Security Constrained Optimal Power Flow via Proximal Message Passing

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***Abstract*—In this paper, we propose a distributed algorithm to solve the Security Constrained Optimal Power Flow (SC-OPF) Problem. We consider a network of devices, each with its own dynamic constraints and objective, subject to reliability constraints across multiple scenarios. Each scenario corresponds to the failure or degradation of a set of devices and has an associated probability of occurrence. The network objective is to minimize the cost of operation of all devices, over a given time horizon, across all scenarios subject to the constraints of transmission limit, upper and lower generating limits, generation-load balance etc. This is a large optimization problem, with variables for consumption and generation for each device, in each scenario. In this paper, we extend the *proximal message passing* framework to handle reliability constraints across scenarios. The resulting algorithm is extremely scalable with respect to both network size and the number of scenarios.**

***Index Terms*—Alternating Direction Method of Multipliers (ADMM), Locational Marginal Price (LMP), Augmented Lagrangian, Shift Factor Matrix**

I. INTRODUCTION

The operation of power grids depends critically on the ability to maintain economic efficiency in the presence of unforeseen events. In this paper, we consider the *security-constrained optimal power flow* (SCOPF) problem, in which devices are connected on a network and there exists a set of *scenarios* — each associated with a given probability of occurrence and corresponding to the failure and/or degradation of a set of devices — over which we must ensure efficient operation of the network. For each scenario, the scenario objective is to minimize the sum of the objective functions associated with that scenario for each device. These objective functions extend over a given time horizon and encode operating costs and constraints for a given device operating under that scenario. Solving the SCOPF generates a *contingency plan* for each device in each scenario. The plans tell us the (real) power flow in each device under nominal system operation, as well as in a set of specified contingencies or scenarios. The contingencies can correspond to failure or degraded operation of a transmission line or generator, or they can correspond to a substantial change in a device. In each scenario the powers for each device must satisfy the network equations (taking into account any failures for that scenario). These powers are constrained in various ways across the scenarios. Generators and loads, for example, might be constrained to maintain their power generation or consumption in any adverse (non-nominal) scenario. Transmission lines, which we model as two-terminal devices, simply work according to the flow equations in any scenario, except one in which they have failed. The goal is to minimize a composite cost function that includes the cost (and constraints) of nominal operation, as well as those associated with operation in any of the (adverse) scenarios. This results in a large optimization problem, since each variable in the network, namely, real power flow, is repeated N times, where N is the number of contingencies. We use a suitable modified version of the message passing algorithm in [11] to solve this problem efficiently. For simplicity, we consider only DC power flows in this paper. The extension to AC power flow involves applying the AC-OPF model from [12], [13] to each scenario and requiring that the phase angles of a given devices are equal across all scenarios in the first time period. In this paper, we extend application of the Proximal Message Passing Algorithm from solving the standard Optimal Power Flow(OPF) Problem to solving the (N-1) Security Constrained OPF (SCOPF). The rest of the paper is organized as follows: In section-II, we give brief literature survey followed by section-III, where we derive the Mathematical Formulations for several different scenarios gradually increasing the level of complexity in our model. Then, in Section-IV, the preceding generalized cases are reformulated into a different framework, $\mathcal{DTN}$ (Devices-Terminals-Nets) Formulation, which is particularly suitable for the ADMM (Alternating Direction Method of Multipliers) [1] Based Proximal Message Passing Algorithm to be applied to the problems. Thereafter, we derive the Proximal Message Passing Algorithm for the scenarios focusing mainly on the Dynamic OPF (D-OPF) Problem in Section-V. Section-VI discusses some of the numerical results. Finally, some concluding remarks are drawn and future works are mentioned in Section-VII.

II. LITERATURE SURVEY AND RELATED WORK

The Optimal Power Flow Problem is at the heart of every kind of Power Systems planning and operations activities. It has been studied for more than half a century now. A recent reference that provides a good summary of the historical development of the problem is [2] by Cain, O'Neill and Castillo. The references cited there also provides good insights into formulation and modeling particularly, of the ACOPF. The

978-1-4799-3960-2/14/$31.00

pioneering work on the Security Constrained OPF (SCOPF) was done by Stott *et al* in [14]. Significant early works on ADMM method during the 70s and 80s were [8], [6], [5] etc. followed by work during the 90's which include [4], [3]. Combining these two fields gives rise to the Distributed Computational methods for OPF problem and significant references in that field include works by Baldick and Kim [9], [10]. Particularly, the last reference provides a good comparison of the distributed methods till the end of 90s.

III. Conventional OPF Formulations

In this section, we consider the conventional or traditional OPF Formulation. We will next introduce the notations and conventions we are going to use for the rest of the paper.

A. Notations and Conventions

We have categorized the entities used in the subsequent formulations into four different groups: Sets, Elements, Index and Parameters.

1) Sets: $\mathcal{D}$: Set of Devices
$\mathcal{T}$: Set of Terminals
$\mathcal{N}$: Set of Nets or Buses
The next three sets form partitions of the set of devices:
$G \subseteq \mathcal{D}$: Set of Generators
$T \subseteq \mathcal{D}$: Set of Transmission Lines
$L \subseteq \mathcal{D}$: Set of Loads
$\mathcal{L} = \{0, 1, 2, ..., |\mathcal{L}|\}$: Set of possible (N-1) Contingencies. The element, 0 indicates the base case.

2) Elements: t: Elements of $\mathcal{T}$
g: Elements of G
D: Elements of L
T: Elements of T
N: Elements of $\mathcal{N}$

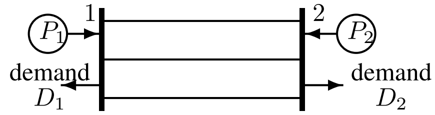
3) Indices: i, j, k: Buses, c: Contingencies, q: Generators, k: Terminals, d: Loads, r: Transmission Lines (in this section, we index them using the bus indices of the ends).

4) Parameters: $R_{ij}, X_{ij}, B_{ij}, Z_{ij} = R_{ij} + jX_{ij}$: Resistance, Reactance, Susceptance and Impedance of the transmission line between buses i and j (for this section) and $R_{T_r}, X_{T_r}, Z_{T_r} = R_{T_r} + jX_{T_r}$: Resistance, Reactance and Impedance of the r^{th} Transmission Line (for the next section)

The variables are the real power P and the bus angles θ (There are no bus angles for DC tie-lines). The following is the convention we follow in order to identify the associations of any particular variable to the sets: $\mathbf{Variable}^{(\mathbf{ContingencyIndex})}_{\mathbf{Net/DeviceElement}_{\mathbf{TerminalNumber}}}$.

