

# Sectoral Technology and Structural Transformation\*

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**February 11, 2013**

## **Abstract**

This paper assesses how structural transformation is affected by sectoral differences in labor-augmenting technological progress, capital intensity, and substitutability between capital and labor. We estimate CES production functions for agriculture, manufacturing, and services on postwar US data and compare them with Cobb–Douglas production functions with different and with equal capital shares. We find that sectoral differences in labor-augmenting technological progress are the main force behind the trends in observed sectoral labor and relative prices. As a result, Cobb–Douglas production functions with equal capital shares (which by construction abstract from differences in capital intensity and the elasticity of substitution) capture the main economic forces behind postwar US structural transformation that originate on the technology side.

*Keywords:* CES production function; Cobb–Douglas production function; structural transformation; elasticity of substitution.

*JEL classification:* O11; O14.

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\*For comments and suggestions, the authors thank Cristiano Cantore, Miguel León–Ledesma, Rachel Ngai, Todd Schoellman and the participants of the Workshops on Structural Transformation at the University of Cagliari and the University of Surrey and the Macro Workshop at ASU. Herrendorf thanks the Spanish Ministry of Education for research support. Valentinyi thanks the Hungarian Scientific Research Fund (OTKA) (Project K-105660-ny).

# 1 Introduction

The reallocation of production factors across the broad sectors agriculture, manufacturing, and services is one of the important stylized facts of growth and development: as economies develop agriculture shrinks, manufacturing first grows and then shrinks, and services grow. A growing recent literature has studied this so called structural transformation and has shown that it has important implications for the behavior of aggregate variables such as output per worker, hours worked, and human capital.<sup>1</sup> This paper is part of a broader research program that asks what economic forces are behind structural transformation. Herrendorf et al. (2013b) addressed the preference aspect of this question and quantified the importance of income effects and substitution effects for changes in the composition of households consumption bundles. In this paper, we focus on the technology aspect of this question. In particular, we ask how important are sectoral differences in technological progress, capital intensity, and the substitutability between capital and labor for structural transformation.

There are two different views in the literature about this question. Most papers on structural transformation use sectoral production functions of the Cobb–Douglas form with capital shares that are equal to the aggregate capital share. The advantage of this way of proceeding is that it is convenient, as sectoral Cobb–Douglas production functions with equal capital shares can be aggregated to an economy–wide Cobb–Douglas production function with the same capital share. However, this way of proceeding assumes away differences in sectoral capital intensity and the substitutability between capital and labor that may imply potentially important economic forces behind structural transformation. To see how these forces operate, suppose first that technological progress is even (i.e., is the same in all sectors) and compare two sectoral production functions that only differ in the relative capital intensity. When the economy is poor and capital is relatively scarce compared to labor, then the price of the output of capital–intensive sector relative to that of the labor sector will be high. As even technological progress takes place, the economy develops and capital becomes less scarce compared to labor and the relative price of the output of the capital–intensive sector will fall. Acemoglu and Guerrieri

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<sup>1</sup>The recent literature started with the papers by Kongsamut et al. (2001) and Ngai and Pissarides (2007). Herrendorf et al. (2013a) provide a review of this literature.

(2008) emphasized this economic force behind structural transformation. Now suppose that technological progress is still even and compare two sectoral production functions that only differ in the elasticity of substitution between capital and labor. As before, when the economy is poor, the relative price of the output of the sector with low substitutability will be high. As even technological progress takes place, the relative price of the output of the sector with low substitutability will fall. Alvarez-Cuadrado et al. (2012) emphasized this economic force behind structural transformation.

In order to assess how quantitatively important the different features of sectoral technology are for structural transformation, we estimate CES production functions for agriculture, manufacturing, and services on postwar US data. We also estimate Cobb–Douglas production functions with sector–specific capital shares and Cobb–Douglas production functions with a common capital share equal to the aggregate capital share. We then endow competitive stand–in firms in each sector with the estimated technologies and ask how well their optimal choices replicate the observed sectoral allocation of labor and the sectoral relative prices. The reason for focusing on sectoral labor is that it is the most widely available measure of sectoral activity, which is most commonly used in the context of structural transformation.

The estimation of the sectoral CES production functions yields the following results. First, labor–augmenting technological progress is fastest in agriculture and slowest in services and the differences in the growth rates are sizeable. Second, agriculture is the most capital–intensive sector and manufacturing is the least capital–intensive sector. Services are more capital intensive than manufacturing because services include the capital–intensive industry owner–occupied housing. Third, capital and labor are most easily substitutable in agriculture and least easily substitutable in services. Moreover, in agriculture capital and labor are more substitutable than in the Cobb–Douglas case and in manufacturing and services they are less substitutable than in the Cobb–Douglas case.<sup>2</sup>

In order to assess how quantitatively important the different features of the estimated sectoral production functions are for structural transformation, we compare the predicted trends

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<sup>2</sup>The finding that in agriculture capital and labor are more substitutable than in the Cobb–Douglas case is consistent with the view that a mechanization wave led to massive substitution of capital for labor in US agriculture during the 1950s and 1960s; see for example Schultz (1964).

in sectoral labor with those of Cobb–Douglas production functions with equal and with different capital shares. It turns out that uneven labor–augmenting technological progress is the dominant force behind these trends. As a result, sectoral Cobb–Douglas production functions with equal capital shares (which by construction abstract from differences in the elasticity of substitution and in capital shares) do a good job at capturing the reallocation of labor across sectors during the process of US structural transformation. The reason for this finding is that the CES production function of agriculture has both the largest relative weight on capital and the largest elasticity of substitution whereas the CES production function of manufacturing has both the smallest relative weight on capital and the smallest elasticity of substitution. Hence, the effects on structural transformation of different relative weights on capital and different elasticities of substitution go in opposite directions and largely cancel each other, leaving the effects of uneven labor–augmenting technological progress as the dominating force behind structural transformation. We also show that similar conclusions hold for relative prices, that is, Cobb–Douglas production functions with equal shares do a good job at predicting the relative prices of sectoral outputs.

This paper falls into a large literature that estimates production functions at the aggregate level, the industry level, or the firm level. Three recent studies most closely related to our work are Antràs (2004), Klump et al. (2007) and León-Ledesma et al. (2010), who revisited the question how substitutable capital and labor are at the level of the aggregate US economy. We adopt the methodology of León-Ledesma et al. (2010) to the level of the three broad sectors that are relevant in the context of structural transformation.

The remainder of the paper is organized as follows. In Section 2 we introduce the concept of value–added production functions. Section 3 discusses the estimation issues that arise and the data that we use. In Section 4, we present the estimation results and in Section 5 we compare the CES production function with the Cobb–Douglas production functions. Section 6.2 discusses the implications of our results for building multi–sector models and section 7 concludes.

