Security-Constrained Unit Commitment With Natural Gas Transmission Constraints

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***Abstract*—The contribution of this paper focuses on the development of a security-based methodology for the solution of short-term SCUC when considering the impact of natural gas transmission system. The proposed methodology examines the interdependency of electricity and natural gas in a highly complex transmission system. The natural gas transmission system is modeled as a set of nonlinear equations. The proposed solution applies a decomposition method to separate the natural gas transmission feasibility check subproblem and the power transmission feasibility check subproblem from the hourly unit commitment (UC) in the master problem. Gas contracts are modeled and incorporated in the master UC problem. The natural gas transmission subproblem checks the feasibility of natural gas transmission as well as natural gas transmission security constraints for the commitment and dispatch of gas-fired generating units. If any natural gas transmission violations arise, corresponding energy constraints will be formed and added to the master problem for solving the next iteration of UC. The iterative process will continue until a converged feasible gas transmission solution is found. A six-bus power system with seven-node gas transmission system and the IEEE 118–bus power system with 14-node gas transmission system are analyzed to show the effectiveness of the proposed solution. The proposed model can be used by a vertically integrated utility or the ISO for the short-term commitment and dispatch of generating units with natural gas transmission constraints.**

***Index Terms*—Benders decomposition, linear programming, mixed-integer programming, natural gas transmission, short-term security-constrained unit commitment.**

NOMENCLATURE

Index:

i	Index of generating unit.
j	Index of compressor.
k	Index of iteration.
s	Index of natural gas supplier.
l	Index of natural gas load.
b	Index of branch in power transmission.
t	Index of hour.
η	Index of natural gas supply contract.
m, n	Node index in natural gas transmission.
p, q	Bus index in power transmission.
ξ	Index of power plant.

Variables:

I	Status indicator of generating units.
P	Generation dispatch of a unit.
SR	Spinning reserve of unit.
SU, SD	Startup and shutdown cost of a unit.
SV_{gas}	Volume of storage facility for power plant.
W	Cost of natural gas contract.
F_{gas}	Natural gas consumption of gas-fired unit.
F_f	Natural gas consumption of compressor.
F_{st}	Startup fuel.
v	Gas delivery amount.
L	Gas load.
π	Pressure of node.
f_{mn}	Natural gas flow from node m to n.
H	Horsepower of compressor.
pf_b	Power flow through branch b.
γ	Control angel of phase shifter.
θ	Bus voltage angle.
$\hat{\boldsymbol{L}}, \boldsymbol{L}$	Gas load at current and next iteration.
$\hat{\boldsymbol{P}}, \boldsymbol{P}$	Power output at current and next iteration.
$\omega(), w()$	Objective function of natural gas and power transmission check subproblems.
$\boldsymbol{\mu}, \lambda$	Vector of dual variables.
$\boldsymbol{x}$	Vector of unknown variables.
$\Delta\boldsymbol{x}$	Difference vector of unknown variables between two iterations.
$\boldsymbol{J}, \boldsymbol{K}$	Jacobian matrix.
$\boldsymbol{A}$	Node-gas provider incidence matrix.
$\boldsymbol{B}$	Node-gas load incidence matrix.
$\boldsymbol{C}$	Bus-generator incidence matrix.
$\boldsymbol{D}$	Bus-electrical load incidence matrix.
$\boldsymbol{E}$	Bus-branch incidence matrix.

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$\boldsymbol{G}$	Gas withdrawing node-compressor incidence matrix.
$\boldsymbol{S_1}, \boldsymbol{S_2}, \boldsymbol{SL}$	Vectors of slack variables.
$g_m(\boldsymbol{\pi}, \boldsymbol{H}, \boldsymbol{v}, \boldsymbol{L})$	Gas mismatch of node m.

Constants:

W_o	Cost of firm natural gas contract.
ρ_{gas}	Price of natural gas.
ρ_{LS}	Penalty price of electricity load shedding.
a_j, b_j, c_j	Parameters of gas consuming function of compressor j.
x_{pq}	Reactance between bus p and q.
$pf_{b,\max}$	Power flow limits of branch b.
$F_{o,\eta}$	Contracted amount of take or pay gas contract.
M	Large number.
NGS	Number of natural gas suppliers.
NGL	Number of natural gas loads.
P_D	Electricity load of power system.
P_L	Estimated loss in power transmission.
P_{LS}	Electricity load shedding.
NT	Ending hour for segment m in startup fuel curve.
NN	Number of nodes in gas transmission system.
SR_r	Required spinning reserve.
NC	Number of compressors.
NB	Number of buses in power transmission.
$N\eta(\xi)$	Number of gas contracts of power plant ξ.
GU	Set of gas-fired generating units.
$GC(m)$	Set of nodes connected with m.
$P_{\min},\ P_{\max}$	Minimum, maximum capacity of a generating unit.
$SV_{gas,\max}$	Upper limit of storage facility for a plant.
$SV_{gas,\min}$	Lower limit of storage facility for a plant.
$R_{\min}, R_{\max}$	Maximum and minimum pressure ratio of a compressor.
$H_{\min}, H_{\max}$	Maximum and minimum horsepower of a compressor.
$L_{\min}, L_{\max}$	Maximum and minimum of natural gas load.
$\pi_{\min}, \pi_{\max}$	Maximum and minimum pressure.
$v_{\min}, v_{\max}$	Maximum and minimum of gas injection.

I. Introduction

According to the published data by the Energy Information Administration (EIA), natural gas-fired units generated 19% of the total U.S. electricity in 2005 [3]. Major attributes of natural gas applied to the electric utility sector are high efficiency, low cost of capital investment, expeditious permitting, and operation flexibility of natural gas-fired combined cycle units in comparison with conventional coal plants.

Gas-fired power plants provide a linkage between natural gas transmission and power transmission systems. Natural gas transmission could affect the security and the economics of power transmission. From the economics point of view, natural gas contracts could affect the commitment, dispatch, and the operation of power systems. From the security point of view, pressure losses, pipeline contingencies, lack of storage or natural gas supply disruptions may lead to forced outages of multiple gas-fired unit or deration of generating capacity which could dramatically increase the operating costs and congestion, and jeopardize the security of power systems [4]. Such conditions necessitate the solution of short-term unit commitment (UC) problem when integrating power transmission, natural gas contracts, as well as natural gas transmission systems as discussed in this paper.

A two-phase nonlinear optimization model was proposed in [5] to model the integrated operation of natural gas and power systems. In [6] and [7] a nonlinear optimization model was proposed by merging the traditional optimal power flow and natural gas transmission constraints. The short-term scheduling of integrated natural gas transmission and hydrothermal power systems was considered in [8] by applying Lagrangian relaxation and dynamic programming. The impact of natural gas transmission charges on power markets was discussed in [9]. The long-term integrated planning was analyzed in [10] as decisions are highly interdependent in natural gas and power transmission systems. The piecewise linear approximation of nonlinear flow-pressure relations was modeled in the UC problem [11].

This paper proposes an integrated model for SCUC with natural gas transmission constraints. The integrated model minimizes power system operating costs by taking into consideration power transmission constraints, natural gas transmission constraints, and natural gas contracts. Benders decomposition is applied to separate the natural gas transmission feasibility check subproblem from the master UC problem and the power transmission feasibility check subproblem as shown in Fig. 1. Constraints related to natural gas contracts are directly included in the master UC problem. When an optimal UC schedule is obtained without violating power transmission constraints, hourly natural gas demands of gas-fired units are then submitted to check the feasibility of natural gas transmission constraints. The natural gas transmission is modeled as a group of nonlinear equations for representing gas node pressures, which can be solved by a Newton–Raphson-like method similar to the Newton–Raphson-based power flows solution. The natural gas transmission feasibility check is formulated as linear programming (LP) problem and solved iteratively. If any violations of natural gas transmission constraints are detected, corresponding energy constraints (Benders cuts) are formed and fed back to

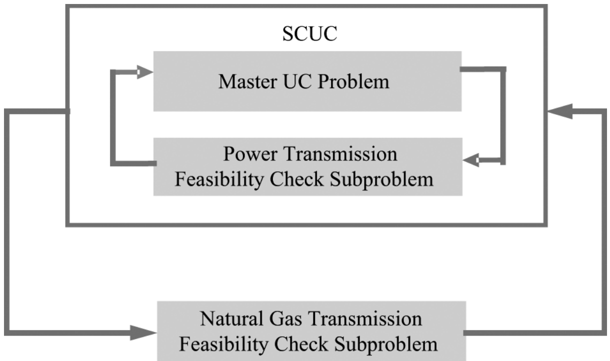

Fig. 1. Decomposition strategy for natural gas and electricity.

the master UC problem for the next iteration of calculation. The cut which represents shortages of natural gas supply or gas transmission congestions would limit the fuel consumption of a group of gas-fired units.

