Seismic focusing by a single planar fracture

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[1] A single plane fracture with an axially symmetric stress distribution behaves as a seismic lens that focuses seismic energy to a beam "waist" at a focal plane. Both phase and amplitude effects on a seismic wave propagating across the fracture contribute to the lensing behavior. Radial gradients in the fracture specific stiffness cause wave refraction through a radially varying group time delay, while the fracture transmission amplitude approximates a Fresnel zone plate. This work demonstrates that a two-dimensional planar fracture, contrasted with three-dimensional geologic structures such as basins and domes, can focus seismic waves. Focusing of seismic waves by fractures should be considered in the interpretation of seismic data from fractured strata with heterogeneous stress distributions. *INDEX TERMS:* 5144 Physical Properties of Rocks: Wave attenuation; 5102 Physical Properties of Rocks: Acoustic properties; 5104 Physical Properties of Rocks: Fracture and flow; 5194 Physical Properties of Rocks: Instruments and techniques; 5199 Physical Properties of Rocks: General or miscellaneous. **Citation:** Oliger, A., D. D. Nolte, and L. J. Pyrak-Nolte, Seismic focusing by a single planar fracture, *Geophys. Res. Lett.*, *30*(5), 1203, doi:10.1029/2002GL016264, 2003.

1. Introduction

[2] Geologic structures such as basins and salt domes can form seismic lenses [*Gao et al.*, 1996; *Muerdter and Ratcliff*, 2001] that affect the arrival times and amplitudes of seismic waves that propagate through the structures. For example, *Gao et al.* [1996] suggested that damage from the Northridge earthquake was enhanced by focusing of seismic waves from a deep convex structure that is bounded by a fault. The hypothesized convex lens structure produced compressional and shear wave amplification two to three times higher in the anomalously high damaged zones than the signals from the areas surrounding these regions.

[3] In this paper, we present experimental evidence and theoretical analysis that demonstrates that a single planar fracture subjected to a non-uniform stress distribution can focus seismic waves. The non-uniform stress distribution causes the fracture specific stiffness to vary as a function of radius along the fracture plane which in turn affects seismic wave propagation across the fracture.

2. Fracture Specific Stiffness

[4] Fracture specific stiffness is defined as the ratio of an increment of stress to the resulting increment of displacement caused by the deformation of the void space in the fracture. Elasticity methods and experimental data both describe how fracture specific stiffness depends on the elastic properties of the rock, how it depends critically on the amount and distribution of contact area in a fracture that arises from two rough surfaces in contact, and how stiffness increases with increasing stress [*Greenwood and Williamson*, 1966; *Gangi*, 1978; *Brown and Scholz*, 1985, 1986; *Hopkins*, 1990].

[5] Several investigators [*Mindlin*, 1960; *Kendall and Tabor*, 1971; *Schoenberg*, 1980; *Kitsunezaki*, 1983; *Pyrak-Nolte and Cook*, 1987; *Pyrak-Nolte et al.*, 1990a, 1990b; *Gu*, 1994; *Nakagawa*, 1998] have examined the role of fracture specific stiffness on wave propagation across and along a fracture. In these investigations, the fracture is modeled as a non-welded contact which is assumed to have negligible thickness compared to the seismic wavelength. The non-welded contact is represented by a set of boundary conditions between two elastic halfspaces. The boundary conditions that describe the non-welded contact are: (1) The normal and shear stresses across the non-welded contact are assumed to be continuous ($\sigma_1 = \sigma_2$); and (2) the displacements are discontinuous by an amount inversely proportional to the specific stiffness of the fracture ($u_2 - u_1 = \sigma/\kappa$), where σ is stress, u is particle displacement, κ is the specific stiffness of the fracture [Pa/m], and subscripts 1 and 2 refer to the elastic media on either side of the fracture. A fracture has both normal and shear components of specific stiffness. From this purely elastic model, the transmission and reflection coefficients, as well as the group time delay, are frequency dependent and depend on the stiffness of the fracture.

