SEGMENTATION OF BRAIN MR IMAGES USING FUZZY SETS AND MODIFIED CO-OCCURRENCE MATRIX

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Abstract

Image segmentation is an indispensable process in the visualization of human tissues, particularly during clinical analysis of magnetic resonance (MR) images. A robust segmentation technique based on fuzzy set theory for brain MR images is proposed in this paper. The histogram of the given image is thresholded according to the similarity between gray levels. The similarity is assessed through second order fuzzy correlation. To calculate the second order fuzzy correlation, a modified co-occurrence matrix is used to extract the local information more accurately. Two parameters - ambiguity and the strength of ambiguity, are introduced to determine the thresholds of the given histogram. The effectiveness of the proposed algorithm, along with a comparison with other methods, has been demonstrated on a set of brain MR images.

1 Introduction

Segmentation is a process of partitioning an image space into some non-overlapping meaningful homogeneous regions. The success of an image analysis system depends on the quality of segmentation [1]. In the analysis of medical images for computer-aided diagnosis and therapy, segmentation is often required as a preliminary stage. Medical image segmentation is a complex and challenging task due to intrinsic nature of the images. The brain has a particularly complicated structure and its precise segmentation is very important for detecting tumors, edema, and necrotic tissues, in order to prescribe appropriate therapy [11]. A series of algorithms for image segmentation based on histogram thresholding can be found in [5,6,12]. Entropy based algorithms have been proposed in [7-9,16].

One of the main problems in medical image segmentation is uncertainty. Some of the sources of this uncertainty include imprecision in computations and vagueness in class definitions. In this background, the possibility concept introduced by the fuzzy set theory has gained popularity in modeling and propagating uncertainty in medical imaging applications. Also, since the fuzzy set theory is a powerful tool to deal with linguistic concepts such as similarity, several segmentation algorithms based on fuzzy set theory are reported in the literature [10,13,16].

In general, all histogram thresholding techniques based on fuzzy set theory work very well when the image gray level histogram is bimodal or multimodal. On the other hand, a great deal of medical images are usually unimodal where, the conventional histogram thresholding techniques perform poorly or even fail. In this class of histograms, unlike the bimodal case, there is no clear separation between object and background pixel occurrences. Thus, to find a reliable threshold, some adequate criterion for splitting the image histogram should be used. A possible one is the use of a measure of similarity or closeness between gray levels. In [10], an approach to threshold the histogram according to the similarity between gray levels has been proposed.

The proposed method is based on a fuzzy measure to threshold the image histogram. The image is thresholded based on a criterion of similarity between gray levels. The second order fuzzy correlation is used for assessing such a concept. The local information of the given image is extracted through modified co-occurrence matrix. The technique proposed here consists of two linguistic variables $\{$bright, dark$\}$ modeled by two fuzzy subsets and a fuzzy region on the gray level histogram. Each of the gray levels of the fuzzy region is assigned to both defined subsets one by one and the second order fuzzy correlation using modified co-occurrence matrix is calculated. The ambiguity of each gray level is determined from the correlations of two fuzzy subsets. Finally, the strength of ambiguity for each gray level is computed. The thresholds of the histogram are determined according to the strength of ambiguity of the gray levels using a nearest mean classifier. Experimental results reported in this paper establish the fact that the proposed method is robust in segmenting brain MR images compared to existing thresholding methods.

The rest of the paper is organized as follows. In Section 2 some basic definitions about fuzzy sets and second order fuzzy correlation along with co-occurrence matrix are introduced. The proposed algorithm for histogram thresholding is presented in Section 3. Experimental results and a comparison with other thresholding methods are presented in Section 4. Concluding remarks are given in Section 5.

2 Fuzzy Sets, Fuzzy Correlation and Co-occurrence Matrix

A fuzzy subset A of the universe X is defined as a collection of ordered pairs

$$A = \{(\mu_A(x), x), \forall x \in X\} \tag{1}$$

where $\mu_A(x)$ ($0 \leq \mu_A(x) \leq 1$) denotes the degree of belonging of the element x to the fuzzy set A. The support of fuzzy set A is the crisp set that contains all the elements of X that have a non-zero membership value in A [4].