The proposed model is for the short-term operation of a large utility in the U.S. that has a large number of gas-fired and dual-fuel generating units. The same method can be applied to the ISO system. In regions where natural gas markets are heavily regulated but power systems are deregulated, natural gas transmission constraints imposed on power market participants such as generating companies (GENCOs) are submitted to electricity markets as energy constraints. In such cases, the proposed integrated SCUC model is a very important tool for the daily scheduling and dispatch of generating units to meet the hourly system load. The proposed model can also be a foundation for the modeling of midterm or long-term operation of integrated natural gas and power transmission systems.

The rest of this paper is organized as follows: Steady-state model of natural gas system is presented in Section III. SCUC model integrating constraints of natural gas system is formulated in Section IV. Section V presents and in detail discusses a six-bus system and the IEEE 118-bus system. The conclusion drawn from the study is provided in Section VI.

II. Model of Natural Gas Transmission System

The natural gas transmission system is one of the largest and most complex nonlinear systems in the world, which is represented by its steady state and dynamic characteristics [13], [14]. The steady-state mathematical model of natural gas transmission system comprised of a group of nonlinear algebraic equations is presented here for SCUC applications.

A. Key Components of Natural Gas System

Fig. 2 depicts the natural gas transmission system from producers to end users that is comprised of natural gas wells, transmission and distribution pipelines, storage facilities, and compressors. From the mathematical modeling viewpoint, these components are categorized into nodes and branches. As state variables, gas pressure is associated with each node while the natural gas flow rate is associated with each branch.

1) Suppliers and Loads in the Gas Transmission System: Natural gas may leave or enter the gas transmission system only through nodes, which represent delivery and receipt points.

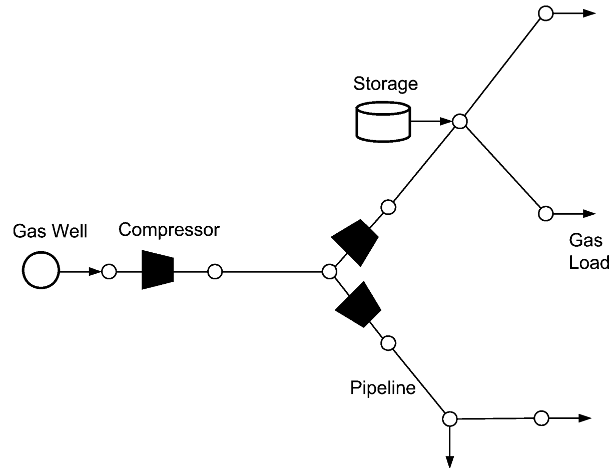

Fig. 2. Natural gas transmission system.

Most of the gas is supplied from gas wells, which are commonly located at remote sites. Supplies are modeled as positive gas injections at related nodes. In this paper, lower and upper limits of gas supply in each period are modeled as

$$v_{s,\min} \le v_s \le v_{s,\max}. \tag{1}$$

The natural gas loads could be residential, commercial, or industrial. Gas-fired power plants represent the most important industrial gas loads, which link electric power and natural gas transmission systems. Gas loads are represented as negative gas injections at related nodes which are modeled as

$$L_{l,\min} \le L_l \le L_{l,\max}. \tag{2}$$

2) Gas Storage: Unlike in electric power systems in which the electricity cannot be stored in large quantities, large sums of natural gas can be injected into natural gas storage facilities at off-peak periods and withdrawn during high demand periods [12]. This makes it possible to maintain a steady flow in a natural gas transmission pipeline even in peak periods or when contingencies occur.

Natural gas storage facilities are categorized based on their capacity and operating parameters that could balance gas transmission according to seasonal, monthly, and daily constraints [25]. The scheduling of natural gas storage facilities is a midterm or long-term optimization problem with min/max storage capacity and injection/extraction rates represented as constraints. However, for the day-ahead scheduling of power systems, we either model a natural gas storage facility as a load or a supplier except for a self-owned storage facility of gas-fired power plants. Their operating status (decision variable that indicate if the storage works as injection or extraction facility) and other parameters (injection or extraction rate limits) as an input data in our model will be sent from a gas company based on its long-term or midterm operation plan.

3) Pipeline Model: The power transmission depends on bus voltage conditions and parameters of transmission lines. By analogy, natural gas transmission, driven by pressures, are dependent on factors such as the length and the diameter of pipelines, operating temperatures and pressures, types of

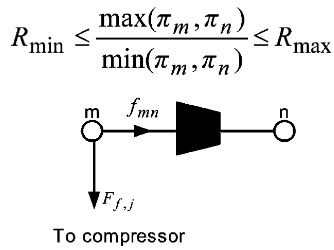

Fig. 3. Modeling of a compressor.

natural gas, altitude change over the transmission path, and the roughness of pipelines. Here, we present certain pipeline characteristics that are mostly used to design or optimize natural gas pipeline systems [15]–[18]. The flow in a natural gas pipeline extending from gas node m to gas node n is modeled as

$$f_{mn} = \operatorname{sgn}(\pi_m, \pi_n) \cdot C_{mn} \sqrt{|\pi_m^2 - \pi_n^2|} \quad (3)$$

$$\operatorname{sgn}(\pi_m, \pi_n) = \begin{cases} 1 & \pi_m \geq \pi_n \\ -1 & \pi_m < \pi_n \end{cases} \quad (4)$$

where C_{mn} is the pipeline constant that depends on temperature, length, diameter, friction, and gas composition.

4) Compressor Modeling: Pressure loss occurs when the natural gas flow encounters a pipeline resistance. In order to compensate for the pressure loss, compressor stations are installed at 50–100 mile intervals along pipelines. Compressor as a branch in the natural gas transmission system is analogous to a phase shifter or transformer in power systems. The natural gas flow through centrifugal compressor will be governed by

$$f_{mn} = \operatorname{sgn}(\pi_m, \pi_n) \frac{H_j}{k_{j2} - k_{j1} \left[\frac{\max(\pi_m, \pi_n)}{\min(\pi_m, \pi_n)} \right]^{\alpha}} \quad (5)$$

where k_{j1}, k_{j2}, and α are empirical parameters corresponding to the compressor design [7], [13], [14]. H_j represents the power of compressor j as a control variable, where

$$H_{min,j} \leq H_j \leq H_{max,j}. \quad (6)$$

The pressure ratio in (5) is restricted within a feasible range in (7) which is based on compressor characteristics. Compressors would consume additional natural gas that can be withdrawn from either inlet or outlet of compressor to drive turbines as illustrated in Fig. 3:

$$R_{min} \leq \frac{\max(\pi_m, \pi_n)}{\min(\pi_m, \pi_n)} \leq R_{max}. \quad (7)$$

The part of natural gas consumed by compressors represents transmission losses of natural gas grid. The amount of consumed natural gas is related to power of compressors given in the following:

$$F_{f,j}(H_j) = c_j + b_j \cdot H_j + a_j \cdot H_j^2. \quad (8)$$

B. Formulation of Node Flow Balance

A steady-state mathematical model of a natural gas transmission system is based on the nodal balance approach that indicates that the natural gas flow injected in a node is equal to the natural gas flowing out of the node. In other words, the natural gas flow mismatch at node m is equal to zero

$$\begin{aligned} g_m(\boldsymbol{v}, \boldsymbol{\pi}, \boldsymbol{H}, \boldsymbol{L}) = & \sum_{s=1}^{NGS} A_{ms} \cdot v_s - \sum_{l=1}^{NGL} B_{ml} \cdot L_l \\ & - \sum_{n \in GC(m)} f_{mn} \\ & - \sum_{j=1}^{NC} G_{mj} \cdot F_{f,j}(H_j) = 0. \end{aligned} \quad (9)$$

By substituting (3), (5), and (8) into (9), a group of nonlinear equations with NN dimensions will be obtained in which there are $NN + NC + NGS + NGL$ variables involving node pressure, horsepower of each compressor, natural gas supply, and natural gas load.

III. Formulation of SCUC With Natural Gas Transmission System

SCUC with natural gas transmission constraints is to determine the hourly UC and dispatch for minimizing the operating cost while meeting the prevailing electricity and natural gas constraints. The outline of the model is described as follows:

Min: System operating cost

$s.t.$

1) Power balance and reserve constraints
2) Individual generator constraints (Including min on/off time, min/max generation, startup/shutdown characteristics, ramp rate limits, etc.)
3) Natural gas contract constraints
4) Gas reserve constraints of gas-fired power plant
5) Power transmission constraints
6) Natural gas transmission constraints

Natural gas constraints are categorized into two types. The first type is modeled based on contracts between utility and natural gas providers. The other natural gas transmission constraints are derived according to natural gas transmission characteristics. The detailed modeling is presented as follows.