[6] Heterogeneity in the fracture specific stiffness has previously been treated as slowly varying relative to the seismic wavelength, enabling the use of simple geometric ray approximations in diffraction theory [*Nihei*, 1989; *Pyrak-Nolte and Nolte*, 1992]. Two phenomena emerge when treating heterogeneity with diffraction theory. First, gradients of the fracture specific stiffness lead to gradients

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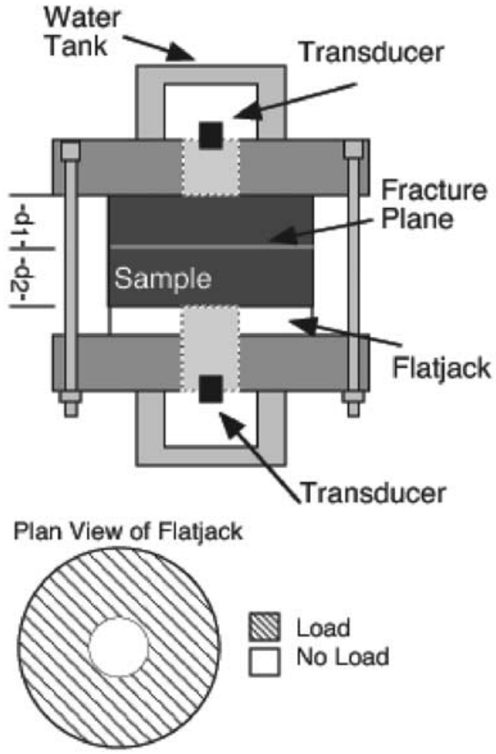

Figure 1. A sketch of the wavefront imaging apparatus and plan view of annular load distribution.

in the group time delay for waves propagating across the fracture. This causes the wave to refract at the fracture. One interesting outcome of this effect is an apparent violation of Snell's Law (but not Fermat's Principle) in which a wave incident normally on the fracture refracts into a finite angle after transmission. Second, varying transmission amplitudes cause wave diffraction.

[7] In this letter we show that these two effects can combine, in the case when stress on a fracture has axial symmetry (with a minimum stress on-axis) to form a seismic lens that concentrates seismic energy along the symmetry axis of the stress field at a distance equal to the focal length of the lens. We demonstrate seismic focussing by a single artificial fracture experimentally in the laboratory using a wavefront imaging technique, and verify the trends with Fresnel diffraction calculations.

3. Experimental Wavefront Imaging

[8] Figure 1 contains a schematic of the experimental set-up. The loading frame consists of two cylindrical stainless steel end caps with a diameter of 0.2794 m and a thickness of 0.0508 m. A central hole (0.0759 m in diameter) in each end-cap enables access to the surface of the sample for seismic imaging and aids in the creation of a non-uniform stress field in the sample. Pressure is applied to the sample by an annular ring brass flat-jack (0.1524 m in diameter by 0.025 m in height) which is attached to a hydraulic pump. In the center of the flat-jack is a hole measuring 0.0710 m in diameter. The annular loading results in the application of a load over an annulus with inner and out diameters of 0.0710 m and 0.1524 m, respectively. The pressure is monitored with an electronic pressure transducer. After placing a sample in the load frame, the end caps are fastened together with eight bolts. Cylindrically-shaped nylon water tanks that are open along the top are attached to the end caps. The tanks hold the water used as the coupling medium between the sample and the acoustic transducers.

[9] Spherically-focused 1 MHz water-coupled piezo-electric transducers and plane-wave 1 MHz water-coupled piezo-electric transducers were used to send and receive compressional waves that are propagated through an aluminum sample containing a single plane synthetic fracture. A wavefront imaging method [*Pyrak-Nolte et al.*, 1999; *Roy and Pyrak-Nolte*, 1997; *Xian et al.*, 2001] was used to investigate the effect of the non-uniform axially-symmetric stress distribution. The source and receiver were focused on the surface of the sample. The receiving transducer was translated over a square region of 0.025 m by 0.025 m centered on the axis of symmetry. The distance between the face of the transducer and the face of the sample was 0.0153 m (the focal length of the transducer) for the spherically focused transducers.