Let $X = [x_{mn}]$ be an image of size $M \times N$ and L gray levels, where x_{mn} is the gray value at location (m, n) in X, $x_{mn} \in G_L = \{0, 1, 2, ..., L-1\}$, - the set of the gray levels, $m = 0, 1, 2, \cdots, M-1$, $n = 0, 1, 2, \cdots, N-1$, and $\mu_X(x_{mn})$ be the value of the membership function in the unit interval $[0, 1]$ which represents the degree of possessing some brightness property $\mu_X(x_{mn})$ by the pixel intensity x_{mn}. By mapping an image X from x_{mn} into $\mu_X(x_{mn})$, the image set X can be written as

$$X = \{(\mu_X(x_{mn}), x_{mn})\} \tag{2}$$

Then, X can be viewed as a characteristic function and μ_X is a weighting coefficient which reflects the ambiguity in X.

A function mapping all the elements in a crisp set into real numbers in $[0, 1]$ is called a membership function. The larger value of the membership function represents the higher degree of the membership. It means how closely an element resembles an ideal element. Membership functions can represent the uncertainty using some particular functions. These functions transform the linguistic variables into numerical calculations by setting some parameters. The fuzzy decisions can then be made. The standard S-function ($S(x_{mn}; a, b, c)$) of Zadeh is as follows:

$$\mu_X(x_{mn}) = \begin{cases} 0 & x_{mn} \leq a \\ 2[\frac{x_{mn}-a}{c-a}]^2 & a \leq x_{mn} \leq b \\ 1 - 2[\frac{x_{mn}-c}{c-a}]^2 & b \leq x_{mn} \leq c \\ 1 & x_{mn} \geq c \end{cases} \tag{3}$$

where $b = \frac{(a+c)}{2}$ is the cross-over point (for which the membership value is 0.5). The shape of S-function is manipulated by the parameters a and c.

2.1 Co-occurrence Matrix

The co-occurrence matrix of the image X is an $L \times L$ dimensional matrix that gives an idea about the transition of intensity between adjacent pixels. In other words, the (i, j)th entry of the matrix gives the number of times the gray level j follows the gray level i (i.e the gray level j is an adjacent neighbor of the level i) in a specific fashion.

Let a denote the (m, n)th pixel in X and b denote one of the eight neighboring pixel of a, - that is, $b \in a_8 =$

$$\{(m, n-1), (m, n+1), (m+1, n), (m-1, n), (m-1, n-1), (m-1, n+1), (m+1, n-1), (m+1, n+1)\}$$

then

$$t_{ij} = \sum_{a \in X, b \in a_8} \delta \tag{4}$$

where

$$\delta = \begin{cases} 1 & \text{if gray level value of } a \text{ is } i \\ & \text{and that of } b \text{ is } j \\ 0 & \text{otherwise} \end{cases}$$

Obviously, t_{ij} gives the number of times the gray level j follows gray level i in any one of the eight directions. The matrix $T = [t_{ij}]_{L \times L}$ is, therefore, the co-occurrence matrix of the image X.

2.2 Second order Fuzzy Correlation

The correlation between two local properties μ_1 and μ_2 (for example, edginess, blurredness, texture, etc.) can be expressed in the following ways [14]:

$$C(\mu_1, \mu_2) = 1 - \frac{4 \sum_{i=1}^{L} \sum_{j=1}^{L} [\mu_1(i,j) - \mu_2(i,j)]^2 t_{ij}}{Y_1 + Y_2} \tag{5}$$

where c_{ij} is the frequency of occurrence of the gray level i followed by j, that is, C is the co-occurrence matrix defined earlier, and

$$Y_k = \sum_{i=1}^{L} \sum_{j=1}^{L} [2\mu_k(i,j) - 1]^2 t_{ij}; \quad k = 1, 2.$$

To calculate correlation between a gray-tone image and its two-tone version, μ_2 is considered as the nearest two-tone version of μ_1. That is,

$$\mu_2(x) = \begin{cases} 0 & \text{if } \mu_1(x) \leq 0.5 \\ 1 & \text{otherwise} \end{cases} \tag{6}$$

For computing the second order fuzzy correlation of an image, represented by a fuzzy set, one needs to choose two pixels at a time and to assign a composite membership value to them. Normally these two pixels are chosen as adjacent pixels. Next subsection presents a two dimensional S-type membership function which represents fuzzy bright image plane assuming higher gray value corresponds to object region.

