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Seeking No War, Achieving No Peace: The Conflict over the Siachen Glacier

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Abstract

This paper models "no war, no peace" situations in a game theoretical framework where two countries are engaged in a standoff over a military sector. The first main objective is to identify rational grounds for such situations and, more precisely, explicit equilibria that leads to such situations. It is demonstrated that both countries get the same payoff from being in this continuous state of perpetual hostility and, moreover, that "no war, no peace" situations can be explained only if the countries perceive an equal measure of military advantage by controlling the area. Given this insight, the second objective of the paper is to provide insights about how "no war, no peace" situations can be resolved. Two different pathways are suggested. The first is idealistic and based on mutual trust whereas the second is based on deterrence meaning that both countries impose a threat of using armed force against the other country in their respective military doctrines.

Keywords: game theory, infinite horizon game, stationary strategies, "no war, no peace", Siachen conflict.

JEL Classification: C73, H56.

1 Introduction

Between 1989 and 2010, at least 32 major peace accords and even more ceasefires were negotiated around the world. Despite this, many of the involved countries and regions experience

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a "no war, no peace" situation, i.e., a situation characterized by continued insecurity, low-level violence, inter-group hostility, and persistence of the factors that sparked and sustained the conflict (Mac Ginty, 2010). Well-known examples include the 1989 post-civil war reconstruction in Lebanon, the 2003 ceasefire agreement in Sri Lanka, the two decade long and ongoing border conflict between Ethiopia and Eritrea, and the never-ending conflict between the nuclear states India and Pakistan over the Siachen Glacier. This paper models "no war, no peace" situations in a game theoretical framework where two countries are engaged in a standoff over a strategically important military sector. Although the conflict over the Siachen Glacier will serve as the leading example, the conclusions that follow from the analysis are not restricted to this particular conflict.

The foundation of the conflict over the Siachen Glacier dates back to 1949 when the Karachi Agreement was signed to establish a ceasefire line in Kashmir following the Indo–Pakistani war of 1947 (Wirsing, 1998). This Agreement did not clearly specify which country that controlled the Siachen Glacier. It only demarcated the ceasefire line till the grid point NJ9842 at the foot of the glacier, and left a vague description that this line was supposed to run "thence north to the glaciers".[1] Thus, this largely inaccessible terrain beyond NJ9842 was not delimited, and so, neither India nor Pakistan had any permanent military presence in the area prior to 1984. The military standoff on the Siachen Glacier began in 1984 when India gained control over the glacier to preempt the seizure of the passes by the Pakistan army.[2] In response to these developments, the Pakistan army initiated an operation to displace the Indian troops on the key passes. This operation led to the first armed confrontation on the glacier on April 25, 1984. The following 19 years, there was multiple of low-level violence confrontations[3] between the Indian and the Pakistan armies in the area. It was not until 2003 that a ceasefire went into effect. However, even after this ceasefire agreement, both India and Pakistan have troops stationed in the area with India having maintained control over the glacier since 1984.

To model a "no war, no peace" situation, an infinite horizon game with two countries is considered.[4] One can think of one of the countries as the incumbent and the other as the challenger (in the conflict over the Siachen Glacier, India plays the role of the incumbent and Pakistan the

[1]See Baghel and Nüsser (2015) and Hussain (2012). In the latter article, the author, who is a retired Pakistani brigadier, posits that this line is interpreted by Pakistan as running east to the glacier.

[2]As discussed in Baghel and Nüsser (2015), Bearak (1999), Hussain (2012) and Fedarko (2003), the Indian fears of such possible seizure were apparently generated by Pakistan's grant of sovereign permission to European expedition and mountaineering teams to visit this area. It is also argued that some intelligence about Pakistan army placing a huge order of heavy winter clothing was interpreted by Indian military planners as preparation for such a seizure.

[3]In fact, it has been estimated that around 97 percent of the military casualties on the Siachen Glacier between 1984 and 2003 not was due to enemy firing but instead a consequence of severe weather conditions, altitude, avalanches, etc. (Ives, 2004)

[4]Infinite horizon war games have recently been considered by, e.g., Debs and Monteiro (2014), Fearon (2004, 2007) and Jackson and Morelli (2009).

role of the challenger). Each country is guided by her military doctrine or, equivalently, by a fundamental set of principles that guide her military forces whenever pursuing national security objectives. Based on their respective doctrines, the countries form their military strategies. Given the deadlock situation which is the very foundation of a “no war, no peace” situation, it is not unreasonable to assume that the countries have non-changing military doctrines (India and Pakistan, for example, have fought over the same sectors in the state of Jammu and Kashmir, four times in seventy years of their co-existence). In the considered game theoretical framework, this means that the countries have stationary strategies. Consequently, both countries, depending on whether they are an incumbent or a challenger, must use the same rule of choosing an action at all times. The actions that the incumbent country chooses between is either to stay or to retreat from the area. After the incumbent country has made her move, the challenger country chooses either to occupy or to not occupy the area. If the incumbent country decides to stay and the challenger country decides to occupy, there will be an armed confrontation.

To formally analyze “no war, no peace” situations, the concept must be defined in terms of military doctrines or, equivalently, in terms of the stationary strategies played by the two countries. Before presenting such definition, we make three observations pertaining to the Siachen conflict and mountain warfare in general. First, it is not unreasonable to suppose that the military doctrine of the incumbent country specifies that the country should stay in the area and maintain control over it. This would, in fact, be consistent with the mountain warfare tactics called *Gebirgskreig*, which was developed during World War I.[5] Indeed, this is also the strategy adopted by India since 1984 when she occupied the Siachen Glacier and became the incumbent in the first period of the game. Second, in line with Gebirgskrieg again, the challenger country may acknowledge the impossible battlefield advantage offered by the peak to the incumbent, and follow the strategy of not occupying as long as the incumbent country stays. This is also the strategy adopted by Pakistan since 1984. Third, given that the challenger country continues to stay in the area in a state of military alert despite the negligible chances of battle, it is reasonable to assume that she would occupy the area if the incumbent country (for some reason) decides to retreat.[6] Given these observations, a “no war, no peace” situation is defined as a steady state where the incumbent country at the start of game chooses the strategy to stay, while the challenger country at the start of game adopts the strategy to not occupy when the incumbent stays, but to occupy when the incumbent retreats. As argued above, such stationary strategies also describe the conflict over the Siachen Glacier.

Given the above definition of a “no war, no peace” situation and the fact that military standoffs often come at huge military and human costs, it is of importance to identify rational grounds for such perpetual hostility and, more precisely, explicit equilibria (Subgame Perfect Nash Equilibria) that lead to “no war, no peace” situations. In the game theoretical framework considered in this paper, two different equilibria that lead to “no war, no peace” situations are analyzed. The

[5]See Baghel and Nüsser (2015).

[6]As before, this would indeed be manoeuvre prescribed by Gerbirgskrieg.

remarkable irony that follows from our analysis is that both the incumbent and the challenger countries, irrespective of their identities, get the same payoff from being in this continuous "no war, no peace" situation. Thus, in trying to be stronger than the other, they end up being the same, but still cannot find reasons to make peace as equals. From the analysis, it is also clear that a "no war, no peace" situation cannot be explained by patience, i.e., the results are independent of the discount factors. Neither can the situation be explained merely by the euphoria a country experiences when defeating its opponent in an armed confrontation. As will be explained in more detail later, a rational explanation to a "no war, no peace" situation is that both countries perceive an equal measure of military advantage by controlling the area.

