 0, & \text{for } C_B \leq C_E. \end{cases} \quad (18)$$

Since $R_B \leq C_B$ can be represented by (13) and C_E can be represented by (14), the fractional equivocation in terms of Eve's distance can be modified as

$$\Delta = \begin{cases} 1, & \text{for } d_E \leq e^{R_B b - R_S b - A} \\ \frac{R_B b - \ln(d_E) - A}{R_S}, & \text{for } e^{R_B b - R_S b - A} < d_E < e^{R_B b - A} \\ 0, & \text{for } e^{R_B b - A} \leq d_E. \end{cases} \quad (19)$$

The expression for the GSOP is given by

$$P_{out} = \mathbb{P}(\Delta < \phi), \quad (20)$$

where ϕ is the minimum value of the fractional equivocation which varies from 0 to 1 ($0 < \phi < 1$). The expression for the GSOP when Eve's distance is uniformly distributed ($\mathcal{U}(0, \bar{d}_E)$) is given by

$$\begin{aligned} P_{out} &= \mathbb{P}(d_E \geq e^{R_B b - A}) + \mathbb{P}(e^{R_B b - R_S b - A} < d_E < e^{R_B b - A}) \\ &\quad \cdot \mathbb{P}\left(\frac{R_B b - \ln(d_E) - A}{R_S} < \phi \mid e^{R_B b - R_S b - A} < d_E < e^{R_B b - A}\right) \\ &= 1 - \frac{e^{R_B b - R_S \phi - A}}{\bar{d}_E}. \end{aligned} \quad (21)$$

To minimize the GSOP subject to $\eta \geq \Gamma$ and $R_B \geq R_S > 0$, the optimization problem can be written as

$$\begin{aligned} \min_{R_B, R_S} \quad & P_{out} = 1 - \frac{e^{R_B b - R_S \phi - A}}{\bar{d}_E}, \\ \text{s.t.} \quad & \eta \geq \Gamma, R_B \geq R_S > 0. \end{aligned} \quad (22)$$

The average fractional equivocation is given as

$$\bar{\Delta} = \mathbb{E}(\Delta). \quad (23)$$

The average fractional equivocation gives an intuitive insight on the overall decoding error probability of Eve and is expressed as

$$\begin{aligned} \bar{\Delta} &= \int_0^\lambda \frac{1}{d_E} dx \\ &\quad + \int_\lambda^{\lambda_1} \frac{1}{d_E} \left(\frac{R_B b - \ln(x) - A}{R_S} \right) dx. \end{aligned} \quad (24)$$

This results in

$$\begin{aligned} \bar{\Delta} &= \frac{\lambda}{d_E} + \left(\frac{R_B b - A}{R_S b \bar{d}_E} \right) (\lambda_1 - \lambda) \\ &\quad - \left(\frac{\lambda_1 \ln(\lambda_1) - \lambda_1 - \lambda \ln(\lambda) + \lambda}{R_S b \bar{d}_E} \right), \end{aligned} \quad (25)$$

where $\lambda = e^{R_B b - R_S b - A}$ and $\lambda_1 = e^{R_B b - A}$. Similar to the optimization problem of the GSOP, the optimization problem

for the maximization of the average fractional equivocation $\bar{\Delta}$ subject to $\eta \geq \Gamma$ and $R_B \geq R_S > 0$ can be expressed as

$$\begin{aligned} \max_{R_B, R_S} \quad & \frac{\lambda}{d_E} + \left(\frac{R_B b - A}{R_S b \bar{d}_E} \right) (\lambda_1 - \lambda) - \frac{\lambda_1 \ln(\lambda_1)}{R_S b \bar{d}_E} \\ & + \frac{\lambda_1}{R_S b \bar{d}_E} + \frac{\lambda \ln(\lambda)}{R_S b \bar{d}_E} - \frac{\lambda}{R_S b \bar{d}_E}, \\ \text{s.t.} \quad & \eta \geq \Gamma, R_B \geq R_S > 0. \end{aligned} \quad (26)$$