For instance, $P^{(c)}_{N_{i_{t_k}}}$ indicates the real power flowing out of the k^{th} terminal, which belongs to the i^{th} net, for the contingency scenario, (c). $P^{(c)}_{T_{r_{t_k}}}$ indicates the real power flowing out of the k^{th} terminal, which belongs to the r^{th} Transmission Line (Device), for the contingency scenario, (c).

In the example in Fig. 1, the terminals of generators 1 and 2 are respectively t_1 and t_5. The terminals of the three transmission lines on Bus-1, which is also net N_1 are $t_{2(I,II,III)}$ and those at Bus-2, which is also net N_2 are $t_{4(I,II,III)}$, whereas those of loads D_1 and D_2 are t_3 and t_6 respectively.



a, b, c: capacity 100 MW/125 MW

Fig. 1: Three transmission lines, a, b, c, joining two buses, 1, and 2.

B. Illustration of Simplest Two Bus Case

First consider the simplest possible case of a two bus system shown in Fig. 1. There are three transmission lines between the two buses with equal impedances and equal power carrying capabilities (100 MW: Continuous and 125 MW: Short time). Assume, that the marginal costs for generating power are \$10/MWh and \$20/MWh for Generators 1 and 2 respectively and they stay the same for the entire range of generating capability. We will try to solve a very simple (N-1) Security constrained dispatch calculation for such a system. We will build the model in steps, increasing the complexity and adding on new constraints for making the analysis more realistic at each step. We will generalize the analysis to arbitrary systems at each step. This approach will help us gain insight as to what exactly is going on physically, as well as understand how the mathematical model is fitting into the pertinent situation. First assume that it is certain that no line outages are ever going to happen and so we can transfer a maximum of 300 MW from Bus-1 to Bus-2. Let load D_2 and D_1 be 500 MW and 300 MW, respectively, and also let both the generators have very high or infinite generating capability. So, Generator-1 will be generating $P_1 = D_1 + 300 = 600MW$, Generator-2 will be generating the remaining 500-300 i.e. 200 MW, and the LMPs (LMP: Locational Marginal Prices for electricity, the incremental cost for generating or providing power at a particular bus, the price at which electricity is traded in the wholesale market) at buses 1 and 2 will be \$10/MWh and \$20/MWh, respectively. The problem for this system can be mathematically formulated as follows, solving which will actually give the same results that we just intuitively examined.

$$\min_{P_1, P_2} C_1(P_1) + C_2(P_2) \tag{1a}$$

$$Subject\ to: \ P_1 + P_2 = D_1 + D_2 \tag{1b}$$

$$|P_1 - D_1| \leq \overline{L} \tag{1c}$$

$$|P_2 - D_2| \leq \overline{L}\ Redundant\ Constraint \tag{1d}$$

where, P_1 and P_2 are real powers produced by Generators-1 and 2 respectively (Decision Variables) and $\overline{L}$ is the maximum power transfer capability of the lines from Bus-1 to Bus-2, which is the sum total of all the three transmission lines between the two buses.

C. Generalization of the Simplest Case to Multi-Bus Systems

The formulation is as follows:

$$\min_{P_i} \sum_{i \in G} C_i(P_i) \tag{2a}$$

$$Subject\ to: \sum_{i \in \mathcal{N}} P_i = \sum_{i \in \mathcal{N}} D_i \tag{2b}$$

$$|\Phi(\mathbf{P} - \mathbf{D})| \leq \overline{\mathbf{L}} \tag{2c}$$

where the bold face letters indicate vectors, Φ is the Shift Factor matrix. From now onward, we will use Φ for the shift factor matrix and $\Phi^{(c)}$ for the shift factor matrix for a particular base-case/contingency scenario. The index $c = 0$ stands for base-case. We can also think of Φ as a $|T| \times |\mathcal{N}|$ matrix with elements ϕ_{ln} which are the ratio between the real power flow on line l and the injection and withdrawal at bus n and the slack bus respectively, where the implicit assumption is that, the linearization of line power flow holds true. Also, $(\mathbf{P}-\mathbf{D})$ is the vector of the real power bus injections. Our next step would be to make the situation a little more realistic by considering the maximum and minimum real power generating limits and so the only other constraints to be added in the above formulations are $P_i^{min} \leq P_i \leq P_i^{max}$, $\forall i \in \mathcal{N}$. (For those buses that do not have a Generator, $P_i^{min} = P_i^{max} = 0$).

D. (N-1) Contingency Constrained OPF for the Two Bus Case: Unequal Capacities and Line Impedances

Moving on to the next level of sophistication in our model, we will now consider incorporation of $(N-1)$ security constraints in our formulation. In the simple two bus case discussed previously, let's, for the sake of simplicity, initially ignore the short term ratings of lines. We are again assuming equal impedances of the lines and identical maximum line flow limits of 100 MW for each line. In order to be secure with respect to a single contingency, now only 200 MW can be transferred from Bus-1 to Bus-2 and so, the remaining (500-200) i.e. 300 MW of load has to be provided by Generator-2. Therefore, being secure with respect to the single outage of the line amounts to evaluating how much power can be transferred if the line is taken out of service, without violating the limit constraints and actually allowing that very quantity of power to flow during pre-contingency. In a more general context, where the impedances as well as the capacities are different and lets assume that the impedances (in this case, the reactances, since we are neglecting the resistances) are X_1, X_2, X_3 with $X_1 \geq X_2 \geq X_3$ (with X_1, X_2, X_3 respectively, the reactances of lines 1, 2 and 3 with capacities a_1, a_2, a_3 (with $a_1 \geq a_2 \geq a_3$, which ensures binding constraints)respectively). Now, the SCOPF is formulated as):

$$\min_{P_1, P_2} C_1(P_1) + C_2(P_2) \tag{3a}$$

$$Subject\ to: \ P_1 + P_2 = D_1 + D_2 \tag{3b}$$

$$Base\ Case: \ \frac{(P_1 - D_1)}{(X_1X_2 + X_2X_3 + X_3X_1)}(X_2X_3) \leq a_1 \tag{3c}$$

$$\frac{(P_1 - D_1)}{(X_1X_2 + X_2X_3 + X_3X_1)}(X_1X_3) \leq a_2 \tag{3d}$$

$$\frac{(P_1 - D_1)}{(X_1X_2 + X_2X_3 + X_3X_1)}(X_1X_2) \leq a_3 \tag{3e}$$

$$Outage\ of\ "c": \ \frac{(P_1 - D_1)}{(X_1 + X_2)}(X_2) \leq a_1 \tag{3f}$$

$$\frac{(P_1 - D_1)}{(X_1 + X_2)}(X_1) \leq a_2 \tag{3g}$$

$$Outage\ of\ "b": \ \frac{(P_1 - D_1)}{(X_1 + X_3)}(X_3) \leq a_1 \tag{3h}$$

$$\frac{(P_1 - D_1)}{(X_1 + X_3)}(X_1) \leq a_3 \tag{3i}$$

$$Outage\ of\ "a": \ \frac{(P_1 - D_1)}{(X_3 + X_2)}(X_3) \leq a_2 \tag{3j}$$

$$\frac{(P_1 - D_1)}{(X_3 + X_2)}(X_2) \leq a_3 \tag{3k}$$

It is to be observed that the particular case of equal line impedance and equal/unequal capacities can be derived from the above model.

