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SEIGNIORAGE, OPERATING RULES AND THE HIGH INFLATION TRAP

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ABSTRACT

A given amount of seigniorage revenue can be collected at either a high or a low rate of inflation. Thus there may be two equilibria when a government finances its deficit by printing money--implying that an economy may be stuck in a high inflation equilibrium when, with the same fiscal policy, it could be at a lower inflation rate.

We show that under rational expectations the high inflation equilibrium is stable and the low inflation equilibrium unstable; under adaptive expectations or lagged adjustment of money balances with rational expectations, it may be the low inflation equilibrium that is stable.

Extending the model to allow for bond as well as money financing of deficits, we show that one of the equilibria disappears if the government sets a nominal anchor for the economy, for instance by fixing the growth rate of money. The dual equilibria and their stability characteristics remain if the government fixes the real interest rate. The existence of dual equilibria is thus a result of the operating rules the government chooses for monetary and fiscal policy.

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SEIGNIORAGE, OPERATING RULES AND THE HIGH INFLATION TRAP.

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A given amount of seigniorage revenue can be collected at either a high or a low rate of inflation. Thus as Sargent and Wallace (1981, 1987) and Liviatan (1984) have shown, there may be two equilibria when a government finances its deficit by printing money. The dual equilibria--a reflection of the Laffer curve--imply that an economy may be stuck in a high inflation equilibrium when, with the same fiscal policy, it could be at a lower inflation rate.[2]

In this paper we first demonstrate the existence of the dual equilibria in a simple model in which money is the only source of deficit financing. We show that under rational expectations the high inflation equilibrium is stable and the low inflation equilibrium unstable; under adapative expectations it may be the low inflation equilibrium that is stable.[3] We then extend the model to allow for the possibility of bond financing of deficits and show that one of the equilibria disappears if the government sets a nominal anchor for the economy, for instance by fixing the growth rate of money. The existence of dual equilibria is thus a result of the operating rules the government chooses for monetary and fiscal policy.

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[1]Bank of Israel (on leave from the Hebrew University) and NBER; and MIT and NBER. We gratefully acknowledge helpful discussions with Rudi Dornbusch and financial assistance from the National Science Foundation and the U.S.-Israel Binational Science Foundation.

[2]By the same fiscal policy we mean the same budget deficit as a percentage of output.

[3]These results are contained in an unpublished note of ours "Expectations and the High Inflation Trap" which has been in circulation in mimeo form since 1984.

The possibility of dual equilibria under pure money financing of the deficit is known from the work cited above. Auernheimer (1973), Evans and Yarrow (1981), and Escude (1985) have shown that the high inflation equilibrium may be stable if expectations adjust rapidly. The contribution of this paper lies in its exposition of the properties of the dual equilibria--which appear to remain relatively little known--and in the extension to bond financing.

I. The Money-Only Model.

In this section we introduce the basic money-only model, the properties of which have been examined by Evans and Yarrow (1981) and Sargent and Wallace (1987). Output (Y) is at the full employment level, and growing at rate n. The government runs a deficit that is a constant proportion of output, d, and finances it entirely by printing high-powered money. It may do this out of choice or because there are no domestic capital markets and no foreign sources of finance.

The demand for high-powered money (H) is assumed to be of the semi-logarithmic (Cagan) form with unitary income elasticity:[4].

(1) $H/PY = h = \exp(-\alpha\pi^{e})$

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[4]This is an empirically relevant specification: its essential property for this paper is that seigniorage revenue first increases and then decreases with correctly anticipated inflation.

where π^e is the expected rate of inflation and P is the price level.

The financing rule for the budget deficit implies (with a dot for the time derivative):[5]

(2) $\dot{H}/PY = d$

Combining (1) and (2) we obtain:

(3) $d = \dot{H}/H \cdot H/PY = \theta h = \theta \exp(-\alpha\pi^e)$

Here θ is the growth rate of high-powered money.

The budget constraint (3) is shown in Figure 1 as the curve GG--a positive (in this case logarithmic) relationship between the expected rate of inflation (π^e) and the growth rate of the monetary base (θ) showing the rate at which the money supply has to be increased to finance the deficit at each level of π^e. The deficit itself is measured as the intercept of GG on the θ-axis. Note that the economy is always on this schedule.

Differentiating equation (1) with respect to time, we get

(4) $\dot{H}/H - \dot{P}/P - \dot{Y}/Y = \theta - \pi - n = -\alpha\dot{\pi}^e$

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[5]We implicitly assume (2) that the deficit is invariant to the inflation rate, thereby omitting the well-known Olivera-Tanzi effect whereby higher inflation increases the deficit. The basic results are unaffected by the inclusion of this effect.

In steady state:

(5) $\pi^{e} = \pi = \theta - n$

The steady state relationship (5) is shown as the 45° line in Figure 1, with intercept on the horizontal axis equal to n. Two potential steady state equilibria are shown in Figure 1, the low inflation equilibrium at A and the high inflation (B) equilibrium. The dual equilibria are a reflection of the Laffer curve: the same amount of inflation revenue may be obtained at either a low or a high inflation rate.

The maximum steady state seigniorage revenue d* is given by:

(6) $d^* = \max_{\{\theta\}} \theta \exp(-\alpha(\theta-n)) = \alpha^{-1} \exp(\alpha n-1).$

The corresponding inflation rate π^* is:[6]

(7) $\pi^* = 1/\alpha - n.$

Depending on the size of the deficit the government wishes to finance, there may be zero, one, or two equilibria. Because the government cannot obtain more than d* in steady state, there is no steady state if $d > d^*$. For $0 < d < d^*$ there are two steady states. For $d = d^*$, and for $d < 0$, there is a unique steady state.

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[6]Similar results are obtained in Friedman (1971).

The existence of two steady state equilibria in the case $0 < d < d^*$ suggests that an economy may find itself at a higher than necessary inflation rate over long periods. An economy with a given budget deficit may find itself at the high inflation equilibrium B, rather than at the low inflation steadte A. Whether this is likely depends on the stability of the respective equilibrium points. We consider two alternative sources of dynamic behavior: adaptive expectations about the inflation rate; and gradual adjustment of real balances towards their target level.

Adaptive Expectations.

Suppose that expectations about the inflation rate respond only to actual inflation:

$$(8) \qquad \dot{\pi}^e = \beta(\pi - \pi^e)$$

The adaptive expectations assumption makes most sense in conditions in which there are no reliable data on the government budget deficit or money growth, which is not unusual in countries with high inflation rates and poorly developed statistical systems. Alternatively, individuals may form expectations adaptively when the government's policy and data announcements command very little credibility.