Furthermore, the average information leakage rate, which gives information about the amount and the rate at which confidential information is leaked to Eve, is given by

$$R_L = \mathbb{E}\{(1 - \Delta)R_S\} = (1 - \bar{\Delta})R_S. \quad (27)$$

Using (25), the average information leakage rate R_L for the case when Eve's distance is uniformly distributed simplifies to

$$\begin{aligned} R_L &= R_S - \frac{\lambda(R_S b + \ln(\lambda) - R_B b + A - 1)}{b \bar{d}_E} \\ &\quad - \frac{\lambda_1(R_B b - \ln(\lambda_1) - A + 1)}{b \bar{d}_E}. \end{aligned} \quad (28)$$

Thus, the optimization problem which minimizes the average information leakage rate R_L subject to $\eta \geq \Gamma$ and $R_B \geq R_S > 0$ is obtained as

$$\begin{aligned} \min_{R_B, R_S} \quad & R_S - \frac{\lambda(R_S b + \ln(\lambda) - R_B b + A - 1)}{b \bar{d}_E} \\ & - \frac{\lambda_1(R_B b - \ln(\lambda_1) - A + 1)}{b \bar{d}_E}, \\ \text{s.t.} \quad & \eta \geq \Gamma, R_B \geq R_S > 0. \end{aligned} \quad (29)$$

The required throughput constraint cannot be achieved when Γ is more than the maximum achievable throughput (when $R_B \geq R_S > 0$). Therefore, to maximize η , where

$$\eta = R_S - \frac{R_S e^{R_B b - A}}{\bar{d}_M}, \quad (30)$$

the optimization problem in (29) can be reformulated as

$$\begin{aligned} \max_{R_B, R_S} \quad & R_S - \frac{R_S e^{R_B b - A}}{\bar{d}_M}, \\ \text{s.t.} \quad & R_B \geq R_S > 0. \end{aligned} \quad (31)$$

For any value of R_S , the partial derivative $\partial\eta/\partial R_B$ is always negative. Thus for maximizing η subject to $R_S > 0$ and $R_B = R_S$, the optimization problem of (31) becomes

$$\begin{aligned} \max_{R_S} \quad & R_S - \frac{R_S e^{R_S b - A}}{\bar{d}_M}, \\ \text{s.t.} \quad & R_S > 0. \end{aligned} \quad (32)$$

Taking the partial derivative of (32) w.r.t. R_S , we get

$$\frac{\partial\eta}{\partial R_S} = 1 - \frac{e^{R_S b - A}}{\bar{d}_M} - \frac{b R_S e^{R_S b - A}}{\bar{d}_M}. \quad (33)$$

By putting $\frac{\partial\eta}{\partial R_S} = 0$, we get the optimal R_S , denoted by R_S^\square , as

$$R_S^\square = \frac{W_0(\bar{d}_M e^{A+1}) - 1}{b}, \quad (34)$$

where $W_0(\cdot)$ denotes the principal branch of the Lambert W function. Substituting (34) in (32), the range of the throughput can be obtained as

$$0 \leq \eta \leq \left(\frac{W_0(\overline{d_M} e^{A+1}) - 1}{b} \right) \left(1 - \frac{e^{W_0(\overline{d_M} e^{A+1}) - 1 - A}}{\overline{d_M}} \right). \quad (35)$$

For the throughput to have the maximum value, the optimum range of the secrecy rate R_S needs to be calculated. The acceptable range for which the throughput is maximum is $R_{s,min} \leq R_S \leq R_{s,max}$. Moreover, the optimum value of Bob's rate for which the throughput of the system is maximized can be obtained from (31) and is given by

$$R_B^* = \frac{1}{b} \left(A + \ln \left(\overline{d_M} - \frac{\overline{d_M} \eta_{max}}{R_S^*} \right) \right), \quad (36)$$

where η_{max} is expressed as

$$\eta_{max} = \left(\frac{W_0(\overline{d_M} e^{A+1}) - 1}{b} \right) \left(1 - \frac{e^{W_0(\overline{d_M} e^{A+1}) - 1 - A}}{\overline{d_M}} \right). \quad (37)$$