E. (N-1) Contingency Constrained OPF for the Generalized Multi-Bus Case: Unequal Capacities and Line Impedances

Hence, the generalized SCOPF can be written down as:

$$\min_{P_i} \sum_{i \in G} C_i(P_i) \tag{4a}$$

$$Subject\ to: \sum_{i \in \mathcal{N}} P_i = \sum_{i \in \mathcal{N}} D_i \tag{4b}$$

$$|\Phi^{(0)}(\mathbf{P} - \mathbf{D})| \leq \overline{\mathbf{L}}^{(0)} \tag{4c}$$

$$|\Phi^{(c)}(\mathbf{P} - \mathbf{D})| \leq \overline{\mathbf{L}}^{(c)} \tag{4d}$$

IV. $\mathcal{DTN}$ Reformulations of the OPF Scenarios

In this section we carry out the reformulation of only the generalized models that we presented in the last section in order for us to be able to solve the problems by the Proximal Message Passing method. In the material that follows, we will group the terms of the objective into three different categories. We will define them for each case. These are:

1)Cost of Generation ($C(P)$): This term consists of the actual total cost of generating real power by the different generators as well as the indicator functions corresponding to the lower and upper generating limits of the different generators. For this term, the real power generated is always considered at the base case.

2)Line Flow Limit Constraint ($F(P)$): This term consists of the sum of the indicator functions corresponding to the

constraints meant for enforcement of the real power flow on the lines being less the maximum allowed, both at the base-case as well as different contingencies.
3)Power-Angle Relation ($\chi(P,\theta)$):This term consists of the sum of the indicator functions corresponding to the relation of the power flow at each end of the lines and the voltage phase angles at the two ends, both at the base-case and the contingencies.

A. $\mathcal{DTN}$ Formulation Applied to the Generalization of the Simplest Case to Multi-Bus Systems

We are considering here the generalization of the simplest two bus case to arbitrary systems and are going to reformulate the OPF equations similar to the paradigm introduced in [11]. Let us introduce the following sign convention that we will follow throughout the rest of the paper: Power coming out of a terminal is positive and going into a terminal is considered negative. Let P_{t_i} refer to the real power coming out of the terminals t_i each of which is associated with exactly one device and one net. Since the loads consume real power, in our two bus case, $P_{D_{1_{t_3}}} = -D_1, P_{D_{2_{t_6}}} = -D_2$. Let a particular net, $N_i \in \mathcal{N}$ has $|N_i|$ number of terminals. The average power mismatch for each net is as follows:

$$\overline{P}_{N_i} = \frac{1}{|N_i|}\sum_{t_k \in N_i \cap \mathcal{T}} P_{N_{i_{t_k}}} = \overline{P}_{N_{i_{t_k}}}$$
$$\forall N_i \in \mathcal{N}, \forall t_k \in N_i \cap \mathcal{T} \tag{5a}$$

Similarly, the phase angle inconsistency equations are:

$$\overline{\theta}_{N_i} = \frac{1}{|N_i|}\sum_{t_k \in N_i \cap \mathcal{T}} \theta_{N_{i_{t_k}}} \tag{6a}$$
$$\tilde{\theta}_{N_{i_{t_k}}} = \theta_{N_{i_{t_k}}} - \overline{\theta}_{N_i}$$
$$\forall N_i \in \mathcal{N}, \forall t_k \in N_i \cap \mathcal{T} \tag{6b}$$

The indicator functions corresponding to the line flow limit constraints are as follows: $\sum_{T_r \in T}\sum_{t_k \in T_r \cap \mathcal{T}} I_{\le}(L_{max_{T_r}} - |P_{T_{r_{t_k}}}|)$.(where $I_{\le}(x) = 0$ if $x \ge 0$ and $= \infty$ otherwise) The indicator functions corresponding to the defining relationship between the power injections on the lines and the phase angles are as follows: $\sum_{T_r \in T}\sum_{t_k, t_{k'} \in T_r \cap \mathcal{T}} I_{=}(P_{T_{r_{t_k}}} + \frac{\theta_{T_{r_{t_k}}} - \theta_{T_{r_{t_{k'}}}}}{X_{T_r}})$.(which, unlike the previously defined indicator functions are zero only when the respective arguments are zero and ∞ otherwise)
The indicator functions corresponding to the Generator maximum and minimum real power generating limits are $\sum_{t_k \in g_q \cap \mathcal{T}, q=1}^{|G|}(I_{\le}(P_{max_{g_q}} - P_{g_{q_{t_k}}}) + I_{\le}(P_{g_{q_{t_k}}} - P_{min_{g_q}}))$ and the Generator Cost functions are of the form $C_{g_q}(P_{g_{q_{t_k}}}) = \alpha_{g_q} {P_{g_{q_{t_k}}}}^2 + \beta_{g_q} P_{g_{q_{t_k}}} + \gamma_{g_q}$. As before, the different terms of the objective function in this case are:
1)Cost of Generation: $C(P) = \sum_{t_k \in g_q \cap \mathcal{T}, q=1}^{|G|}(C_{g_q}(P_{g_{q_{t_k}}}) + I_{\le}(P_{max_{g_q}} - P_{g_{q_{t_k}}}) + I_{\le}(P_{g_{q_{t_k}}} - P_{min_{g_q}}))$
2)Line Flow Limit Constraint: $F(P) = \sum_{T_r \in T}\sum_{t_k \in T_r \cap \mathcal{T}} I_{\le}(L_{max_{T_r}} - |P_{T_{r_{t_k}}}|)$
3)Power-Angle Relation: $\chi(P,\theta) = \sum_{T_r \in T}\sum_{t_k, t_{k'} \in T_r \cap \mathcal{T}} I_{=}(P_{T_{r_{t_k}}} + \frac{\theta_{T_{r_{t_k}}} - \theta_{T_{r_{t_{k'}}}}}{X_{T_r}})$
With all the above components, the reformulated OPF for this particular case can written as:

$$\min_{P_{t_k}, \theta_{t_k}} C(P) + F(P) + \chi(P,\theta) \tag{7a}$$
$$Subject\ to:\ \overline{P}_{N_{i_{t_k}}} = 0, \tilde{\theta}_{N_{i_{t_k}}} = 0, \forall N_i \in \mathcal{N}, \forall t_k \in \mathcal{T} \tag{7b}$$