1) Objective Function: The objective function (10) is composed of fuel costs (utilities) or bids (ISO) for producing electric power and startup and shutdown costs of individual units over the scheduling horizon:

$$\begin{aligned} Min \Bigg\{ & \sum_{\eta} W_{o,\eta} + \sum_t \sum_{\eta} W_{\eta t} + \sum_t \sum_{i \notin GU} \\ & \cdot [F_{c,i}(P_{it}) \cdot I_{it} + SU_{it} + SD_{it}] + \sum_t \rho_{LS} \cdot P_{LS} \Bigg\}. \end{aligned} \quad (10)$$

2) Power Balance and Reserve Constraints:

$$\begin{aligned} & \sum_{i \in GU} P_{it} \cdot I_{it} + \sum_{i \notin GU} P_{it} \cdot I_{it} \\ & = P_{D,t} + P_{L,t} - P_{LS,t}, \quad \forall t \end{aligned} \quad (11)$$

$$\sum_{i \in GU} SR_{it} \cdot I_{it} + \sum_{i \notin GU} SR_{it} \cdot I_{it} \geq SR_{r,t}, \quad \forall t. \quad (12)$$

3) Individual Generator Constraints: Generating units will have individual constraints for restricting ramp up/down, min up/down time, emission, max/min capacity, and so on. A single-cycle natural gas unit is modeled as a thermal unit. Either the mode or the component model [22] may be used for modeling a combined-cycle unit. The model for fuel switching units is provided in [23] and [24]. The detailed modeling of individual generating unit constraints is discussed in [1].

4) Natural Gas Contract Constraints: Changes to natural gas regulations in the United States, which began in 1985, have offered new alternatives to natural gas transmission companies, locational distribution companies, and large end users [21], [27]. The changes have encouraged such companies to unbundle merchant services which allow buyers to transport natural gas through pipelines. The natural gas transmission open access allows purchasers to buy natural gas directly from various sources such as natural gas producers and independent marketers. Large users may buy transportation services from natural gas transmission companies.

Transportation services are divided into no-notice, firm, and interruptible. In general, natural gas supply contracts with interruptible transportation services can be interrupted with little notice and penalties [27]. The gas-fired generators have preferred interruptible transportation contracts which are more economical than firm services for supplying electricity at lower prices in competitive electricity markets. Since the gas turbine operation would require high gas pressure, electric generators are more susceptible than other gas loads to pressure losses. Therefore, natural gas supply contracts for gas-fired generating units are relatively less expensive with a lower service priority. Hence, in emergencies, gas-fired generating units are curtailed first especially in winters when gas locational distribution companies would buy the majority of pipeline capacity for supplying their own customers.

In this paper, we model natural gas supply contracts and transportation services together. A utility may sign up natural gas supply contracts with suppliers as take-or-pay or flexible contracts. Here we assume that each supply contract is corresponding to a priority level of service. The price of a natural gas supply contract has already included costs of transportation services. For take-or-pay natural gas supply contracts, the total cost is constant (13). Take-or-pay natural gas supply contracts are to be consumed regardless of cost:

$$W_{\eta} = W_{o,\eta}. \tag{13}$$

The sum of natural gas usage cannot exceed the contracted amount stated as

$$\sum_{t} F_{gas,\eta t} \le F_{o,\eta}. \tag{14}$$

For flexible natural gas supply contract price is usually higher than that of take-off-pay contact with the same transportation services. The total cost depends on the natural gas usage stated as

$$W_{\eta t} = (\rho_{gas,\eta}) \cdot F_{gas,\eta t}. \tag{15}$$

Every natural gas contract is considered as a natural gas load with a certain service priority at hour t which is given as

$$L_{lt} = F_{gas,\eta t}. \tag{16}$$

5) Natural Gas Reserve Constraints of Gas-Fired Plants: Some gas-fired power plant may construct a self-owned storage tank so that it can reserve a few of natural gas to be used in peak hours. The natural gas balance constraint related to a gas contract is given as

$$\sum_{i \in GU(\xi)} (F_{gas,it} + F_{st,it}) = SV_{gas,p(t-1)} - SV_{gas,pt} + \sum_{\eta=1}^{N\eta(\xi)} F_{gas,\eta t}. \tag{17}$$

The self-owned storage is usually small which is scheduled by power plants. The volume of storage facility is governed by

$$SV_{gas,p,\min} \le SV_{gas,pt} \le SV_{gas,p,\max}. \tag{18}$$

6) Power Transmission Constraints:

$$\begin{aligned} \boldsymbol{E} \cdot \boldsymbol{pf} &= \boldsymbol{C} \cdot \hat{\boldsymbol{P}} - \boldsymbol{D} \cdot \boldsymbol{P_L} \\ pf_b &= \frac{\theta_p - \theta_q - \gamma_{pq}}{x_{pq}} \quad (p, q \in b), \quad \forall b \\ |pf_b| &\le pf_{b,\max}, \quad \forall b \\ \boldsymbol{\gamma}_{\min} &\le \boldsymbol{\gamma} \le \boldsymbol{\gamma}_{\max} \\ \theta_{ref} &= 0. \end{aligned} \tag{19}$$

7) Natural Gas Transmission Constraints: Equations (1)–(9).

IV. SCUC Solution With Natural Gas Transmission System

Fig. 4 depicts the SCUC flowchart with natural gas transmission constraints. Benders decomposition is applied to decompose the original large-scale optimization problem into a master problem and several subproblems, based on the LP duality theory. The master problem is a mixed-integer program and subproblems are linear programs [19], [20]. The process is described as follows.

The ISO or the utility operator would execute the day-ahead SCUC to determine the UC schedule that would satisfy power transmission constraints. Meanwhile, natural gas transmission operators provide the ISO with the information on industrial and residential gas load forecasting with service priorities, natural gas transmission parameters, and planned outage of natural gas pipelines. The ISO will check the feasibility of natural gas transmission system for serving expected natural gas loads based on available natural gas resources. If the natural gas check subproblem is infeasible, natural gas usage constraints for gas-fired power plants will be formed and added to the master UC problem for the daily UC rescheduling. The iterative process between UC and natural gas transmission feasibility check subproblem will continue until the feasibility of natural gas transmission flow is obtained. The ISO or utility will determine the hourly generation UC and dispatch in the day-ahead market accordingly. The corresponding natural gas consumption data will be transmitted to the natural gas company for evaluation.

A. Master UC Problem

It is represented by (10)–(18).

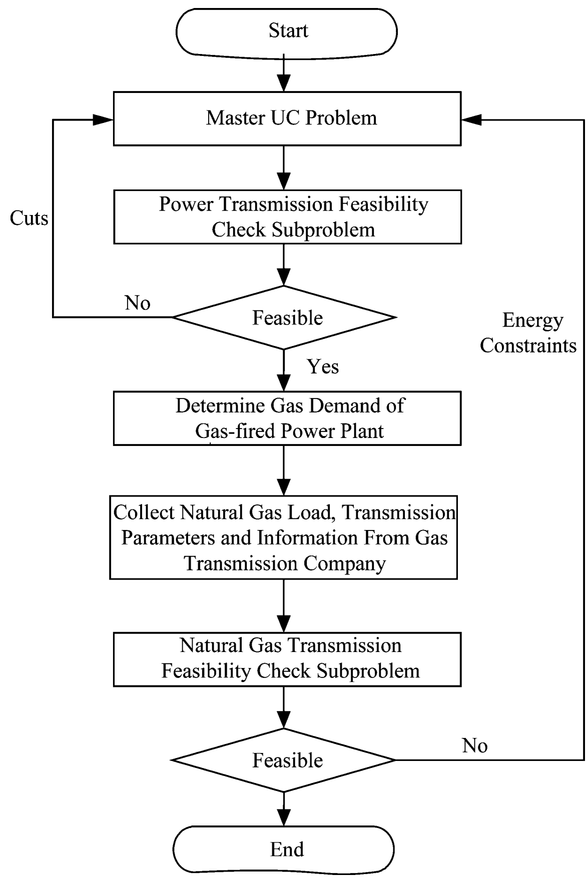

Fig. 4. Flowchart of SCUC with natural gas transmission constraints.

B. Power Transmission Check Subproblem

The subproblem for power transmission check conducts the security analysis based on the UC solution. The solution of subproblem may be parallelized because hourly power transmission constraints are not coupled. The power transmission model can be either DC or AC. The AC model is presented in [18]. Here, for the sake of focusing on our proposed natural gas subproblem, we present a DC model. The subproblem minimizes the sum of bus power imbalances subjected to the relaxed DC transmission formulation (20). The subproblem mitigates transmission violations and iterates with UC via power transfer distribution factors (PTDFs) or Benders cuts.