2.3 2D S-type Membership Function

The 2D S-type membership function reported in [15] assigns a composite membership value to a pair of adjacent pixels as follows. For a particular threshold b,

1) (b, b) is the most ambiguous point, that is, the boundary between object and background. Therefore its membership value for the fuzzy bright image plane is 0.5.
2) If one object pixel is followed by another object pixel, then its degree of belonging to object region is greater than 0.5. The membership value increases with increase in pixel intensity.
3) If one object pixel is followed by one background pixel or vice versa, the membership value is less than or equal to 0.5, depending on the deviation from the boundary point (b, b).
4) If one background pixel is followed by another background pixel, then its degree of belonging to object

region is less than 0.5. The membership value decreases with decrease of pixel intensity.

Instead of using fixed bandwidth (Δb), we take the parameters of S-type membership function as follows [10]:

$$b = \frac{\sum_{i=p}^{q} x_i \cdot h(x_i)}{\sum_{i=p}^{q} h(x_i)} \tag{7}$$

$$\Delta b = \max\{|b - (x_i)_{min}|, |b - (x_i)_{max}|\} \tag{8}$$

$$c = b + \Delta b; \quad a = b - \Delta b \tag{9}$$

where $h(x_i)$ denotes the image histogram and x_p and x_q are the limits of the subset being considered. The quantities $(x_i)_{min}$ and $(x_i)_{max}$ represent the minimum and maximum gray levels in the current set for which $h((x_i)_{min}) \neq 0$ and $h((x_i)_{max}) \neq 0$. Basically, the crossover point b is the mean gray level value of the interval $[x_p, x_q]$. With the function parameters computed in this way, the S-type membership function adjusts its shape as a function of the set elements.

3 Proposed Algorithm

The proposed method for segmentation of brain MR images consists of three phases:

1) modification of co-occurrence matrix defined earlier;
2) measure of ambiguity for each gray level x_i; and
3) measure of strength of ambiguity.

Each of the three phases is elaborated next one by one.

3.1 Modification of Co-occurrence Matrix

In general, for a given image consists of object on a background, the object and background each have a unimodal gray-level population. The gray levels of adjacent points interior to the object, or to the background, are highly correlated, while across the edges at which object and background meet, adjacent points differ significantly in gray level. If an image satisfies these conditions, its gray-level histogram will be primarily a mixture of two unimodal histograms corresponding to the object and background populations, respectively. If the means of these populations are sufficiently far apart, their standard deviations are sufficiently small, and they are comparable in size, the image histogram will be bimodal.

In medical imaging, the histogram of the given image is in general unimodal. One side of the peak may display a shoulder or slope change, or one side may be less steep than the other, reflecting the presence of two peaks that are close together or that differ greatly in height. The histogram may also contain a third, usually smaller, population corresponding to points on the object-background border. These points have gray levels intermediate between those of the object and background; their presence raises the level of the valley floor between the two peaks, or if the peaks are already close together, makes it harder to detect the fact that they are not a single peak. As the histogram peaks are close together and very unequal in size, it may be difficult to detect the valley between them. This paper presents a method of producing a transformed co-occurrence matrix in which the valley is deeper and is thus easier to detect. In determining how each point of the image should contribute to the transformed co-occurrence matrix, this method takes into account the rate of change of gray level at the point, as well as the point's gray level (edge value); that is, the maximum of differences of average gray levels in pairs of horizontally and vertically adjacent 2-by-2 neighborhoods [2]. If Δ is the edge value at a given point, then

$$\begin{aligned}\Delta = \frac{1}{4}\max\{&|x_{m-1,n} + x_{m-1,n+1} + x_{m,n} + x_{m,n+1} \\ &- x_{m+1,n} - x_{m+1,n+1} - x_{m+2,n} - x_{m+2,n+1}|, \\ &|x_{m,n-1} + x_{m,n} + x_{m+1,n-1} + x_{m+1,n} \\ &- x_{m,n+1} - x_{m,n+2} - x_{m+1,n+1} - x_{m+1,n+2}|\}\end{aligned}$$

According to the image model, points interior to the object and background should generally have low edge values, since they are highly correlated with their neighbors, while those on the object-background border should have high edge values. Thus if we produce a co-occurrence matrix of the gray levels of points having low edge values only, the peaks should remain essentially same, since they correspond to interior points, but valley should become deeper, since the intermediate gray level points on the object-background border have been eliminated.