B. $\mathcal{DTN}$ Formulation Applied to the (N-1) Contingency Constrained Generalized Multi-Bus Case: Unequal Capacities and Unequal Line Impedances

The average net real power imbalance for the base case as well as the contingencies are as follows (similar to the ones presented before in section 4.3):

$$\overline{P}_{N_i}^{(c)} = \frac{1}{|N_i|}\sum_{t_k \in N_i \cap \mathcal{T}} P_{N_{i_{t_k}}}^{(c)} = \overline{P}_{N_{i_{t_k}}}^{(c)}$$
$$\forall N_i \in \mathcal{N}, \forall t_k \in N_i \cap \mathcal{T}, \forall (c) \in \mathcal{L} \tag{8a}$$

and the phase consistency constraints for the base case as well as contingencies are as follows:

$$\overline{\theta}_{N_i}^{(c)} = \frac{1}{|N_i|}\sum_{t_k \in N_i \cap \mathcal{T}} \theta_{N_{i_{t_k}}}^{(c)} \tag{9a}$$
$$\tilde{\theta}_{N_{i_{t_k}}}^{(c)} = \theta_{N_{i_{t_k}}}^{(c)} - \overline{\theta}_{N_i}^{(c)}$$
$$\forall N_i \in \mathcal{N}, \forall t_k \in N_i \cap \mathcal{T}, \forall (c) \in \mathcal{L} \tag{9b}$$

The components of the objective function are as follows:
1)Cost of Generation (At Base Case):
$C(P^{(0)}) = \sum_{t_k \in g_q \cap \mathcal{T}, q=1}^{|G|}(C_{g_q}(P_{g_{q_{t_k}}}^{(0)}) + I_{\le}(P_{max_{g_q}} - P_{g_{q_{t_k}}}^{(0)}) + I_{\le}(P_{g_{q_{t_k}}}^{(0)} - P_{min_{g_q}}))$
2)Line Flow Limit Constraint ((N-1) Secure):
$F(P^{(c)}) = \sum_{(c) \in \mathcal{L}}\sum_{T_r \in T}\sum_{t_k \in T_r \cap \mathcal{T}} I_{\le}(\overline{L}_{T_r}^{(c)} - |P_{T_{r_{t_k}}}^{(c)}|)$
3)Power-Angle Relation ((N-1) Secure):
$\chi(P^{(c)}, \theta^{(c)}) = \sum_{(c) \in \mathcal{L}}\sum_{T_r \in T}\sum_{t_k, t_{k'} \in T_r \cap \mathcal{T}} I_{=}(P_{T_{r_{t_k}}}^{(c)} + \frac{\theta_{T_{r_{t_k}}}^{(c)} - \theta_{T_{r_{t_{k'}}}}^{(c)}}{X_{T_r}^{(c)}})$
So, the reformulated OPF Problem for this case is as follows:

$$\min_{P_{t_k}^{(c)}, \theta_{t_k}^{(c)}} C(P^{(0)}) + F(P^{(c)}) + \chi(P^{(c)}, \theta^{(c)}) \tag{10a}$$
$$Subject\ to:\ \overline{P}_{N_{i_{t_k}}}^{(c)} = 0, \tilde{\theta}_{N_{i_{t_k}}}^{(c)} = 0, \forall N_i \in \mathcal{N}, \forall t_k \in \mathcal{T}, \forall (c) \in \mathcal{L} \tag{10b}$$
$$\sum_{T_r \in N_i \cap T} P_{N_{i T_{r_{t_k}}}}^{(c)} = \sum_{T_r \in N_i \cap T} P_{N_{i T_{r_{t_k}}}}^{(0)}, \forall (c) \in \mathcal{L}, \forall N_i \in \mathcal{N} \tag{10c}$$

The last constraint is for enforcing the condition that within a particular dispatch interval, the pre-and post-contingency bus injections for the transmission lines are the same and the only things that change are the line flows, as long as we consider outage of only transmission elements and are in the DC-OPF regime.

V. ADMM Based Proximal Message Passing Algorithm for the SCOPF Problem

In this section, we will present the ADMM Based Proximal Message Passing iterations for only two of the models presented in the previous sections: The Generalization to the Multi-Bus Case with and without the (N-1) Contingency Constraint. For the sake of completeness, we present here a brief summary of the ADMM based Proximal Message Passing Algorithm as applied to OPF problem, but for details we refer the readers to [11] and [1].

A. Message passing algorithm

In this section, we describe the message passing algorithm used to solve the SC-OPF. We begin by assuming that all device objective functions are convex, closed, and proper (CCP) functions. We then derive a distributed, message passing algorithm using operator splitting and the alternating directions method of multipliers (ADMM) [1]. This algorithm has guaranteed convergence for CCP functions, is fully decentralized, and is robust.

B. Consensus form SC-OPF

Before applying ADMM to solve the SC-OPF, we first replicate the power plans $P \in \mathbf{R}^{|\mathcal{T}|\times(|\mathcal{L}|+1)}$ by introducing a copy, $z \in \mathbf{R}^{|\mathcal{T}|\times(|\mathcal{L}|+1)}$, of the plans. We then solve the *consensus form SC-OPF*:

$$\begin{array}{ll} \text{minimize} & f(P) \\ \text{subject to} & \bar{z} = 0 \\ & P = z. \end{array} \tag{11}$$

Where $\bar{z}$ is the arithmetic mean of z associated with a particular net. Because of the consensus constraint, when we solve the consensus form SC-OPF, the optimal solution will agree with the solution of the original SC-OPF. We introduce the indicator function $g(z) = \mathcal{I}_{\{z|\bar{z}=0\}}(z)$, which is 0 whenever $\bar{z} = 0$ and $+\infty$ otherwise (if the power balance constraint is violated). Because $\bar{z}$ is the average power at each net, the set $\{z \mid \bar{z} = 0\}$ can be written as $\bigcap_{N_i\in\mathcal{N}}\{z \mid \bar{z}_{N_i} = 0\}$, where $\bar{z}_{N_i}$ is the average power at net N_i; then,

$$g(z) = \sum_{N_i\in\mathcal{N}} g_{N_i}(z) = \sum_{N_i\in\mathcal{N}} \mathcal{I}_{\{z|\bar{z}_{N_i}=0\}}(z).$$