This R_B^* is different for different R_S^* . Now, according to (22), in order to minimize the GSOP, we need to maximize

$$F_1 = R_B b - R_S \phi - A. \quad (38)$$

Therefore, by substituting (36) in (38) and differentiating w.r.t. R_S , we get

$$\phi = \frac{\overline{d_M} \eta_{max}}{R_S (\overline{d_M} R_S - \overline{d_M} \eta_{max})}. \quad (39)$$

Using ϕ , the optimal secrecy rate R_{s1}^* which minimizes the GSOP is obtained as

$$R_{s1}^* = \frac{\eta_{max} \phi + \sqrt{\eta_{max}^2 \phi^2 + 4\phi \eta_{max}}}{2\phi}. \quad (40)$$

Thus, Bob's optimal rate R_{B1}^* can be obtained by substituting (40) in (36). Similarly, the optimal secrecy rate R_{s2}^* and Bob's optimal rate R_{B2}^* which maximize the average fractional equivocation $\overline{\Delta}$, subject to $R_{s,min} \leq R_S \leq R_{s,max}$ can be obtained after solving the maximization problem as given in (26). The optimization problem in (26) can be rewritten as

$$\begin{aligned} \max_{R_S} & \frac{e^{R_B b - A} (1 - e^{-R_S b})}{b R_S \overline{d_E}} \\ \text{s.t.} & R_{s,min} \leq R_S \leq R_{s,max}. \end{aligned} \quad (41)$$

Here maximizing $\overline{\Delta}$ requires maximizing

$$F_2 = \frac{e^{R_B b - A} (1 - e^{-R_S b})}{b R_S \overline{d_E}}. \quad (42)$$

Now, for any R_s , Bob's optimal rate R_{B2}^* which maximizes the throughput and the average fractional equivocation is same as that obtained in (36). By putting R_{B2}^* in (41), the optimization problem which maximizes the average fractional equivocation $\overline{\Delta}$ subject to $R_{s,min} \leq R_S \leq R_{s,max}$ can be modified as

$$\begin{aligned} \max_{R_S} & \frac{(\overline{d_M} R_S - \overline{d_M} \eta_{max}) (1 - e^{-R_S b})}{b R_S^2 \overline{d_E}} \\ \text{s.t.} & R_{s,min} \leq R_S \leq R_{s,max}. \end{aligned} \quad (43)$$

From (43), it can be observed that a closed form expression for R_{S2}^* is mathematically intractable. Thus, obtaining R_{S2}^* becomes a numerical optimization problem which can be solved by implementing the golden section search (GSS) technique. Furthermore, based on (29), the minimization of the average information leakage rate is accomplished by maximizing the average fractional equivocation, since the solution R_{S3}^* to the optimization problem is mathematically intractable. Thus, the optimal secrecy rate R_{S3}^* which minimizes the average information leakage rate can be obtained by employing the GSS technique.

Remark: Note that the above analysis for the uniform case can be extended to the non-uniform case also by simply modifying the expressions for the transmission probability (P_{tx}), the outage probability (P_{out}), the average fractional equivocation ($\overline{\Delta}$), and the average information leakage rate (R_L). Based on these newfound expressions, one can obtain the optimal secrecy and Bob's transmission rates which minimize P_{out} , maximize $\overline{\Delta}$, and minimize R_L of the system.

IV. NUMERICAL RESULTS

Based on the mathematical analysis presented in the previous section, in this section, we present the numerical results for the proposed system model to validate the effect of optimal secrecy rate and optimal transmission rate for Bob that would minimize generalized secrecy outage, maximize average fractional equivocation and finally minimize average information leakage rate respectively. The optimum range of throughput constraint as obtained by (35) is $0 \leq \eta \leq 0.16$ bits/s with $\alpha = 0.01 s^{-1}$ and $D = 500 \mu m^2/s$. Using the η range, we first analyze the optimal secrecy rate that optimizes various secrecy performance metrics.