[10] The synthetic fractures were created by placing two aluminum right cylinders in contact. The fracture was oriented perpendicular to the axis of the cylinders. All of the samples had a diameter of 0.1524 m. The total lengths of the samples are given in Table 1, along with the distances along the axis of the sample from the source plane (surface of the sample) to the fracture, d_1, (see Figure 1) and from the fracture to the receiver plane (surface of the sample), d_2, for the fracture samples. Non-uniform stress fields were achieved by the annular loading configuration, and the magnitude of the variation of stress along the fracture plane was altered by using samples of different lengths. In the imaging region (0.025 m $\times$ 0.025 m), the radial stress distribution on the fracture plane caused by the annular load produces a minimum in stress in the center of the fracture plane which increases with distance from the center.

[11] Figures 2a–2d shows the amplitudes of the received seismic waves (represented by color scale) as a function of time (in microseconds) and position (horizontal axis in millimeters) at 7 MPa for (a) an intact sample and (b–d) the fractured samples. The figure shows that the energy is concentrated along the sample axis compared with the intact sample (Figure 2a), that shows uniform energy distributed over the receiving face. The energy in each of the fractured samples arrives later at the center of the first-arrival wavefront, indicating that the receiver face rests in front of the focal plane. The exception is for the case where d_2 equals 114 mm (Figure 2d) when the first arriving energy arrives as a plane wave over the diameter of the sample, indicating that the receiver face rests at the focal plane of the seismic

Table 1. Total Sample Length and the Distances Along the Axis of the Sample From the Source Plane to the Fracture, d_1, and From the Fracture to the Receiver Plane, d_2

Sample	Total Length (mm)	d_1 (mm)	d_2 (mm)
S76	76	–	–
S3838	76	38	38
S3876	114	38	76
S114114	228	114	114

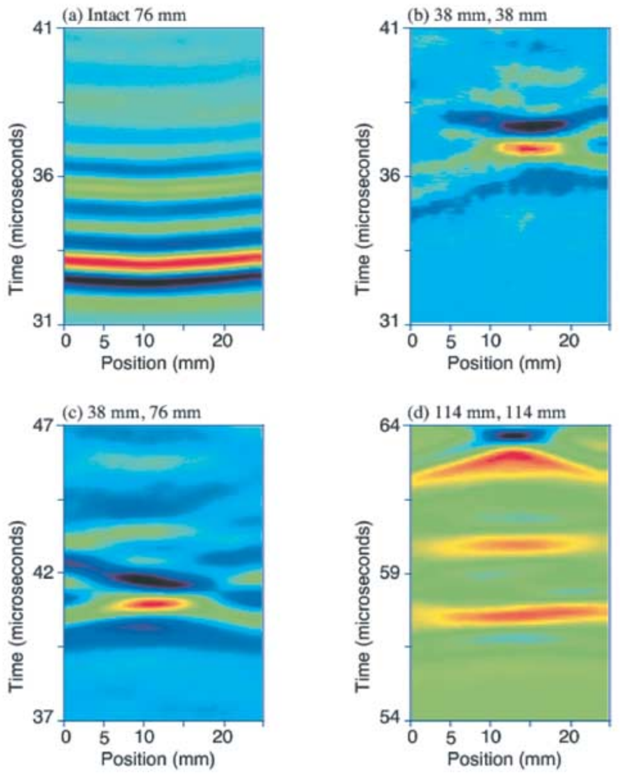

Figure 2. Experimental results showing the amplitude (represented by color scale; red-high, blue-low) as a function of time (in microseconds) and position (horizontal axis in millimeters) at 7 MPa. The numbers at the top of each image represent d_1 and d_2. d_1 represents the distance from the plane of the source to the plane of the fracture, and d_2 represents the distance from the fracture to the plane of the receiver.

lens. Hence, we observe that the fracture in this case acts as a lens with a focal length of approximately 114 mm.

4. Fresnel Diffraction Analysis

[12] A seismic plane wave with a Gaussian intensity profile incident at a fracture plane has the transmitted complex amplitude (in the geometric ray approximation)

$$f(r) = \frac{\exp(-r^2/\mathrm{w}_0^2)}{1 + i\frac{\omega Z}{2\kappa(r)}} \tag{1}$$

where w_0 is the beam waist at the fracture plane, ω is the frequency of the wave, Z is the seismic impedance of the medium (equal on both sides of the fracture), and $\kappa(r)$ is the radially varying fracture specific stiffness. The transmitted amplitude propagating to the receiver plane is calculated using Fourier analysis by convolving f(r) with the impulse-response function of the free-space propagation transfer function t_2(x, y) [*Saleh and Teich*, 1991].