Since the summands in the last expression only involve each net N_i separately, $g(z)$ separates across nets completely

$$g(z) = \sum_{N_i\in\mathcal{N}} \mathcal{I}_{\{z_{N_i}|\bar{z}_{N_i}=0\}}(z_{N_i}).$$

C. ADMM and the prox-project message passing algorithm

We apply ADMM to solve the SC-OPF by first forming the (scaled) augmented Lagrangian,

$$\mathcal{L}(P,z,u) = f(P) + g(z) + (\rho/2)\|P - z + u\|_2^2,$$

where $u = (1/\rho)y$ is the scaled dual variable y associated with the consensus constraint $P = z$. We obtained the augmented Lagrangian by completing the squares. ADMM is then

$$\begin{aligned} P^{(\nu+1)} &= \operatorname*{argmin}_P \left(f(P) + (\rho/2)\|P - z^{(\nu)} + u^{(\nu)}\|_2^2\right) \\ z^{(\nu+1)} &= \operatorname*{argmin}_z \left(g(z) + (\rho/2)\|P^{(\nu+1)} - z + u^{(\nu)}\|_2^2\right) \\ u^{(\nu+1)} &= u^{(\nu)} + (P^{(\nu+1)} - z^{(\nu+1)}). \end{aligned}$$

where the superscript is an iteration counter. Because of our problem structure, we can further simplify ADMM. The P-updates separate across devices and

$$P_d^{(\nu+1)} = \operatorname*{argmin}_{P_d} \left(f_d(P_d) + (\rho/2)\|P_d - z_d^{(\nu)} + u_d^{(\nu)}\|_2^2\right)$$

for all $d \in \mathcal{D}$. Furthermore, the z-updates separate across nets and z_{N_i}-update is just a Euclidean projection on to the set $\bar{z}_{N_i} = 0$ and can be solved analytically, so

$$z_{N_i}^{(\nu+1)} = P_{N_i}^{(\nu+1)} + u_{N_i}^{(\nu)} - \bar{P}_{N_i}^{(\nu+1)} - \bar{u}_{N_i}^{(\nu)}.$$

Substituting this expression for z_{N_i} in to the u-update—which also splits across nets—we obtain the **proximal message passing algorithm**:

1) *Proximal plan updates.*

$$P_d^{(\nu+1)} = \mathbf{prox}_{f_d,\rho}(P_d^{(\nu)} - \bar{P}_d^{(\nu)} - u_d^{(\nu)}), \quad \forall d \in \mathcal{D}.$$

2) *Scaled price updates.*

$$u_{N_i}^{(\nu+1)} := u_{N_i}^{(\nu)} + \bar{P}_{N_i}^{(\nu+1)}, \quad \forall N_i \in \mathcal{N},$$

where the proximal function for a function g is given by

$$\mathbf{prox}_{g,\rho}(v) = \operatorname*{argmin}_x (g(x) + (\rho/2)\|x - v\|_2^2).$$

This algorithm alternates between evaluating prox functions (in parallel) on each device and performing price updates on each net. This algorithm has the following three properties:

a) Convergence.: With mild conditions on device objective functions f_d—namely, that they are closed, convex, and proper—and provided a feasible solution exists, the following properties of our algorithm hold.

1) *Residual convergence.* $\bar{P}^{(\nu)} \to 0$ as $\nu \to \infty$,
2) *Objective convergence.* $\sum_{d\in\mathcal{D}} f_d(P_d^{(\nu)}) + \sum_{N_i\in\mathcal{N}} g_{N_i}(P_{N_i}^{(\nu)}) \to f^\star$ as $\nu \to \infty$,
3) *Dual variable convergence.* $\rho u^{(\nu)} = y^{(\nu)} \to y^\star$ as $\nu \to \infty$,

where $f^\star$ is the optimal value for the (convex) SC-OPF, and $y^\star$ are the optimal dual variables (prices). A proof of these conditions can be found in [1]. Convergence of our algorithm guarantees that, if message passing is run long enough, power balance will be satisfied by $P^{(\nu)}$ to any desired accuracy.

b) Distributed.: As long as each device has the ability to access the average power imbalance for the nets it shares with its neighbors, this algorithm can be completely decentralized. Then, the algorithm consists of each device planning for each contingency and a broadcast of plans to its neighbors.

D. Stopping criterion

We can define primal and dual residuals for the prox-project message passing algorithm:

$$r^{(\nu)} = \bar{P}^{(\nu)}, s^{(\nu)} = \rho\left((P^{(\nu)} - \bar{P}^{(\nu)}) - (P^{(\nu-1)} - \bar{P}^{(\nu-1)})\right).$$

Here $P^{(\nu)}$ is interpreted as a power plan. A simple terminating criterion for prox-project message passing is when

$$\|r^{(\nu)}\|_2 \leq \epsilon^{\text{pri}}, \qquad \|s^{(\nu)}\|_2 \leq \epsilon^{\text{dual}},$$

where ϵ^{pri} and ϵ^{dual} are, respectively, primal and dual tolerances.

E. Choice of ρ

The value of the algorithm parameter ρ can greatly affect the convergence rate of the message passing algorithm. There are no known methods for choosing the optimal value of ρ *a priori*, except in certain special cases [7]. For more details on ρ selection, consult [1].

F. Implementation of proximal functions

Each device is responsible for implementing its proximal function. In general, evaluating the proximal function requires solving an optimization problem. The complexity of solving this optimization problem depends on the structure of the local problem. In the case of SC-OPF, the variables are the local power plans P_d and any other private variables. At most, the variables are coupled through the base case $P_d^{(0)}$. This results in an arrow structure in the KKT system of the local optimization problem. This kind of structure can be exploited and solved with linear complexity. If the power plans do not couple through the base case, then the local problem is completely separable across the contingencies. Because of this simple structure in the local SC-OPF problems on each device, we can quickly and efficiently evaluate the proximal functions for each device.

