Sediment Transport Around a Mound
Breakwater: the Toe Erosion Problem

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Abstract

The combined action of waves (incident and reflected) and the alongbreakwater current driven by wave energy dissipation inside an infinitely long porous breakwater under oblique wave attack (Baquerizo and Losada, 1998), is taken into account to study the formation of bars in front of the structure and the sediment transport at the toe of it. The main features of the bars are explained in terms of the incident wave characteristics, remarking the wave incidence angle dependence, and the structural and hydrodynamic properties of the breakwater. Results may explain the existence of erosion/deposition patterns at the toe of the structure.

Introduction

Some coastal structures, designed to protect an area from wave action are built with quarry or rip-rap. Generally speaking, they reflect part of the incident energy modifying the wave field in the neighborhood of the structure. Moreover, part of the energy is dissipated inside the porous medium due to pore friction.

On sandy beds, it is known that wave action may induce the erosion of the toe of the breakwater, producing the failure of the structure. However, sometimes it is found that sand accumulates at the toe, filling up the structure and reducing its functionality as a wave dissipator.

Dalrymple et al. (1991) obtained the wave field in front of and inside a porous vertical structure when a monochromatic wave impinges obliquely on it. On the other hand, Baquerizo and Losada (1998) showed that for waves approaching obliquely an infinitely long porous structure, wave energy dissipation inside the pores drives an alongbreakwater current inside the structure that is transferred to the water regions by

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turbulent diffusion. Moreover, they studied the combined action of waves and the current to study the sediment transport patterns in front of the porous structure showing that these mechanisms explain the formation of bars parallel to the breakwater, producing in some cases the erosion of the toe of the structure.

In this paper, the shape of the bars and the conditions under which erosion/deposition occurs at the toe of the structure are analyzed in terms of the characteristics of the incident wave and the structural properties of the breakwater. The reflection coefficient and the phase lag between the incident and reflected waves is presented first, next the drift velocity and the alongbreakwater current profiles are analyzed, finally the sediment transport patterns are obtained and the tendency of the bed is studied.

Theoretical Background

Let us consider an infinitely long vertical porous structure of width b, in water of constant depth, h, and a monochromatic wave train that impinges obliquely on it with an angle of incidence θ. The origin of the reference frame is in the seaward face of the
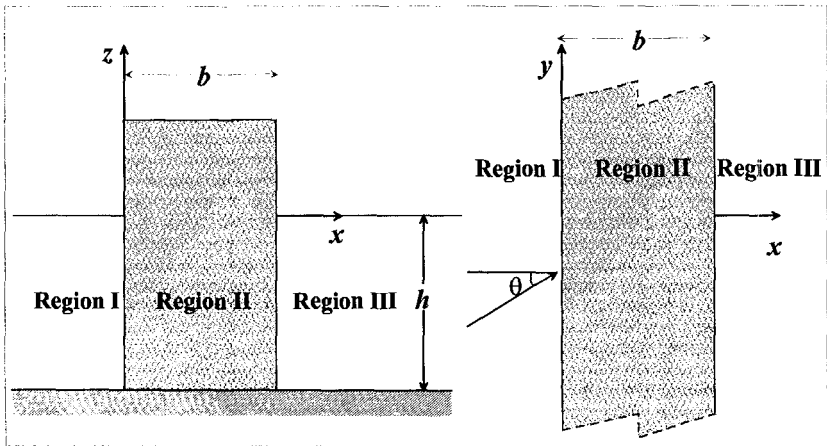
structure with the x-axis normal to it, the y-axis lying along the breakwater and the z-axis pointing upwards (see Fig. 1).

Figure 1. Definition sketch showing waves approaching an infinitely long vertical porous breakwater at an oblique angle.

The presence of the breakwater modifies the incident wave field. In the first region, part of the incident wave energy is returned in the seaward direction. Some of the remaining energy, transmitted to the porous structure is dissipated because of friction losses within the pores. Inside the breakwater, part of the wave energy is reflected

because of the discontinuity at the leeward face of the structure and some is transmitted to the third region.

Wave field inside and outside the structure

Under plane wave assumption, Dalrymple et al. (1991) obtained the velocity potentials in each region. The corresponding water surface elevation can be expressed as:

$$\eta_\alpha(x,y,t) = a_\alpha(x)e^{-i(\vec{k}_\alpha \cdot \vec{x} - \sigma t)} \quad \alpha = I,II,III \tag{1}$$

where the amplitude of the oscillations, $a_\alpha(x)$, in each region $\alpha = I,II,III$ is constant along lines parallel to the breakwater.