## 2 Value-added Production Functions

We start with the question of whether to write production functions in gross-output form or in value-added form. Since value added equals the difference between gross output and intermediate inputs, the difference between the two possibilities is whether one counts everything that the sector produces (“gross output”) or whether one counts only what the sector produces beyond the intermediate inputs that it uses (“value added”). To see the issues involved in this question, it is useful to start with the aggregate production function. In a closed economy, GDP equals value added by definition. Therefore, GDP  $G$  is ultimately produced by combining domestic capital  $K$  and labor  $L$ , and we can write the aggregate production function as a value-added production function:

$$G = H(K, L)$$

where  $H$  has the usual regularity conditions. In an open economy, GDP is in general not equal to domestic value added anymore because of imported intermediate inputs. Therefore, GDP is ultimately produced with domestic capital, labor, and imported intermediate inputs  $Z$ :

$$G = H(K, L, Z)$$

While imported intermediate inputs are often abstracted from, they can be quantitatively important, in particular in small open economies that have few natural resources.

Turning now to sectoral production functions, the question which production function to use arises even in a closed economy. The reason for this is that all sectors use intermediate inputs from other sectors, and so sectoral output practically never equals sectoral value added. Therefore, it is natural to start with a production function for gross output. Denoting the sector index by  $i \in \{a, m, s\}$ , the production function for sectoral gross output can be written as:

$$G_i = H_i(K_i, L_i, Z_i)$$

The question then arises under which conditions a value-added production functions  $F_i(., .)$

exist such that sectoral value added is given by:

$$Y_i \equiv \frac{P_{gi}G_i - P_{zi}Z_i}{P_{yi}} = F_i(K_i, L_i)$$

where  $P_{gi}$ ,  $P_{zi}$ , and  $P_{yi}$  denote the prices of gross output, intermediate inputs, and value added, all expressed in current dollars.

Sato (1976) answered that question by showing that a value added production function exists if there is perfect competition and if the other input factors are separable from intermediate inputs, that is, the gross-output production function is of the form

$$G_i = H_i(F_i(K_i, L_i), Z_i) \quad (1)$$

where  $H_i$  and  $F_i$  have the usual regularity conditions (i.e., they are continuously differentiable and concave in each input factor and the Inada conditions hold). To see Sato's argument, consider the problem of a stand-in firm that takes prices and gross output as given and chooses capital, labor, and intermediate inputs to minimize its costs subject to the constraint that it produces the given output:

$$\min_{K_i, L_i, Z_i} R_i K_i + W_i L_i + P_{zi} Z_i \quad \text{s.t.} \quad H_i(F_i(K_i, L_i), Z_i) \geq G_i \quad (2)$$

where  $R_i$  and  $W_i$  denote the rental rates for capital and labor, both expressed in current dollars. The first-order conditions to this problem imply:

$$P_{yi} = \lambda_i \frac{\partial H_i(F_i(K_i, L_i), Z_i)}{\partial Y_i} \quad (3)$$

$$R_i = \lambda_i \frac{\partial H_i(F_i(K_i, L_i), Z_i)}{\partial Y_i} \frac{\partial F_i(K_i, L_i)}{\partial K_i} \quad (4)$$

$$W_i = \lambda_i \frac{\partial H_i(F_i(K_i, L_i), Z_i)}{\partial Y_i} \frac{\partial F_i(K_i, L_i)}{\partial L_i} \quad (5)$$

where  $\lambda_i$  is the multiplier on the constraint. Substituting the first equation into the second and

third equation gives:

$$R_i = P_{yi} \frac{\partial F_i(K_i, L_i)}{\partial K_i} \quad (6)$$

$$W_i = P_{yi} \frac{\partial F_i(K_i, L_i)}{\partial L_i} \quad (7)$$

Using that the envelope theorem implies that the multiplier on the constraint equals the price of value added  $P_{yi}$ , it is straightforward to show that these are the first-order conditions to the problem of a stand-in firm that takes prices and value added as given and chooses capital and labor to minimize its costs subject to the constraint that it produces the given value added:

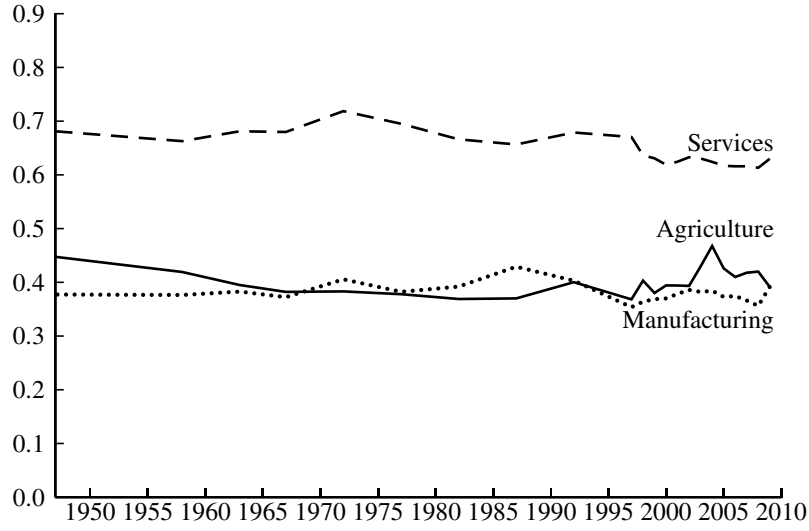
$$\min_{K_i, L_i} R_i K_i + W_i L_i \quad \text{s.t.} \quad F_i(K_i, L_i) \geq Y_i \quad (8)$$

The question remains if condition (1) holds in the data. A sufficient condition is that the sectoral production function is Cobb–Douglas between value added and intermediate inputs:

$$G_i = [F_i(K_i, L_i)]^{\eta_i} Z_i^{1-\eta_i} \quad (9)$$

In this case, perfect competition implies that the share of intermediate inputs is constant over time. Figure 1 plots the intermediate good shares for the post-war US. We can see that none of them has a pronounced trend. We take that to mean that the functional form (9) is a reasonable approximation when one is interested in secular trends in the US, which is what the literature on structural transformation focuses on. We will therefore proceed under the assumption that sectoral value-added production functions exist.