G. Proximal Message Passing for Generalization of the Simplest Case to Multi-Bus Systems

A slightly reformulated version of the $\mathcal{DTN}$ equations from the last section, which allows us to apply the Proximal Message Passing Algorithm is presented here:

$$\min_{P_{t_k},\theta_{t_k}} C(P) + F(P) + \chi(P,\theta) + \sum_{N_i \in \mathcal{N}} (\overline{I}(z_{N_i t_k}) + \tilde{I}(\xi_{N_i t_k})) \tag{12a}$$

$$Subject\ to:\ P_{t_k} = z_{t_k}, \theta_{t_k} = \xi_{t_k}, \forall N_i \in \mathcal{N}, \forall t_k \in \mathcal{T} \tag{12b}$$

where $\overline{I}(z_{N_i t_k})$ and $\tilde{I}(\xi_{N_i t_k})$ are indicator functions of the sets $\{z_{t_k}|\overline{z}_{N_i t_k}\}$ and $\{\xi_{t_k}|\bar{\xi}_{N_i t_k}\}$ respectively.

1) Iterates for Generators: They consist of the update equations for the real power and voltage-phase angles of the generator terminals and are as follows:

$$\begin{aligned}(P_{g_q t_k}^{(\nu+1)}, \theta_{g_q t_k}^{(\nu+1)}) = argmin_{P_{g_q t_k},\theta_{g_q t_k}}[C_{g_q t_k}(P_{g_q t_k}, \theta_{g_q t_k}) + \\ I_{\leq}(P_{max_{g_q}} - P_{g_{q_{t_k}}}) + I_{\leq}(P_{g_{q_{t_k}}} - P_{min_{g_q}}) + \\ \frac{\rho}{2}(||P_{g_{q_{t_k}}} - z_{g_q t_k}^{(\nu)} + u_{g_q t_k}^{(\nu)}||_2^2 + ||\theta_{g_{q_{t_k}}} - \xi_{g_q t_k}^{(\nu)} + v_{g_q t_k}^{(\nu)}||_2^2)], \\ \forall g_q \in G, t_k \in \mathcal{T} \cap G\end{aligned} \tag{13a}$$

Here, ν, $(\rho)(u_{t_k})$ and $(\rho)(v_{t_k})$ are the iteration count, dual variable for power balance and dual variable for phase consistency constraints respectively.ρ is the penalty parameter of the Augmented Lagrangian term.

2) Iterates for Transmission Lines: They consist of the update equations for the real power and voltage-phase angles of the Transmission Line terminals, which are two terminal devices and are as follows:

$$\begin{aligned}&(P_{T_r t_k}^{(\nu+1)}, \theta_{T_r t_k}^{(\nu+1)}) \\ &= argmin_{P_{T_r t_k},\theta_{T_r t_k}}[\sum_{k,k' \in \mathcal{T} \cap T_r} (\overline{L}_{T_r} - |P_{T_{r_{t_k}}}| + \\ &I_{=}(P_{T_{r_{t_k}}} + \frac{\theta_{T_{r_{t_k}}} - \theta_{T_{r_{t_{k'}}}}}{X_{T_r}}) + \\ &\frac{\rho}{2}(||P_{T_{r_{t_k}}} - z_{T_r t_k}^{(\nu)} + u_{T_r t_k}^{(\nu)}||_2^2 + ||\theta_{T_{r_{t_k}}} - \xi_{T_r t_k}^{(\nu)} + v_{T_r t_k}^{(\nu)}||_2^2))], \\ &\forall T_r \in T, t_k \in \mathcal{T} \cap T\end{aligned} \tag{14a}$$

3) Iterates for Loads: They consist of the update equations for the real power and voltage-phase angles of the loads (which have constant real power consumption) and are as follows:

$$\begin{aligned}P_{D_d t_k}^{(\nu+1)} &= P_{D_d t_k}^{(\nu)} \\ \theta_{D_d t_k}^{(\nu+1)} &= argmin_{\theta_{D_d t_k}}[\frac{\rho}{2}(||\theta_{D_{d_{t_k}}} - \xi_{D_d t_k}^{(\nu)} + v_{D_d t_k}^{(\nu)}||_2^2)], \\ &\forall D_d \in L, t_k \in \mathcal{T} \cap L\end{aligned} \tag{15a}$$

4) Iterates for Nets: We are writing here just the analytical forms already derived in [11].

$$\forall N_i \in \mathcal{N}, \forall t_k \in \mathcal{T} \cap N_i$$

$$z_{N_i t_k}^{(\nu+1)} = u_{N_i t_k}^{(\nu)} + P_{N_i t_k}^{(\nu+1)} - \overline{u}_{N_i t_k}^{(\nu)} - \overline{P}_{N_i t_k}^{(\nu+1)} \tag{16a}$$

$$\xi_{N_i t_k}^{(\nu+1)} = \overline{v}_{N_i t_k}^{(\nu)} - \overline{\theta}_{N_i t_k}^{(\nu+1)} \tag{16b}$$

$$u_{N_i t_k}^{(\nu+1)} = u_{N_i t_k}^{(\nu)} + (P_{N_i t_k}^{(\nu+1)} - z_{N_i t_k}^{(\nu+1)}) \tag{16c}$$

$$v_{N_i t_k}^{(\nu+1)} = v_{N_i t_k}^{(\nu)} + (\theta_{N_i t_k}^{(\nu+1)} - \xi_{N_i t_k}^{(\nu+1)}) \tag{16d}$$

In the above, all the devices update their variables in parallel. Then all the nets update the first two variables in parallel and then update the next two in parallel. It is to be observed here that, each $P_{N_i t_k}$ actually comes from the updates from the devices in the previous set of updates, because each of them is actually the real power output/consumption of the respective device having the same terminal in the particular net. Using the above equations, the prox-functions and the proximal message passing algorithm for this case can be written as follows:

$$\begin{aligned}&(P^{(\nu+1)}_{g_qt_k}, \theta^{(\nu+1)}_{g_qt_k})\\ &= \mathbf{prox}_{C(P),\rho}(P^{(\nu)}_{g_qt_k} - \overline{P}^{(\nu)}_{g_qt_k} - u^{(\nu)}_{g_qt_k}, \overline{v}^{(\nu-1)}_{g_qt_k} + \overline{\theta}^{(\nu)}_{g_qt_k} - v^{(\nu)}_{g_qt_k}),\\ &\forall g_q \in G \end{aligned} \tag{17a}$$

$$\begin{aligned}&(P^{(\nu+1)}_{T_rt_k}, \theta^{(\nu+1)}_{T_rt_k}, P^{(\nu+1)}_{T_rt_{k'}}, \theta^{(\nu+1)}_{T_rt_{k'}})\\ &= \mathbf{prox}_{F+\chi,\rho}(P^{(\nu)}_{T_rt_k} - \overline{P}^{(\nu)}_{T_rt_k} - u^{(\nu)}_{T_rt_k}, \overline{v}^{(\nu-1)}_{T_rt_k} + \overline{\theta}^{(\nu)}_{T_rt_k} - v^{(\nu)}_{T_rt_k}),\\ &\forall T_r \in T \end{aligned} \tag{17b}$$