The general Benders decomposition method will form both infeasibility and optimality cuts. We update upper and lower bounds until the gap between the bounds is less than a given value. For particular engineering applications, (10) will be fully included in the master problem. The subproblems will check constraints (19) and produce infeasibility cuts (21) which are fed back to the master problem if any hourly values of objective function (20) are nonzero:

$$
\begin{aligned}
Min \quad & w(\hat{\boldsymbol{P}}) = \sum_{p=1}^{NB} (S1_p + S2_p) \\
S.t. \quad & \boldsymbol{E}\cdot\boldsymbol{pf} = \boldsymbol{C}\cdot\hat{\boldsymbol{P}} - \boldsymbol{D}\cdot\boldsymbol{P_L} + \boldsymbol{S}1 - \boldsymbol{S}2 \quad \lambda \\
& pf_b = \frac{\theta_p - \theta_q - \gamma_{pq}}{x_{pq}} \quad (p,q\in b), \quad \forall b \\
& pf_{b,\max} \le pf_b \le pf_{b,\max}, \quad \forall b \qquad (20) \\
& \boldsymbol{\gamma}_{\min} \le \boldsymbol{\gamma} \le \boldsymbol{\gamma}_{\max} \\
& \theta_{ref} = 0
\end{aligned}
$$

$$
w(\boldsymbol{P}) = w(\hat{\boldsymbol{P}}) + \sum_{i=1}^{NG} \mu_{it}(P_{it} - \hat{P}_{it}) \le 0. \qquad (21)
$$

C. Natural Gas Transmission Feasibility Check Subproblem

In the natural gas transmission (9), the number of variables is more than the number of equations and the solution would require fixing no more than $NC+NGS+NGL$ variables in iterative nonlinear optimization techniques. The Newton–Raphson method is applied to solve the nonlinear equations. If $\boldsymbol{x}$ represents the vector of unknown variables, the iterative equations are given as

$$
\begin{cases} \boldsymbol{J}_k \Delta\boldsymbol{x}^{(k)} = -\boldsymbol{g}(\boldsymbol{x}^{(k)}) \\ \boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} + \Delta\boldsymbol{x}^{(k)} \end{cases} \qquad (22)
$$

where $\boldsymbol{J}^{(k)}$, the Jacobian matrix in the kth iteration, is given as

$$
\boldsymbol{J}^{(k)} = \boldsymbol{Dg}(\boldsymbol{x}^{(k)}) = \left.\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}^{(k)}}. \qquad (23)
$$

The nonzero elements of Jacobian matrix are given as

$$
\frac{\partial g_m}{\partial v_s} = A_{ms} \qquad (24)
$$

$$
\frac{\partial g_m}{\partial L_l} = -B_{ms} \qquad (25)
$$

$$
\frac{\partial g_m}{\partial \pi_m} = -\sum_{n\in m} \frac{\partial f_{mn}}{\partial \pi_m} \qquad (26)
$$

$$
\frac{\partial g_m}{\partial \pi_n} = -\frac{\partial f_{mn}}{\partial \pi_n} \qquad (27)
$$

$$
\frac{\partial g_m}{\partial H_j} = -\sum_{n\in GC(m)} \frac{\partial f_{mn}}{\partial H_j} - \sum_{j=1}^{NC} G_{mj}\cdot(2a_j + b_j). \qquad (28)
$$

Based on (3)–(8), we would calculate the partial derivative of natural gas flow of each branch with respect to the corresponding pressure and horsepower of compressor. The detailed formulations of $\partial f_{mn}/\partial \pi_m$, $\partial f_{mn}/\partial \pi_n$ and $\partial f_{mn}/\partial H_j$ are given in (41)–(48) in Appendix B. If π, H, v, L represent the variables, the modified Jacobian matrix is $\begin{bmatrix} \boldsymbol{J}_\pi^{(k)} & \boldsymbol{J}_H^{(k)} & \boldsymbol{A} & -\boldsymbol{B} \end{bmatrix}$. The iterative formulation is represented in (29). The Jacobian matrix is highly sparse and sparse matrix techniques including sparse storages, triangular factorization, fast forward (backward) substitution are used to solve (29) which results from the Newton–Raphson method:

$$
\begin{aligned}
&\begin{bmatrix} \boldsymbol{J}_\pi^{(k)} & \boldsymbol{J}_H^{(k)} & \boldsymbol{A} & -\boldsymbol{B} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{\pi}^{(k)} \\ \Delta\boldsymbol{H}^{(k)} \\ \Delta\boldsymbol{v}^{(k)} \\ \Delta\boldsymbol{L}^{(k)} \end{bmatrix} = -\boldsymbol{g}(\boldsymbol{\pi}, \boldsymbol{H}, \boldsymbol{v}, \boldsymbol{L}) \\
&\begin{bmatrix} \boldsymbol{\pi}^{(k+1)} \\ \boldsymbol{H}^{(k+1)} \\ \boldsymbol{v}^{(k+1)} \\ \boldsymbol{L}^{(k+1)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\pi}^{(k)} \\ \boldsymbol{H}^{(k)} \\ \boldsymbol{v}^{(k)} \\ \boldsymbol{L}^{(k)} \end{bmatrix} + \begin{bmatrix} \Delta\boldsymbol{\pi}^{(k)} \\ \Delta\boldsymbol{H}^{(k)} \\ \Delta\boldsymbol{v}^{(k)} \\ \Delta\boldsymbol{L}^{(k)} \end{bmatrix}.
\end{aligned} \qquad (29)
$$

The natural gas transmission feasibility check problem (30)–(37) would check whether the available gas resources and controllable compressors could satisfy natural gas transmission limits as well as natural gas demands of gas-fired units committed by the master UC problem. The nonnegative natural gas load shedding variables $\boldsymbol{SL}$ are added to ensure that the optimization problem is feasible. From a physical viewpoint, such slack variables represent virtual natural gas load shedding at each delivery point to eliminate mismatches.

The objective is to minimize the sum of slack variables. Constraints (31)–(37) represent pressure limits on each node, maximum/minimum gas delivery mass of each node, horsepower limit, and pressure ratio range of compressors, respectively. Since natural gas flow equations are nonlinear, we use the successive LP to solve the problem iteratively. The Jacobian matrix is the same as (23). Hence

$$\text{Min}\ \omega(\hat{\boldsymbol{L}}) = \sum_{l=1}^{NGL} (SL_l) \quad (30)$$

s.t.

$$[\boldsymbol{J}_\pi \quad \boldsymbol{J}_{\boldsymbol{H}} \quad \boldsymbol{A} \quad -B] \begin{bmatrix} \Delta\boldsymbol{\pi} \\ \Delta\boldsymbol{H} \\ \Delta\boldsymbol{v} \\ \Delta L \end{bmatrix} = -\boldsymbol{g}(\boldsymbol{\pi}, \boldsymbol{H}, \boldsymbol{v}, \hat{L}) \quad \boldsymbol{\mu} (31)$$

$$\boldsymbol{SL} + \Delta\boldsymbol{L} = 0 \quad (32)$$

$$-\hat{\boldsymbol{L}} \leq \Delta\boldsymbol{L} \leq 0 \quad (33)$$

$$\boldsymbol{\pi}_{\min} \leq \boldsymbol{\pi} + \Delta\boldsymbol{\pi} \leq \boldsymbol{\pi}_{\max} \quad (34)$$

$$\boldsymbol{v}_{\min} \leq \Delta\boldsymbol{v} + \boldsymbol{v} \leq \boldsymbol{v}_{\max} \quad (35)$$

$$\boldsymbol{H}_{min} \leq \boldsymbol{H} + \Delta\boldsymbol{H} \leq \boldsymbol{H}_{max} \quad (36)$$

$$\boldsymbol{R}_{min} \cdot (\boldsymbol{\pi}_n + \Delta\boldsymbol{\pi}_n) \leq \boldsymbol{\pi}_m + \Delta\boldsymbol{\pi}_m \leq \boldsymbol{R}_{max} \cdot (\boldsymbol{\pi}_n + \Delta\boldsymbol{\pi}_n). \quad (37)$$

The iterative solution process of natural gas transmission feasibility check subproblem is discussed as follows.