Fig.3 shows the plot of the optimal secrecy rate R_S^* versus the throughput constraint η . As shown in the plot the values of different optimal secrecy rate parameters (R_{S1}^* , R_{S2}^* and R_{S3}^*) are obtained by minimizing GSOP, maximizing average fractional equivocation and minimizing average information leakage rate respectively. Since R_{S1}^* , R_{S2}^* and R_{S3}^* are distinct from each other, they have different optimal ranges. As shown in the figure, the optimal range of R_{S1}^* which minimizes GSOP is 0.0142 to 0.4780 bits/s, while to maximize average fractional equivocation the optimal range of R_{S2}^* is 0.0100 to 0.3316 bits/s, and finally in order to minimize average information leakage rate the optimal range of R_{S3}^* is 0.10 to 0.6684 bits/s. Moreover, the optimal transmission rate for Bob (R_B^*) as represented by (36) is distinct for all the three optimization scenarios and is represented as R_{B1}^* , R_{B2}^* and R_{B3}^* . From the figure it can also be noted that the distinct values of R_{S1}^* , R_{S2}^* and R_{S3}^* is the primary reason for the distinct values of the optimal transmission rates of Bob (R_{B1}^* , R_{B2}^* and R_{B3}^*). Thus from the plot, it is evident that by employing different secrecy metrics to evaluate the secrecy performance, the optimal design parameters are different.

The variation of optimal secrecy rate, R_{s1}^* , which basically minimizes the GSOP, is shown in Fig.4 as a function of throughput for different values of fractional equivocation, ϕ . From the figure it can be observed that as the level of ϕ

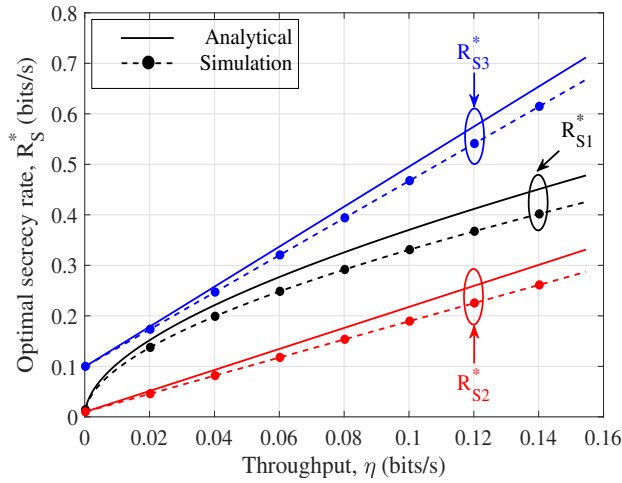


Fig. 3. Optimal secrecy rate versus throughput for different secrecy performance metrics. The other parameters are $\phi = 1$, $\bar{d}_M = 50\mu\text{m}$ and $\bar{d}_E = 50\mu\text{m}$.

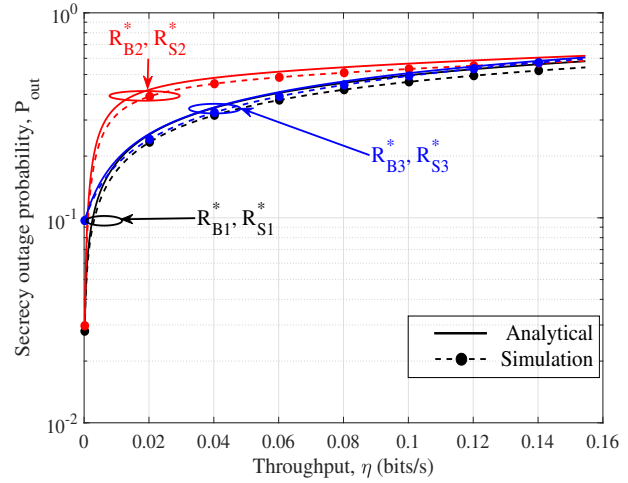


Fig. 5. Secrecy outage probability versus throughput. The other parameters are $\phi = 1$, $\bar{d}_M = 50\mu\text{m}$, and $\bar{d}_E = 50\mu\text{m}$.