[13] In our numerical simulations, we assume a specific stiffness that is minimum at the center of the fracture and increases quadratically as a function of radius from the center. The quadratic dependence is motivated by the non-uniform stress distribution in the radial direction and is meant primarily to illustrate seismic focusing. It is not intended to model the data in detail. We investigated the effects of phase (refraction) and amplitude (Fresnel zone plate) separately and then in combination to assess how each effect contributes to the seismic lensing. For illustrative purposes, the fracture specific stiffness at the center was chosen to be 1×10^{13} [Pa/m] and to increase quadratically as a function of radius with a coefficient 5×10^{13} [Pa/m^3].

[14] Figure 3a shows the diffraction results for an intact sample. The distance along the z axis is in centimeters. Figure 3b shows the effect of the amplitude-only obtained by using the absolute value of the fracture transfer function in equation (1). The fracture plane itself is not shown because the Fesnel approximation is not valid closer than approximately 2 wavelengths to the fracture plane. The low stress at the center of the fracture suppresses the wave amplitude for short distances after the fracture, but the wave diffraction around the central transmission minimum produces a constructive wave maximum along the central axis at a distance 25 cm from the fracture plane.

[15] Figure 3c shows the effect of the phase-only by choosing unity amplitude in the transfer function but retaining the phase. The low stress at the center produces the greatest group time delay, with a decreasing delay with increasing radius from the central axis. This radial variation in the group delay produces a converging wavefront that produces a beam waist at a focal distance of 10 cm from the fracture plane. Figure 3d shows the combined effect of both amplitude and phase. The focal distance in this case is approximately 15 cm from the fracture plane.

[16] Further numerical studies have investigated the dependence of the focal length of the seismic lens on the average value of the stiffness in addition to the second derivative of the stiffness with respect to radius. The strongest focusing occurs under the condition $\omega Z/\bar{\kappa} \approx 1$ where $\bar{\kappa}$ is the average fracture specific stiffness. The group time delay for this condition is most sensitive to radial variation in stiffness. It is important to point out that fracture specific stiffness may vary laterally by an order-of-magni-

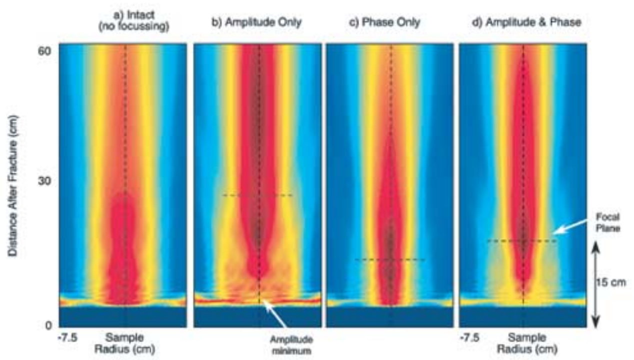

Figure 3. Theoretical diffraction results for 0.5 MHz showing (a) an intact sample, (b) the effect of amplitude only, (c) the effect of the phase only, and (d) the complete effect of both amplitude and phase. The wavelength is 1 cm and the fracture specific stiffness at the center is 1×10^{13} [Pa/m].

tude over one or several wavelengths caused by geologic overburden or stresses in the field, or induced by localized stresses in laboratory investigations [*Hildyard and Young*, 2002]. This condition of strong heterogeneity may require an approach that goes beyond Fresnel diffraction theory to explain detailed behavior.

5. Conclusions

[17] In conclusion, we have experimentally and numerically demonstrated for the first time that a single planar fracture under axially symmetric stress can focus seismic energy at a focal distance along the central axis. This may have important consequences for the interpretation of seismic data from fractured strata with heterogeneous stress distributions.