1) Calculate modified Jacobian matrix $\boldsymbol{J}$ and initial natural gas node mismatch vector $-\boldsymbol{g}(\boldsymbol{\pi}, \boldsymbol{H}, \boldsymbol{v}, \hat{\boldsymbol{L}})$ based on the initial gas load, system states, and generation schedules of natural gas providers.
2) Use LP to minimize the objective function (30) and calculate changes in state and control variables of the natural gas transmission system $\Delta\pi$, ΔH, Δv, and ΔL. If the difference between current and previous iterative changes is less than a specified threshold ε, stop the process. Otherwise, go to Step 3.
3) Update state and control variables. Calculate elements of Jacobian matrix and go back to Step 2:

$$\begin{bmatrix} \boldsymbol{\pi}^{(k+1)} \\ \boldsymbol{H}^{(k+1)} \\ \boldsymbol{v}^{(k+1)} \\ \boldsymbol{L}^{(k+1)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\pi}^{(k)} \\ \boldsymbol{H}^{(k)} \\ \boldsymbol{v}^{(k)} \\ \boldsymbol{L}^{(k)} \end{bmatrix} + \begin{bmatrix} \Delta\boldsymbol{\pi}^{(k)} \\ \Delta\boldsymbol{H}^{(k)} \\ \Delta\boldsymbol{v}^{(k)} \\ \Delta\boldsymbol{L}^{(k)} \end{bmatrix}.$$

Once the above iterative process is completed, a nonnegative objective function (30) that is larger than the specified tolerance means that nodal gas suppliers and loads cannot provide a feasible natural gas flow solution. Otherwise, an energy constraint (Benders cut) given in (38) will be formed:

$$\omega(\boldsymbol{L}) = \omega(\hat{\boldsymbol{L}}) + \sum B_{lm}^T \cdot \mu_m \cdot (L_l - \hat{L}_l) \leq 0 \quad (38)$$

$$\omega(F_{gas,\eta}) = \omega(\hat{F}_{gas,\eta}) + \sum B_{lm}^T \cdot \mu_m \cdot (F_{gas,\eta} - \hat{F}_{gas,\eta}) \leq 0. \quad (39)$$

Here, $\omega(\hat{\boldsymbol{L}})$ is equal to the sum of slack variables in the current iteration. A positive $\omega(\hat{\boldsymbol{L}})$ corresponds to a positive natural gas node mismatch $-\boldsymbol{g}(\boldsymbol{\pi}, \boldsymbol{H}, \boldsymbol{v}, \hat{\boldsymbol{L}})$. $\boldsymbol{\mu}$ is the dual variable corresponding to (30). $\boldsymbol{\mu}$ captures the sensitivity of $\omega(\hat{\boldsymbol{L}})$ to right hand mismatch $-\boldsymbol{g}(\boldsymbol{\pi}, \boldsymbol{H}, \boldsymbol{v}, \hat{\boldsymbol{L}})$ which is derived by solving the dual problem of successive LP. So $\boldsymbol{\mu}$ can capture the mismatch of natural gas node and feed the information back to the UC problem. By substituting (16) into (38), we derive energy constraints (39) which is added to the master UC problem for the next iterative solution of UC.

Here, Benders cuts are generated from the solution of a successive approximation of a nonlinear equation. Pipeline equations may make the subproblems nonconvex in feasible sets. So if the initial operating point of natural gas problem is not close enough to the global optimal points, the final solution of SCUC with natural gas constraints may result in a local optimal solution. However, we may try different initial points of natural gas flow to find the best possible solution. Reference [26] introduces a set of initial points $(\pi_m,\ \pi_n)$ to linearize (3) which can replace the nonlinear function with linear inequality for each pipeline. For any given pipeline flow, only one of the inequality constraints, namely the one that approximates the flow best, will be binding. For a pipeline network without compressors, (30)–(37) represent a LP problem, rather than a successive LP, when applying linearized inequality constraints [26]. The improvement can potentially enhance the quality of optimal solution with the same computing time.

V. Case Studies

We apply two case studies consisting of a six-bus power system with seven-node gas system and the IEEE 118-bus system with 14-node natural gas system to illustrate the performance of our SCUC with natural gas transmission constraints. We assume that 1 kilo-cubic feet of natural gas can generate 1 MBtu of energy in both cases.

A. Six-Bus System

The six-bus system, depicted in Fig. 5, has three gas-fired units, five transmission lines, and two tap-changing transformers. The characteristics of generators, buses, transmission lines, and tap-changing transformers and the hourly load distribution over the 24-h horizon are given in Tables X–XV in Appendix A, respectively. The unit startup and shutdown costs are assumed negligible and equal to zero in this case. The seven-node natural gas system is given in Fig. 6, which has one compressor, five pipelines, two natural gas suppliers, and five natural gas loads. The natural gas transmission parameters are listed in Tables XVII–XXI in Appendix A.

According to Fig. 6, natural gas loads 1, 5, 3 correspond to the gas-fired units 1, 2, 3, which are determined by the hourly generation dispatch of generating units. The units have interruptible

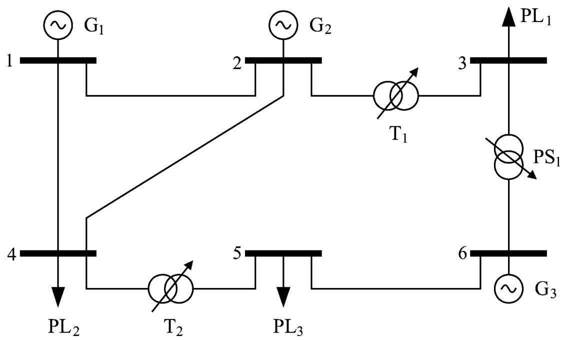

Fig. 5. Six-bus power system.

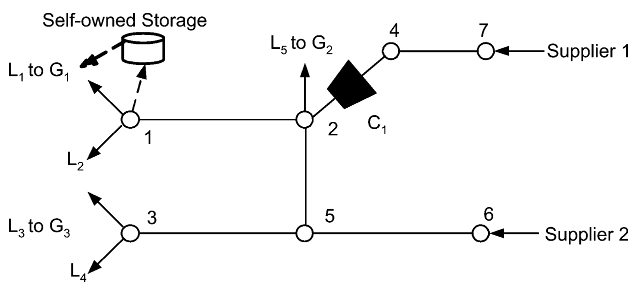

Fig. 6. Seven-node natural gas system.

TABLE I
HOURLY SCHEDULE OF CASE 1 OF SIX-BUS SYSTEM

Daily production cost: $509,572	
Unit	**Hours (0-24)**
1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
2	1 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 0 0 0
3	0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0

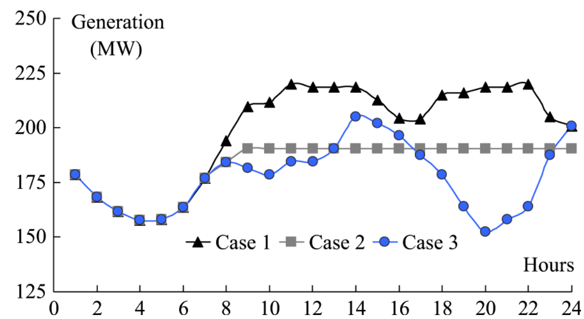

Fig. 7. Hourly dispatch of unit 1 in Cases 1–3.

TABLE II
HOURLY SCHEDULE OF CASE 2 OF SIX-BUS SYSTEM

Daily production cost: $550,399	
Unit	**Hours (0-24)**
1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
2	1 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
3	0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

contracts with the natural gas company. Gas loads 2 and 4 represent equivalent loads consumed by other natural gas users. Here, we assume that natural gas loads 2 and 4 have a higher priority than gas-fired power plants, which means any natural gas transmission infeasibility will lead to the natural gas load shedding of gas-fired power plants. The hourly sums of natural gas loads 2 and 4 are given in Tables XXII and XXIII.

In order to discuss the efficiency of the proposed approach as well as the impact of natural gas transmission system on SCUC results, we consider the following five cases:

Case 1: SCUC without natural gas transmission constraints
Case 2: SCUC with natural gas transmission constraints
Case 3: Impact of fluctuating natural gas loads
Case 4: Impact of natural gas pipeline outages
Case 5: Impact of natural gas storage on power plants

Theses cases are discussed as follows.

1) Case 1: SCUC Without Natural Gas Transmission Constraints: We calculate the hourly UC solution in 24 h by considering DC transmission constraints and ignoring gas transmission constraints. The hourly commitment schedule is shown in Table I in which the hour 0 represents the initial condition. The peak electricity load is at hour 17 when units 1, 2, 3 are dispatched at 204.11 MW, 37.01 MW, and 20 MW, respectively. The daily operating cost is $509 572. In this Case, expensive generating units 2 and 3 are not committed at certain hours in order to minimize the daily operating cost.