Substituting for π in (8) from (4) we obtain

$$(9) \qquad \dot{\pi}^* = (1-\alpha\beta)^{-1}\beta(\theta-n-\pi^*)$$

where $\theta = d\exp(\alpha\pi^*)$ from (3). We examine the implications of equation (9) using Figure 1.

When expectations adjust sufficiently slowly that $\beta < 1/\alpha$ we get $\dot{\pi}^* > 0$ for all points below the 45° line ($\pi^* < \theta - n$) and $\dot{\pi}^* < 0$ for all points above the 45° line. This implies that A is a stable equilibrium and B an unstable equilibrium (see arrows along GG in Figure 1). With the slow adjustment of expectations, the economy will converge to point A from any point to the left of B. The economy moves away from the point B, if it starts in the vicinity of that point.

If the economy ever finds itself to the right of B, it degenerates into hyperinflation. Here the government prints money at an ever-increasing rate, always printing sufficiently faster that expectations never catch up t
o actual inflation. Although the monetary base ultimately becomes very small, the government is printing money so fast that it is able to finance its deficit.

Consider now the effect of an increase in the deficit when the economy is in a steady state at point A. The GG curve shifts to GG′. The increase in the deficit to d′ implies an instantaneous jump in the growth rate of money (and of the actual inflation rate) from A to C, and a gradual further upward movement of θ (and π) from C to A′ as inflationary expectations adjust upwards and the government has to print money more rapidly as the monetary base shrinks.[7]

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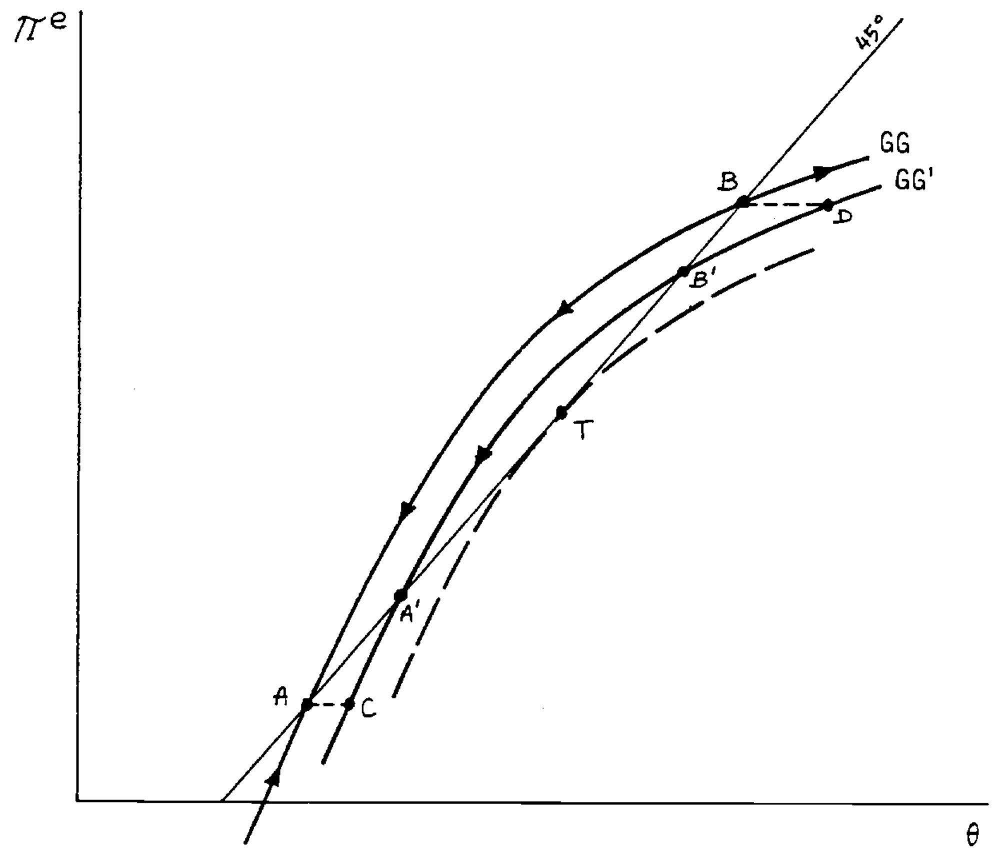
[7]The adaptive expectations assumption is that the expected inflation rate does not jump.

FIGURE 1



Note that as d increases more and more, there comes a stage at which d exceeds d* and the GG curve shifts beyond the point of tangency (T). Now there is no steady state, and inflation continues increasing indefinitely. This is a second case of hyperinflation, differing from that described above (when the economy moves along GG to the right of B) in that there is no steady state equilibrium.

High-Inflation Equilibrium: The stability properties of A and B reverse when the coefficient of adaptation β is sufficiently large that $\beta > 1/\alpha$: the high-inflation point B becomes the stable equilibrium and A is unstable.[8]

This upper stable equilibrium point has unintuitive comparative steady state properties. An increase in the deficit leads to a reduction in the steady state inflation rate. This can be seen in the move from B to B' when the deficit rises, shifting GG to GG'. This unusual result is entirely a result of the Laffer curve property that at B an increase in the steady state inflation rate reduces seigniorage.

The dynamics of the transition from B to B' start with a move from B to D at the time the deficit increases. With the expected rate of inflation and therefore real balances given, the growth rate of money increases from B to D at the moment of the change in the deficit. This increase in money growth is accompanied by a reduction in the inflation rate. The expected inflation rate therefore begins to fall, the demand

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[8]Bruno (1986) explores the possibility of dual stable equilibria. Interpreting β as a measure of the speed of the economy's response to inflation, β is likely to be low when the inflation rate is low and high when inflation is high. That implies that both the low and high inflation equilibria may be stable.

for real balances increases, and the government can print money less rapidly. The economy moves gradually from D to B′.

There is no very good intuitive explanation for the initial fall in the inflation rate. Given that the economy is on the wrong side of the Laffer curve, a decline in the inflation rate is needed to generate more revenue when the deficit rises. That is why the economy has to end at B′. For it to move in that direction from B, given the expectations adjustment equation (9), the inflation rate has to start falling. That is easy to see. Less easy to see though is the process by which the rapid adjustment of expectations ensures stable adjustment towards equilibrium. We return to this issue when we examine adjustment under rational expectations.

Rational Expectations.