More generally, we can compute a weighted co-occurrence matrix in which points having low edge values are counted heavily, while points having high values are counted less heavily. If $|\Delta|$ is the edge value at a given point, then Equation 4 becomes

$$t_{ij} = \sum_{a \in X, b \in a_8} \frac{\delta}{(1 + |\Delta|^2)} \tag{10}$$

This gives full weight (1) to points having zero edge value and negligible weight to high edge value points.

3.2 Measure of Ambiguity $\mathcal{A}$

The aim of the proposed method is to threshold the gray level histogram by splitting the image histogram into multiple crisp subsets, using second order fuzzy correlation previously defined. First, let us define two linguistic variables {dark, bright} modeled by two fuzzy subsets of X, denoted by A and B, respectively. The fuzzy subsets A and B are associated with the histogram intervals $[x_{min}, x_p]$ and $[x_q, x_{max}]$ respectively, where x_p and x_q are the final and initial gray level limits for these subsets, and x_{min} and x_{max} are the lowest and highest gray levels of the image respectively. Then, the ratio of the cardinalities of two fuzzy subsets A and B is given by,

$$\beta = \frac{n_A}{n_B} = \frac{|\{x_{min}, x_{min+1}, \cdots, x_{p-1}, x_p\}|}{|\{x_q, x_{q+1}, \cdots, x_{max-1}, x_{max}\}|} \tag{11}$$

Next, we calculate $C_A(x_{min} : x_p)$ and $C_B(x_q : x_{max})$, where $C_A(x_{min} : x_p)$ is the second order fuzzy correlation of fuzzy subset A and its two-tone version; and $C_B(x_q : x_{max})$ is the second order fuzzy correlation of fuzzy subset B and its two-tone version using modified co-occurrence matrix. Since the key of the proposed method is the comparison of fuzzy correlations, we have to normalize those measures. This is done by computing a normalizing factor α according to the following relation

$$\alpha = \frac{C_A(x_{min} : x_p)}{C_B(x_q : x_{max})} \tag{12}$$

To obtain the segmented version of the gray level histogram, we add to each of the subsets A and B a gray level x_i picked up from the fuzzy region and form two fuzzy subsets $\acute{A}$ and $\acute{B}$ which are associated with the histogram intervals $[x_{min}, x_i]$ and $[x_i, x_{max}]$, where $x_p < x_i < x_q$. Then, we calculate $C_{\acute{A}}(x_{min} : x_i)$ and $C_{\acute{B}}(x_i : x_{max})$. The ambiguity of the gray value of x_i is calculated as follows

$$\mathcal{A}(x_i) = 1 - \frac{|C_{\acute{A}}(x_{min} : x_i) - \alpha \cdot C_{\acute{B}}(x_i : x_{max})|}{(1+\alpha)} \tag{13}$$

Finally, applying this procedure for all gray levels of the fuzzy region, we calculate the ambiguity of each gray level. The process is started with $x_i = x_p + 1$, and x_i is incremented one by one until $x_i < x_q$. The ratio of the cardinalities of two modified fuzzy subsets $\acute{A}$ and $\acute{B}$ at each iteration is being modified accordingly

$$\acute{\beta} = \frac{n_{\acute{A}}}{n_{\acute{B}}} = \frac{|\{x_{min}, x_{min+1}, \cdots, x_{i-1}, x_i\}|}{|\{x_i, x_{i+1}, \cdots, x_{max-1}, x_{max}\}|} > \beta \tag{14}$$

Unlike [10], in proposed method as x_i is incremented one by one, the value of $\acute{\beta}$ also increases. Fig.1 represents the