2) Case 2: SCUC With Natural Gas Transmission Constraints: We apply the SCUC schedule given in Case 1 and incorporate natural gas transmission constraints. When we ignored natural gas constraints, the gas demand of unit 1 was 2952 kcf/h at hour 17, and the gas load 2 at the same node was 4000 kcf/h. However, the natural gas transmission limit of pipeline 1 which is 6765 kcf/h is violated because the minimum pressure at the natural gas node 1 and the maximum pressure at the natural gas node 2 are 105 and 170 Psig, respectively. Since the gas load 2 at hour 17 has the higher priority, the natural gas load 1 corresponding to unit 1 is slacked and the dual variable $\boldsymbol{\mu}$ is calculated after solving the successive LP. According to (38), a Benders cut (energy constraints) given by (40) is created which indicates that a gas load shedding of 187.1 kcf/h will take place at the natural gas load 1. The new UC solution in the master problem results in a new generation dispatch of 190.43 MW for unit 1 at hour 17:

$$\omega(\hat{\boldsymbol{L}}) = 187.1 + [1 \cdot L_{1,t=17}(P_{1,t=17})] - 1 \cdot 2952 \leq 0. \quad (40)$$

In this case, the natural gas transmission feasibility check subproblem also encounters natural gas flow violations at hours 8–16 and 18–24. SCUC, with a single iteration between the master UC problem and natural gas transmission feasibility check subproblem, commits the expensive unit 2 at hours 10, 11, and 22 as well as the expensive unit 3 at hours 8 and 9. Fig. 7 shows that the generation dispatch of cheaper unit 1 at hours 8–24 is curtailed. Table II shows that the daily operating cost of $550 399 is higher than that of Case 1.

3) Case 3: Impact of Fluctuating Natural Gas Loads: There is a higher chance of natural gas flow congestion or pressure loss if the residential gas load and electricity generation would peak simultaneously. In this case and thereafter, we use the natural gas load data given inTable XXIII in Appendix A instead of a constant load presented in Case 2. New SCUC scheduling results are given in Table III with an operating cost of $553 850

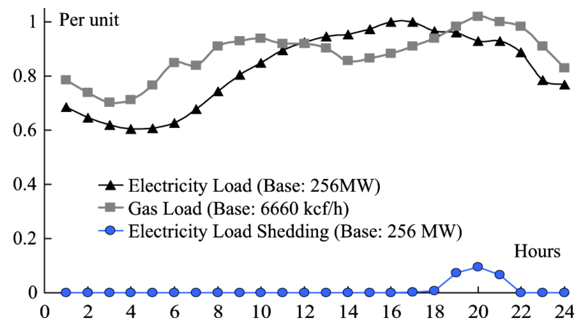

Fig. 8. Case 3 results.

TABLE III
HOURLY SCHEDULE OF CASE 3 OF SIX-BUS SYSTEM

Daily production cost: $553,850	
Electric load shedding: 62.63 MW	Load shedding cost: $62,630
Unit	**Hours (0-24)**
1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
2	1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
3	0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0

TABLE IV
HOURLY SCHEDULE OF CASE 4 OF SIX-BUS SYSTEM

Daily production cost: $496,873	
Electric load shedding: 737 MW	Load shedding cost: $737,000
Unit	**Hours (0-24)**
1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0
2	1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
3	0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

which is higher than those in Cases 1 and 2. The SCUC solution resorts to 62.63 MW of electric load shedding during hours 17–21 to deal with natural gas shortages. The total load shedding cost is $62 650 since the load shedding price is $1000/MW. Fig. 8 demonstrates that both electric and natural gas loads are high when load shedding is deemed necessary. Accordingly, hourly variations of residential and industrial natural gas load can affect the security and economics of power systems.

4) Case 4: Impact of Natural Gas Pipeline Outages: The potential outages of natural gas system will become a critical issue when the total installed capacity of natural gas-fired generating units increases. In this case, the outage of natural gas pipeline 2 between nodes 2 and 5 at hours 21–24 will lead to the curtailment of generation unit 1 as shown in Table IV.

Here unit 1 is the cheapest one; however, unit 2 is more likely to stay online longer than unit 1 during outage hours. In this case, the minimum generation of unit 1 is 100 MW which cannot be sustained when there is a natural gas shortage. At the same time, unit 1 is larger than other generating units which makes it impossible for other units to compensate the generation gap when unit 1 is on outage at hours 21–24. Accordingly, a 737 MW of electric load shedding will occur, which is an undesirable option in power systems. This case shows that natural gas pipeline outages may have a significant impact on the power system security and economics.

5) Case 5: Impact of Natural Gas Storage on Power Plants: We consider an additional storage facility owned by gas-fired

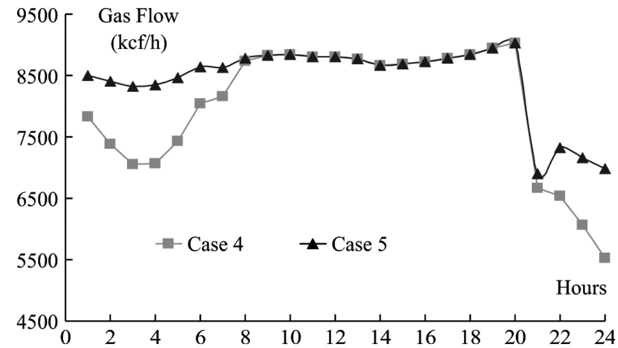

Fig. 9. Natural gas pipeline flows in Cases 4 and 5.

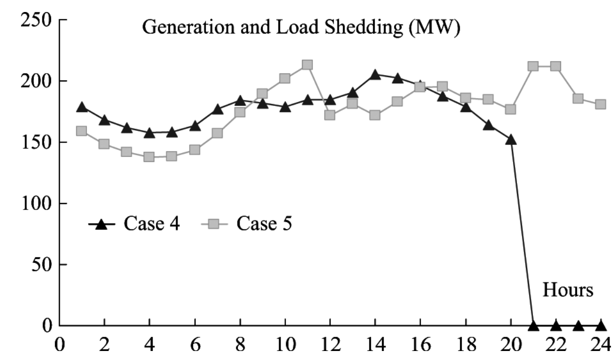

Fig. 10. Hourly dispatch of unit 1 in Cases 4 and 5.

power unit 1 as shown in Fig. 6. The storage parameters are listed in Table XXIV. With storage, the additional natural gas transported through pipeline 1 during off-peak hours is stored for peak hours or when pipeline outages occur. Table V gives the new daily UC schedule with a production cost of $557 351. This new schedule will not require any curtailments of electric loads at hours 21–24. The reasons are given as follows. First, after the seven-node natural gas transmission is divided into two subsystems, the supplier 2 cannot provide gas fuels to units 2 and 1. By comparing with Case 4, we learn that unit 3 is kept online to make use of supplier 2 by decreasing the production of generating units 1 and 2. Meanwhile, the most expensive unit 2 is not committed at hours 9–11 and 22–24, which indicates that the additional available gas can be used to produce power by the most efficient unit 1 or stored in storage. Fig. 9 shows the hourly natural gas transmission through pipeline 1 in Cases 4 and 5, respectively. It is clear that pipeline 1 transports more gas to the storage and unit 1 at hours 1–8 and 21–24. Fig. 10 shows the hourly dispatch of unit 1 in Cases 4 and 5. Fig. 11 shows the hourly gas volume in storage. The natural gas storage can improve the security and economics of constrained power systems by supplying additional gas reserves to power plants. The information on cuts and CPU times is given in Table VI.

B. IEEE 118-Bus System

A modified IEEE 118-bus system is used to study the SCUC with natural gas transmission constraints. The system has 54 fossil units, 12 gas-fired combined cycle units, seven hydro units, 186 branches, 14 capacitors, nine tap-changing transformers, and 91 demand sides. The peak load of 7300 MW

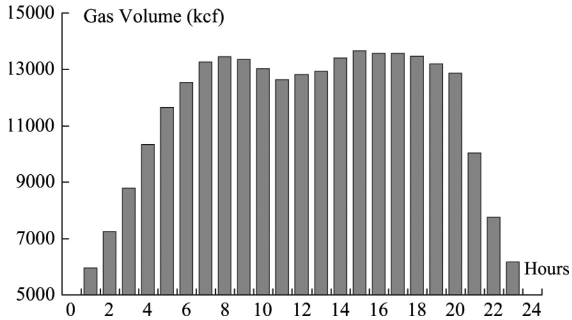

Fig. 11. Hourly gas volume of storage in Case 5.

TABLE V
HOURLY SCHEDULE OF CASE 5 OF SIX-BUS SYSTEM

Daily production cost: $557,351	
Total electricity load shedding: 0 MW	
Unit	**Hours (0-24)**
1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
2	1 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 0 0 0
3	0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

TABLE VI
INFORMATION OF GENERATED CUTS AND ITERATION OF SIX-BUS SYSTEM

6 Bus system	Case 1	Case 2	Case 3	Case 4	Case 5
Total Electric Network Cuts	10	10	10	10	10
Total Gas Cuts	0	16	21	21	25
Number of Iterations of Electric Network Loop	2	3	4	4	6
Number of Iterations of Gas Loop	0	2	3	3	5

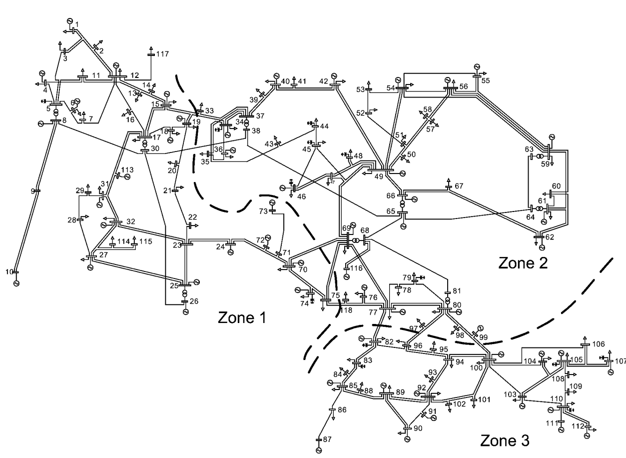

Fig. 12. 118-bus power system.

occurs at hour 21. The power transmission topology is shown in Fig. 12.