The latter conclusion will also be helpful in providing insights about how "no war, no peace" situations can be resolved. This paper adopts the natural view that a "no war, no peace" situation is resolved when both countries agree to retreat whenever being in the position of an incumbent country and, furthermore, not to occupy whenever being in the position of a challenger country if the incumbent country decides to retreat. In such event, the area gets mutually designated to be no man's land, presumably through a sovereign treaty, and both countries will, consequently, cease to have their armies stationed in the area. Hence, a first step to obtain a resolution is that both countries must change their military doctrines, i.e., their stationary strategies. However, this alone will not lead to an equilibrium outcome. As will be demonstrated, such a change in the military doctrines must also be accompanied with substantial peace activism in order for the considered strategy profiles to constitute an equilibrium.

Two different strategy profiles that resolve the "no war, no peace" situation in combination with peace activism are identified, and both of them have been observed to play key roles in historical conflicts. The first is idealistic in the sense that a resolution is achieved without imposing any threats of retaliation. Instead, it relies on mutual trust and good faith as, e.g., the solution to the Argentine–Brazilian détente of the late 1970s (Oelsner, 2007). The second is based on deterrence meaning that both countries imposes a threat of using armed force against the other country in their respective military doctrines. This type of doctrine was popularized in the aftermath of World War II when the rivalry between United States and Soviet Union was at its peak.

While the earliest economic analysis of war can be traced back to Schelling (1966), there has been a spurt in theoretical studies of war in recent years.[7] In this emerging literature, the paper that is most closely related to our paper is Yared (2010). In his paper, conflicts between sovereign states are (exactly as in our paper) modelled as a discrete time dynamic game. However, as will be explained next, the paper by Yared (2010) differs from our paper in many aspects.

In Yared (2010), conflicts are modeled as a game between two countries in a setting where both countries agree and acknowledge that one is mightier than the other in terms of military and economic resources. Their engagement essentially translates into determination of a kind of protection offering or "concession" that must be made by the weaker state in time period to

[7]Some such papers are Baliga and Sjöström (2004), Chassang and Miquel (2010), Fearon (1995), Leventoğlu and Slantchev (2007), Powell (1999, 2004), and Schwarz and Sonin (2007).

escape war being thrust on it. A state variable determines whether the weaker state can or cannot afford to make such concessions. The periods where suitable offerings are made to avert, are labelled as periods of peace. Further, the decision to inflict a war is allowed to be probabilistic as well as history dependent. In fact, as mentioned in Yared (2010), their model of war closely follows the repeated game with imperfect monitoring setup of Abreu et al. (1990). In addition, Yared (2010) uses the notion of subgame perfection with correlated strategies in presence of an exogenous public randomization device, and shows that any such conflict must lead to war on equilibrium path that is followed by periods that may have peace with positive probabilities, which in turn may be followed by wars with positive probabilities and so on. The paper refers to this kind of war as a "temporary war", which it considers different from another kind of war called "total war" where both countries fight every period for eternity. Interestingly, the model shows that under certain conditions, the latter may constitute an equilibrium path.

We find that there are situations where the model of Yared (2010) is difficult to motivate. For example, the infinite horizon of model suggests that the concessions are inexhaustible, almost like payment of protection money in the example of Barbary Wars during 1805–1815 cited by Yared (2010). However, modern conflicts rarely involve payment of protection money or regular placatory concessions under the threat of war. The international relations architecture, under the leadership of United Nations, ensures that such extortion-like tactics are not successful. And so, most modern conflicts between sovereign states, since World War II, are better classified as an assortment of border disputes where countries are at a disagreement in terms of occupation of land or water (e.g., India–China, India–Pakistan, and China–Philippines). Further, in almost all these cases, no country considers herself weaker than the other while formulating strategies to deal with such conflicts (e.g., North Korea–South Korea, India–China, and India–Pakistan). To accommodate for these realities, we present a completely new model that explicitly accounts for the militarized sector underlying the conflict and keep both countries symmetric in all respects other than the military strategic value of this sector.

More importantly, Yared (2010) considers decisions to inflicting war to be probabilistic. In this day and age of advanced biological, chemical and nuclear weapons that are capable of exterminating great swathes of population at the smallest of triggers both as war casualty as well as collateral damage, it is unlikely that military doctrines undertake the decision to go to war in a probabilistic manner.[8] Further, the long duration of conflicts like Soviet–US, India–Pakistan, India–China, North Korea–South Korea, etc., suggest that military doctrines are reasonably inflexible over time and are better modelled as stationary strategies than history dependent ones. Hence, in our model, the focus is on military doctrines, which invoke pure strategies that are stationary in nature.

As described in the above, Yared (2010) allows for peaceful periods where one country accepts the oppressive hegemony of the other by making placatory payment. We believe that a

[8]In recent years, it may be argued that North Korea is a probable exception.

better description of peace would be a state of nature where military doctrines of competing countries are in suitable agreement (perhaps, in terms of a bilateral treaty) that allows both countries to eliminate *any* chance of armed confrontation in the relevant area at all future time periods. Accordingly, our model envisages peace as a steady state where both countries can disengage from their conflict, and devote these resources to generating positive outcomes for their respective countries for all future time periods.

Finally, Schwarz and Sonin (2007) analyse *brinksmanship* in conflicts where parties have no commitment power pre-dates Yared (2010). They use a dynamic continuous time setting to analyze conflicts where one party is admittedly stronger than the other, and uses this asymmetric power structure to extract concessions. While this is a very interesting paper that encompasses several possible conflicts of modern world, for the same reasons stated above, we believe that some of their modelling assumptions (e.g., concessions) are not suitable to be applied to sovereign conflicts that generate wars.

The remaining part of the paper is outlined as follows. Section 2 introduces the game theoretical framework. The theoretical findings are stated in Section 3. Section 4 concludes the paper. All proofs are delegated to the Appendix.

2 The Game Theoretical Framework

This section introduces the game theoretical framework together with a number of important concepts and definitions.

2.1 Preliminaries

Consider an infinite horizon game $\mathcal{G}$ with two players, denoted by 1 and 2. The players should be thought of as two countries involved in a conflict over a sector of crucial military significance that, at most, one of them can control. Let $\mathcal{G}^i$ denote the infinite horizon game where country i controls the military sector in the first time period of the game, i.e., in time period $t = 1$. The game $\mathcal{G} \in \{\mathcal{G}^1, \mathcal{G}^2\}$ is defined conditionally on which country that controls the military sector at $t = 1$, i.e., $\mathcal{G} = \mathcal{G}^i$ if and only if country i controls the military sector at $t = 1$. This paper focuses, without loss of generality, on the game $\mathcal{G} = \mathcal{G}^1$.

A stage game is played in each time period $t \geq 1$ and it is defined on the basis of the incumbent country at the beginning of time period t. Formally, the stage game defined by G^i_t means that country i is the incumbent country in the beginning of time period t. In stage game G^i_t, the incumbent country i has the option to either stay in the sector or retreat from the sector. After the incumbent country i has made her move, the challenger country $j \neq i$ chooses either to occupy or not occupy the sector. Note that even if a country is the incumbent in time period t, it is not necessarily the case that the very same country is the incumbent in some other time period $t' > t$. This depends on the actions chosen by the players. If, for example, the incumbent country decides to retreat and the challenger country replies by occupying in time period t, then the roles of the two counties are reversed in time period $t+1$. By assumption, both countries can perfectly observe each others moves in each time period. A formal description of all the actions possible is presented below.