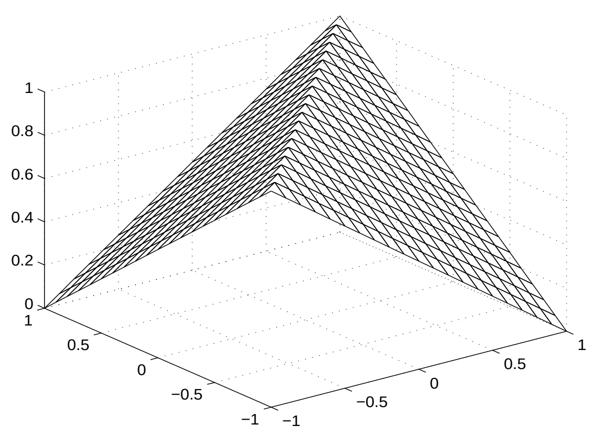

Fig. 1. Ambiguity $\mathcal{A}$ of the gray level x_i as a function of correlations of two fuzzy sets for $\alpha = 1.0$

ambiguity $\mathcal{A}(x_i)$ of the gray level x_i as a function of fuzzy correlations of two modified fuzzy subsets $\acute{A}$ and $\acute{B}$ for $\alpha = 1.0$. In other words, we calculate the ambiguity by observing how the introduction of a gray level x_i of the fuzzy region affects the similarity measure among gray levels in each of the modified fuzzy subsets $\acute{A}$ and $\acute{B}$. The ambiguity $\mathcal{A}$ is maximum for the gray level x_i in which the correlations of two modified fuzzy subsets are equal. The threshold level (T) for segmentation corresponds to gray value with maximum ambiguity $\mathcal{A}$. That is,

$$\mathcal{A}(T) = \max \ \ \arg\{\mathcal{A}(x_i)\}; \ \ \forall \ \ x_p < x_i < x_q \tag{15}$$

To find out multiple thresholds corresponding to multiple segments, we next introduce strength of ambiguity.

3.3 Strength of Ambiguity $\mathcal{S}$

In this subsection, the strength of ambiguity ($\mathcal{S}$) of each gray level x_i is calculated as follows.

Let, the difference of the gray levels between the current gray level x_i and the gray level x_j that is the closest gray level on the left-hand side whose ambiguity value is larger than or equal to the current ambiguity value is given by

$$\Delta L(x_i) = \begin{cases} x_i - x_j & \text{if } \mathcal{A}(x_j) \geq \mathcal{A}(x_i) \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

Similarly, the difference of the gray levels between the current gray level x_i and the gray level x_k that is the closest gray level on the right-hand side whose ambiguity value is larger than or equal to current ambiguity value is given by

$$\Delta R(x_i) = \begin{cases} x_k - x_i & \text{if } \mathcal{A}(x_k) \geq \mathcal{A}(x_i) \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

The strength of ambiguity of the gray level x_i is given by

$$\mathcal{S}(x_i) = \mathcal{D}(x_i) \times \Delta\mathcal{A}(x_i) \tag{18}$$

where $\mathcal{D}(x_i)$ is the absolute distance of the gray level x_i and $\Delta\mathcal{A}(x_i)$ is the difference of ambiguities of gray levels x_i and x_m which is given by

$$\Delta\mathcal{A}(x_i) = \mathcal{A}(x_i) - \mathcal{A}(x_m) \tag{19}$$

1) If $\Delta L(x_i) = 0$ and $\Delta R(x_i) = 0$, it means that the current gray level x_i has the highest ambiguity value; then,

$$\mathcal{D}(x_i) = \max(x_i - x_{min}, x_{max} - x_i) \tag{20}$$

and x_m is the gray level with smallest ambiguity value between x_{min} and x_{max}.

2) If $\Delta L(x_i) \neq 0$ and $\Delta R(x_i) = 0$, then,

$$\mathcal{D}(x_i) = \Delta L(x_i) \tag{21}$$

and x_m is the gray level with smallest ambiguity value between x_i and x_j.

3) If $\Delta R(x_i) \neq 0$ and $\Delta L(x_i) = 0$, then,

$$\mathcal{D}(x_i) = \Delta R(x_i) \tag{22}$$

and x_m is the gray level with smallest ambiguity value between x_i and x_k.