Seawards of the structure, the amplitude of the water surface elevation is:

$$\begin{aligned} a_I(x) &= \left(|A_1|^2 + |B_1|^2 + 2|A_1||B_1|\cos(2k_a\cos\theta x + \varphi)\right)^{1/2} \\ &= |A_1|\left(1 + |R|^2 + 2|R|\cos(2k_a\cos\theta x + \varphi)\right)^{1/2} \end{aligned} \tag{2}$$

where A_1 is the amplitude of the incident wave, $R = |R|e^{i\phi}$ is the reflection coefficient of the structure and $B_1 = A_1 R$ is the amplitude of the wave reflected at the seaward face of the structure. $a_I(x)$ varies periodically with x, with a characteristic wavelength $L_b = L/2\cos(\theta)$, showing lines of maximum (quasi antinodes) and minimum (quasi nodes) amplitude, whose distance to the breakwater depends on the phase lag between the incident and the reflected trains.

Inside the structure, the amplitude is:

$$a_{II}(x) = |s - if|\left(|A_2|^2 e^{2q_I x} + |B_2|^2 e^{-2q_I(x-b)} + 2|A_2||B_2|e^{q_I b}\cos(q_R(2x-b) + \varphi_{A_2} - \varphi_{B_2})\right)^{1/2} \tag{3}$$

where $A_2 = |A_2|e^{i\varphi_{A_2}}$ is the amplitude of the wave transmitted to the structure and $B_2 = |B_2|e^{i\varphi_{B_2}}$ is the amplitude of the wave reflected at the leeward face of the breakwater. s and and f are parameters describing the porous medium and q_R, q_I are, respectively, the real and imaginary parts of the wave number inside the coastal structure (see Dalrymple et al.,1991; Losada et al.,1993 for details). The waves transmitted at $x = 0$ and reflected at $x = b$ are dissipated as they propagate inside the structure and consequently, the oscillations of the amplitude of the water surface elevation inside the porous medium decrease with x. As the characteristic length of the oscillations is again $L_b = L/2\cos(\theta)$, they can be observed only for breakwater widths larger than L_b.

Alongbreakwater current

Wave energy dissipation inside the pores produces a variation, in the acrossbreakwater direction, of the radiation stress, S_{xy}, inside the porous medium (Méndez, 1997) driving a current, $V(x)$, that flows parallel to the structure (Baquerizo and Losada, 1998). Due to the turbulence induced by temporal and spatial fluctuations of the velocity flow inside the pores, the current is transferred to the water regions.

The velocity profile shows a maximum inside the structure, close to the seaward face of it, and decreases towards 0 far from the structure. The magnitude of the velocity profile depends on the structural properties of the breakwater and on the characteristics on the incident wave.

Summarizing, in the proximity of the structure, particularly at the toe of the structure, there are the combined action of waves and the alongbreakwater current, which may provide an efficient mechanism for initiating and transporting the sediment.

Sediment transport

Using Bailard's formula (1981), the sediment transport patterns under the influence of waves and currents are obtained in terms of the total velocity, $\vec{u}_t$, at the bottom, which results from the superposition of the orbital velocity, $\vec{u}_o$, and a small perturbation which does not depend on time, $\vec{U}$. Notice that the orbital velocity is due to the incident and the reflected waves.

Far from the structure, $\vec{U}$ is the drift velocity, $\vec{U}_d$, at the top of the bottom boundary layer. Close to the structure, both alongbreakwater current, $V(x)\hat{j}$, and drift velocity have to be considered. Although V may modify the mean motion of the boundary layer, in a first approach, as $|U_d|$ and V are both very small compared to the orbital motion, they may be linearly superimposed without significant error.

Bed morphology in front of the coastal structure

Dalrymple et al. (1991) showed that the reflection coefficient on a porous structure depends on the angle of incidence and that there is a value, called the Brewster angle, for which the modulus of the reflection coefficient is minimum. That means that the same breakwater may behave as a highly reflective or as a low reflective structure depending on the angle of wave attack. Baquerizo and Losada (1998) showed that the drift velocity patterns in front of the structure and the alongbreakwater current also depend on the breakwater reflectivity. This property justifies the choice of two case studies representing a moderately permeable and dissipative breakwater, and an almost impermeable and fully reflective one, with special attention in the effect of the angle of incidence. Table 1 summarizes the principal characteristics of the breakwater and wave conditions for the two cases, which will be referred to Structure A and Structure B, respectively.