2.2 Strategies

Both countries choose a stationary strategy. This type of strategy depends on the current state of the infinite horizon game $\mathcal{G}$, namely, whether country 1 or 2 is the incumbent. More precisely, a stationary strategy of any country i, for each time period t, prescribes one action when the country is the incumbent country and two actions when the country is the challenger country (one each for the two possible actions that an incumbent country can undertake). Consequently, the strategies of the countries do not vary with time. This is not an unlikely restriction given the non-changing military doctrines with respect to the type of standoffs investigated in this paper. For example, in the above described conflict over the Siachen Glacier between India and Pakistan, India captured the Siachen Glacier in 1984 and has stayed in this inhospitable terrain ever since. In all these years, there has not been a full fledged battle in the area between India and Pakistan even though the armies of both these countries has been stationed in the region. Thus, the Siachen conflict can be viewed as the game $\mathcal{G}^{\text{India}}$ whose first time period is at 1984.

The strategy of country i is described as an ordered pair where the first entry prescribe an action if country i is an incumbent, and the second entry prescribes a pair of actions if country i is a challenger. To simplify the notation, the following abbreviations will be used. In any stage game G_t^i, the strategies of the incumbent country, stay or retreat, are denoted by s or r, respectively. Similarly, the challenger strategies are referred to as:

- (o, o) occupy when the incumbent stays, and occupy when incumbent retreats,
- (o, no) occupy when incumbent stays, and not occupy when incumbent retreats,
- (no, o) not occupy when incumbent stays, and occupy when incumbent retreats,
- (no, no) not occupy when incumbent stays, and not occupy when incumbent retreats.

Thus, the strategy $[s, (no, o)]$ of country i means that she stays whenever she is the incumbent, and she occupies if and only if the other country j retreats whenever she is the challenger. Let $S := \{s, r\} \times \{(o, o), (o, no), (no, o), (no, no)\}$ be the set of all possible pure strategies that any country can take in the game $\mathcal{G}$. As mentioned earlier, we restrict our attention to pure strategies only, because it is unlikely that countries randomize in matters of armed conflict.

2.3 Payoffs

To explain the payoff structure that arises from the above described strategies, let P_i denote the strictly positive value of controlling the military sector for country i. To reflect that the battlefield advantage of controlling the military sector may differ between the two countries, we allow for the possibility that $P_1 \neq P_2$. Furthermore, maintaining troops in the military sector comes at a cost $c > 0$. Note that we presume identical perceptions of cost of maintaining troops on the peak across countries. This is a reasonable assumption because countries engaged in such conflicts (e.g., India and Pakistan or North Korea and South Korea), typically, consider themselves no less than the other, both in terms of valour as well as resources at disposal.

The considered strategy space gives rise to four different payoff pairs that may be realized in any given stage game G_t^i for time period t. These payoff pairs are described next together with the implied stage games for time period $t + 1$.

- *The incumbent country i stays and the challenger country j occupies.* In this situation, there will be an armed confrontation. The outcome is modelled as a probabilistic event where the incumbent country i wins the battle with probability p while the challenger country j wins the battle with probability $1 - p$. The country $l \in \{i, j\}$ that wins the battle gets payoff $P_l + W$. The first component of the payoff reflects that the winning country now controls the sector and, consequently, experience the positive value P_l of controlling the sector. The second component $W > 0$ represents the value from winning the war (e.g., nationalistic euphoria that emerges upon a military victory) net of the costs associated with the battle. The country that looses the battle gets payoff 0. The stage game played in period $t + 1$ is given by G_{t+1}^l.

- *The incumbent country i stays and the challenger country j chooses to not occupy.* In this situation, the incumbent country i maintains control over the military sector and, consequently, gets payoff $P_i - c$. The challenger country j, on the other hand, gets payoff $kd > 0$ where $d > 0$ and $k > 1$. Here, the assumptions on the constants k and d imply that a country gets a higher payoff when of being in the position of a challenger and not fighting a battle (to get payoff $kd > 0$), rather than fighting and loosing a battle (and get payoff 0). The stage game played in period $t + 1$ is given by G_{t+1}^i.

- *The incumbent country i retreats and the challenger country j occupies.* In this situation, the challenger country j gets payoff $P_j - c$ since country j then controls the military sector. The incumbent country i gets a payoff $d > 0$. The assumptions on the constants d and k now imply that a country gets a higher payoff of being in the position of a challenger and not fight a battle (and get payoff kd) rather than retreating from the sector only to see the rival occupying it (and get payoff d). Both these situations are, however, better than losing a battle (and get payoff 0). The stage game played in period $t + 1$ is given by G_{t+1}^j.

- *The incumbent country i retreats and the challenger country j chooses to not occupy.* In this situation, the military sector would be designated to be no man's land since none of the countries occupy the military sector. This would, consequently, put an end to the "no war, no peace" situation. Note that we do not envisage realization of this outcome of a stage game to be the end of $\mathcal{G}^1$. Instead, this outcome is considered to be a steady (stationary) state, presumably backed by sovereign agreements, that de-escalates the sector for all times in future, leading to a peace payoff of $H > 0$ to both countries at each time t in future.

Both countries are assumed to be expected utility maximizers and so, the instantaneous payoff they receive in case of an armed conflict is simply the expected payoff from the ensuing lottery. The countries also discount future payoffs at the rate $\delta \in (0, 1)$.

2.4 Solution Concept and Regularity Assumptions

We look for asymptotic Subgame Perfect Nash Equilibria (SPNE) in the above described infinite horizon game $\mathcal{G}^1$. Such an equilibrium can be interpreted as a strategy profile that constitutes a SPNE for all $\delta \in (\rho, 1)$ for some fraction $\rho \in (0, 1)$. Thus, we are interested in the equilibria that are realized when rational countries are sufficiently patient in their conduct of international relations. The pair of equilibrium payoffs for any subgame $\mathcal{G}^i$ is denoted by $(\bar{x}_1^i(\delta), \bar{x}_2^i(\delta))$. This effectively means that we are interested in conflict outcomes where countries are in no hurry to resolve the "no war, no peace" situation. Given the general tendency of parties in such conflicts to sustain their hostilities for long periods of time (e.g., the three decade long and ongoing conflict over the Siachen Glacier between India and Pakistan), this perspective seems natural.

So far, no assumptions has been imposed on the parametric values in the payoff structure. However, the make sure that the model isn't too poorly behaved, the following two regularity conditions will be assumed in the remaining part of the paper.

Assumption 1. Assume the following regularity conditions:

(i) $p \in \left(\max\left\{\frac{1}{2}, \eta\right\}, 1\right)$ where $\eta := \max\left\{\frac{c+W}{P_i+W}, 1 - \frac{kd}{P_i+W}\right\}_{i=1,2}$,

(ii) $P_i + W > 2kd$ for each $i \in \{1, 2\}$.

The first regularity condition (i) reflects the conventional military wisdom that an army situated in a sector of crucial military strategic significance has an advantage over the challenger, and, more precisely, that the probability of winning a battle when being in the position of an incumbent is always larger than 50 percent. This assumption can also be defended in the context of the conflict over the Siachen Glacier since an accepted rule of thumb for war on level ground is that the "attacker needs a local advantage of at least 3:1 in combat power to break trough a defender's front" (Mearsheimer, 1989, p.54). Attacking a prepared enemy position in mountain terrain, however, requires a greater ratio of attacking soldiers to defending soldiers than a war conducted on level ground. The second regularity condition (ii) ensures that both countries find it much more gratifying to defeat the other in battle, than to bide time in trying to capture the military sector without any armed confrontation.