4) If $\Delta L(x_i) \neq 0$ and $\Delta R(x_i) \neq 0$, then,

$$\mathcal{D}(x_i) = \min(\Delta L(x_i), \Delta R(x_i)) \tag{23}$$

and x_m is the gray level with smallest ambiguity value between x_i and x_p, where x_p is the adjacent peak location of the current gray level x_i, where

$$x_p = \begin{cases} x_k & \text{if } \mathcal{A}(x_j) \geq \mathcal{A}(x_k) \\ x_j & \text{otherwise} \end{cases} \tag{24}$$

The thresholds are determined according to the strengths of ambiguity of the gray levels using a nearest mean classifier [3]. If the strength of ambiguity of gray level x_t is the strongest, then x_t is declared to be the first threshold. In order to find other thresholds, $\mathcal{S}(x_t)$ and those strengths of ambiguity which are less than $\mathcal{S}(x_t)/10$ are removed. Then, the mean (M) of strengths of ambiguity is calculated. Finally, the minimum mean distance is calculated as follows

$$D(x_s) = \min |\mathcal{S}(x_i) - M|; \quad x_p < x_i < x_q \tag{25}$$

where x_s is the location which has the minimum distance with M. The strengths that are larger than or equal to $\mathcal{S}(x_s)$ are also declared to be thresholds.

The detailed experimental results reported next validate the framework we have set to segment brain MRI with feasible computation.

4 Implementations and Experimental Results

In this section we present the results of different thresholding methods for segmentation of brain MR images. Above 100 MR images with different size and 16 bit gray levels are tested with different methods using C language in LINUX environment having machine configuration Pentium IV, 3.2 GHz, 1 MB cache, and 1 GB RAM. All the medical images are brain MRI.

It is seen that the choice of n_A, n_B and β is critical (Relation 11). As n_A and n_B increase, the computational time decreases, resulting in non-acceptable segmentation. However, for a good segmentation, the typical value of β is 1.0 and $n_A = n_B = 10$.

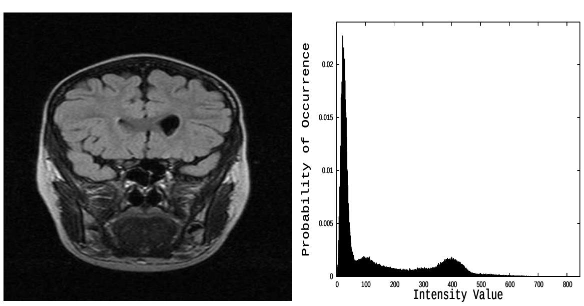

Fig. 2. Original image and corresponding histogram

Let us explain the proposed method by Fig.2-4. Fig.2 shows the original MR images and their gray value histograms, while Fig.3 represents the fuzzy second order correlations $C_{\acute{A}}(x_{min} : x_i)$ and $C_{\acute{B}}(x_i : x_{max})$ of two modified fuzzy subsets $\acute{A}$ and $\acute{B}$ with respect to the gray level x_i of the fuzzy region and the ambiguity of each gray level x_i. The values of α and β are also

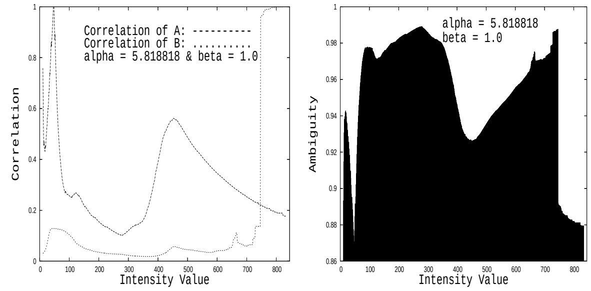

Fig. 3. Correlations of two fuzzy subsets and measure of ambiguity

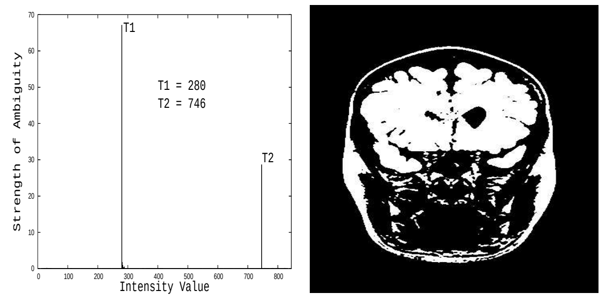

Fig. 4. Strength of ambiguity and segmented image (proposed)

given here. Fig.3 depicts the strength of ambiguity of each gray level x_i and the segmented image of the proposed method. The thresholds are determined according to the strength of ambiguity.