In most respects the case of rational expectations, or perfect foresight can be represented as the limiting case when $\beta \to \infty$ or $\pi = \pi^*$ always (divide both sides of (8) by β and let $\beta \to \infty$). The dynamics of actual (and expected) inflation will then be represented by

$$(10) \qquad \dot{\pi} = \alpha^{-1}(\pi + n - \theta)$$

There are now many rational expectations equilibria. With deficit d, all points on GG are rational expectations equilibria. Usually point A would be identified as <u>the</u> rational expectations equilibrium, on the grounds that the inflation rate would explode if the economy started anywhere else. But in this case any path starting to the right of A converges to point B. Point B is thus a stable rational expectations equilibrium. Without a learning process, or some other means of generating dynamics, there is no clear principle by which one equilibrium is to be chosen over another.[9]

The one respect in which the rational expectations equilibrium is not the limiting case of adaptive expectations is that of the initial condition. The adaptive expectations formulation does not allow π^e to jump, whereas under rational expectations it is assumed that the economy can move discretely from one equilibrium to the next. For instance under rational expectations there is no reason the economy could not move from A to A′ at once.[10]

Consider in particular the effects of an increase in the budget deficit when expectations are rational and the economy is at the high inflation equilibrium B. In the adaptive expectations case we knew the economy moved to D because π^e was held constant. But with rational expectations it is not clear where the economy will move. It could possibly move to point D, with in this case the actual and expected

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[9]Marcet and Sargent (1987) use a least squares learning mechanism to form expectations of next period's price level in a model with a money demand function similar to that of Cagan. (See DeCanio (1979) for a particularly clear example of the use of least squares learning). They show that the low inflation equilibrium is stable for a range of initial expected inflation rates; for higher initial expected inflation rates there is no equilibrium with learning, even in conditions where the higher rational expectations equilibrium is stable. The results may be interpeted in light of the property that the least squares predictor of the inflation rate is always less than the maximum inflation rate observed so far. The generality of their results is not clear, for their money demand function unlike Cagan's permits real balances and seigniorage revenue to go negative, properties on which their proofs depend.

[10]We have benefitted from reading a Comment on this and related points by Mario Henrique Simonsen.

inflation rates (which are equal, where they are defined) both increasing initially, along with the growth rate of money. Then the economy moves with lower inflation to B′. We can say in this case that the reduction in inflation takes place because that is what is needed for consistency with portfolio equilibrium. But the economy could as well have moved to some other point on GG′.

The adaptive expectations restriction that π^* be a state variable is reasonable in situations where the deficit and the rate of money printing are not known. But if there is information on the deficit, and if individuals understand the link between inflation and the deficit, the expected rate of inflation might change discretely when the deficit changes.

Comparative Dynamics.

Returning to the adaptive expectations case, we examine the effects of changes in several empirically relevant variables. For purposes of the discussion, we assume the economy is initially in the low inflation equilibrium.

A fall in the exogenous growth rate, n, for instance due to a productivity slowdown, shows in the figure as a leftward shift of the 45° line. This has the same qualitative effect as a rise in d. A fall in the growth rate of output implies that the government has to accelerate the printing of money if it desires to continue extracting the same relative real resources from seigniorage.[11]

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[11]See Melnick and Sokoler (1984) for an analysis along these lines in the Israeli context after 1973.

One interesting aspect of this relationship is that a 1% change in the growth rate of output implies a greater than 1% increase in the inflation rate. The result is that

(11) $d\pi/dn = -(1-\alpha\theta)^{-1}$

Accordingly seemingly small changes in the growth rate of output may be associated with large changes in the inflation rate.

A second type of disturbance is a downward shift in the demand function for base money, for example as a result of the introduction of new financial asset that is a close money substitute.[12] From equation (3) we see that $\theta = d/h$, and thus a fall in h (for given π^*) has the same effect as a rise in d, shifting the GG curve rightward.

Finally, we note that the model can be extended to the case in which output supply is variable. Suppose we have a Lucas-type output supply function:[13]

(12) $Y = Y_o e^{nt}(P/P^*)^{\gamma}$

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[12]Here again the Israeli case of the introduction of dollar linked liquid bank deposits (Patam) after 1977 may serve as a good example. Another, but probably less relevant, historical case is Hungary after World War II (Bomberger and Makinen, 1983).

[13]This can be obtained from an ordinary short-run supply function Y=Y(W/P,K), translated into rates of change, and writing the change in the nominal wage as a function of expected inflation. The supply function can also be extended to include raw material inputs.

where P is the actual and P^* the expected price level. Then, differentiating with respect to time, we obtain

$$(13) \qquad \dot{Y}/Y = n + \gamma(\pi-\pi^*)$$

Substituting (13) into (4), and using (8):

$$(9)' \qquad \dot{\pi}^* = (1+\gamma-\alpha\beta)^{-1}\beta[\theta-n-\pi^*]$$

The fact that output supply is positively affected by a rise in unanticipated inflation $(\pi-\pi^*)$ has a stabilizing effect on the model, enhancing the likelihood that A is a stable equilibrium, with the stability condition now requiring $(1+\gamma-\alpha\beta) > 0$. The stabilizing influence arises from the fact that an increase in the inflation rate calls forth an increase in the demand for money as the level of output rises.

Lagged Adjustment of Real Balances.

An alternative source of dynamics is lagged adjustment of real balances. Specifically, assume that real balances are adjusted according to:

$$(14) \qquad (\dot{H/PY}) = \varphi((H/PY)^* - (H/PY))$$

where $(H/PY)^*$ indicates desired real balances, given by the demand function (1).[14]

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[14]Note that this form of adjustment function implicitly assumes that nominal balances adjust one-for-one with the price level and output, but adjust only partially in response to differences between the desired and actual ratios of real balances to output.

Assuming rational expectations, and imposing the government budget constraint (2), we obtain