3 Theoretical Findings

As already explained in the above, this paper investigates "no war, no peace" situations in a game theoretical framework where two countries are engaged in a standoff over a strategically important military sector. The game theoretical framework was introduced in the previous section and the reader should have the conflict over the Siachen Glacier in mind when thinking about the model. More precisely, and as mentioned earlier, India captured the Siachen Glacier in 1984, apparently to pre-empt such a move by Pakistan, and has stayed in this inhospitable terrain ever since. In all these years, there has not been a full fledged battle in the area between India and Pakistan even if the armies of both these countries has been stationed in the area at inconceivable human and economic costs.[9]

To formally analyze "no war, no peace" situations, the concept must be defined in terms of the game theoretical framework presented in the previous section. To arrive at such a definition we invoke the aforementioned three features of mountain warfare derived from *Gebirgskrieg*. In particular, we note that "no war, no peace" situations are reflections of military objectives of: (i) taking control of peaks, (ii) retain control by stationing of active military posts, and (iii) never launching a frontal assault on any peak to obtain control.[10] These objective translate into both countries playing the strategy $(s, (no, o))$.

However, in this paper, we use consider a weaker definition of a "no war, no peace" situation where the incumbent country in time period $t = 1$ chooses strategy s and the challenger country in period $t = 1$ adopt the strategy $[s, (no, o)]$. Note that a strategy profile based on these premisses will never lead to an armed confrontation as countries play stationary strategies. Further, this definition of a "no war, no peace" situation also leaves room for different actions for the incumbent country in time period $t = 1$ whenever this country is in the position of a challenger. That is, the definition only states that the incumbent country in time period $t = 1$ must adopt a strategy of type $[s, a]$ for some $a \in \{(o, o), (o, no), (no, o), (no, no)\}$.

Our first result captures the tragedy of the status quo in such situations. It shows that any such situation can be justified as a game theoretic steady state among rational players only if at least one of the countries prefers to be a challenger forever than being incumbent forever. In context of the conflict over Siachen Glacier, this means that the unending quasi-competition for the glacier makes sense only if either of India and Pakistan (or both of them) has no inherent desire to posses the peak, irrespective of its perceived military value or associated costs. In the language of the considered game, the theorem states that if there exists an equilibrium path of $\mathcal{G}^1$ where "no war, no peace" situation reigns, then at least one of the countries prefer to be a

[9]While most of this human cost is borne by loss of soldiers to avalanches and HAPE (high altitude pulmonary edema); the monetary cost, at least on the Indian side, has been recorded to be an astounding $500,000$–$750,000$ US dollars per day. See Baghel and Nüsser (2015) and Bearak (1999).

[10]An interesting example of the third point appears in Baghel and Nüsser (2015), where Italian forces mined under Austrian positions stationed on peaks to explode charges in the battle of Dolomite during World War I.

challenger than an incumbent in all stage games.

Theorem 1. *Fix any $i = 1, 2$. If there exists an SPNE σ that never leads to a peaceful resolution or an armed confrontation on equilibrium path in $\mathcal{G}^i$, then there exists a country j such that $P_j - c \leq kd$.*

Thus, Theorem 1 emphasizes that the essence of a "no war, no peace" situation can only be a deeply entrenched fear of being cheated over territorial control, which forces countries to disregard the obvious incentives to vacate the peak.

3.1 Rational Basis for "No War, No Peace" Situations

Given that instances of "no war, no peace" situations has been observed across the world over time, it is important to identify rational grounds for such hostility and, more precisely, explicit equilibria that lead to "no war, no peace" situations, which can then be used to identify possible resolutions to "no war, no peace" situations.

The following two results shed some light on this. The first of these equilibria is the symmetric equilibrium σ' where $\sigma'_i := [s, (no, o)]$ for both countries i. The behaviour underscored by this strategy combination is one of deep distrust, where no country attacks the other to capture the sector but waits for a chance to occupy it unchallenged.

Theorem 2. *The strategy profile σ' is an SPNE if and only if $P_i - c = kd \geq H$ for each $i \in \{1, 2\}$.*

A second equilibrium σ'' that leads to a "no war, no peace" situation is the strategy profile where $\sigma''_1 := [s, (no, no)]$ and $\sigma''_2 := [s, (no, o)]$. In this asymmetric equilibrium, the two countries have doctrinal differences, but perpetual hostility plays out, in the sense that the incumbent country would prefer to make peace by not occupying if there arises a chance to do so, while the challenger country would perceive this as a weakness and, consequently, be prepared to snatch the sector whenever the incumbent retreats. However, this equilibrium requires very special conditions where the stage challenger payoff must not only equal to the stage incumbent payoff but also the stage payoff from peaceful resolution.

Theorem 3. *The strategy profile σ'' is an SPNE if and only if $P_i - c = kd = H$ for each $i \in \{1, 2\}$.*

One conclusion that follows from Theorems 2 and 3 is that σ' and σ'' constitute equilibria only if $P_1 = P_2$, i.e., only if both countries perceive an equal measure of the military advantage of controlling the sector. As a result, the remarkable irony that follows from these results is that both the incumbent and the challenger countries, irrespective of their identities, get the same payoff from being in this continuous "no war, no peace" situation. Thus, in trying to be stronger than the other, they end up being the same, but still cannot find reasons to make peace as equals.

Theorems 1–3 provide couple of interesting insights about "no war, no peace" situations. First is the observation is that the equilibria described in Theorems 2 and 3 are independent of the discount factor, i.e., the results hold for any $\delta \in (0,1)$. Hence, one cannot criticize countries that end up in a "no war, no peace" situation on grounds of being short or far-sighted when they play out these equilibria. Second is the observation that existence of the equilibria described in Theorems 2 and 3 does *not depend* on the extent of patriotic euphoria W that would ensue from battlefield victory over the other as long as there is sufficient value to possessing the peak. These two insights together indicate that a "no war, no peace" situation cannot be viewed merely as a political exercise aimed at domestic constituencies. More precisely, the "no war, no peace" situation crucially depends on that both countries perceive an equal measure of military advantage to controlling the sector, that is, P_1 being equal to P_2. Given this conclusion, it follows that a "no war, no peace" situation may not endure if $P_1 \neq P_2$. Hence, the equilibria presented in Theorems 2 and 3 are not very robust as they depend on particular arrangement of parameters that may not be practically realized. Hence, one may be hopeful escaping such bad equilibria and settling into a peace equilibrium that resolves the perpetual hostility deadlock if situations where $P_1 \neq P_2$ can be engineered. Such engineering is discussed next.

3.2 Resolutions to "No War, No Peace" Situations

This section aims to shed some light on possible resolutions to "no war, no peace" situations. There can, obviously, be several different definitions of what the notion of resolution means in the context of "no war, no peace" situations. This paper adopts the view that the conflict is resolved whenever the incumbent in a stage game chooses to retreat and the challenger does not occupy. In such situations, the sector is designated to be a no man's land and, consequently, the two countries will not have their armies stationed in the area. We expect such a situation to translate into a sovereign agreement among the countries to de-escalate the sector forever.