The natural gas transmission is composed of 14 nodes, 12 pipelines, and two compressors as shown in Fig. 13. The test data for the 118-bus power system and 14-node gas system are given in motor.ece.iit.edu/data/Gastranssmion_118_14test.xls.

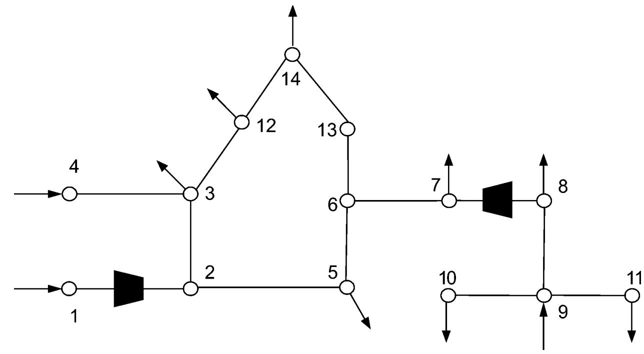

Fig. 13. 14-node gas transmission system.

TABLE VII
HOURLY UNIT COMMITMENT OF CASE 1 FOR THE 118-BUS SYSTEM

Daily production cost: $ 1936329	
Unit	**Hours (0-24)**
1001-1003	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1004-1005	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1006	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1007	1 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1008-1009	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1010-1011	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1012	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0
1013	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1014	1 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1015	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1016	1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1017-1018	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1019	1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1020-1021	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1022-1023	1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1024-1025	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1026	1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1027-1029	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1030-1033	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1034	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1
1035-1036	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1037	1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1038	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1039-1040	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1041-1042	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1043-1045	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1046-1047	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1048	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0
1049-1050	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1051	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0
1052	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1053	1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1054	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
4001	0 1 1 3 3 3 3 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
4002	0 0 0 0 0 2 2 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
4003	0 0 0 0 0 0 0 1 1 3 3 4 4 4 4 4 4 4 4 4 4 4 4 4 4
4004	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 3 3 4 4 4 4 4 4
4005-4006	0 2 2 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
4007-4009	0 2 2 2 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
4010-4011	0 2 2 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
4012	0 2 2 2 2 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
6001	0 1 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
6002	0 1 1 0 0 0 0 0 1 1 1 1 1 1 0 1 1 1 1 1 0 0 1 0 0
6003	0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
6004	0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
6005	0 1 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
6006	0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 1 1 1 1 1 1 1 1 0 0
6007	0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

TABLE VIII
GENERATED CUTS AND ITERATIONS OF 118-BUS SYSTEM

Iteration Index of Gas Transmission Loop	**1**	**2**	**3**	**4**	**5**	**6**
Number of Iterations of Power Transmission Loop in Each Iteration of Natural Gas Loop	8	2	3	1	1	1
Power Transmission Cuts in Each Iteration of Natural Gas Transmission Loop	68	2	2	0	0	0
Gas Cuts in Each Iteration of Natural Gas Transmission Loop	24	22	17	15	4	0

Table VII presents the base case SCUC without natural gas transmission constraints with a daily operating cost of $1 936 329. The execution time is 60 s on a 2.6-GHz personal computer. Since combined-cycle gas units demonstrate a better efficiency, generating units 4001–4012 are committed at certain hours to serve the hourly load while the expensive fossil units such as 1013, 1047, and 1052 are not committed.

In order to check if the gas flow is feasible, we use the initial UC solution to examine the hourly natural gas transmission feasibility check subproblem(30)–(37). In the subproblem, gas load violations occur in nodes 5, 10–11, 14, which show that the natural gas required by combined-cycle units cannot be transported to related nodes by the gas transmission system in base case operating conditions. Accordingly, natural gas demands are slacked and energy constraints are fed back to the master UC problem. The information on generated cuts and iterations of natural gas and power transmission loops are listed in Table VIII. After six iterations between SCUC and natural gas feasibility check subproblems, we obtain the generation dispatch and hourly UC results shown in Table IX. In Table IX, combined-cycle units 4003, 4005, and 4010 are off because of gas shortages. Furthermore, the distribution of combined-cycle units has changed as compared to that in the base case. The daily operating cost is $1 965 738 which is higher than that of the base case.

VI. CONCLUSIONS

This paper proposes a mathematical model and its solution for SCUC with natural gas transmission constraints. At the base case, natural gas transmission is modeled by a group of nonlinear equations. Benders decomposition is applied to separate the natural gas transmission subproblem from the master UC problem and the power transmission subproblem. The decomposition would avoid the computational complexity when solving the proposed large-scale optimization problem. Successive LP is applied to solve the natural gas transmission feasibility check subproblem. The LP duality theory is applied to generate energy constraints corresponding to natural gas transmission violations, which are added to the master UC problem for rescheduling the hourly UC. The tests on the six-bus power system linking with the seven-node gas system and the IEEE 118-bus systems with the 14-node gas system demonstrate the effectiveness of the proposed scheduling approach.

TABLE IX
HOURLY UNIT COMMITMENT OF CASE 2 FOR THE 118-BUS SYSTEM

Daily production cost: $ 1965738

Unit	**Hours (0-24)**
1001-1003	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1004-1005	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1006	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1007	1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1008-1009	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1010-1011	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1012-1013	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0
1014	1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1015	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1016	1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1017-1018	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1019	1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1020-1021	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1022-1023	1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1024-1025	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1026	1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1027-1029	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1030-1033	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1034	1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1035-1036	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1037	1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1038	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1039-1040	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1041-1042	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1043-1045	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1046	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1047-1048	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1
1049-1050	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1051-1052	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1
1053	1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1054	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
4001	0 1 1 3 3 3 3 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
4002	0 0 0 0 2 2 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
4003	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
4004	0 0 0 0 0 2 2 4 4 4 4 4 4 4 4 4 4 4 4 3 3 4 4 4 4
4005	0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
4006	0 2 2 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
4007-4009	0 2 2 2 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
4010	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 2 2 1 1 0 0 0
4011	0 2 2 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
4012	0 2 2 2 2 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
6001	0 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
6002	0 1 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 0 0
6003	0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
6004	0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
6005	0 1 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
6006	0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 1 1 0 1 1 1 1 1 0 0
6007	0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

APPENDIX A

TABLE X
PARAMETERS OF GENERATORS

Unit	af (MBtu/ MW^2h)	Bf (MBtu/ MWh)	cf (MBtu/ h)	Contract Gas Price ($/MBtu)	Initial MW
G1	0.0004	13.51	176.95	6.2345	150
G2	0.001	32.63	129.97	6.2305	50
G3	0.005	17.70	137.41	6.231	0

TABLE XI
PARAMETERS OF GENERATORS

Unit	Pmin (MW)	Pmax (MW)	Ramp (MW/h)	Min On (h)	Min Off (h)	Initial hour
G1	100	220	55	4	4	4
G2	10	100	50	2	3	2
G3	10	20	20	1	1	-1

TABLE XII
PARAMETERS OF POWER TRANSMISSION BRANCH

Branch	From Bus	To Bus	R (p.u.)	X (p.u.)	Flow Limit (MW)
Line 1	1	2	0.005	0.17	200
Line 2	1	4	0.003	0.258	100
Line 3	2	4	0.007	0.197	100
Line 4	5	6	0.002	0.14	100
TF1	2	3	0	0.037	100
TF2	4	5	0	0.037	100
PS1	3	6	0.0005	0.018	100

TABLE XIII
PARAMETERS OF TAP-CHANGING TRANSFORMER AND PHASE-SHIFTER

Line No.	From Bus	To Bus	Tap Min	Tap Max	Angle min	Angle Max
TF1	2	3	1.0204	1.0753	0	0
TF2	4	5	1.0204	1.0753	0	0
PS1	3	6	0	0	-30	30

TABLE XIV
DISTRIBUTION FACTOR OF ELECTRICITY LOAD

Load	1	2	3
Bus No.	3	4	5
Factor	0.2	0.4	0.4
Price of Load Shedding ($/MW)	1000	1000	1000