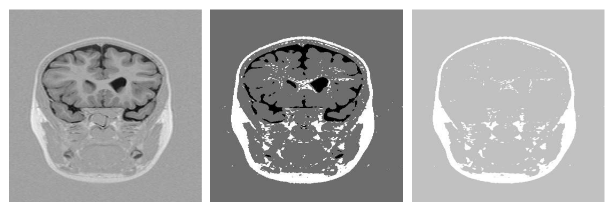

Fig. 5. Image-761; Proposed-761; and IOAC-761

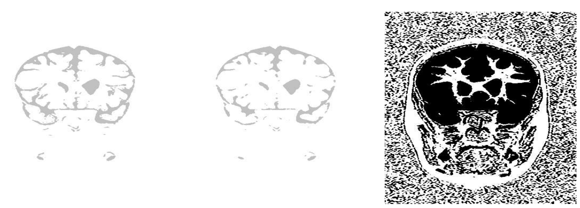

Fig. 6. 1DFC-761; Entropy-761; and Otsu-761

In Fig.5-10, the original MR images, the segmented images using proposed method, the segmented results with index of area coverage (IOAC) [13], 1-D fuzzy correlation (1DFC), conditional entropy [7-9], and Otsu [5] are shown. Next we provide the information about thresholds of different methods.

Sl.	Thresholds					
No.	Proposed	Otsu	Entropy	1DFC	2DFC	IOAC
761	1929, 2067	2045	1928	1965	1954	2070
763	1946, 2069	1822	1924	1959	1949	2104
788	1905, 2069	1841	1928	1935	1951	2154

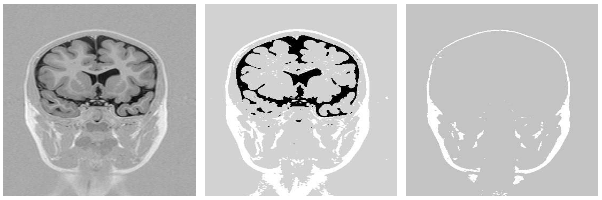

Fig. 7. Image-763; Proposed-763; and IOAC-763

Fig. 8. 1DFC-763; Entropy-763; and Otsu-763

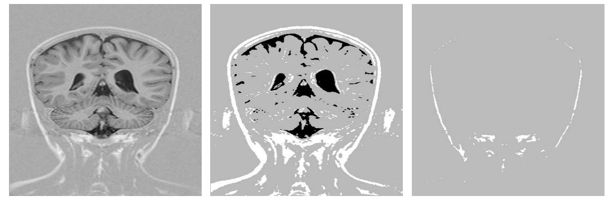

Fig. 9. Image-788; Proposed-788; and IOAC-788

Fig. 10. 1DFC-788; Entropy-788; and Otsu-788

Unlike existing thresholding methods, the proposed scheme can detect multiple segments of the objects if there exists. All the results reported in this paper clearly establish the fact that the proposed method is robust in segmenting brain MR images compared to existing thresholding methods. None of the existing thresholding methods could generate as consistently good segments as the proposed algorithm. Also, some of the existing methods have failed to detect object regions.

5 Conclusion

A robust segmentation technique based on the fuzzy set theory for segmentation of brain MRI has been presented in this paper. The histogram threshold is determined according to the similarity between gray levels. The fuzzy framework is used to obtain a mathematical model of such a concept. The edge information of each pixel location is incorporated to modify the co-occurrence matrix. The threshold determined in this way avoids local minima. This characteristic represents an attractive property of the proposed method. From the experimental results, the proposed algorithm produces a segmented images more promising than do the conventional methods.

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