Two different strategy profiles that resolve a "no war, no peace" situation will be discussed in this section. The first is idealistic in the sense that a resolution is achieved without imposing any threats of retaliation. Instead, it relies on mutual trust and good faith. The second is based on deterrence meaning that both countries imposes a threat of using armed force against the other country. However, as we argue below, both these resolutions must be accompanied by substantial peace activism in order to be successful.

The first resolution strategy profile mirrors the common idea that trust and good faith can lead to resolutions of conflicts. For example, Oelsner (2007, p.257) reports the Argentine–Brazilian détente of the late 1970s and the determination to build a zone of positive peace in Latin America's southern cone, and writes:

> "... over the last six decades, some regions have overcome the security dilemma and states have constructed peaceful relationships based on mutual trust and confidence, resembling friendship at the interstate levels."

Such an ideal situation is described in our setting by the strategy profile $\tilde{\sigma}$ where $\tilde{\sigma}_i := [r, (no, no)]$ for both countries i. However, as the next result shows, such optimism will be realized if and only if both countries find peace "sufficiently attractive". More precisely, $\tilde{\sigma}$ can constitute an equilibrium only as a result of aggressive peace activism that underlines the returns to peace and excruciating costs to bear in this standoff. In other words, while mutual trust is indispensable to adopt the strategy profile $\tilde{\sigma}$, a greater amount of public debate and engagement is needed to generate a consensus that $P_i - c \leq H$ for both countries i.

Theorem 4. The strategy profile $\tilde{\sigma}$ is an SPNE if and only if $P_i - c \leq H$ for each $i \in \{1, 2\}$.

The second kind of resolution of a "no war, no peace" situation is based on the idea that both countries adopt a military doctrine founded on mutually assured destruction. Even though this concept has been discussed at least since the Franco–Prussian War in the 1870s[11], it was popularized in the aftermath of World War II when the rivalry between United States and Soviet Union heated up. It was also then when John Von Neumann's game theoretical analysis became a tool to analyze global nuclear war which, eventually, led to the doctrine of mutually assured destruction. This doctrine can be seen as a form of Nash equilibrium, since no country, once they have achieved a sufficiently large arsenal, has rational reasons to initiate a conflict or to disarm.

In "no war, no peace" situations, a doctrine that resembles mutually assured destruction is a scenario of cautious peace where each country incorporates a threat of war in her doctrine by publicly adopting positions which call for occupation, and hence, armed conflict, whenever she faces an incumbent who does not retreat. In order for this scenario to resolve the conflict, and consequently designate the sector to be no man's land, the strategies must be described by the strategy profile σ^* where $\sigma_i^* := [r, (o, no)]$ for both countries i. However, this retaliation strategy must not only be accompanied with with aggressive peace activism (as the above described idealistic equilibrium $\tilde{\sigma}$) but also with bolstering military and technical capabilities which gives the incumbent country an invincible position of strength. The latter is captured by a "sufficiently high" probability of winning the battle for the incumbent country (in fact a stronger condition than the one in Assumption 1(i)). Another way to paraphrase this condition is that one cannot hope to achieve a peaceful resolution of the conflict only by peace activism, both countries would rationally find reasons to enter peace negotiations only when the degree of invincibility of an incumbent is sufficiently high. In this sense, the ever increasing military expenditures in both India and Pakistan may not be as inimical for peace as thought conventionally.[12]

[11] The English author Wilkie Collins discussed this concept in the context of the Franco–Prussian War. See the article "Wilkie Collins and Mutually Assured Destruction" which is available at the website of the *Wilkie Collins Society* (`http://wilkiecollinssociety.org/newsletter-spring-2009`). Retrieved 22 December, 2017.

[12] Between 2007 to 2009, India's defence budget increased from 24 billion USD to 40 billion USD. Observing this, Pakistan increased its defence budget by around 32 percent. *Source:* `www.indiatoday.in`, article "Indian Army's Cold Start doctrine: All you need to know". Retrieved 21 September, 2017.

Theorem 5. The strategy profile σ^* is an SPNE if and only if, for each $i \in \{1,2\}$, $P_i - c \leq H$ and $1 - \frac{kd}{P_i+W} \leq p$.

4 Conclusions

One of the innovations in this paper is to model "no war, no peace" situations as infinite horizon games where the involved countries are guided my their respective military doctrines. These doctrines are, by assumption, constant over time (India, for example, had the same military doctrine between 1947 and 2017). This in turn, implies that players can be assumed to play stationary strategies. Given this, it has been demonstrated that a "no war, no peace" situation indeed can constitute an equilibrium. However, these type of equilibria are not very robust as their existence rests on very particular arrangements of parameters and, more precisely, on the fact that both countries perceive an equal measure of military advantage to controlling the military sector. As explained in this paper, this conclusion also hints on two possible pathways toward a resolution of "no war, no peace" situations.

Most assumptions in this paper are intuitive in nature. Perhaps the most difficult assumption to motivate is the regularity condition that both countries find more gratifying to defeat the other in battle, than to bide time in trying to capture the military sector without any confrontation. Whether or not this is a too strong assumption can be debated. Naturally, more research is needed not only to find natural and realistic assumptions, but also to fully understand the rational behind "no war, no peace" situations and possible resolutions to them. This paper and the findings in it should, therefore, be seen as one piece in a larger puzzle that not yet is fully understood.

Appendix: Proofs

A well-known result that will be used repeatedly in the proofs is that, in any dynamic game, a strategy profile is a SPNE if and only if it satisfies the one-stage deviation principle. A strategy profile satisfies the one-stage deviation principle if neither country (i.e., player) can increase their payoff by deviating unilaterally from their strategy in any single stage game and then returning to the specified strategy thereafter. The subgame $\mathcal{G}^1$ is, without loss of generality, considered throughout the Appendix.

Theorem 1. Fix any $i = 1, 2$. If there exists an SPNE σ that never leads to a peaceful resolution or an armed confrontation on equilibrium path in $\mathcal{G}^i$, then there exists a country j such that $P_j - c \leq kd$.

Proof. Note first that any two stage games G_t^i and $G_{t'}^i$ are identical for a given $i \in \{1,2\}$ and any two distinct time periods t and t'. This follows since the game $\mathcal{G}^i$ is stationary for both i and the stage payoffs do not change over time. Hence, to simplify notation, the subscript t will be dropped in the remaining part of the proof and stage games are, consequently, denoted by G^1 and G^2.

Without loss of generality, we establish the result for an equilibrium σ that implies a path without armed confrontation or peaceful resolution in $\mathcal{G}^1$. Note that any such equilibrium path may either lead to (a) the terminal history where country 1 never retreats, or (b) the terminal history where country 1 retreats and country 2 occupies at $t = 1$. In the remaining part of the proof, suppose an arbitrary fixed positive fraction $\rho \in (0, 1)$ for which σ qualifies as equilibrium.[13]

Case (a). In this case, country 1, who is the incumbent at time period $t = 1$, is prescribed by σ to stay. Given the assumption that there is no armed confrontation on the equilibrium path arising out of play of σ, this action by country 1 is followed by the action of "not occupying" by country 2 at time $t = 1$. Because the countries play stationary strategies, these actions are repeated in all time periods of the game, and so, 1 remains an incumbent forever while 2 remains a challenger forever. Consequently, $\bar{x}_1^1(\delta) = \frac{P_1 - c}{1-\delta}$ and $\bar{x}_2^1(\delta) = \frac{kd}{1-\delta}$. Consider now the one-stage deviation of country 2 at $t = 1$ where 2 chooses to occupy. Such action would lead to an armed conflict and, consequently, give country 2 an expected payoff equal to:

$$p[0 + \delta \bar{x}_2^1(\delta)] + (1-p)[P_2 + W + \delta \bar{x}_2^2(\delta)]. \tag{1}$$

From the assumption that σ is a SPNE, it follows that the expected payoff in condition (1) must be less that or equal to $\bar{x}_2^1(\delta)$ or, equivalently, after some rearranging, that:

$$(1-p)(1-\delta)\delta \bar{x}_2^2(\delta) \leq (1 - p\delta)kd - (1-p)(1-\delta)(P_2 + W). \tag{2}$$

Note first that the above condition must hold for all $\delta \in (\rho, 1)$ by our definition of equilibrium. Note next that $\bar{x}_2^2(\delta)$ is the only unknown value in condition (2). By construction, $\bar{x}_2^2(\delta)$ can take four different values, and these values depend on the kind of actions prescribed by σ to the countries in stage game G^2. These four possibilities will be discussed in the following cases (a.i)–(a.iv) to establish that the condition $P_2 - c \leq kd$ must hold.