(15) $d = \varphi \exp(-\alpha\pi) + (\pi+n-\varphi)h$

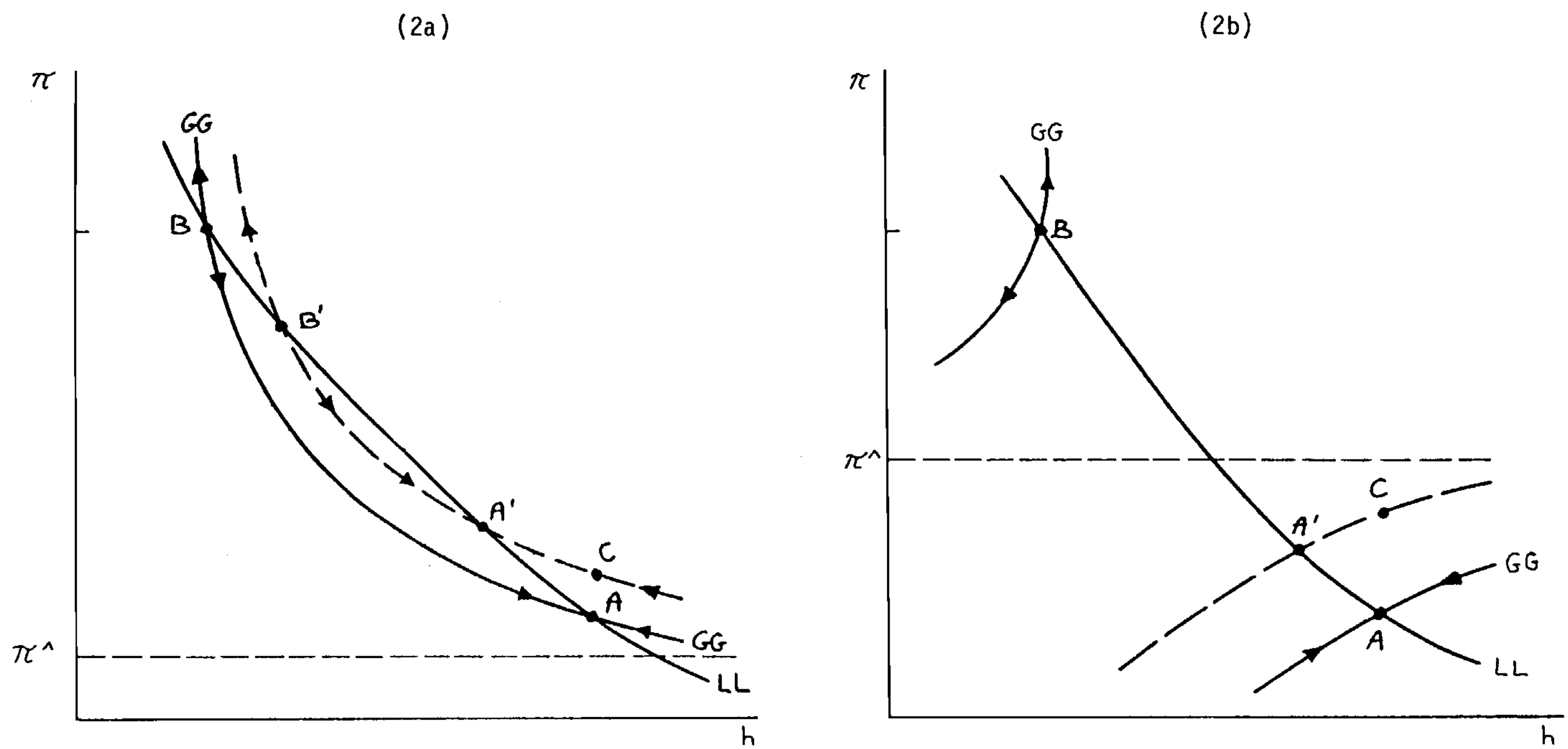
The relationship between the inflation rate and real balances implied by (15) is:

(16) $d\pi/dh = -[h(1-\alpha\varphi)]^{-1}(\pi+n-\varphi)$

The demand for money function (1) and the budget constraint (15) are plotted in Figures 2 and 3, as the LL and GG loci respectively. Several types of intersection are possible, depending on the values of the adjustment parameters. In both figures we assume $d < d^*$ and thus show two possible steady states, at A and B. This implies that the inflation rate at the upper equilibrium (B) exceeds the revenue maximizing rate, $(1/\alpha)-n$.

In Figure 2, we assume that $1 > \alpha\varphi$, so that the denominator of (16) is positive. This is clearly analagous to the assumption $1 > \alpha\beta$ in the adaptive expectations model, with the assumption being that adjustment of real balances is relatively slow. The second inequality is determined by the inflation rate $\pi^\wedge$ at which the numerator of (16) is equal to zero.

FIGURE 2



In Figure 2(a) we assume that $\hat{\pi}$ is below the inflation rate π_A at the low inflation equilibrium. In this case A is a stable equilibrium and B unstable. In Figure 2(b) $\hat{\pi}$ is between the inflation rates at the high and low equilibria.[15] Once again the low inflation equilibrium is stable, and the high inflation equilibrium unstable.

Consider now an increase in the deficit. The low inflation equilibrium moves from A to A'. On impact, with real balances the state variable, the economy moves in each figure to point C. In Figure 2(a) the inflation rate jumps, and then gradually rises to its new higher steady state level, with real balances falling in the process. In Figure 2(b) the increase in the deficit raises the inflation rate initially, but it then falls as the economy moves to the new equilibrium.

Figure 3 shows the adjustment pattern when $1 < \alpha\beta$, and $\hat{\pi} < \pi^B$.[16] The low inflation equilibrium is unstable and the high inflation equilibrium stable. We thus find again that a very high adjustment speed makes the high inflation equilibrium stable--but this time with the difference that expectations are rational.[17]

<u>Summary</u>: Both the comparative steady state and dynamic behavior of the economies examined in this section are sometimes unusual. The

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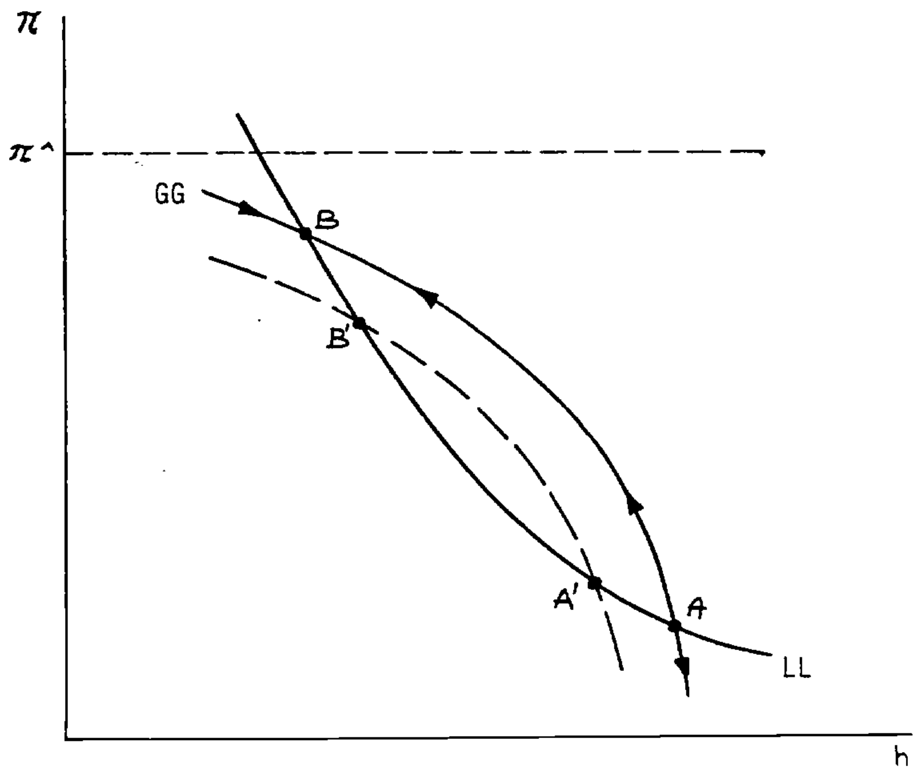
[15]It is not possible, given $1 > \alpha\beta$, for $\hat{\pi}$ to exceed π_B.