**Figure 1: Intermediate Inputs Shares in the US**



Source: Input–Output Tables for the United States, Bureau of Economic Analysis

### 3 Estimating Sectoral Production Functions

#### 3.1 Deriving the system to estimate

We restrict our attention to the class of CES production functions:

$$Y_{it} = A_i \left[ \theta_i (K_{it})^{\frac{\sigma_i-1}{\sigma_i}} + (1 - \theta_i) (\exp(\gamma_i t) L_{it})^{\frac{\sigma_i-1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i-1}} \quad (10)$$

where  $i \in \{a, m, s\}$  denotes the sector,  $A_i$  is TFP,  $\theta_i$  is the relative weight of capital,  $\sigma_i$  is the elasticity of substitution, and  $\gamma_i$  is the growth rate of labor–augmenting technological progress.<sup>3</sup>

León-Ledesma et al. (2010) show that for estimation purposes it is advantageous to reparameterize this production function in normalized form:

$$Y_{it} = F_i(K_{it}, L_{it}) = \xi_i \bar{Y}_i \left[ \bar{\theta}_i \left( \frac{K_{it}}{\bar{K}_i} \right)^{\frac{\sigma_i-1}{\sigma_i}} + (1 - \bar{\theta}_i) \left( \frac{\exp(\gamma_i(t - \bar{t})) L_{it}}{\bar{L}_i} \right)^{\frac{\sigma_i-1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i-1}} \quad (11)$$

<sup>3</sup>In contrast, Jorgenson et al. (1987) estimated translog production functions for 45 disaggregate US industries during 1948–79. Although we recognize that translog production functions have many advantages in empirical work, we focus on CES production functions here because they are more common in multi–sector growth models.



where  $\bar{Y}_i$ ,  $\bar{K}_i$  and  $\bar{L}_i$  are the geometric averages of output, capital and labor over the sample period;  $\bar{t}$  is the arithmetic average of the time index; and  $\xi_i$  is an auxiliary parameter close to unity. The advantage of working with the normalized form (11) instead of (10) is that  $\bar{\theta}_i$  equals the average capital share in sector  $i$ . This means that the value of  $\bar{\theta}_i$  can be obtained from the data directly and independently of the estimated value of  $\sigma_i$ . In contrast,  $\theta_i$  depends on  $\sigma_i$ , and so identification may be an issue when one estimates the two parameters together.

We assume that each sector has a stand-in firm, which behaves competitively and takes as given sectoral value added, the sectoral interest rate and wage when it chooses sectoral capital and labor to minimize its costs subject to the constraint that it produces at least the given sectoral output. Denoting the price of value added in sector  $i$  by  $P_{yit}$  and the real interest rate and real wage in sector  $i$  by

$$r_{it} \equiv \frac{R_{it}}{P_{yit}}, \quad w_{it} \equiv \frac{W_{it}}{P_{yit}} \quad (12)$$

we can write the problem of the stand-in firm as:

$$\min_{K_{it}, L_{it}} r_{it}K_{it} + w_{it}L_{it} \quad \text{s.t.} \quad F_i(K_{it}, L_{it}) \geq Y_{it} \quad (13)$$

The first-order conditions to this problem imply

$$r_{it} = \frac{\bar{\theta}_i \bar{Y}_i}{\bar{K}_i} \xi_i^{\frac{\sigma_i-1}{\sigma_i}} \left( \frac{Y_{it} \bar{K}_i}{\bar{Y}_i K_{it}} \right)^{\frac{1}{\sigma_i}} \quad (14)$$

$$w_{it} = \frac{(1 - \bar{\theta}_i) \bar{Y}_i}{\bar{L}_i} \xi_i^{\frac{\sigma_i-1}{\sigma_i}} \exp\left( \frac{\sigma_i}{\sigma_i - 1} \gamma_i (t - \bar{t}) \right) \left( \frac{Y_{it} \bar{L}_i}{\bar{Y}_i L_{it}} \right)^{\frac{1}{\sigma_i}} \quad (15)$$

Taking logs of (11) and (14)–(15) and rearranging, we arrive at a system of three equations for

each sector:

$$\log\left(\frac{Y_{it}}{\bar{Y}_i}\right) = \frac{\sigma_i}{\sigma_i - 1} \log\left[\bar{\theta}_i \left(\frac{K_{it}}{\bar{K}_i}\right)^{\frac{\sigma_i-1}{\sigma_i}} + (1 - \bar{\theta}_i) \left(\frac{\exp(\gamma_i(t - \bar{t}))L_{it}}{\bar{L}_i}\right)^{\frac{\sigma_i-1}{\sigma_i}}\right] + \log(\xi_i) \quad (16)$$

$$\log(r_{it}) = \log\left(\bar{\theta}_i \frac{\bar{Y}_i}{\bar{K}_i}\right) + \frac{1}{\sigma_i} \log\left(\frac{Y_{it}}{\bar{Y}_i} \frac{\bar{K}_i}{K_{it}}\right) + \frac{\sigma_i - 1}{\sigma_i} \log(\xi_i) \quad (17)$$

$$\log(w_{it}) = \log\left((1 - \bar{\theta}_i) \frac{\bar{Y}_i}{\bar{L}_i}\right) + \frac{1}{\sigma_i} \log\left(\frac{Y_{it}}{\bar{Y}_i} \frac{\bar{L}_i}{L_{it}}\right) + \frac{\sigma_i - 1}{\sigma_i} [\gamma_i(t - \bar{t}) + \log(\xi_i)] \quad (18)$$

We observe  $Y_{it}/\bar{Y}_i$ ,  $L_{it}/\bar{L}_i$ ,  $K_{it}/\bar{K}_i$ ,  $w_{it}$ ,  $r_{it}$ , and  $\bar{\theta}_i$ . Specifically,  $w_{it}$  is the part of value added that goes to labor divided by the product of sectoral labor and the sectoral price level and  $r_{it}$  is the part of value added that does not go to labor divided by the product of sectoral capital (which includes sectoral land) and the sectoral price level.  $\bar{\theta}_i$  is the share of capital income in sector  $i$ 's value added, which we calculate according to the method of Gollin (2002).

We estimate  $\sigma_i$ ,  $\gamma_i$ , and  $\xi_i$  from the equations (16)–(18) for the aggregate economy and the three sectors using three-stage least squares with an AR(1) error structure.<sup>4</sup> For the aggregate economy this results in a three-equation system, and for the sectoral estimation in a nine-equation system with three equations for each of the three sectors. By estimating the three sectors together, we allow for the possibility that error terms across equations and sectors are correlated. Several right-hand side variables are endogenous. To deal with that, we follow León-Ledesma et al. (2010) and use as instrumental variables the one-period lagged values (appropriate to each sector or the aggregate economy) of the log rental rate on capital, log real wage, log normalized output, log normalized capital, and log normalized labor. Additionally, we include a time trend with the instruments for equations (16) and (18) because it is an exogenous right-hand side variable in both equations.

## 3.2 Data

For output, we use the BEA's "GDP-by-Industry" tables 1947–2010, which contain value added at current prices and quantity indexes of value added by industries according to the North American Industrial Classification (NAICS). We define sectors in the obvious way: agriculture

<sup>4</sup>Including one lag is sufficient to ensure that the innovations to the errors are white noise.

comprises farms, fishing, forestry; manufacturing comprises construction, manufacturing, and mining; and services comprise all other industries (i.e. government, education, real estate, trade, transportation, etc.). Real output for sector  $i$ ,  $Y_{it}$ , is defined by the sector's value added expressed in 2005 prices.