	A_1(m)	h(m)	T(s)	b(m)	f^1	ε^2
Structure A:	0.5	5	10	10	0.8	0.45
Structure B:	0.5	5	10	10	50	0.25

[1] friction factor of the porous medium
[2] porosity of the porous medium

Table 1. Incident wave and breakwater characteristics

For Structure A, the reflection coefficient varies from to $R=53$ at $\theta=0°$ to $R=0.04$ at the Brewster angle, $\theta_B=70°$, then it increases. The phase lag between the incident and the reflected trains is very small for $\theta<\theta_B$. At θ_B it changes rapidly until it achieves a constant value $\phi\approx\pi$ for angles larger than θ_B (Fig.2 a). This means that for angles smaller than θ_B the antinodal lines will be at $x_n=-nL_b$, $n=0,1,2,...$ whereas for higher values they will be at $x_n=-nL_b-L_b/2$, $n=0,1,2,...$. For the almost impermeable breakwater (Case B), the structure behaves always like a highly reflective structure and the Brewster angle converges asymptotically to $\theta=\pi/2$. The phase lag is close to 2π and therefore, the antinodal lines will be at $x_n=-nL_b$, $n=0,1,2,...$
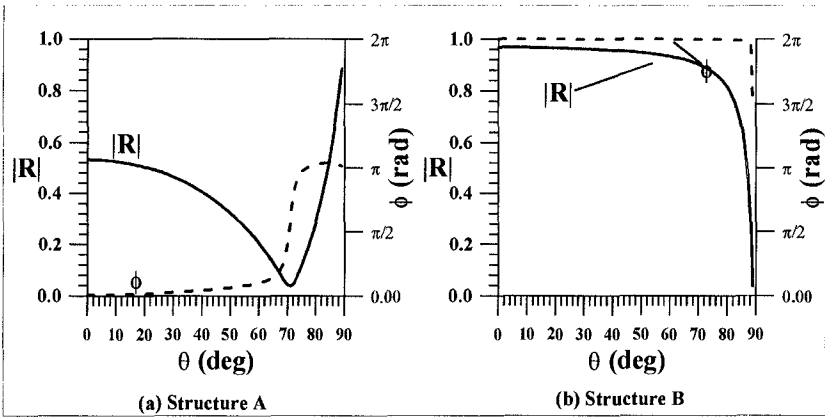

(a) Structure A   (b) Structure B

Figure 2. Magnitude and phase of the reflection coefficient

The drift velocity $\vec{U}_d=U_d\hat{i}+V_d\hat{j}$ at the top of the boundary layer in a three-dimensional small-amplitude oscillatory flow was obtained by Hunt and Johns (1963) and applied later by Carter et al. (1973) to a field resulting from the superposition of an incident wave and a reflected wave with normal incidence. For the wave field in front of the structure, $\vec{U}_d$ is a function uniform in y, varying periodically with the distance to the breakwater. The velocity component in the $y-$ direction, V_d, is in phase with the wave amplitude, that is, it is maximum at, $x_n=-nL_b-\phi L_b/(2\pi)$, $n=0,1,2,3,...$ The component in the $x-$ direction, U_d, has a lag of $-L_b/4$ with respect to V_d with maximum values at $x_n-L_b/4$. V_d is always a positive function, whereas U_d changes the sign for values of the reflection coefficient above a threshold value which is about $R_t=0.82$.

Fig. 3 shows, for Cases A and B, the $x-$ and the $y-$ component of the drift velocity as well as the alongbreakwater current profile obtained for $\theta=45°$. For Case A and small angles of incidence, U_d and V_d are of the same order of magnitude, and as θ increases, the $y-$ component of the drift velocity dominates over the $x-$ component. The resulting sediment transport patterns (Fig. 4 a) are essentially in the direction of propagation of the incident wave, except at the antinodes where the angle with respect to the $x-$ axis increases slightly. For Case B, the drift velocity in the $y-$ direction is larger

than U_d and, because it corresponds to a highly reflective structure with reflection coefficients above R_t, U_d changes the sign. The resulting sediment transport converge towards the antinodes and diverges at the nodes (Fig 4 b).
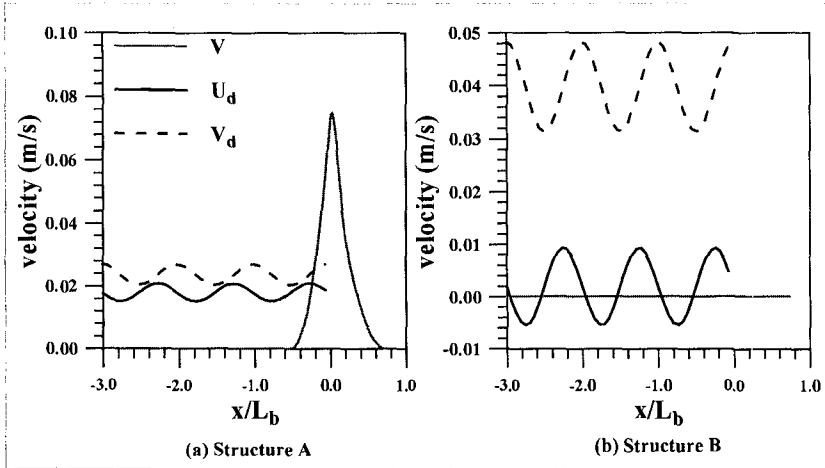