[16]We omit the figure for the case $1 < \alpha\beta$ and $\hat{\pi} > \pi_B$. Its stability properties are the same as those in Figure 3, but the adjustment path is not identical.

[17]To avoid cluttering the figures we show mainly local dynamics in Figures 2 and 3. The budget constraint becomes vertical at points on the locus $[h = \alpha\theta\ \exp(-\alpha\pi)]$, which in Figure 2 lies to the left of LL and in Figure 3 to the right of LL.

FIGURE 3



comparative steady state results around the high inflation equilibrium are a result of the economy being on the wrong side of the Laffer curve. The dynamic results typically make sense when adjustment--of either expectations or money balances, or indeed other nominal variables--is slow, and seem counterintuitive when adjustment is fast. This is because intuition relates to goods market behavior: an increase in the growth rate of money increases demand and therefore inflation. But there is always in monetary models another source of dynamics, most clearly seen in the no adjustment lag, rational expectations model: that is the dynamics needed if there is to be portfolio equilibrium. An initial increase in the growth rate of money may reduce the nominal interest rate through the portfolio effect: to validate the implied reduction in expected inflation, the inflation rate has then to fall--as it does around the high inflation equilibrium. This second source of dynamics is typically to blame for apparently unusual adjustment patterns.

II. Bond and Money Financing of Deficits.

Assume now that the government can finance its deficit either by borrowing from the central bank (increasing H, as before) or by the sale of bonds to the public at real interest rate r. We thus assume that the government finances itself through indexed rather than nominal bonds; this assumption is of no consequence when expectations are rational, but may affect the stability analysis when expectations are adaptive. The government budget constraint accordingly becomes

$$\dot{H}/P + \dot{B} - rB = G-T = dY \tag{17}$$

Here G is government purchases, T is taxes, and B is the stock of indexed bonds. We assume that the primary (non-interest) deficit is a constant proportion, d, of output.

Denoting by $b = B/Y$ the ratio of bonds to output, and by $v = V/Y$ the ratio of wealth to income, the wealth constraint is[18]

$$v = b + h \tag{18}$$

The demand function for real balances is assumed to be:

$$h = v.\exp(-\alpha(\pi^e+r)) \tag{19}$$

with the demand function for bonds implied by (18) and (19).

Assuming exogenous output and no investment, goods market equilibrium obtains when

$$Y = c(r)V-c_1T + G \tag{20}$$

where consumption is assumed to be an increasing function of marketable wealth (V), a decreasing function of the interest rate ($c'(r) < 0$), and a decreasing function of taxes.[19]

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[18]We assume that government bonds are regarded as net wealth, and omit the analysis of the Ricardian equivalence case.

[19]The assumption $c'(r) < 0$ is consistent with the results that would be obtained if consumption were explicitly a function of permanent income which, with income exogenous, is a declining function of the real interest rate.

Goods market equilibrium therefore implies the value of wealth

(21) $v = (1+c_1t-g)/c(r) = v(\underset{+}{r},\underset{-}{g},\underset{+}{t})$

Here t and g are ratios of upper case letters to output. Wealth in (21) is determined by the requirement of goods market equilibrium at an exogenous level of output: for instance, because an increase in r reduces demand, wealth has to increase to restore equilibrium.[20] We specialize (21) by assuming[21]

(21)' $v = (1+c_1t-g).r^{\gamma}$

The government budget constraint can be rewritten as[22]

(17)' $\theta h + \dot{b} + nb = d + rb$

In steady state, $\dot{b} = 0$, $\theta = \pi + n$, and $\pi = \pi^e$. The steady state budget constraint is thus

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[20]The simplifying assumption in (21) is that the value of wealth that clears the goods market is independent of the inflation rate. If, for instance, there is a Lucas supply curve, then goods market equilibrium will be affected by both the actual and expected inflation rates and the dynamic analysis that follows would be affected.

[21]The constant elasticity form of c(r) is generally convenient, but does imply that wealth would be zero at a zero real interest rate.

[22]If bonds were nominal instead of indexed, the difference between the actual and expected rates of inflation would appear in the government budget constraint.

(17)" $(\pi+r)h = d + (r-n)v$

The steady state budget constraint is plotted as DD in Figure 4, in (r,π) space. The slope of the steady state locus is

(22) $dr/d\pi = [b+\alpha ih-(d.\gamma/r)]^{-1}\{h(1-\alpha(\pi+r)\}$

Here i is the nominal interest rate. The numerator of (22) is positive up to the inflation rate $\pi^* = (1/\alpha-r)$, and then becomes negative: up to π^*, increases in the steady state inflation rate increase seigniorage. The sign of the denominator depends on the magnitude of γ, the wealth elasticity of consumption demand. If γ is large, an increase in the interest rate may increase wealth by so much that the deficit can be financed with a lower inflation rate.