An additional issue arises with measuring value added in agriculture. As standard in national income accounting, NIPA reports "Rent paid to nonoperator landlords" as part of value added of the real estate industry. Since both the labor and capital that generate this rent are reported as input factors in agriculture, consistency requires us to treat this rent as part of value added of agriculture. Therefore, we add "Rent paid to nonoperator landlords" (as reported by the BEA in NIPA Table 7.3.5 "Farm Sector Output, Gross Value Added, and Net Value Added") to the value added of agriculture and subtract it from the value added of services.

Turning to inputs, we calculate the capital stocks by sector from the BEA's "Fixed Asset" tables 1947–2010, which contain the year–end current cost and quantity index in 2005 prices of the net stock of fixed assets. Fixed assets are constructed according to NAICS. Since the BEA fixed assets only includes reproducible capital, we add the value of farm land from the USDA to the fixed assets in agriculture.<sup>5</sup> Given that the data report year–end capital stocks, we calculate the capital stocks during period  $t$  as the geometric averages of the relevant year–end capital stocks in  $t$  and  $t - 1$ .<sup>6</sup>

We calculate labor input by sector from two data sources. The BEA's "GDP–by–Industry" tables 1947–2010 follow the NAICS classification consistently, but report only full- and part-time employees by industry. The BEA's "Income–and–Employment–by–Industry" tables 1948–2010 again change the industry classification: they use SIC72 during 1947–1987, SIC87 during 1987–1997, and NAICS during 1998–2010, but they contain much more detailed information about hours worked by full-time and part-time employees by industry; full-time equivalent employees by industry; self-employed persons by industry; and persons engaged in production by

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<sup>5</sup>The data are from "Land in farms" and "Farm real estate values" tables of the "U.S. and State Farm Income and Wealth Statistics" tables from the U.S. Department of Agriculture (USDA). The data includes the quantity of land in acres and the value of land per acre.

<sup>6</sup>Since the BEA publishes neither the value added nor the capital stock data for the sectors as we define them, we have to construct these aggregates from the underlying BEA data ourselves. Since the BEA calculates real quantities with the chain–weighted method, they are not additive. We use the so called cyclical expansion procedure to aggregate real quantities; see Appendix B for a description of this method.

industry. To construct sectoral hours worked, we use the GDP-by-Industry Tables to the maximum extent possible and the Income-and-Employment-by-Industry Tables to the minimum extent possible. In particular, we merge the two data sources as follows:

$$\begin{aligned} \text{Self-emp}_{NAICS} &= \frac{\text{Self-emp}_{SIC}}{\text{Part- and full-time emp}_{SIC}} \text{Part \& full-time emp}_{NAICS} \\ \text{Full-time eq emp}_{NAICS} &= \frac{\text{Full-time eq emp}_{SIC}}{\text{Part \& full-time emp}_{SIC}} \text{Part \& full-time emp}_{NAICS} \\ \text{Hours full-time eq emp}_{NAICS} &= \frac{\text{Hours full-time eq emp}_{SIC}}{\text{Full-time eq emp}_{SIC}} \text{Full-time eq emp}_{NAICS} \\ \text{Hours persons engaged}_{NAICS} &= \text{Hours full-time eq emp}_{NAICS} \\ &\quad + \frac{\text{Hours full-time eq emp}_{NAICS}}{\text{Full-time eq emp}_{NAICS}} \text{Self-emp}_{NAICS} \end{aligned}$$

Labor input for sector  $i$ ,  $L_{it}$ , is defined as hours worked in sector  $i$  constructed above.

We also need the rental prices of the production factors, which for sector  $i$  are defined as:

$$r_{it} = \frac{\theta_{it} Y_{it}}{K_{it}} \quad w_{it} = \frac{(1 - \theta_{it}) Y_{it}}{L_{it}}$$

where  $\theta_{it}$  is the share of capital income in sector's  $i$  value added in period  $t$ . We already described the construction of  $Y_{it}$ ,  $K_{it}$  and  $L_{it}$  from the data, so we only need to describe the construction of the capital share in value added. We use the BEA's "Components-of-Value-Added-by-Industry" Tables 1947–2010 as follows: "compensation of employees" is labor income; "gross operating surplus minus proprietors' income" is capital income; proprietors' income is split into capital and labor income according to above shares. In the case of agriculture, we add "Rent paid to nonoperator landlords" to "gross operating surplus" since rent is capital income. An issue arises because the industry classification changes over time in these tables. In particular, SIC72 applies to 1947–1987, SIC87 applies to 1987–1997, and NAICS applies to 1998–2010. We calculate the sectoral capital shares for each of the three subperiods and assume that the same capital share also applies to the corresponding NAICS classifications. Since our three sectors are fairly aggregated, this is not big issue here.

**Table 1: Estimation Results**

	Aggregate	Agriculture	Manufacturing	Services
$\sigma$	0.69** (0.022)	1.09** (0.032)	0.77** (0.015)	0.64** (0.015)
$\gamma$	0.017** (0.001)	0.092** (0.001)	0.020** (0.002)	0.012** (0.002)
$\xi$	1.02** (0.018)	0.98** (0.010)	0.95** (0.046)	1.04** (0.053)
$\bar{\theta}$	0.33	0.61	0.29	0.34

Standard errors in parentheses; \*\*  $p < 0.01$

## 4 Estimation Results

The estimation results are summarized in Table 1.<sup>7</sup> We find that capital and labor are most substitutable in agriculture and least substitutable in services. In agriculture capital and labor are more substitutable than in the Cobb–Douglas case, which is consistent with the view that a mechanization wave led to massive substitution of capital for labor in agriculture after World War II. In manufacturing and services capital and labor are less substitutable than Cobb–Douglas. On the aggregate, we find that capital and labor are less substitutable than Cobb–Douglas, which is consistent with the previous results of Antràs (2004), Klump et al. (2007) and León-Ledesma et al. (2010).

Labor–augmenting technological progress is fastest in agriculture and slowest in services and the differences in the growth rates of technological progress are sizeable: in agriculture technological progress grew by 9.2% per year, whereas in manufacturing it grew by 2.0% and in services it grew by just 1.2%; these growth rates result in an average of 1.7% annual growth of aggregate labor–augmenting technological progress. Since these numbers appear rather large,

<sup>7</sup>The appendix contains further information that shows that the fit (as measured by mean–squared error) is good. Moreover, it reports multivariate Ljung–Box Q–statistics, which test for autocorrelation in the residuals, and do not reject the null hypothesis of no residual autocorrelations. To conserve space we only report the test statistics up to two lags, but the existence of higher order autocorrelation is also strongly rejected.

it is useful to remember two qualifications. First, the growth in TFP implied by these numbers is smaller because the labor share is smaller than one. Second, we have used measures of raw sectoral labor that do not take into account sectoral human capital. Increases in sectoral human capital then show up as an increases in labor–augmenting technological progress.

The fact that technological progress is slowest in services while the share of value added produced in services is growing is sometimes referred to as Baumol “disease”, because Baumol (1967) was the first to point out that these two facts imply decreasing growth rates of real GDP. Moreover, if the current trends of structural transformation continue, then services will dominate the economy in the limit, and so aggregate labor-augmenting technological progress will fall from its 1.7% post-war average to the lower 1.2% post-war average for services.