$$\begin{aligned}&(P^{(\nu+1)}_{D_dt_k}, \theta^{(\nu+1)}_{D_dt_k})\\ &= \mathbf{prox}_{-D,\rho}(\overline{v}^{(\nu-1)}_{D_dt_k} + \overline{\theta}^{(\nu)}_{D_dt_k} - v^{(\nu)}_{D_dt_k}), \forall D_d \in L \end{aligned} \tag{17c}$$

$$u^{(\nu+1)}_{N_it_k} = u^{(\nu)}_{N_it_k} + \overline{P}^{(\nu+1)}_{N_it_k}, \forall N_i \in \mathcal{N} \tag{17d}$$

$$v^{(\nu+1)}_{N_it_k} = \tilde{v}^{(\nu)}_{N_it_k} + \tilde{\theta}^{(\nu+1)}_{N_it_k}, \forall N_i \in \mathcal{N} \tag{17e}$$

H. Proximal Message Passing for (N-1) Contingency Constrained Generalized Multi-Bus Case: Unequal Capacities and Unequal Line Impedances

The slightly reformulated $\mathcal{DTN}$ equations from previous section are:

$$\begin{aligned}&\min_{P^{(c)}_{t_k}, \theta^{(c)}_{t_k}} C(P^{(0)}) + F(P^{(c)}) + \chi(P^{(c)}, \theta^{(c)})\\ &+ \sum_{(c)\in\mathcal{L}} \sum_{N_i\in\mathcal{N}} (\overline{I}(z^{(c)}_{N_it_k}) + \tilde{I}(\xi^{(c)}_{N_it_k})) \end{aligned} \tag{18a}$$

$$\begin{aligned}&Subject\ to:\ P^{(c)}_{t_k} = z^{(c)}_{t_k}, \theta^{(c)}_{t_k} = \xi^{(c)}_{t_k}, \forall N_i \in \mathcal{N}, \forall t_k \in \mathcal{T},\\ &\forall (c) \in \mathcal{L} \end{aligned} \tag{18b}$$

It is to be observed here that the power balance and the phase consistency constraints need to be satisfied for each and every contingency scenario. The different update equations of the Proximal Message Passing Algorithm in this case are as follows:

1) Iterates for Generators: They consist of the update equations for the real power output and voltage-phase angles of the generator terminals for both the base case and the different (N-1) contingency scenarios and are as follows:

$$\begin{aligned}&(P^{(0)(\nu+1)}_{g_qt_k}, \theta^{(c)(\nu+1)}_{g_qt_k}) = argmin_{P^{(0)}_{g_qt_k}, \theta^{(c)}_{g_qt_k}}[C_{g_qt_k}(P^{(0)}_{g_qt_k}, \theta^{(c)}_{g_qt_k})\\ &+ I_{\le}(P^{max}_{g_q} - P^{(0)}_{g_{q_{t_k}}}) + I_{\le}(P^{(0)}_{g_{q_{t_k}}} - P^{min}_{g_q}) +\\ &\sum_{(c)\in\mathcal{L}} (\frac{\rho}{2})(||P^{(0)}_{g_{q_{t_k}}} - z^{(c)(\nu)}_{g_qt_k} + u^{(c)(\nu)}_{g_qt_k}||_2^2 +\\ &||\theta^{(c)}_{g_{q_{t_k}}} - \xi^{(c)(\nu)}_{g_qt_k} + v^{(c)(\nu)}_{g_qt_k}||_2^2)],\\ &\forall g_q \in G, t_k \in \mathcal{T} \cap G \end{aligned} \tag{19a}$$

2) Iterates for Transmission Lines:

$$\begin{aligned}&(P^{(c)(\nu+1)}_{T_rt_k}, \theta^{(c)(\nu+1)}_{T_rt_k}, P^{(c)(\nu+1)}_{T_rt_{k'}}, \theta^{(c)(\nu+1)}_{T_rt_{k'}})\\ &= argmin_{P^{(c)}_{T_rt_k}, \theta^{(c)}_{T_rt_k}}[\sum_{k,k'\in\mathcal{T}\cap T_r} (I_{\le}(\overline{L}^{(c)}_{T_r} - |P^{(c)}_{T_{r_{t_k}}}|) +\\ &I_{=}(P^{(c)}_{T_{r_{t_k}}} + \frac{\theta^{(c)}_{T_{r_{t_k}}} - \theta^{(c)}_{T_{r_{t_{k'}}}}}{X^{(c)}_{T_r}}) +\\ &\frac{\rho}{2}(||P^{(c)}_{T_{r_{t_k}}} - z^{(c)(\nu)}_{T_rt_k} + u^{(c)(\nu)}_{T_rt_k}||_2^2 +\\ &||\theta^{(c)}_{T_{r_{t_k}}} - \xi^{(c)(\nu)}_{T_rt_k} + v^{(c)(\nu)}_{T_rt_k}||_2^2))]\\ &\forall T_r \in T, t_k \in \mathcal{T} \cap T, (c) \in \mathcal{L} \end{aligned} \tag{20a}$$

3) Iterates for Loads: They consist of the update equations for the real power and voltage-phase angles of the loads (which have constant real power consumption) and are as follows:

$$\begin{aligned}&P^{(c)(\nu+1)}_{D_dt_k} = P^{(c)(\nu)}_{D_dt_k} = -D_{d_{t_k}}\\ &\theta^{(c)(\nu+1)}_{D_dt_k} = argmin_{\theta^{(c)}_{D_{d_{t_k}}}}[\frac{\rho}{2}(||\theta^{(c)}_{D_{d_{t_k}}} - \xi^{(c)(\nu)}_{D_dt_k} + v^{(c)(\nu)}_{D_dt_k}||_2^2)],\\ &\forall D_d \in L, t_k \in \mathcal{T} \cap L, (c) \in \mathcal{L} \end{aligned} \tag{21a}$$

4) Iterates for Nets: We are writing here just the analytical forms already derived in [11].