The sign of the numerator of (22), which depends on the strength of the interest elasticity of saving, is crucial in determining patterns of dynamic adjustment. In Figure 4 we show the steady state budget constraint DD under the more plausible assumption that the numerator is positive, i.e. when the interest elasticity of saving is small, implying that an increase in the interest rate requires increased offsetting inflation revenue. With that assumption, the DD locus in Figure 4 first rises and then beyond π^* has a negative slope. Figure 5 shows the DD locus when γ is large: in that case the DD locus is U-shaped.[23]

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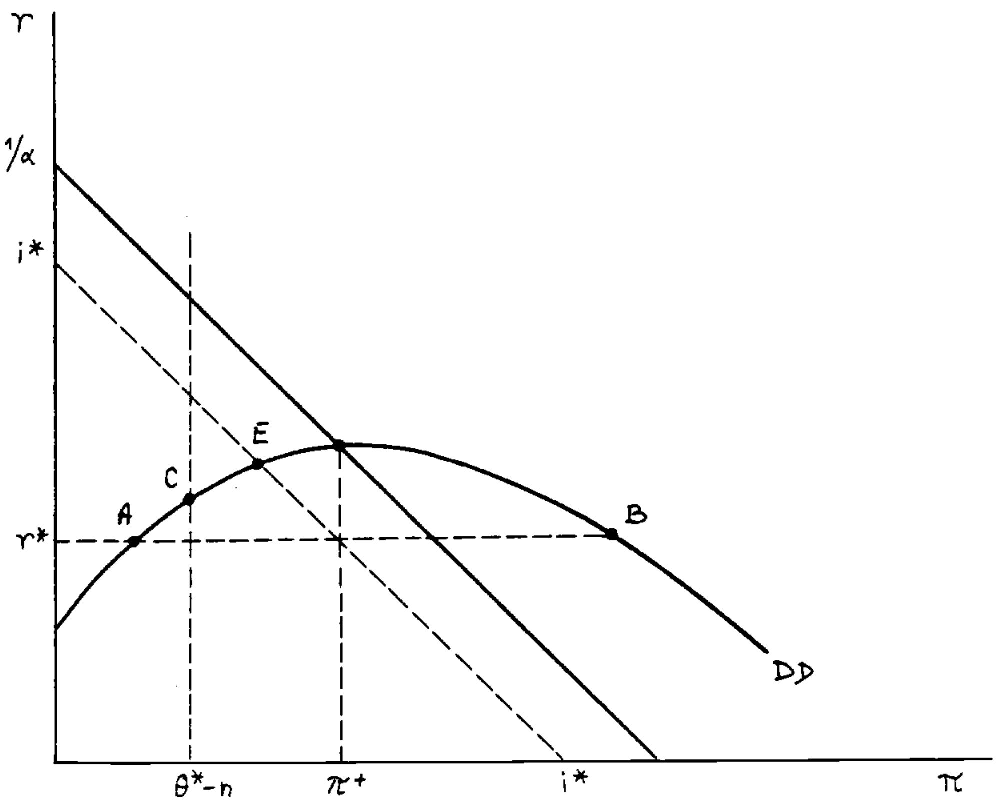
[23]While DD unambiguously has the indicated shape when γ is small, we have not been able to tie down the shape of DD in Figure 5. We assume in the remainder of this section that around equilibria, DD in the case where γ is high, has the shape shown in Figure 5.

FIGURE 4



Steady States and the Nominal Anchor.

Using Figures 4 and 5 we examine the steady state results of three possible deficit financing policies. Suppose first that the government fixes the real interest rate, at r*. This corresponds to a policy in which monetary policy is used to maintain the real interest rate. Operationally, the Treasury sells whatever amount of bonds it can at interest rate r* and leaves residual financing to the central bank. As in Section I, there are dual--low- and high-inflation--equilibria, at A and B.

Alternatively, the central bank can fix the growth rate of money and leave the Treasury to finance the remainder of the deficit by bond sales. Fixing the growth rate at θ^* results in a unique equilibrium at C, with inflation rate equal to θ^*-n.[24] This is a situation in which there is strict control over the nominal amount of credit the central bank provides the Treasury, which has to borrow to finance any extra needs.

Finally, the central bank could keep the nominal interest rate constant.[25] Thus steady states lie along the line with slope of -45° on which $(\pi+r)$ is constant. In Figure 4, there is only one possible equilibrium with a constant nominal interest rate, at point E. That result certainly holds for $\gamma = 0$,[26] i.e. when wealth is constant, and

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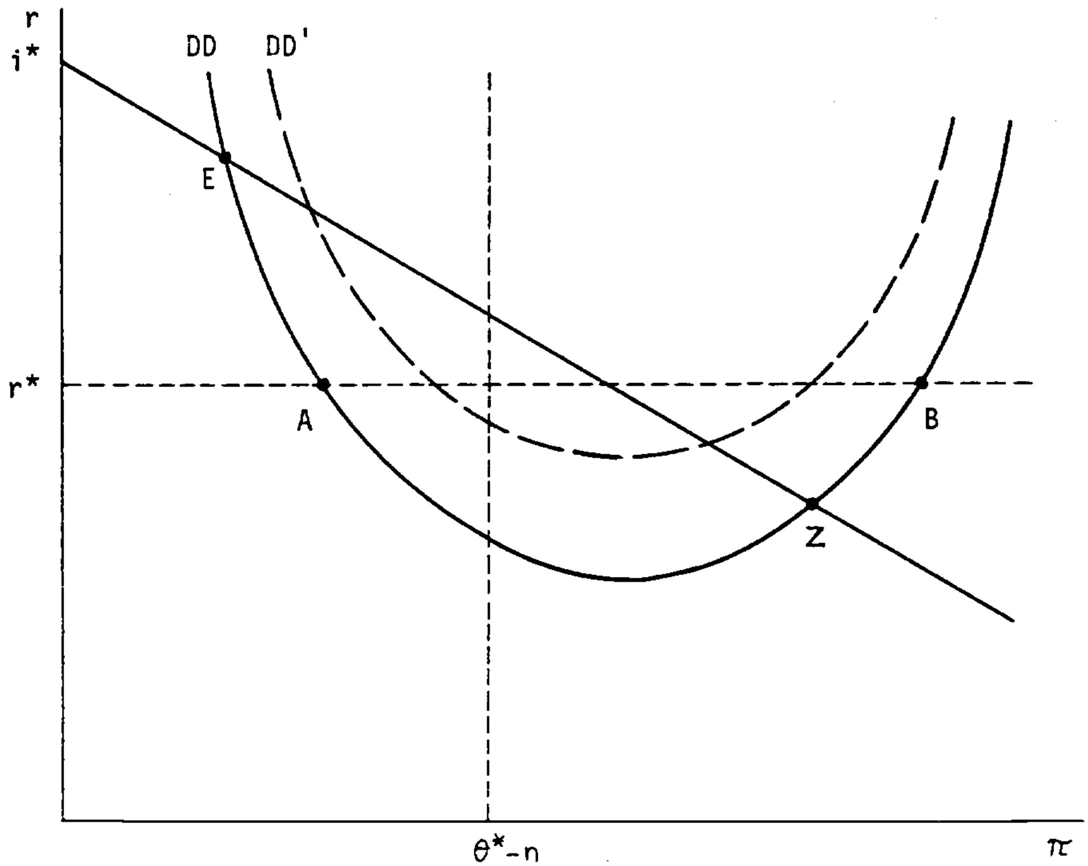
[24]As in the money-only model, the deficit may be too large to be financed at all.