The last row of Table 1 reports  $\bar{\theta}$ , that is, the average capital share in the post war period. We can see that the aggregate capital share comes out as the standard value of 1/3, and that the sectoral capital shares differ from that standard value. However, while the agricultural capital share is considerably larger than the aggregate capital share, the capital shares in manufacturing and services are fairly close to the aggregate capital share. The capital share in agriculture is much larger than the other two capital shares because capital includes land and agriculture is land intensive. The capital share in services is larger than in manufacturing because the capital–intensive industry owner–occupied housing is part of services.

## **5 Sectoral Technology and Structural Transformation**

### **5.1 Sectoral production functions**

In this section, we evaluate the implications of the different features of sectoral technology for structural transformation. To this end, we compare the unrestricted CES production functions that we have estimated above with two restricted CES production functions: (i) we impose  $\sigma_i = 1$  which results in a Cobb–Douglas production function with possibly different capital shares; (ii) we impose  $\sigma_i = 1$  and  $\bar{\theta}_i = \bar{\theta}$ , which results in Cobb–Douglas production functions with a common capital share equal to the aggregate capital share. We write these three functional

forms as follows:

$$Y_{it} = \left[ \bar{\theta}_i \left( \frac{\xi_i \bar{Y}_i}{\bar{K}_i} K_{it} \right)^{\frac{\sigma_i-1}{\sigma_i}} + (1 - \bar{\theta}_i) \left( \frac{\xi_i \bar{Y}_i \exp(\gamma_i(t - \bar{t}))}{\bar{L}_i} L_{it} \right)^{\frac{\sigma_i-1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i-1}}$$

$$Y_{it} = \left( \frac{\bar{Y}_i}{\bar{K}_i} K_{it} \right)^{\bar{\theta}_i} \left( \frac{\bar{Y}_i \exp(\gamma_i(t - \bar{t}))}{\bar{L}_i} L_{it} \right)^{1-\bar{\theta}_i}$$

$$Y_{it} = \left( \frac{\bar{Y}_i}{\bar{K}_i} K_{it} \right)^{\bar{\theta}} \left( \frac{\bar{Y}_i \exp(\gamma_i(t - \bar{t}))}{\bar{L}_i} L_{it} \right)^{1-\bar{\theta}}$$

To simplify the notation, we define (where  $\xi_i = 1$  in the Cobb–Douglas cases):

$$A_{ik} \equiv \frac{\xi_i \bar{Y}_i}{\bar{K}_i}, \quad A_{ilt} \equiv \frac{\xi_i \bar{Y}_i \exp(\gamma_i(t - \bar{t}))}{\bar{L}_i}$$

and write:

$$Y_{it} = \left[ \bar{\theta}_i (A_{ik} K_{it})^{\frac{\sigma_i-1}{\sigma_i}} + (1 - \bar{\theta}_i) (A_{ilt} L_{it})^{\frac{\sigma_i-1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i-1}} \quad (19)$$

$$Y_{it} = (A_{ik} K_{it})^{\bar{\theta}_i} (A_{ilt} L_{it})^{1-\bar{\theta}_i} \quad (20)$$

$$Y_{it} = (A_{ik} K_{it})^{\bar{\theta}} (A_{ilt} L_{it})^{1-\bar{\theta}} \quad (21)$$

To obtain the parameters the Cobb–Douglas production functions, we set  $\bar{\theta} = 1/3$ ,  $\bar{\theta}_a = 0.54$ ,  $\bar{\theta}_m = 0.29$ , and  $\bar{\theta}_s = 0.34$ . This leaves  $\gamma_i$  to estimate. We drop equations (17)–(18) and estimate the output equations (16) jointly for the three sectors where we parameterize  $A_k$  and  $A_l$  in the same way as in the case in the CES and again assume AR(1) error terms. Table 2 reports the resulting average annual growth rates of labor–augmenting technological progress. To put them into perspective, it is useful to calculate the implied growth rates of TFP. For the two Cobb–Douglas production functions, they are obtained as  $\exp(\gamma_i \bar{\theta}_i)$ . It is not clear how to calculate TFP for the CES production function, so we don't attempt to do this here. Table 3 shows the growth rates for TFP. They are sizeable compared to what other studies find; see for example Jorgenson et al. (1987). The reason for this is that we have not taken into account improvements in the quality of sectoral labor (e.g., through increases in years of schooling and experience). In our estimation, such improvements show up as labor–augmenting technological

**Table 2: Average Annual Growth Rates of Labor–augmenting Technological Progress (in %)**

	Aggregate	Agriculture	Manufacturing	Services
CES	1.7	9.2	2.0	1.3
CD with $\bar{\theta}_i$	1.8	9.3	2.1	1.5
CD with $\bar{\theta}$	1.8	6.1	2.1	1.5

**Table 3: Average Annual Growth Rates of TFP (in %)**

	Aggregate	Agriculture	Manufacturing	Services
CD with $\bar{\theta}_i$	1.2	3.5	1.5	1.0
CD with $\bar{\theta}$	1.2	4.1	1.4	1.0

progress.

## 5.2 Sectoral labor allocations

We now turn to the sectoral labor allocations that result from the optimal choices of stand-in firms with are endowed with these production functions. Solving the first-order conditions to the firm problem, (14)–(15), for sectoral labor, we obtain for each functional form:

$$L_{it} = \left( \bar{\theta}_i \left( \frac{1 - \bar{\theta}_i}{\bar{\theta}_i} \frac{A_{ilt} r_{it}}{A_{ik} w_{it}} \right)^{1-\sigma_i} + (1 - \bar{\theta}_i) \right)^{\frac{\sigma_i}{1-\sigma_i}} \frac{Y_{it}}{A_{ilt}} \quad (22)$$

$$L_{it} = \left( \frac{1 - \bar{\theta}_i}{\bar{\theta}_i} \frac{A_{ilt} r_{it}}{A_{ik} w_{it}} \right)^{\bar{\theta}_i} \frac{Y_{it}}{A_{ilt}} \quad (23)$$

$$L_{it} = \left( \frac{1 - \bar{\theta}}{\bar{\theta}} \frac{A_{ilt} r_{it}}{A_{ik} w_{it}} \right)^{\bar{\theta}} \frac{Y_{it}}{A_{ilt}} \quad (24)$$

It is worth to take a moment and build intuition for how the different features of technology affect the allocation of labor across the three broad sectors. The term  $Y_{it}/A_{ilt}$  is common to the right–hand sides because more labor–augmenting technological progress implies that less labor



is needed to produce the given quantity  $Y_{it}$  of sectoral value added. The other right-hand-side terms differ among the different functional forms. It is easiest to start with the Cobb–Douglas cases. The term  $[(1 - \bar{\theta}_i)/\bar{\theta}_i]^{\bar{\theta}_i}$  is decreasing in  $\bar{\theta}_i$  and captures that a sector with a larger capital share receives less labor than a sector with a smaller capital share. The term  $[(A_{ilt}r_{it})/(A_{ik}w_{it})]^{\bar{\theta}_i}$  captures that an increase in the relative rental rate of capital to labor (where both rental rates are expressed relative to the relevant  $A$ 's) leads to a decrease in the sectoral capital–labor ratio and an increase in sectoral labor, and that this increase is larger when the sectoral capital share is larger.