Figure 3. Drift velocity and alongbreakwater current profiles

For Structure A, the alongbreakwater current at the face of the structure is of the same order of magnitude than the drift velocity (see Fig. 3 a), and the maximum values of the alongbreakwater current profile are achieved for $\theta \approx 50°$. For structure B the alongbreakwater current is negligible compared to the drift velocity.

Fig. (5) shows, for Cases A and B, the evolution with the angle of incidence of the local time variation of the bed. For Case A, it can be seen that the tendency of the bed is to form bars parallel to the breakwater almost sinusoidal in shape, with a distance between crests of L_b, and a distance from the first crest to the structure of $L_b/4$. The higher bars are expected to develop for small angles of incidence. As the angle of incidence increases, the height of the bars decreases and the effect of alongbreakwater current becomes more important. Consequently, it is expected that the bed erodes at the toe of the breakwater.

For Case B the bed develops bars with double peak crests and deep troughs. Again, the higher bars are expected to be for small angles of incidence. The distance between troughs is L_b and the first bar through is $L_b/2$ away from the structure. For all angles of incidence, sediment accumulates at the toe of the structure.

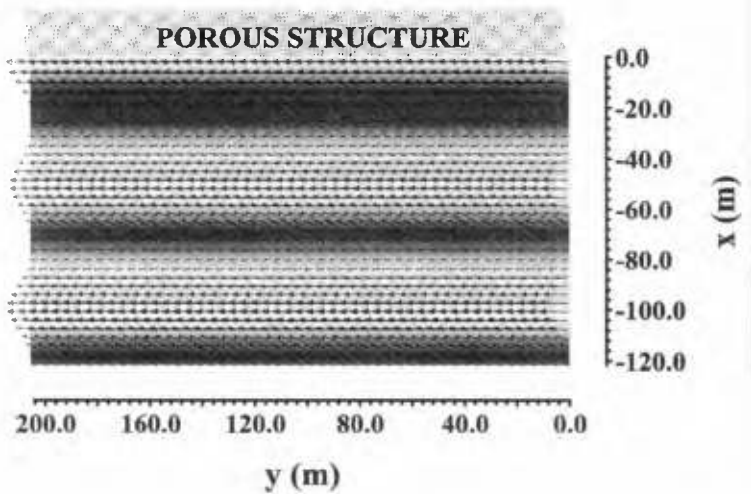**(a) Structure A**
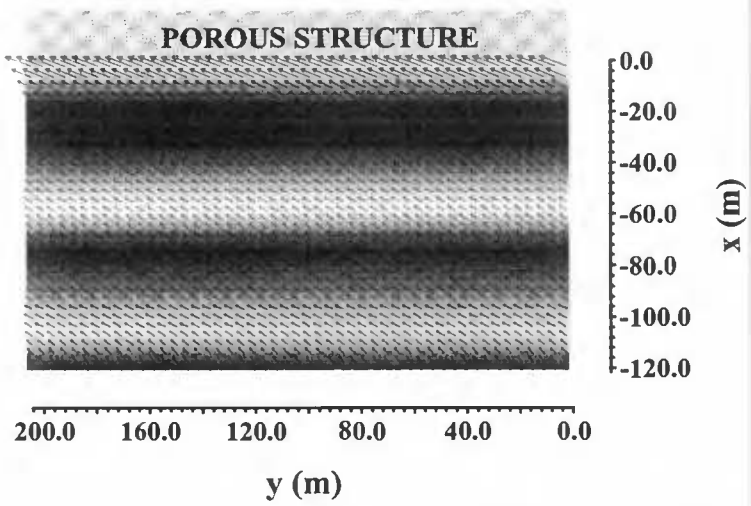
(b) Structure B