Case (a.i) *Country* 2 *retreats and country* 1 *chooses not to occupy in* G^2. Note that this case pertains to the situation where the supposed equilibrium σ leads to a resolution of the conflict in stage game G^2 leading to $x_2^2(\delta) = \frac{H}{1-\delta}$. Imputing this value in condition (2), as δ converges to 1 in limit, it follows that:

$$H \leq kd. \tag{3}$$

Now, in subgame G^2, the one-stage deviation by country 2 of staying (instead of retreating as prescribed by σ) would give a payoff of:

$$P_2 - c + \delta x_2^2(\delta) = P_2 - c + \frac{\delta H}{1-\delta}, \tag{4}$$

[13] Recall that any equilibrium reported in this paper is an asymptotic one.

if σ prescribes country 1 to not occupy when country 2 stays in G^2, and:

$$p\left(P_2 + W + \delta x_2^2(\delta)\right) + (1-p)(0 + \delta x_2^1(\delta)) = p\left(P_2 + W + \frac{\delta H}{1-\delta}\right) + (1-p)\frac{\delta kd}{1-\delta}, \quad (5)$$

if σ prescribes country 1 to occupy when country 2 stays in G^2.

Since our notion of equilibrium implies that no one-stage deviation is profitable, as δ converges to 1 in limit, equation (4) implies that $P_2 - c \leq H$, and so, by equation (3) we get that $P_2 - c \leq kd$. Arguing similarly, by equation (5) we get that:

$$(1-\delta)p(P_2 + W) + p\delta H + (1-p)\delta kd \leq H,$$

which implies that:

$$(1-\delta)p(P_2 + W) + (1-p)\delta kd \leq H(1-p\delta).$$

The latter inequality, when taken together with condition (3), implies that $(1-\delta)p(P_2 + W) + (1-p)\delta kd \leq kd(1-p\delta)$, which is equivalent to the condition $p(P_2 + W) \leq kd$. Now, by Assumption 1(ii), $kd < \frac{P_2+W}{2}$, we get that $p < \frac{1}{2}$ which contradicts Assumption 1(i). Therefore, σ must not prescribe country 1 to occupy when country 2 stays in G^2, and so, only the equations (3) and (4) can hold implying that $P_2 - c \leq kd$.

Case (a.ii) *Country* 2 *retreats and country* 1 *chooses to occupy in* G^2. Note that the equilibrium corresponding to this case are: $\bar{x}_2^2(\delta) = d + \delta\bar{x}_2^1(\delta) = \frac{d+(k-1)\delta d}{1-\delta}$ and $\bar{x}_1^2(\delta) = P_1 - c + \delta\bar{x}_1^1(\delta) = \frac{P_1-c}{1-\delta}$. Consider the off-equilibrium path node in G^2 where 2 stays. At this node, if country 1 chooses to occupy, there will be an armed confrontation leading to an expected payoff:

$$p(0 + \delta\bar{x}_1^2(\delta)) + (1-p)[P_1 + W + \delta\bar{x}_1^1(\delta)] = \frac{\delta(P_1 - c) + (1-\delta)(1-p)(P_1 + W)}{1-\delta}. \quad (6)$$

If, on the other hand, country 1 chooses not to occupy, the consequent expected payoff equals:

$$kd + \delta\bar{x}_1^2(\delta) = \frac{\delta(P_1 - c) + (1-\delta)kd}{1-\delta}. \quad (7)$$

By Assumption 1 and conditions (6) and (7), country 1 gets a greater payoff by not occupying whenever country 2 stays in G^2 under this case. Hence, the one-stage deviation of country 2 where she stays in the stage game G^2 will not cause an armed conflict, and so gives her a payoff of $P_2 - c + \delta\bar{x}_2^2(\delta)$. In order for σ to constitute an equilibrium, this payoff should be no greater than $\bar{x}_2^2(\delta)$ for all $\delta \in (\rho, 1)$ or, equivalently (recalling that $\bar{x}_2^2(\delta) = \frac{d+(k-1)\delta d}{1-\delta}$):

$$P_2 - c \leq d + (k-1)\delta d \text{ for all } \delta \in (\rho, 1).$$

As before, this condition implies that $P_2 - c \leq kd$ as δ converges to 1 in limit.

Case (a.iii) *Country* 2 *stays and country* 1 *chooses to occupy in* G^2. This situation leads to an armed confrontation in stage game G^2. From (2) and our supposition of σ being an equilibrium, the following condition must hold for all $\delta \in (\rho, 1)$:

$$\gamma(\delta) := (1-p)[(1-\delta)(1-p\delta)(P_2+W)+\delta(1-\delta)p(P_2+W)+(1-p)\delta^2 kd]-(1-p\delta)^2 kd \leq 0.$$

Note that $\gamma(1) := \lim_{\delta\to 1}\gamma(\delta) = 0$. Further, taking the first derivative of $\gamma(\delta)$ with respect to δ, letting $\delta \to 1$, and simplifying yields:

$$\gamma'(1) = (1-p)[2kd - (P_2 + W)].$$

From Assumption 1(ii), it then follows that $\gamma'(1) < 0$. This together with $\gamma(1) := \lim_{\delta\to 1}\gamma(\delta) = 0$ imply that $\gamma(\delta)$ cannot be non-positive for all $\delta \in (\rho, 1)$. Hence, we arrive at a contradiction, and so, Case (a.iii) cannot prevail.

Case (a.iv) *Country* 2 *stays and country* 1 *chooses not to occupy in* G^2. This situation gives country 2 an equilibrium payoff: $\bar{x}_2^2(\delta) = \frac{P_2-c}{1-\delta}$, and so, (2) implies that:

$$(1-p)\delta(P_2 - c) \leq (1-p\delta)kd - (1-p)(1-\delta)(P_2 + W), \text{ for all } \delta \in (\rho, 1).$$

Hence, as δ converges to 1 in limit, we get $P_2 - c \leq kd$.