When the economy is poor, the economy–wide capital–labor ratio is low and the relative rental rate of capital to labor is high, implying that a sector with a larger capital share receives relatively less labor. As the economy develops, the capital–labor ratio increases and the relative rental rate of capital to labor decreases, implying an increase in the relative labor of this sector. This is the mechanism that Acemoglu and Guerrieri (2008) emphasized.

For the case of the CES production functions, there is an additional substitution effect: if the elasticity of substitution is larger than one, a higher rental rate of capital relative to labor leads to larger reduction of the capital–labor ratio than in the Cobb–Douglas case; if the elasticity of substitution is smaller than one, a higher rental rate of capital relative to labor leads to smaller reduction of the capital–labor ratio than in the Cobb–Douglas case. Hence, when the economy is poor and the relative rental rate of capital to labor is high, a sector with a smaller elasticity of substitution receives relatively less labor. As the economy develops, the relative rental rate of capital to labor decreases, implying an increase in the relative labor in this sector. This is the mechanism that Alvarez-Cuadrado et al. (2012) emphasized.

Figure 2 plots the labor allocations that are implied by equations (22)–(24) when we plug in the estimated parameter values for  $\sigma_i$  and  $\bar{\theta}_i$  and the data values of the exogenous variables  $A_{ik}$ ,  $A_{ilt}$ ,  $Y_{it}$ ,  $r_{it}$ , and  $w_{it}$ . Note that we have normalized hours worked in 1948 to one. We can see that all three functional forms do a reasonable job at capturing the secular changes in sectoral hours worked. In particular, the CES and the Cobb Douglas with different capital shares perform nearly identical. The only noticeable difference between the two is that the CES form does somewhat better at mimicking the short–run fluctuations in the manufacturing

sector. The Cobb Douglas with equal capital shares does somewhat worse, in particular in manufacturing and in agriculture. The reason for this is that it misses that manufacturing has a larger labor share and agriculture has a smaller labor share than the aggregate. As a result, the Cobb Douglas with equal labor shares systematically allocates too little labor to manufacturing and too much labor to agriculture. Compared to the other two functional forms, manufacturing hours predicted by the Cobb Douglas with equal shares are therefore lower and agricultural hours are higher. Nonetheless, even the Cobb Douglas with equal shares gets the main secular trends of hours mostly right.

The reason for why the Cobb–Douglas production function with equal shares gets the main secular trends of hours mostly right is that the CES production function of agriculture has both the largest relative weight on capital and the largest elasticity of substitution whereas the CES production function of manufacturing has both the smallest relative weight on capital and the smallest elasticity of substitution. Hence, the effects on structural transformation of different relative weights on capital and different elasticities of substitution go in opposite directions and partly cancel each other, leaving the effects of uneven labor–augmenting technological progress as the dominating force behind structural transformation.

### 5.3 Relative prices

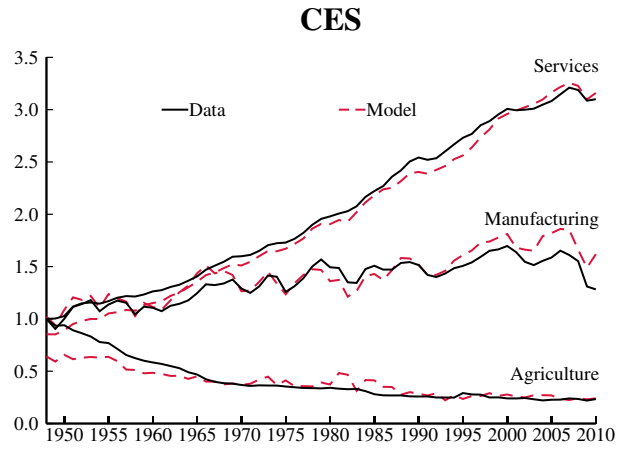
We continue with the relative prices of sectoral value added that each of the three functional forms implies under the maintained assumption that the sectoral stand-in firm behave competitively. The first-order conditions to the firm problem (13) imply that the real wage  $w_{it}$  equals the marginal product of labor. Hence, cost minimization implies that the price of sector  $i$ 's value added relative to services is given by:

$$P_{it} = \frac{P_{it}}{P_{st}} = \frac{W_{it}}{W_{st}} \frac{MPL_{st}}{MPL_{it}}$$

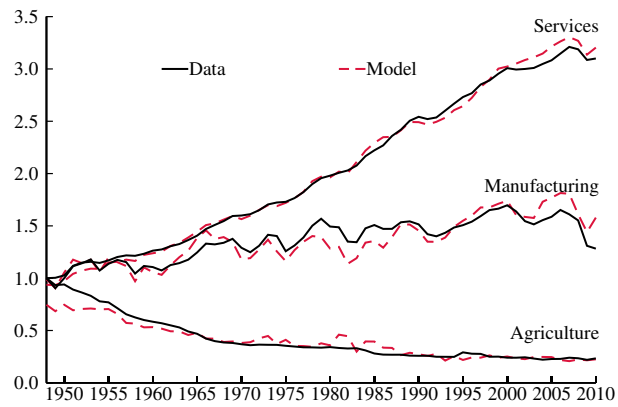
While we observe the nominal wages  $W_{it}$  and  $W_{st}$  in the data, the model implies the values for the marginal products  $MPL_{it}$  and  $MPL_{st}$ .

Figure 3 reports the results that the three functional forms imply for the relative prices. We

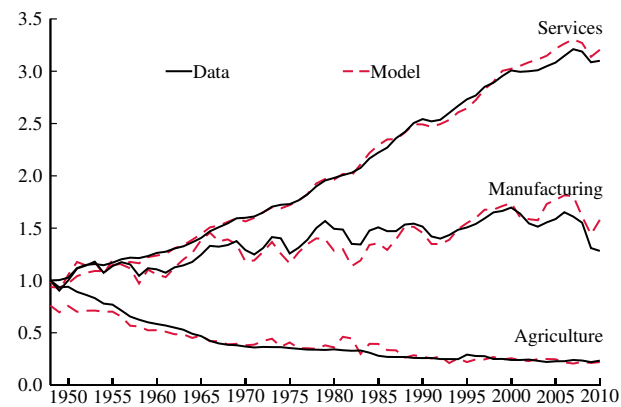
**Figure 2: Hours Worked (Data=1 in 1948)**



**Cobb Douglas with Different Capital Shares**



**Cobb Douglas with Same Capital Shares**



can see that they all do reasonably well with respect to the relative price of agriculture. In contrast, the CES does worst with respect to the relative price of manufacturing and the two Cobb Douglas perform nearly identically well.