$$\forall N_i \in \mathcal{N}, \forall t_k \in \mathcal{T} \cap N_i, \forall (c) \in \mathcal{L}$$

$$z^{(c)(\nu+1)}_{N_it_k} = u^{(c)(\nu)}_{N_it_k} + P^{(c)(\nu+1)}_{N_it_k} - \overline{u}^{(c)(\nu)}_{N_it_k} - \overline{P}^{(c)(\nu+1)}_{N_it_k} \tag{22a}$$

$$\xi^{(c)(\nu+1)}_{N_it_k} = \overline{v}^{(c)(\nu)}_{N_it_k} + \overline{\theta}^{(c)(\nu+1)}_{N_it_k} \tag{22b}$$

$$u^{(c)(\nu+1)}_{N_it_k} = u^{(c)(\nu)}_{N_it_k} + (P^{(c)(\nu+1)}_{N_it_k} - z^{(c)(\nu+1)}_{N_it_k}) \tag{22c}$$

$$v^{(c)(\nu+1)}_{N_it_k} = v^{(c)(\nu)}_{N_it_k} + (\theta^{(c)(\nu+1)}_{N_it_k} - \xi^{(c)(\nu+1)}_{N_it_k}) \tag{22d}$$

In the above, as before, not only do all the devices update their variables in parallel, but also, except the generators, all devices have associated with them the base-case and the contingency scenarios, each of which in turn update their respective variables in parallel as well. Then all the nets and the base-case/contingency scenarios associated with them update the first two set of variables in parallel and then update the next two in parallel. For this case, the prox messages and the

Proximal Message Passing Algorithm is as follows:

$$(P_{g_q t_k}^{(0)(\nu+1)}, \theta_{g_q t_k}^{(c)(\nu+1)}) = \mathbf{prox}_{C(P^{(0)}),\rho}(P_{g_q t_k}^{(0)(\nu)} - \overline{P}_{g_q t_k}^{(c)(\nu)} - u_{g_q t_k}^{(c)(\nu)}, \overline{v}_{g_q t_k}^{(c)(\nu-1)} + \overline{\theta}_{g_q t_k}^{(c)(\nu)} - v_{g_q t_k}^{(c)(\nu)}), \forall g_q \in G, \forall (c) \in \mathcal{L} \quad (23a)$$

$$(P_{T_r t_k}^{(c)(\nu+1)}, \theta_{T_r t_k}^{(c)(\nu+1)}, P_{T_r t_{k'}}^{(c)(\nu+1)}, \theta_{T_r t_{k'}}^{(c)(\nu+1)}) = \mathbf{prox}_{F(P^{(c)})+\chi(P^{(c)},\theta^{(c)}),\rho}(P_{T_r t_k}^{(c)(\nu)} - \overline{P}_{T_r t_k}^{(c)(\nu)} - u_{T_r t_k}^{(c)(\nu)}, \overline{v}_{T_r t_k}^{(c)(\nu-1)} + \overline{\theta}_{T_r t_k}^{(c)(\nu)} - v_{T_r t_k}^{(c)(\nu)}), \forall T_r \in T, \forall (c) \in \mathcal{L} \quad (23b)$$

$$(P_{D_d t_k}^{(c)(\nu+1)}, \theta_{D_d t_k}^{(c)(\nu+1)}) = \mathbf{prox}_{-D,\rho}(\overline{v}_{D_d t_k}^{(c)(\nu-1)} + \overline{\theta}_{D_d t_k}^{(c)(\nu)} - v_{D_d t_k}^{(c)(\nu)}), \forall D_d \in L, \forall (c) \in \mathcal{L} \quad (23c)$$

$$u_{N_i t_k}^{(c)(\nu+1)} = u_{N_i t_k}^{(c)(\nu)} + \overline{P}_{N_i t_k}^{(c)(\nu+1)}, \forall N_i \in \mathcal{N}, \forall (c) \in \mathcal{L} \quad (23d)$$

$$v_{N_i t_k}^{(c)(\nu+1)} = \tilde{v}_{N_i t_k}^{(c)(\nu)} + \tilde{\theta}_{N_i t_k}^{(c)(\nu+1)}, \forall N_i \in \mathcal{N}, \forall (c) \in \mathcal{L} \quad (23e)$$

VI. Simulation Studies and Results

We have carried out our numerical simulations on a computer with an Intel (R) Xeon (R) CPU E5-2670 with a clocking frequency of 2.6 GHz, having 1.5 GB RAM, running Windows 7 Enterprise. We have used MATLAB R 2013a with CVX installed on it in order to write the code. So far, we have only done a purely serial implementation of the algorithm with the parameters, ρ set to 1 and ϵ^{pri} to 0.001. For the sake of brevity, in this paper, we are mentioning the results of only the two bus system (which we introduced in Section-II). This will help us compare the simulation results with the ones we obtained intuitively, so that it validates our approach. Again, for the purposes of this simulation, we have assumed all the transmission lines having equal capacities of 100 MW and reactances of 0.15 Ω. For the SCOPF case, we have assumed outage of just line-3. Also, our lower and upper generating limits for both Generators 1 and 2 were 0 MW and 10000 MW respectively. Here are the most important parameters:

A. *Simple Two Bus OPF*

Time required to solve:231.8139 s; Number of Iterations: 122; Generator 1: 600.0003 MW (600 MW), Generator 2: 199.9986 MW, (200 MW)Line Flow on each line: 100 MW (100 MW); Scaled Dual Variable for Power balance (LMP) on Bus-1: 9.9729 (10)$/MWh, Bus-2: 20.002 (20)$/MWh and the primal tolerance at the solution: 2.8267×10^{-4}. The values in the parentheses above are the ones which we determined earlier intuitively and they match very well with the simulated ones.

B. *Two Bus SCOPF*

Time required to solve:639.7596 s; Number of Iterations: 193; Generator 1: 500.0008 MW (500 MW) at base case, Generator 2: 299.999 MW, (300 MW) at base case, Line Flow on each line: 66.667 MW (66.667 MW) at base case; Line Flow on each line: 100 MW (100 MW) at post contingency case;Scaled Dual Variable for Power balance on Bus-1: 7.4857 $/MWh, Bus-2: 7.4787 $/MWh at base case and Bus-1: 2.4744 $/MWh, Bus 2: 12.49 $/MWh at post-contingency. The primal tolerance at the solution: 2.57×10^{-4}. Figures within parentheses indicate intuitively determined values as before. In the above case, the sum of base case and post-contingency scaled dual variables give the actual LMPs.

VII. Conclusion & Future Work

In this paper, we looked at the extension of the Alternating Direction Method of Multipliers(ADMM) Based Proximal Message Passing Algorithm from solving simple OPF Problems to solving SCOPF Problems which are secure to (N-1) Contingencies. We have specifically considered outages of Transmission Lines in this paper. We have presented the algorithm and looked at the numerical results pertaining to a simple systems. In our future work, we will be extending the method to implement multiple dispatch time look-ahead calculation with emphasis to post-fault thermal limit restoration and also we will be implementing the fully distributed/multithreading and peer-to-peer computations.

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