Case (b). In this case, σ prescribes country 1, who is the incumbent at time $t = 1$, to retreat, and prescribes country 2 to occupy at time $t = 1$. Hence, $\bar{x}_1^1(\delta) = d+\delta\bar{x}_1^2(\delta)$ and $\bar{x}_2^1(\delta) = P_2-c+\delta\bar{x}_2^2$, and country 2 is the incumbent at time $t = 2$ on the equilibrium path arising out of play of σ. Note that unlike Case (a), now G^2 is a stage that is reached on the equilibrium path implied by σ in $\mathcal{G}^1$. And so, σ must not imply a peaceful resolution or armed confrontation in G^2 (or equivalently in $\mathcal{G}^2$). Therefore, now $\bar{x}_2^2(\delta)$ can only take two different values, and as before, these values depend on the kind of actions prescribed by σ to the countries in stage game G^2. We analyze these possibilities as separate cases (b.i)–(b.ii) below, and show that $P_2 - c \leq kd$.

Case (b.i) *Country* 2 *retreats and country* 1 *chooses to occupy in* G^2. In this situation, it follows that $\bar{x}_1^2(\delta) = P_1 - c + \delta\bar{x}_1^1(\delta)$ where $\bar{x}_1^1(\delta) = d + \delta(P_1 - c + \delta\bar{x}_1^1(\delta))$. From these two conditions, the following must hold:

$$\bar{x}_1^2(\delta) - \bar{x}_1^1(\delta) = \frac{P_1 - c - d}{1+\delta}. \tag{8}$$

Consider next the unilateral one-stage deviation by country 1 in the stage game G^1 at time period $t = 1$, where country 1 stays instead of retreating. This action will lead to an armed confrontation if, at this off-equilibrium path, country 2 chooses to occupy. Now, occupying would give country 1 an expected payoff $p\delta\bar{x}_1^2(\delta) + (1-p)[P_1 + W + \delta\bar{x}_1^1(\delta)]$ whereas not occupying would give country 1 payoff $kd + \delta\bar{x}_1^2(\delta)$. The difference in payoff between occupying and not occupying for country 1 can thus be written as:

$$\gamma(\delta) := (1-p)(P_1 + W) - kd - \delta(1-p)[\bar{x}_1^2(\delta) - \bar{x}_1^1(\delta)]. \tag{9}$$

From conditions (8) and (9), it now follows that:

$$\gamma(1) := \lim_{\delta\to 1}\gamma(\delta) = \frac{(1-p)(P_1+2W+c+d)-2kd}{2}.$$

Consequently, $\gamma(1)<0$ by Assumption 1(ii). Therefore, one can safely claim that for sufficiently high values of δ, the equilibrium σ dictates that country 1 will not occupy when country 2 decides to stay in stage game G^2. This implies that for such high values of δ, the unilateral one-stage deviation where country 2 stays, will not lead to an armed conflict. Therefore, country 2 will get payoff $P_2-c+\delta\bar{x}_2^2(\delta)$ which, by our supposition of σ being an equilibrium, must be less than or equal to $\bar{x}_2^2(\delta)$ for all these high values of δ. This condition holds only if $\frac{P_2-c}{1-\delta}\leq\frac{d+\delta(P_2-c)}{1-\delta^2}=\bar{x}_2^2(\delta)$ for all δ that are sufficiently large. And so, in the limit as δ converges to 1, we get that $P_2-c\leq d$, which in turn implies that $P_2-c\leq kd$ (as $k>1$).

Case (b.ii), *Country* 2 *stays and country* 1 *chooses not to occupy in* G^2. Using almost identical arguments as in the proof of Case (a.iv), it can be demonstrated that $P_1-c\leq kd$. □

Theorem 2. The strategy profile σ' is an SPNE if and only if $P_i-c=kd\geq H$ for each $i\in\{1,2\}$.

Proof. Fix an $i=1,2$. To prove necessity, suppose that σ' is an equilibrium for the game $\mathcal{G}^i$. Then there exists some $\rho\in(0,1)$ such that the strategy profile σ' is a SPNE for all $\delta\in(\rho,1)$. Furthermore, for all $\delta\in(\rho,1)$, it holds that $\bar{x}_1^1(\delta)=P_1-c+\delta\bar{x}_1^1(\delta)$ and $\bar{x}_1^2(\delta)=kd+\delta\bar{x}_1^2(\delta)$, implying that $(\bar{x}_1^1(\delta),\bar{x}_2^1(\delta))=\left(\frac{P_1-c}{1-\delta},\frac{kd}{1-\delta}\right)$. By identical arguments, it follows that $(\bar{x}_1^2(\delta),\bar{x}_2^2(\delta))=\left(\frac{kd}{1-\delta},\frac{P_2-c}{1-\delta}\right)$.

Recall next that each country i makes a move either (i) as an incumbent, or (ii) as a challenger responding to the other country j choosing to stay, or (iii) as a challenger responding to the other country j choosing to retreat. Further, by stationarity, the action choices at each of these nodes must not vary over time. Because σ' is a SPNE, by assumption, the one-stage deviation property requires that the following inequalities, that correspond to cases (i)–(iii), must hold for all $\delta\in(\rho,1)$:

(i) $[d+\delta\bar{x}_1^2(\delta)]\leq\bar{x}_1^1(\delta)\;\Leftrightarrow\;(1+\delta(k-1))d\leq P_1-c,$

(ii) $[p\delta\bar{x}_1^2(\delta)+(1-p)(P_1+W+\delta\bar{x}_1^1(\delta))]\leq\bar{x}_1^2(\delta)\;\Leftrightarrow\;\frac{(1-\delta)(1-p)(P_1+W)+\delta(P_1-c)(1-p)}{1-p\delta}\leq kd,$

(iii) $\frac{H}{1-\delta}\leq[P_1-c+\delta\bar{x}_1^1(\delta)]\;\Leftrightarrow\;H\leq P_1-c.$

Because the above inequalities must hold for all $\delta\in(\rho,1)$, they must obviously hold in limit as $\delta\to 1$. But this implies that $kd=P_1-c\geq H$. By similar arguments, it can be demonstrated that $kd=P_2-c\geq H$.

Now, from the proof of necessity above, it can easily be seen that parameter restrictions $P_i-c=kd\geq H,\forall\, i=1,2$ and Assumption 1 imply that σ' satisfies the one-stage deviation property for all possible values of δ. Hence, the proof of sufficiency follows. □

Theorem 3. The strategy profile σ'' is an SPNE if and only if $P_i - c = kd = H$ for each $i \in \{1, 2\}$.

Proof. Fix an $i = 1, 2$. To prove necessity, suppose that σ'' is an equilibrium for the game $\mathcal{G}^i$. Then there exists some $\rho \in (0, 1)$ such that the strategy profile σ'' is a SPNE for all $\delta \in (\rho, 1)$. Furthermore, for all $\delta \in (\rho, 1)$ and for all $i \neq j \in \{1, 2\}$, it holds that $\bar{x}_i^i(\delta) = \frac{P_i - c}{1-\delta}$ and $\bar{x}_j^i(\delta) = \bar{x}_i^j(\delta) = \frac{kd}{1-\delta}$. As in the proof of Theorem 2, each country i makes a move either (i) as an incumbent, or (ii) as a challenger responding to the other country j choosing to stay, or (iii) as a challenger responding to the other country j choosing to retreat. Because σ'' is a SPNE, by assumption, the one-stage deviation property requires that the following inequalities, that correspond to cases (i)–(iii), must hold for country 1 for all $\delta \in (\rho, 1)$:

(1.i) $[d + \delta \bar{x}_1^2(\delta)] \leq \bar{x}_1^1(\delta) \;\Leftrightarrow\; (1 + \delta(k-1))d \leq P_1 - c,$