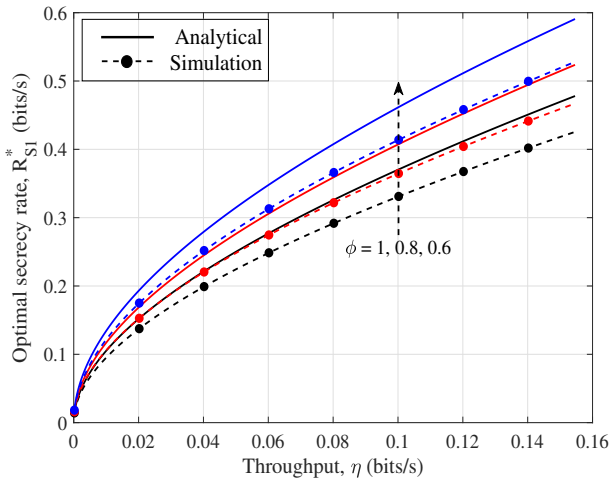


Fig. 4. Optimal secrecy rate versus throughput for GSOP for different fractional equivocation (ϕ). The other parameters are $\bar{d}_M = 50\mu\text{m}$ and $\bar{d}_E = 50\mu\text{m}$.

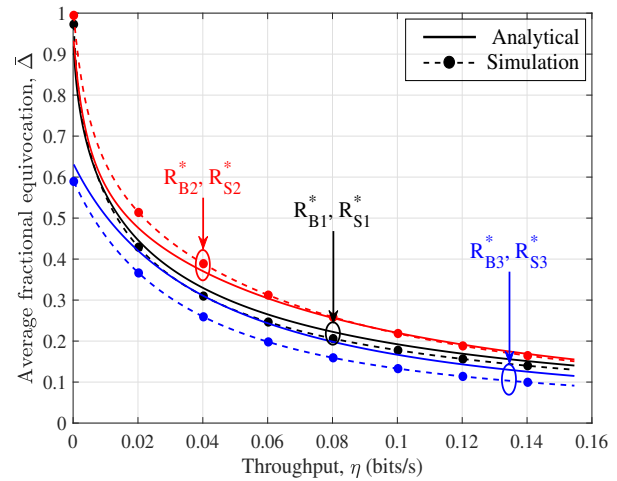


Fig. 6. Average fractional equivocation versus throughput. The other parameters are $\phi = 1$, $\bar{d}_M = 50\mu\text{m}$, and $\bar{d}_E = 50\mu\text{m}$.

decreases, R_{S1}^* increases minimizing the GSOP. Furthermore, based on the analytical results obtained in the preceding section, we have obtained three different optimal design parameter pairs: R_{B1}^*, R_{S1}^* are the optimal design parameters for minimizing GSOP, R_{B2}^*, R_{S2}^* are the optimal design parameters for maximizing average fractional equivocation, and R_{B3}^*, R_{S3}^* are the optimal design parameters that minimize average information leakage rate.

The effect of secrecy outage probability with variation in throughput constraint, for different values of optimal design parameter pairs (R_{B1}^*, R_{S1}^*) , (R_{B2}^*, R_{S2}^*) and (R_{B3}^*, R_{S3}^*) is shown in Fig.5. It can be observed from the SOP plot that increasing the throughput constraint increases the SOP of the system. This is primarily because of the fact that as the throughput of the system is increased, the probability of the particle getting absorbed at Eve increases. Furthermore, the effect of different optimal rate design parameters on the SOP

plot can also be observed in the figure.