TABLE XV
ELECTRICITY LOAD DATA OF 24 h

Hour	Load (MW)	Hour	Load (MW)	Hour	Load (MW)	Hour	Load (MW)
1	175.19	7	173.39	13	242.18	19	245.97
2	165.15	8	190.40	14	243.60	20	237.35
3	158.67	9	205.56	15	248.86	21	237.31
4	154.73	10	217.20	16	255.79	22	227.14
5	155.06	11	228.61	17	256	23	201.05
6	160.48	12	236.10	18	246.74	24	196.75

TABLE XVI
PARAMETERS OF NODES IN GAS TRANSMISSION SYSTEM

Node No.	Min-Pressure (Psig)	Max-Pressure (Psig)
1	105	150
2	140	170
3	150	195
4	70	100
5	150	200
6	160	240
7	100	140

TABLE XVII
PARAMETERS OF GAS PIPELINE

Index	From Node	To Node	C (kcf/Psig)
Pipe 1	1	2	50.6
Pipe 2	2	5	37.5
Pipe 3	5	6	45.3
Pipe 4	3	5	43.5
Pipe 5	4	7	50.1

TABLE XVIII
PARAMETERS OF NATURAL GAS COMPRESSOR

Index	Inlet Node	Outlet Node	α	K_1	K_2	Rmin	Rmax
C1	4	2	0.25	0.165	0.1	1.6	2.45

TABLE XIX
PARAMETERS OF NATURAL GAS COMPRESSOR

Index	af (MBtu/ MW^2h)	bf (MBtu/ MWh)	Cf (MBtu/h)	Node	Hmin	Hmax
C1	0	0.2	50	2	400	600

TABLE XX
PARAMETERS OF NATURAL GAS SUPPLIER

Supplier No.	Node No.	Min Output (kcf/h)	Max Output (kcf/h)
1	7	5300	5300
2	6	1000	6000

TABLE XXI
PARAMETERS OF GAS LOAD

Load No.	Node No.	Distribution Factor	Service Priority
1	1	Gas consumption of G_1	low
2	1	2/3 of Residential gas load	high
3	3	Gas consumption of G_3	low
4	3	1/3 of Residential gas load	high
5	2	Gas consumption of G_2	low

$$\left.\frac{\partial f_{mn}}{\partial \pi_m}\right|_{\pi_m^+} = \lim_{\Delta\pi \to 0^+} \frac{C_{mn}[\mathrm{sgn}(\pi_m + \Delta\pi, \pi_n)\sqrt{(\pi_m+\Delta\pi)^2 - \pi_n^2} - \mathrm{sgn}(\pi_m, \pi_n)\sqrt{\pi_m^2 - \pi_n^2}]}{\Delta\pi}$$

$$= \lim_{\Delta\pi \to 0^+} \frac{C_{mn}\sqrt{2\pi_m \cdot \Delta\pi + \Delta\pi^2}}{\Delta\pi} = +\infty$$

$$\left.\frac{\partial f_{mn}}{\partial \pi_m}\right|_{\pi_m^-} = \lim_{\Delta\pi \to 0^-} \frac{C_{mn}[\mathrm{sgn}(\pi_m + \Delta\pi, \pi_n)\sqrt{\pi_m^2 - (\pi_n+\Delta\pi)^2} - \mathrm{sgn}(\pi_m, \pi_n)\sqrt{\pi_m^2 - \pi_n^2}]}{\Delta\pi}$$

$$= \lim_{\Delta\pi \to 0^-} \frac{-C_{mn}\sqrt{-2\pi_m \cdot \Delta\pi - \Delta\pi^2}}{\Delta\pi} = \lim_{\substack{\Delta\delta \to 0^+ \\ \Delta\delta = -\Delta\pi}} \frac{C_{mn}\sqrt{2\pi_m \cdot \Delta\delta - \Delta\delta^2}}{\Delta\delta} = +\infty$$

TABLE XXII
RESIDENTIAL GAS LOAD DATA IN CASE 2

Hour	Gas Load (kcf)
1-24	6000

TABLE XXIII
RESIDENTIAL GAS LOAD DATA OF 24 hIN CASE 5

Hour	Gas Load (kcf/h)	Hour	Gas Load (kcf /h)	Hour	Gas Load (kcf /h)	Hour	Gas Load (kcf/h)
1	5220	7	5580	13	6000	19	6540
2	4920	8	6060	14	5700	20	6780
3	4680	9	6180	15	5760	21	6660
4	4740	10	6240	16	5880	22	6540
5	5100	11	6120	17	6060	23	6060
6	5640	12	6120	18	6240	24	5520

TABLE XXIV
PARAMETERS OF SELF-OWNED STORAGE FACILITY

SV at hour 0 (kcf)	SVmin (kcf)	SVmax (kcf)	SV at hour 24 (kcf)
5000	5000	15000	5000

APPENDIX B

For natural gas pipelines:

If

$$\pi_m > \pi_n > 0, f_{mn} = C_{mn}\sqrt{\pi_m^2 - \pi_n^2}$$

then

$$\partial f_{mn}/\partial \pi_m = C_{mn}\pi_m/\sqrt{\pi_m^2 - \pi_n^2}.$$

If

$$\pi_n > \pi_m > 0, f_{mn} = -C_{mn}\sqrt{\pi_n^2 - \pi_m^2}$$

then

$$\partial f_{mn}/\partial \pi_m = C_{mn}\pi_m/\sqrt{\pi_n^2 - \pi_m^2}.$$

If $\pi_m = \pi_n > 0$, see the equations at the top of the page. In these equations in order to avoid numerical instability, we assign a large constant M or $-$M when π_m and π_n are very close. Hence

$$\frac{\partial f_{mn}}{\partial \pi_m} = \begin{cases} M, & \left|\pi_m^2 - \pi_n^2\right| < \varepsilon \\ \frac{C_{mn}\pi_m}{\sqrt{|\pi_m^2 - \pi_n^2|}} & \left|\pi_m^2 - \pi_n^2\right| \geq \varepsilon. \end{cases} \quad (41)$$

Similarly

$$\frac{\partial f_{mn}}{\partial \pi_n} = \begin{cases} -M, & \left|\pi_m^2 - \pi_n^2\right| < \varepsilon \\ \frac{-C_{mn}\pi_n}{\sqrt{|\pi_m^2 - \pi_n^2|}} & \left|\pi_m^2 - \pi_n^2\right| \geq \varepsilon. \end{cases} \quad (42)$$

For compressors: If node m is the inlet of compressor, then $\pi_n > \pi_m > 0$ (or $1 < R_{\min} \leq \pi_n/\pi_m \leq R_{\max}$)

$$f_{mn} = \frac{H_j}{k_{j1}\left[\frac{\pi_n}{\pi_m}\right]^\alpha - k_{j2}}.$$

Therefore

$$\frac{\partial f_{mn}}{\partial \pi_m} = \frac{\alpha H_j k_{j1} \frac{\pi_n^\alpha}{\pi_m^{\alpha+1}}}{\left[k_{j1}\left(\frac{\pi_n}{\pi_m}\right)^\alpha - k_{j2}\right]^2} \quad (43)$$

$$\frac{\partial f_{mn}}{\partial \pi_n} = \frac{-\alpha H_j k_{j1} \frac{\pi_n^{\alpha-1}}{\pi_m^\alpha}}{\left[k_{j1}\left(\frac{\pi_n}{\pi_m}\right)^\alpha - k_{j2}\right]^2} \quad (44)$$

$$\frac{\partial f_{mn}}{\partial H_j} = \frac{1}{k_{j1}\left(\frac{\pi_n}{\pi_m}\right)^\alpha - k_{j2}}. \quad (45)$$

If node n is the inlet of compressor, then $\pi_m > \pi_n > 0$ (or $1 < R_{\min} \leq \pi_m/\pi_n \leq R_{\max}$)

$$f_{mn} = \frac{-H_j}{k_{j1}\left[\frac{\pi_m}{\pi_n}\right]^\alpha - k_{j2}}.$$

Thus

$$\frac{\partial f_{mn}}{\partial \pi_m} = \frac{\alpha H_j k_{j1} \frac{\pi_m^{\alpha-1}}{\pi_n^\alpha}}{\left[k_{j1}\left(\frac{\pi_m}{\pi_n}\right)^\alpha - k_{j2}\right]^2} \quad (46)$$

$$\frac{\partial f_{mn}}{\partial \pi_n} = \frac{-\alpha H_j k_{j1} \frac{\pi_m^\alpha}{\pi_n^{\alpha+1}}}{\left[k_{j1}\left(\frac{\pi_m}{\pi_n}\right)^\alpha - k_{j2}\right]^2} \quad (47)$$

$$\frac{\partial f_{mn}}{\partial H_j} = \frac{-1}{k_{c1}\left(\frac{\pi_m}{\pi_n}\right)^\alpha - k_{c2}}. \quad (48)$$

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