[25]In this model, in which the demand for money is proportional to wealth, that policy requires the ratio h/b to be constant.

[26]With i fixed at i*, the steady state government budget constraint is $d = [i^* \exp(-\alpha i^*)+n-r]v$ which is satisfied at a unique value of r for constant v. The possibility of multiple equilibria even with fixed i arises from the property $v'(r)>0$.

FIGURE 5



continues to hold for small values of $\bar{Y}$. As Figure 5 shows, with a high interest elasticity of saving, there may be two equilibria with a constant nominal interest rate, at E and Z.

Figures 4 and 5 illustrate the importance of a nominal anchor. With a constant real interest rate, the steady state inflation rate may take two values. With a constant growth rate of money, there is only one possible steady state inflation rate. The constant nominal interest rate policy is intermediate, allowing only one equilibrium when $\bar{Y}$ is small but two equilibria otherwise.

Comparative Steady States: Consider now the steady state effects of an increase in the deficit. In the goods market, as of a given interest rate, wealth declines. With lower wealth, and a higher deficit, more inflation revenue is needed to balance the budget at a given interest rate. In Figure 4, the DD curve would shift down (not shown). With a constant rate of money growth and hence inflation, the real interest rate falls. The fall is a result of the requirement of budget financing, but the dynamic adjustment around the equilibrium is not necessarily stable as we see below. At constant real interest rate, the inflation rate rises around A and falls around B. With constant nominal interest rate, the inflation rate rises.

In Figure 5 we show the effects of an increase in the deficit when the interest effect on saving is strong. In that case the DD curve is U-shaped, and an increase in the deficit shifts the U up to DD'. As of a given inflation rate, the real interest rate rises. As of a given real or nominal interest rate, the low inflation rate rises and the high rate falls.

An increase in the nominal interest rate increases the ratio of money to bonds. This raises the low equilibrium inflation rate in Figure 4, but reduces it in Figure 5. Figure 4 thus confirms the now common result that bond financing of deficits is inflationary; the Figure 5 result is an exception to this rule.

There remains the question of the dynamic stability of the economy under the alternative policy choices. Relative to the money only model, the model with bonds includes an extra source of potential instability through the effects of rising interest payments associated with bond finance. It contains a potential stabilizing force in the effects of the interest rate on saving.

Dynamics.

We shall in each case examine dynamic behavior under the assumptions of rational and adaptive expectations. We start with the constant real interest rate rule.

Constant Real Interest Rate: Under rational expectations there can be no dynamics if the real interest rate is pegged. Given the level of wealth--determined by the real interest rate--there are only two inflation rates consistent with the government budget deficit.[27] The reason this case differs from the rational expectations money only model is that there was in that case no offsetting change in bonds when real balances change.

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[27]Under the assumption of constant wealth, the budget constraint becomes $(\pi+r)\exp(-\alpha(\pi+r)) = d + (r-n)v$, which is satisfied by at most two inflation rates.

Under adaptive expectations, dynamics are again quite simple. Figure 6 illustrates. With constant wealth the budget constraint is the same as the steady state constraint (17)" except that π and π^* may differ. Thus

$$(17)''' \quad (\pi+r)\exp(-\alpha\pi^*)v = d + (r-n)v$$

The GG curve in (π,π^*) space thus intersects the 45° line as shown, with at most two equilibria (as is implied also in Figures 4 and 5), with the lower inflation equilibrium stable and the upper equilibrium unstable.

An increase in the deficit reduces wealth (from (21)). Under the assumptions represented in Figure 4 the higher deficit moves the GG curve down, shifting the two equilibria from A to A′ and B to B′ respectively. A policy decision to increase the real interest rate has a similar effect. But recall the assumption that an increase in the interest rate on balance requires an increase in inflation revenue to maintain budget balance. If saving is highly interest elastic, it is possible that an increase in the real interest rate could shift the GG curve up, reducing the equilibrium low inflation rate.[28]

An increase in the deficit results in an immediate increase in the inflation rate as the rate of money growth rises, and then in a continuing rise in both actual and expected inflation.

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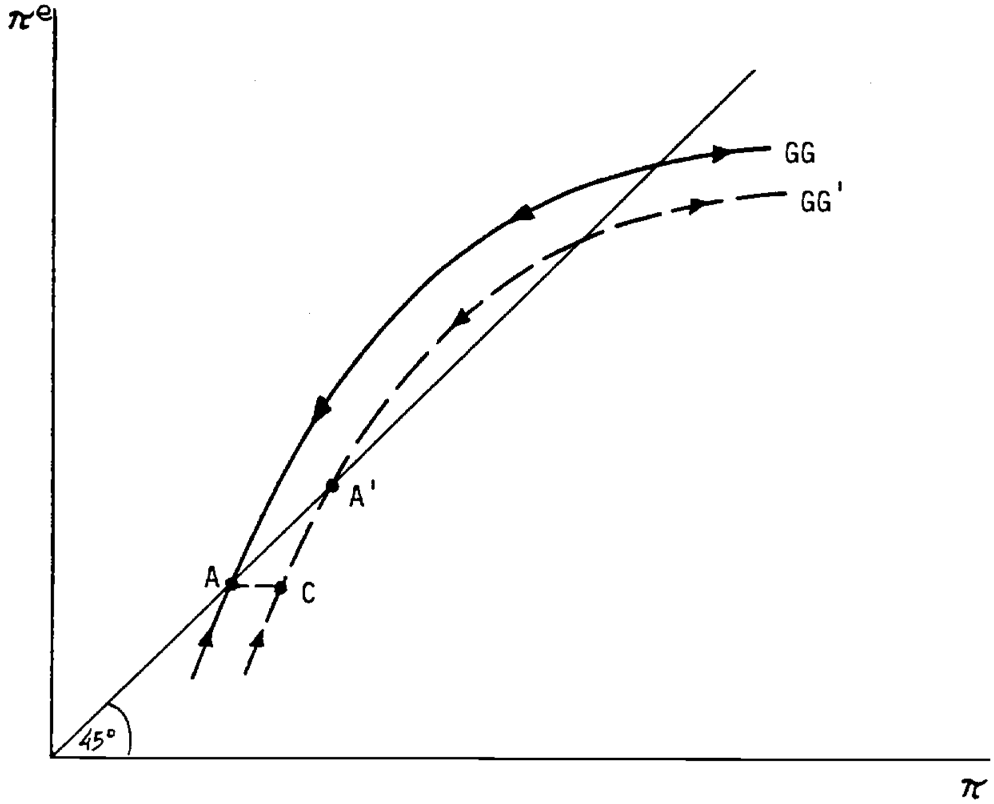
[28]In this case the DD curve is U-shaped as in Figure 5.