Fig.6 presents the plot for average fractional equivocation as a function of throughput for different optimal rate design parameter pairs (R_{B1}^*, R_{S1}^*) , (R_{B2}^*, R_{S2}^*) , and (R_{B3}^*, R_{S3}^*) . From the figure, it is evident that increasing throughput decreases average fractional equivocation. The plot also shows that though the transmission with R_{B2}^* and R_{S2}^* maximizes average fractional equivocation, the system's performance also suffers from higher SOP at the same time.

The plot showing average information leakage rate versus throughput for different design parameter pairs (R_{B1}^*, R_{S1}^*) , (R_{B2}^*, R_{S2}^*) , and (R_{B3}^*, R_{S3}^*) , is shown in Fig.7. From the plot, it can be easily inferred that increasing throughput causes the average information leakage rate to increase. This is perhaps because of the fact that increasing throughput increases the particle's probability of getting absorbed at Eve. Additionally, transmission with R_{B3}^* and R_{S3}^* not only increases the average information leakage rate but it also simultaneously increases

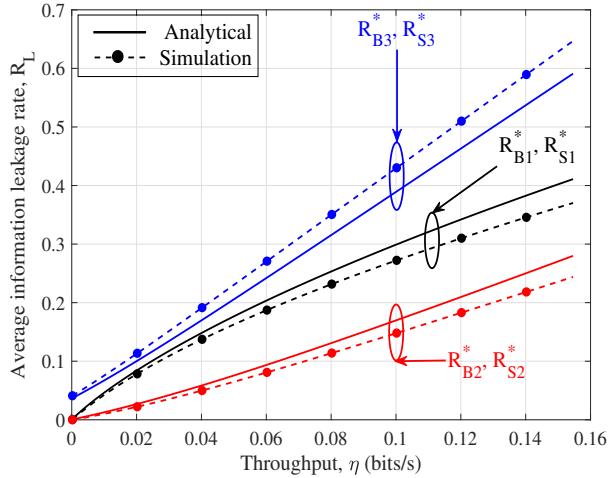


Fig. 7. Average information leakage rate versus throughput. The other parameters are $\phi = 1$, $\bar{d}_M = 50\mu\text{m}$, and $\bar{d}_E = 50\mu\text{m}$.

SOP and decreases average fractional equivocation, respectively.

From the Fig.5, Fig.6 and Fig.7 it is also apparent that using R_{B1}^* and R_{S1}^* as the optimal design parameter not only minimizes GSOP but it also leads to a significant loss when the system's requirement is to maximize average fractional equivocation or to minimize average information leakage. Similarly, using R_{B2}^* and R_{S2}^* as the optimal design parameter for the system's design undoubtedly maximizes average fractional equivocation and minimizes the average information leakage, but it significantly deteriorates in terms of GSOP. Lastly, using R_{B3}^* and R_{S3}^* as the optimal design parameter, the system not only suffers from lower fractional equivocation, but it also suffers from higher average information leakage rate. Thus from the observations, it is evident that for a particular optimal design parameter value, there is always a trade-off between various secrecy performance metrics of the system.

V. CONCLUSION

In this paper, we have investigated the optimal secrecy and Bob's optimal transmission rates in order to minimize the GSOP, maximize the average fractional equivocation, and minimize the average information leakage rate, respectively. Expressions for optimal values of design parameters which characterize the performance of the system were obtained and analyzed for various secrecy performance metrics. Based on the optimal design parameters, we obtained the corresponding numerical results. From the numerical results, it can be inferred that there is always a trade-off between different optimal design parameters which not only minimize the secrecy outage probability and maximize the average fractional equivocation but also minimize the average information leakage rate.

This analysis of secrecy optimization will help bridge the gap between theoretical and practical aspects of the secrecy in MC systems. We expect this analysis to serve as a basis for our future research involving more complex, multi-particle MC systems.

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