(1.ii) $[p\delta \bar{x}_1^2(\delta) + (1-p)(P_1 + W + \delta \bar{x}_1^1(\delta))] \leq \bar{x}_1^2(\delta) \;\Leftrightarrow\; \frac{(1-\delta)(1-p)(P_1+W)+\delta(P_1-c)(1-p)}{1-p\delta} \leq kd,$

(1.iii) $[P_1 - c + \delta \bar{x}_1^1(\delta)] \leq \frac{H}{1-\delta} \;\Leftrightarrow\; P_1 - c \leq H.$

By similar arguments, the following inequalities must hold for country 2 for all $\delta \in (\rho, 1)$:

(2.i) $\frac{H}{1-\delta} \leq \bar{x}_2^2 \;\Leftrightarrow\; H \leq P_2 - c,$

(2.ii) $[p\delta \bar{x}_2^1(\delta) + (1-p)(P_2 + W + \delta \bar{x}_2^2(\delta))] \leq \bar{x}_2^1(\delta) \;\Leftrightarrow\; \frac{(1-\delta)(1-p)(P_2+W)+\delta(P_2-c)(1-p)}{1-p\delta} \leq kd,$

(2.iii) $\frac{H}{1-\delta} \leq \frac{P_2-c}{1-\delta} \;\Leftrightarrow\; H \leq P_2 - c.$

Because the inequalities (1.i)–(1.iii) and (2.i)–(2.iii) must hold for all $\delta \in (\rho, 1)$, they must obviously hold in limit as $\delta \to 1$. But this implies that $kd \leq P_1 - c \leq H \leq P_2 - c \leq kd$, and so, the necessity part of the proof follows.

As before, from the proof of necessity above, it can easily be seen that parameter restrictions $P_i - c = kd \geq H, \forall\, i = 1, 2$ and Assumption 1 imply that σ' satisfies the one-stage deviation property for all possible values of δ. Hence, the proof of sufficiency follows. $\square$

Because the proofs of Theorems 4 and 5 are almost identical, only latter of them is proved in this Appendix.

Theorem 5. The strategy profile σ^* is an SPNE if and only if, for each $i = 1, 2$, $P_i - c \leq H$ and $1 - \frac{kd}{P_i + W} \leq p$.

Proof. Fix an $i = 1, 2$. To prove necessity, suppose that σ^* is an equilibrium for the game $\mathcal{G}^i$. Then there exists some $\rho \in (0, 1)$ such that the strategy profile σ^* is a SPNE for all $\delta \in (\rho, 1)$. Note next that, in this case, the game ends at time period $t = 1$. Furthermore, for all $\delta \in (\rho, 1)$ and for all $i \neq j \in \{1, 2\}$, it holds that $\bar{x}_i^j(\delta) = \frac{H}{1-\delta}$. As in the proof of Theorem 2, each country i makes a move either (i) as an incumbent, or (ii) as a challenger responding to the other country j choosing to stay, or (iii) as a challenger responding to the other country j choosing to retreat. Because σ^* is a SPNE, by assumption, the one-stage deviation property requires that the following inequalities, that correspond to cases (i)–(iii), must hold for each country $i \in \{1, 2\}$ for all $\delta \in (\rho, 1)$:

(i) $P_i - c + \delta \bar{x}_i^i(\delta) \leq \bar{x}_i^i(\delta) \;\Leftrightarrow\; P_i - c \leq H$,

(ii) $\delta p \bar{x}_i^j(\delta) + (1-p)(P_i + W + \delta \bar{x}_i^i(\delta)) \leq \bar{x}_i^j(\delta) \;\Leftrightarrow 1 - \frac{kd}{P_i+W} \leq p$,

(iii) $P_i - c + \delta \bar{x}_i^i(\delta) \leq \bar{x}_i^i(\delta) \;\Leftrightarrow P_i - c \leq H$.

The necessity result now follows directly from the above inequalities.

Finally, from the proof of necessity above, it can easily be seen that parameter restrictions $P_i - c \leq H, \forall\, i = 1, 2$ and $1 - \frac{kd}{P_i+W} \leq p$ imply that σ' satisfies the one-stage deviation property for all possible values of δ. Hence, the proof of sufficiency follows. □

References

Abreu, D., D. Pearce, and E. Stacchetti (1990). Toward a theory of discounted repeated games with imperfect monitoring. *Econometrica 58*(5), 1041–1063.

Baghel, R. and M. Nüsser (2015). Securing the heights: The vertical dimension of the Siachen conflict between India and Pakistan in the Eastern Karakoram. *Political Geography 48*, 24–36.

Baliga, S. and T. Sjöström (2004). Arms races and negotiations. *The Review of Economic Studies 71*(2), 351–369.

Bearak, B. (1999). The coldest war; frozen in fury on the roof of the world. `www.nytimes.com/1999/05/23/world/the-coldest-war-frozen-in-fury-on-the-roof-of-the-world.html`. Accessed: 16th April, 2018.

Chassang, S. and G. P. Miquel (2010). Conflict and deterrence under strategic risk. *The Quarterly Journal of Economics 125*(4), 1821–1858.

Debs, A. and N. P. Monteiro (2014). Known unknowns: Power shifts, uncertainty, and war. *International Organization 68*(1), 1–31.

Fearon, J. D. (1995). Rationalist explanations for war. *International Organization 49*(3), 379–414.

Fearon, J. D. (2004). Why do some civil wars last so much longer than others? *Journal of Peace Research 3*(41), 275–301.

Fearon, J. D. (2007). Fighting rather than bargaining. In *Annual Meetings of the American Political Science Association*.

Fedarko, K. (2003). The coldest war. `www.outsideonline.com/1912811/coldest-war`. Accessed: 16th April, 2018.

Hussain, J. (2012). The fight for siachen. `tribune.com.pk/story/368394/the-fight-for-siachen`. Accessed: 16th April, 2018.

Ives, J. (2004). *Himalayan perceptions: Environmental change and the well-being of mountain peoples*. Routledge Studies in Physical Geography and Environment.

Jackson, M. O. and M. Morelli (2009). Strategic militarization, deterrence and wars. *Quarterly Journal of Political Science 4*(4), 279–313.

Leventoğlu, B. and B. L. Slantchev (2007). The armed peace: A punctuated equilibrium theory of war. *American Journal of Political Science 51*(4), 755–771.

Mac Ginty, R. (2010). No war, no peace: Why so many peace processes fail to deliver peace. *International Politics 47*(2), 145–162.

Mearsheimer, J. J. (1989). Assessing the conventional balance: The 3:1 rule and its critics. *International Security 13*(4), 54–89.

Oelsner, A. (2007). Friendship, mutual trust and the evolution of regional peace in the international system. *Critical Review of International Social and Political Philosophy 10*(2), 257–279.

Powell, R. (1999). *In the shadow of power: States and strategies in international politics*. Princeton University Press.

Powell, R. (2004). Bargaining and learning while fighting. *American Journal of Political Science 48*(2), 344–361.

Schelling, T. C. (1966). *Arms and influence*. Yale University Press.

Schwarz, M. and K. Sonin (2007). A theory of brinkmanship, conflicts, and commitments. *The Journal of Law, Economics & Organization 24*(1), 163–183.

Wirsing, R. (1998). War or peace on the line of control? The India–Pakistan dispute over Kashmir turns fifty. *Boundary and Territory Briefing 2*(5), 1–40.

Yared, P. (2010). A dynamic theory of war and peace. *Journal of Economic Theory 145*(5), 1921–1950.