FIGURE 6



A downward shift in the demand for money function will in this case result in a higher inflation rate around the low inflation equilibrium. Given wealth, the shift from money is also a shift into bonds, thereby implying an increase in the interest bill faced by the government, and thus relative to the money-only model, a larger increase in the inflation rate.

Constant Money Growth: With constant money growth and rational expectations, dynamics are totally unstable unless interest rate effects on saving and thus wealth are large. We now write the budget constraint in the form

$$(23) \qquad d + (r-n)b = \theta h + \dot{v} - \dot{h}$$

To see the role of interest rate effects on wealth, consider first setting $\gamma = 0$, so that wealth is determined in the goods market and is independent of the interest rate. Then setting $\dot{v} = 0$ in (23)′, and using the definition of h, the budget constraint becomes

$$(24) \qquad d + (r-n)v = (\pi+r)\exp(-\alpha(\pi+r)).v$$

The DD curve retains the shape seen in Figure 4, and all dynamics take place on that curve. The steady state occurs at a given inflation rate, and motion around that steady state is unstable. Thus with rational expectations and no wealth effects the economy either goes immediately to its steady state with constant rate of inflation, determined by the rate of money growth, or fails to reach a steady state.

When interest rate effects on saving are included, $\dot{v}$ in (23) is no longer zero. Instead, with rational expectations, dynamics in the model are described by two equations:

$$(25) \qquad \gamma\dot{r}/r = (d/v) + (r-n) - (\pi+r)\exp(-\alpha(\pi+r))$$

$$(26) \qquad \alpha\dot{\pi} = \pi+n-\theta + ((\gamma/r)-\alpha)\dot{r}$$

Examining local stability conditions, the trace of the characteristic matrix is positive[29]. The characteristic determinant is given by

$$(27) \qquad \text{Det.} = (\alpha\gamma v)^{-1}[rb+\alpha rih-\gamma d]$$

This is negative if the DD curve is U shaped as in Figure 5, in which case there can be a saddle point approach to equilibrium. Otherwise the equilibrium is an unstable focus.

<u>Adaptive Expectations</u>: In the presence of bonds, the stability conditions under adaptive expectations and constant money growth are quite different from those in the money only model. With expectations adjusting according to (4), and from the budget constraint (23) and differentiating the demand for money function, we obtain the two dynamic equations:

$$(28) \qquad \dot{\pi}^* = D^{-1}\,\beta\{\theta-n-\pi^* + (\alpha-(\gamma/r))(r/\gamma v)[d+(r-n)v-h(r+\pi^*)]\}$$

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[29] It is equal to $[(r-n) + (b/\alpha v) + (r/\gamma)]$.

where $D = (1-\alpha\beta) + (\alpha-(\gamma/r))(rh/\gamma v)$

(29) $\dot{r}/r = (\gamma vD)^{-1}\ \{(1-\alpha\beta)[d+(r-n)v-rh] - h[\theta-n-\alpha\beta\pi^{*}]\}$

We present the local stability conditions around the unique steady state. The trace and determinant of the characteristic matrix are

(30) $$\mathrm{Tr.} = \frac{r(b+\alpha ih-\alpha\beta v) - \gamma[d+b\beta+\alpha\beta(r-n)v]}{\alpha rh + \gamma(b-\alpha\beta v)}$$

(31) $$\mathrm{Det.} = \frac{-(\alpha hi+b) + \gamma d/r}{\alpha rh + \gamma(b-\alpha\beta v)}$$

Consider first the case in which $\gamma = 0$, i.e. saving is not interest-responsive and equilibrium wealth is therefore determined by the condition of goods market equilibrium. Then the determinant is unambiguously negative, indicating the roots are of opposite sign, and therefore that the equilibrium is a saddle point. Thus the introduction of adaptive expectations alone does not affect the stability of the system.[30]

For $\alpha\beta$ small, and $r>n$, increases in γ, the interest elasticity of saving, move the economy towards stability. For γ sufficiently large, the trace becomes negative and the determinant positive, thus

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[30]Indeed, for $\gamma=0$, the coefficient β does not appear in the expression for the determinant.

ensuring local stability of the equilibrium. Note that it takes both adaptive expectations and a positive interest elasticity of saving for stability of equilibrium when the growth rate of money is held constant.

The money and bonds model under both constant real interest rate and constant growth rate of money assumptions produces different results from the money only model. Stability is far more problematic, the speed of adjustment of expectations is less significant, and the role of the interest elasticity of saving becomes more central. Dynamics under the constant nominal interest rate policy can be shown to be quite similar to those with a constant real interest rate.

III. Concluding Comments.

The possibility of dual equilibria in inflationary economies raises the possibility that an economy may find itself stuck at a high inflation equilibrium when, with the same fiscal policy, a low inflation equilibrium is attainable. Such dual equilibria are a result of the failure to adopt a nominal anchor for the economy, and thus can be prevented by a change in policy operating rules.

In the simplest money only model, the high inflation equilibrium is stable under rational expectations, despite its unattractive comparative steady state properties. This stability carries over when expectations are adaptive but adjust very fast, or when real balances are adjusted with a short lag. It is possible that these high inflation traps disappear for more detailed specifications of the information structure, but that remains to be seen.

When the asset menu is expanded to include real bonds, the nature of the equilibria and dynamic properties of the model are seen to depend strongly on the government's policy choices. If the real interest rate is held fixed, dual equilibria remain, and dynamic adjustment is stable under slow adaptive expectations around the low inflation equilibrium. The similarity with the results of the money only model extend to the stability of the upper equilibrium under rational expectations and rapidly adaptive expectations.

The dual equilibrium problem can be avoided by a policy that fixes the growth rate of money. In that case, stability of the equilibrium becomes more problematic than it is with alternative policies, and depends on the interest elasticity of saving as well as the speed of adjustment of expectations.

The model can also be extended to the open economy. Assume that the sole sources of budget deficit finance are money printing and foreign borrowing. The fundamental dual equilibrium result remains if the government attempts to fix the real exchange rate, and disappears if the growth rate of money or nominal rate of depreciation are held fixed by monetary policy. In such cases, the rational expectations equilibrium is saddle point stable, while the inflationary process is stable under a fixed nominal rate of exchange depreciation with slow adaptive expectations. Similar results obtain when as a matter of policy the nominal exchange rate is adjusted adaptively to the inflation rate.

The results of this paper reinforce the view that avoidance of unbalanced budgets can play a major role in maintaining macroeconomic stability.

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