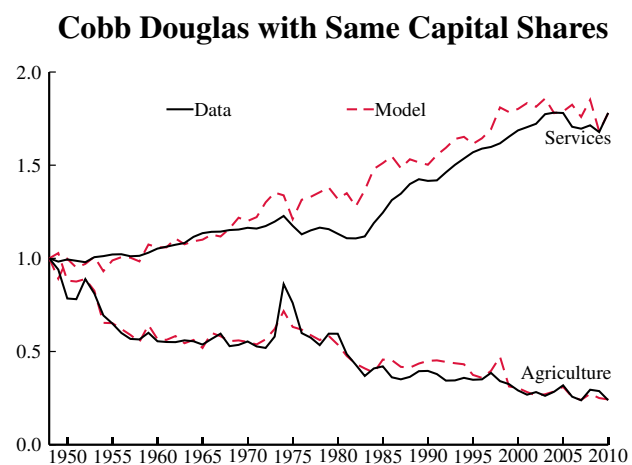
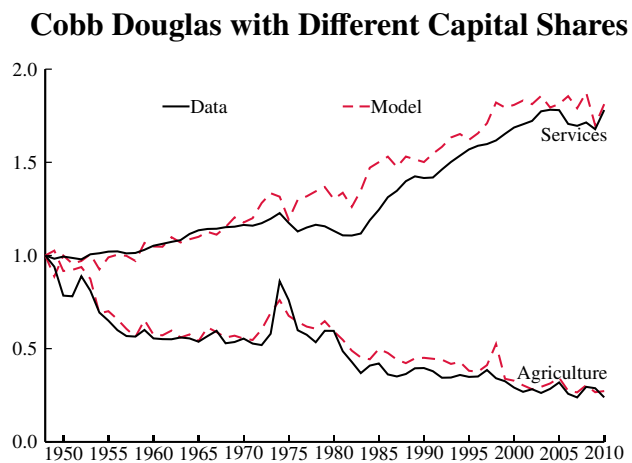
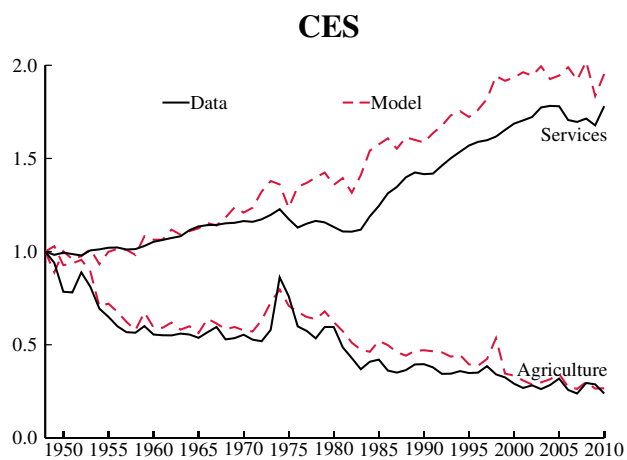
## **6 Implications for Building Multi–sector Models**

### **6.1 Equalizing marginal value products**

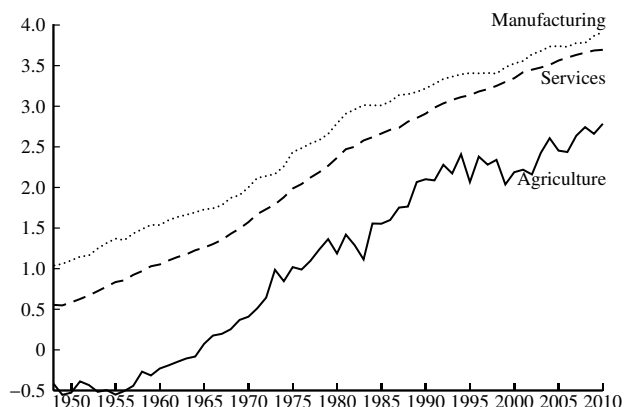
Many builders of multi–sector models assume that the marginal value products of each primary factor of production (here capital and labor) are equalized across sectors. A set of assumptions that implies this is: (i) competitive firms rent each factor of production in a common factor market at a common nominal rental rate; (ii) each factor of production can be moved across sectors without any frictions or costs. Unfortunately, it turns out that in the US the nominal rental rates are not equalized across sectors. Figure 4 shows that the marginal value product is somewhat higher in manufacturing than in services, and is much lower in agriculture than in the other two sectors. Given this evidence, our estimation strategy of system (16)–(18) has been to use the *observed* nominal rental rates and prices of sectoral value added instead of imposing that nominal rental rates are equalized across sectors.

The previous paragraph raises the question, in which way, if any, our estimated sectoral production functions may be used in multi–sector models that equalize marginal value products across sectors. The answer is that in order to incorporate our estimated production functions in a multi–sector model, one needs to add a reason for the difference in the marginal value products across sectors. In the case of labor, the most obvious reason is differences in sectoral human capital that reflect difference in innate ability, experience, and years of schooling like in Jorgenson et al. (1987) or Herrendorf and Schoellman (2012). The latter paper, for example, found that average sectoral human capital is lower in agriculture than in the rest of the US economy, and that the difference accounts for almost all of the difference in nominal wages. This implies that per efficiency unit of labor the average nominal wages were roughly equal in agriculture and the rest of the US economy during the last thirty years. In the case of capital, obvious reasons for the difference in the marginal value products across sectors are unmeasured

**Figure 3: Sectoral Prices Relative to Manufacturing (Data=1 in 1948)**



**Figure 4: Sectoral Marginal Value Products of Labor (in logs)**



quality differences in the measured stock of sectoral capital and unmeasured parts of the stock of capital; see Jorgenson et al. (1987) and McGrattan and Prescott (2005) for further discussion.

## 6.2 Value-added versus final-expenditure production functions

So far, we have focused on value-added production functions. While this is a natural starting point when one studies the technology side of structural transformation, Herrendorf et al. (2013b) pointed out that one can also interpret the sectoral outputs as final goods that are consumed or invested. In this subsection we discuss the implications of our results for models of structural transformation that interpret sectoral outputs in this way.

Before we delve into the details, an example may be helpful. Consider a household which derives utility from the three consumption categories agriculture, manufacturing, and services. Herrendorf et al. (2013b) pointed out that one can take two different perspectives on what these categories are: the value-added perspective and the final-goods perspective. The value-added perspective breaks the household's consumption into the value-added components from the three sectors and assigns each value-added component to a sector. For example, if the household consumes a cotton shirt, then the value added of producing raw cotton goes to agriculture, the value added of processing to manufacturing, and the value added of distribution to services. This means that the consumption categories in the utility function of the household are

the value added that is produced in the three sectors agriculture, manufacturing, and services. In contrast, the final-goods perspective assigns each consumption good to one of the three consumption categories. The cotton shirt, for example, would typically be assigned to manufacturing. This means that the consumption categories in the utility function of the household become final-goods categories. This dramatically changes the meaning of the three sectors, as the manufacturing sector now produces the entire cotton shirt, implying that it combines the value added from the different industries that is required to produce the cotton shirt.

