# Segmenting Planar Superpixel Adjacency Graphs w.r.t. Non-planar Superpixel Affinity Graphs - Supplementary Material - 

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## 1 Derivation of the Dual of the Decomposition

We now derive the dual of the Lagrangian decomposition. We begin by writing the decomposition below.

$$
\begin{align*}
\max _{\theta^{p m c}, \psi} & 1^{T}\left(\theta^{\mathrm{P}}-\theta^{p m c}-Y \psi\right)+1^{T}\left(\psi+\theta^{\mathrm{NP}}\right)  \tag{1}\\
\text { subject to } & Z \theta^{p m c} \geq 0  \tag{2}\\
& -W \psi \geq \theta^{\mathrm{NP}}  \tag{3}\\
& \theta^{\mathrm{P}}-\theta^{p m c}-Y \psi \leq 0  \tag{4}\\
& \min \left(\left[0,-\theta^{\mathrm{P}}\right]\right) \leq-\theta^{p c m}-Y \psi  \tag{5}\\
& \psi \geq 0 \tag{6}
\end{align*}
$$

The dual of this decomposition for the planar multicut problem is derived in [1]. In order to be closer to the form of [1], we make the following adjustments to the notation. We define $\phi=\min \left(\left[0,-\theta^{\mathrm{P}}\right]\right)$ and $\hat{\lambda}=-\theta^{p m c}-\phi$. Notice that $\theta^{p m c}=-\hat{\lambda}-\phi$. We now rewrite the decomposition using $\hat{\lambda}$ and $\phi$.

$$
\begin{array}{ll}
\max _{\hat{\lambda}, \psi} & 1^{T}\left(\theta^{\mathrm{P}}+\hat{\lambda}+\phi-Y \psi\right)+1^{T}\left(\psi+\theta^{\mathrm{NP}}\right) \\
\text { subject to } & Z(-\hat{\lambda}-\phi) \geq 0 \\
& -W \psi \geq \theta^{\mathrm{NP}} \\
& \theta^{\mathrm{P}}+\hat{\lambda}+\phi-Y \psi \leq 0 \\
& \phi \leq \hat{\lambda}+\phi-Y \psi \\
& \psi \geq 0 \tag{12}
\end{array}
$$

We now move terms to different sides of the relevant inequalities. This facilitates the writing of the LP as a Lagrangian.

[^0]\[

$$
\begin{align*}
\max _{\hat{\lambda}, \psi} & 1^{T}\left(\theta^{\mathrm{P}}+\hat{\lambda}+\phi-Y \psi\right)+1^{T}\left(\psi+\theta^{\mathrm{NP}}\right)  \tag{13}\\
\text { subject to } & 0 \leq-Z \hat{\lambda}-Z \phi  \tag{14}\\
& 0 \leq-\theta^{\mathrm{NP}}-W \psi  \tag{15}\\
& 0 \leq-\theta^{\mathrm{P}}-\phi+Y \psi-\hat{\lambda}  \tag{16}\\
& 0 \leq \hat{\lambda}-Y \psi  \tag{17}\\
& \psi \geq 0 \tag{18}
\end{align*}
$$
\]

We now write the above LP as a Lagrangian.

$$
\begin{align*}
\min _{\gamma, \omega, \beta, \delta \geq 0} \max _{\hat{\lambda}, \psi \geq 0} & 1^{T}\left(\theta^{\mathrm{P}}+\hat{\lambda}+\phi-Y \psi\right)+1^{T}\left(\psi+\theta^{\mathrm{NP}}\right)  \tag{19}\\
& +\gamma^{T}(-Z \hat{\lambda}-Z \phi)  \tag{20}\\
& +\omega^{T}\left(-\theta^{\mathrm{NP}}-W \psi\right)  \tag{21}\\
& +\beta^{T}\left(-\theta^{\mathrm{P}}-\phi+Y \psi-\hat{\lambda}\right)  \tag{22}\\
& +\delta^{T}(\hat{\lambda}-Y \psi) . \tag{23}
\end{align*}
$$

Now we group the terms that have components of $\psi$ and $\hat{\lambda}$ together. This facilitates the creation of new constraints.

$$
\begin{align*}
\min _{\gamma, \omega, \beta, \delta \geq 0} & 1^{T}\left(\theta^{\mathrm{P}}+\phi\right)+1^{T} \theta^{\mathrm{NP}}-\gamma^{T} Z \phi-\omega^{T} \theta^{\mathrm{NP}}+\beta^{T}\left(-\theta^{\mathrm{P}}-\phi^{T}\right)  \tag{24}\\
& +\left(1^{T}-\gamma^{T} Z-\beta^{T}+\delta^{T}\right) \hat{\lambda}  \tag{25}\\
& +\left(-1^{T} Y+1^{T}-\omega^{T} W+\beta^{T} Y-\delta^{T} Y\right) \psi \tag{26}
\end{align*}
$$

Since $\hat{\lambda}$ is unbounded and $\psi$ is non-negative, the LP with the following constraints is equivalent to the Lagrangian above.

$$
\begin{align*}
\min _{\gamma, \omega, \beta, \delta \geq 0} & 1^{T}\left(\theta^{\mathrm{P}}+\phi\right)+1^{T} \theta^{\mathrm{NP}}-\gamma^{T} Z \phi-\omega^{T} \theta^{\mathrm{NP}}+\beta^{T}\left(-\theta^{\mathrm{P}}-\phi^{T}\right)  \tag{27}\\
\text { subject to } & \gamma^{T} Z+\beta^{T}=1^{T}+\delta^{T}  \tag{28}\\
& \omega^{T} W \geq \beta^{T} Y-1^{T} Y+1^{T}-\delta^{T} Y . \tag{29}
\end{align*}
$$

Except for transposes of some terms, this is the final form of the Lagrangian decomposition used in the main manuscript.

## References

1. J. Yarkony. MAP inference in Planar Markov Random Fields with Applications to Computer Vision. PhD thesis, University of California, Irvine, 2012.

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