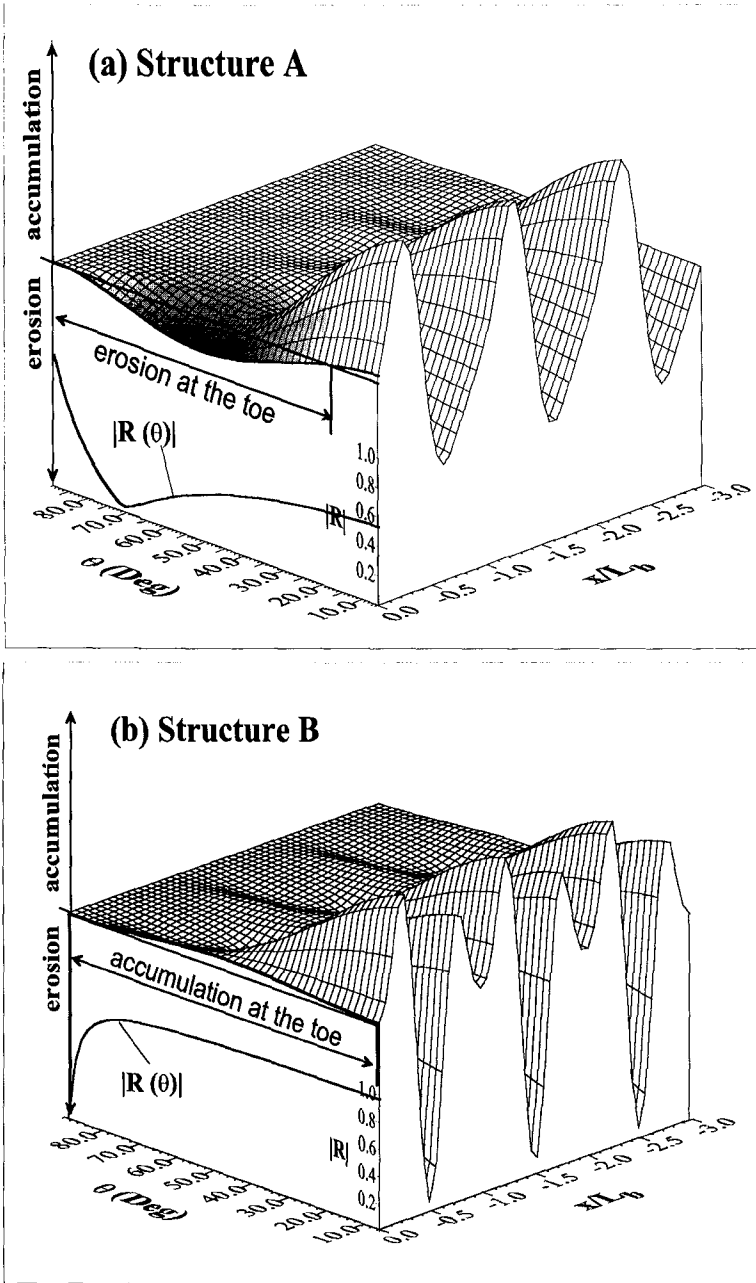Figure 4. Sediment transport patterns in front of the breakwater

Figure 5. Local time variation of the bed in front of the breakwater

Conclusions

The combined action of waves (incident and reflected) and the alongbreakwater current, driven by wave energy dissipation inside a porous structure, is considered to study the sediment transport patterns and the associated local time variation of the bed in front of an infinitely long porous breakwater of finite width when a monochromatic wave train impinges obliquely on it.

The modulation of the $x-$ and the $y-$ components of the drift velocity with the distance to the breakwater is found to produce sediment transport patterns that explain the formation of bars parallel to the breakwater whose characteristics depend on the characteristic of the incident wave train (wave number and angle of wave attack) and on the reflection coefficient of the structure. Moreover, the alongbreakwater current modifies the sediment transport in the neighborhood of the breakwater.

For a moderately permeable and dissipative structure, the reflectivity depends strongly on the angle of incidence and the bars are expected to be almost sinusoidal in shape, with a distance between crests of $L_b = L/(2\cos\theta)$. For small angles of incidence, the effect of the alongbreakwater current is negligible and the first bar crest is likely to be $L_b/4$ away from the structure. For larger angles of incidence, the alongbreakwater erodes the bed in the vicinity of the breakwater; this effect is maximum at $\theta \approx 50°$.

For an almost impermeable and fully reflective structure, the alongbreakwater current is negligible compared to the drift velocity and has no significant effect on the sediment transport patterns. The $x-$ component of the drift velocity changes the sign, and as a result, the sediment transport converges towards the antinodes and diverges at the nodes. The tendency of the bed is to form bars with two peaks and deep troughs, with a distance between troughs of L_b and the first through located $L_b/2$ away from the structure. It is expected that sand accumulates at the toe of the structure.

Acknowledgements

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