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References

Brown, S. R., and C. H. Scholz, Closure of random surfaces in contact, *J. Geophys. Res.*, *90*, 5531, 1985.
Brown, S. R., and C. H. Scholz, Closure of rock joints, *J. Geophys. Res.*, *91*, 4939, 1986.
Gangi, A. F., Variation of whole- and fractured-porous-rock permeability with confining pressure. Int, *J. Rock Mech. Min. Sci. Geomech. Abstr.*, *15*, 249, 1978.
Gao, S., H. Liu, P. M. Davis, and L. Knopoff, Localized amplification of seismic waves and correlation with damage due to the Northridge earthquake: Evidence for focusing in Santa Monica, *Bull. Seismol. Soc. Am.*, *86*, S209, 1996.
Greenwood, J. A., and J. B. P. Williamson, Contact of nominally flat surfaces, *Proc. R. Soc. London, Ser.*, 300–319, 1966.
Gu, B., Interface waves on a fracture in rock, Ph.D. thesis, Univ. of Calif., Berkeley, 1994.
Hildyard, M. W., and R. P. Young, Modelling seismic waves around underground openings in fractured rock, *Pure Appl. Geophys.*, *159*, 247, 2002.
Hopkins, D. L., The effect of surface roughness on joint stiffness, aperture, and acoustic wave propagation, Ph.D. thesis, Univ. of Calif., Berkeley, 1990.
Kendall, K., and D. Tabor, An ultrasonic study of the area of contact between stationary and sliding surfaces, *Proc. R. Soc. London, Ser. A.*, *323*, 321, 1971.
Kitsunezaki, C., Behavior of plane waves across a plane crack, *J. Min. Coll. Akita Univ., Ser. A*, *3*, 173, 1983.
Mindlin, R. D., Waves and vibrations in isotropic planes, in *Structural Mechanics*, edited by J. W. Goodier and W. J. Hoff, Pergammon, New York, 1960.
Muerdter, D., and D. Ratcliff, Understanding subsalt illumination through ray-trave modeling, part 3, Salt ridges and furrows, and the impact of acquisition orientation, *Leading Edge*, *20*, 803, 2001.
Nakagawa, S., Acoustic resonance characteristics of rock and concrete containing fractures, Ph.D. thesis, Univ. of California, Berkeley, 1998.
Nihei, K. T., Modeling elastic waves in fractured fock with the Kirchhoff method, M. S. thesis, Univ. of Calif., Berkeley, 1989.
Pyrak-Nolte, L. J., and N. G. W. Cook, Elastic interface waves along a fracture, *Geophys. Res. Lett.*, *11*, 1107, 1987.
Pyrak-Nolte, L. J., and D. D. Nolte, Frequency dependence of fracture stiffness, *Geophys. Res. Lett*, *3*, 325, 1992.
Pyrak-Nolte, L. J., L. R. Myer, and N. G. W. Cook, Transmission of seismic waves across single natural fractures, *J. Geophys. Res.*, *95*, 8617, 1990a.
Pyrak-Nolte, L. J., L. R. Myer, and N. G. W. Cook, Anisotropy in seismic velocities and amplitudes from multiple parallel fractures, *J. Geophys. Res.*, *95*, 11,345, 1990b.
Pyrak-Nolte, L. J., B. L. Mullenbach, X. Li, D. D. Nolte, and A. S. Grader, Detecting sub-wavelength layers and interfaces in synthetic sediments using seismic wave transmission, *Geophys. Res. Lett.*, *26*, 127, 1999.
Roy, S., and L. J. Pyrak-Nolte, Observation of a distinct compressional-mode interface wave on a single fracture, *Geophys. Res. Lett.*, *24*, 173, 1997.
Saleh, B. E. A., and M. C. Teich, *Fundamentals of Photonics*, 120 pp., John Wiley, New York, 1991.
Schoenberg, M., Elastic wave behavior across linear slip interfaces, *J. Acoust. Soc. Am.*, *5*, 1516, 1980.
Xian, C. J., L. J. Pyrak-Nolte, and D. D. Nolte, Compressional waves guided between parallel fractures, Int, *J. Rock Mech. Min. Sci. Geomech. Abstr.*, *38*, 765, 2001.

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