Although the sectoral production functions under the two perspectives are very different objects, we emphasize that they are two representations of the *same* underlying data, which are linked through intricate input-output relationships. To see the implications of this, it is useful to think that at a first approximation the sectoral output under the final-goods perspective are some weighted average of the sectoral value added from the value-added perspective. This implies that the properties of the production function under the final-goods perspective are a weighted average of the properties of the properties of the production functions under the value-added perspective. Valentinyi and Herrendorf (2008) showed that as a result the capital shares of industry gross output tend to be closer to the aggregate capital share than the capital shares of industry value added. This suggests that the sectoral capital shares under the final-goods perspective should be closer to the aggregate capital share than the sectoral capital shares under the value-added perspective. We conjecture that a similar argument applies also to the elasticity of substitution, that is, for a given sector the elasticity of substitution is closer to one under the final-goods perspective than under the value-added perspective.

These arguments suggest that under the final-goods perspective the sectoral production functions are closer to the Cobb-Douglas production function with a common capital share than under the value-added perspective. Since we have shown above that the Cobb-Douglas production functions with a common capital share do a reasonable job at capturing sectoral employment and relative prices under the value-added perspective, this suggests that they will also do a reasonable job under the final-goods perspective. Note that since the aggregate capital share is the same under both perspectives, it is straightforward to parameterize the Cobb-Douglas production functions with a common capital share under the final-goods perspective.

## 7 Conclusion

In this paper, we have assessed the technological forces behind structural transformation, i.e., the reallocation of production factors across agriculture, manufacturing, and services. In particular, we have asked how important for structural transformation are sectoral differences in labor-augmenting technological progress, the elasticity of substitution between capital and labor, and the intensities of capital. We have estimated CES production functions for agriculture, manufacturing, and services on postwar US data. We have found that differences in labor-augmenting technological progress are the predominant force behind structural transformation. As a result, sectoral Cobb–Douglas production functions with equal capital shares (which by construction abstract from differences in the elasticity of substitution and in capital shares) do a reasonably good job of capturing the main trends of US structural transformation.

In this paper, we have restricted our attention to the postwar US. It is also of interest to extend this analysis to a larger set of countries, in particular to situations which feature a larger range of real incomes. This will be useful in assessing the extent to which one can account for the process of structural transformation with stable sectoral technologies.

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## Appendix A

**Table 4: Root Mean Square Errors – Equations (16)–(18)**

Specification	log(r)			log(w)			log(Y)		
	Agr	Man	Ser	Agr	Man	Ser	Agr	Man	Ser
C-D (equal)	-	-	-	-	-	-	0.083	0.026	0.010
C-D (unequal)	-	-	-	-	-	-	0.086	0.025	0.010
CES	0.057	0.048	0.019	0.048	0.026	0.011	0.087	0.025	0.010

**Table 5: Multivariate Ljung-Box Q-Statistics**

Specification	# of Lags	degrees freedom	Q-statistic	p-value
C-D (equal)	1	9	13.210	0.153
	2	18	20.989	0.280
C-D (unequal)	1	9	14.434	0.108
	2	18	21.975	0.233
CES	1	81	102.368	0.055
	2	162	176.596	0.205

**Table 6: Root Mean Square Errors – Labor Allocation and Relative Prices**

Specification	Labor Allocation			Relative Prices	
	Ag	Man	Ser	Ag	Ser
C-D (equal)	0.044	0.106	0.056	0.054	0.090
C-D (unequal)	0.063	0.104	0.050	0.057	0.087
CES	0.148	0.101	0.085	0.051	0.121

## Appendix B: Approximate Aggregation of Chained Quantity Indices

Chain indices relate the value of an index number to its value in the previous period. In contrast, fixed–base indices relate the value of an index number to its value in a fixed base period. While chain indices are preferable to fixed–base indices when prices change considerably over time, using them may lead to problems because real quantities are not additive in general, that is, the real quantity of an aggregate does not equal the sum of the real quantities of its components. In practice, this becomes relevant when one is interested in the real quantity of an aggregate, but the statistical agencies only report the real quantities of the components of this aggregate. This appendix explains how to construct the real quantity of the aggregate according to the so called cyclical expansion procedure.

Let  $Y_{it}$  be the nominal value,  $y_{it}$  the real value,  $Q_{it}$  the chain–weighted quantity index, and  $P_{it}$  the chain–weighted price index for variable  $i \in \{1, \dots, n\}$  in period  $t$ . Let  $t = b$  be the base year for which we normalize  $Q_{ib} = P_{ib} = 1$ . The nominal and real values of variable  $i$  in period  $t$  are then given by:

$$Y_{it} = P_{it} \frac{Q_{it}}{Q_{ib}} Y_{ib} = P_{it} Q_{it} Y_{ib},$$

$$y_{it} = \frac{Y_{it}}{P_{it}} = Q_{it} Y_{ib}.$$

Let  $Y_t = \sum_{i=1}^n Y_{it}$  and suppose that the statistical agency reports  $y_{it}$ ,  $Q_{it}$  and  $P_{it}$  for all components

$i$  but not  $y_t$ ,  $Q_t$  and  $P_t$ . Since in general  $y_t \neq \sum_i y_{it}$ , we need to find a way of calculating  $y_t$ .

We start by approximating  $Q_t$  using the “chain–summation” method:<sup>8</sup>

$$\frac{Q_t}{Q_{t-1}} = \sqrt{\frac{\sum_i P_{it-1} y_{it}}{\sum_i P_{it-1} y_{it-1}} \frac{\sum_i P_{it} y_{it}}{\sum_i P_{it} y_{it-1}}}.$$

Using this expression iteratively, we obtain  $Q_t$  as:

$$Q_t = \frac{Q_t}{Q_{t-1}} \frac{Q_{t-1}}{Q_{t-2}} \cdots \frac{Q_{b+1}}{Q_b} Q_b = \frac{Q_t}{Q_{t-1}} \frac{Q_{t-1}}{Q_{t-2}} \cdots \frac{Q_{b+1}}{Q_b},$$

where the last step used the normalization  $Q_b = 1$ . The real value and the price in period  $t$  then follow as:

$$\begin{aligned} y_t &= Q_t Y_b, \\ P_t &= \frac{Y_t}{Q_t Y_b}. \end{aligned}$$

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<sup>8</sup>This is only an approximation because sums like  $\sum_i P_{it-1} y_{it}$  are not directly observable and the statistical agency typically uses more disaggregate categories than  $i \in \{1, \dots, n\}$  to calculate them.