Seismic response of structures on embedded foundations

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Abstract

The potential importance of soil structure interaction effects on the seismic response of structures has been long recognized. The nature of the interaction phenomena, including the modification of the seismic waves by the foundation's geometry (kinematic interaction) and the increase in the structure's effective flexibility due to the foundation and the surrounding soil (inertial interaction) have also been detailed in a number of papers. The possibility of having a reduction of the seismic motions for embedded foundations has been, however, a subject of continued controversy. The general topic of soil structure interaction for structures with embedded foundations is revisited in this paper, reviewing the basic concepts with some emphasis on approximate solutions which allow to develop a better feeling for the behavior of the solution.

1 Introduction

The interaction between a structure and its underlying soil under seismic excitation has been the subject of considerable interest and controversy for over 25 years. The effect is particularly important when dealing with massive and stiff structures, such as nuclear power plants, supported on relatively soft soils. It is not surprising, therefore, that most of the research conducted on this topic was in fact related to the seismic design of nuclear power plants. Two general approaches were developed for seismic soil structure interaction analyses: a direct solution in which the complete soil-structure system is modeled and solved in a single step, and a three step or substructure approach. In the direct solution it is necessary to determine first a motion at the free surface of the soil or at any other location (for example a hypothetical rock outcrop). This motion is normally determined using a deconvolution process. In the substructure approach it is necessary to: 1) determine the seismic motion of the foundation without any structure, and in most cases without any mass; 2)

determine the dynamic stiffnesses of the foundation as functions of frequency for a steady state harmonic excitation; 3) perform the dynamic analysis of the structure supported on the dynamic stiffness matrix computed in the second step and subjected to the motions obtained in the first. Step 1 is often referred to as the kinematic interaction analysis. It represents the change in the characteristics of the base motion due to the geometry of the foundation. The modification of the dynamic characteristics of the structure due to the flexibility of the foundation (the addition of the foundation's stiffness matrix) is referred to as the inertial interaction. Once the dynamic analysis of the combined structure-foundation system is performed the motion at the base of the structure will be different from that used as input. The modification is due to the additional deformations of the soil caused by the base shears, axial forces and overturning moments resulting from the inertia forces in the structure.

When using the direct approach kinematic and inertial interaction effects are automatically combined and it is not possible to isolate their individual contributions to the final results. This combination is necessary when trying to account for nonlinear effects (nonlinear material behavior of the soil or nonlinear contact between the foundation and the soil). It is possible in the substructure approach to bypass the computation of the foundation motions without structure (step 1) computing instead the motions at the interface of the foundation in the free field (without any excavation for an embedded foundation) and a second dynamic stiffness matrix relating forces and displacements at selected points along that interface (also without excavation). It is, however, much more meaningful to carry out step 1 as outlined above, computing the foundation motions and inspecting them for correctness.

In this paper, kinematic and inertial interaction effects are considered separately in order to assess their relative importance for structures on embedded foundations. This implies a linear solution with the superposition of the different effects. Nonlinear soil behavior must be simulated through the use of equivalent material properties consistent with the expected level of strains. There are now a number of computer programs available for linear dynamic soil structure interaction analyses with any desired degree of accuracy. The use of approximate solutions for the actual design and final analyses of important structures is therefore very hard to justify. Approximate methods are, however, of value in preliminary designs, to assess the importance of interaction effects and thus the need for more sophisticated or rigorous analyses, and for parametric studies to better understand the behavior of the solution. They will be used for this last purpose in this paper.

2 Formulation

Let us consider a homogeneous or layered half space without any excavation and the same soil profile once the excavation has taken place, as shown in Fig. 1. Assume that the surface of the excavation (bottom surface and sidewalls) is discretized using either boundary elements, finite elements, or any other discretization scheme, and let the subscript i refer to a generic node along the surface. Assume further that the same surface, discretized mesh and nodes are drawn on the free field (half space without excavation). Let then, $u_0(x, y, z)$ represent the vector of displacements (u, v, w) at any point (x, y, z)along the future excavation's surface in the free field, due to seismic waves propagating at any angle of incidence from the bottom of the soil deposit

 V_{i0} be the vector of corresponding displacements at node *i*

 $t_0(x, y, z)$ represent the vector of tractions acting on the surface of the future excavation at any point in the free field, due to the same train of waves

 F_{i0} be the vector of equivalent nodal forces at node i

Let finally V_0 and F_0 denote the vectors of displacements and equivalent forces at all the nodes *i* along the surface of the future excavation in the free field. V_0 and F_0 would be obtained from standard analyses of wave propagation in layered media.

If V_f are the corresponding displacements along the nodes *i* of the excavation, once this has been performed and S_f is the dynamic stiffness matrix of the soil for the excavation (relating forces and displacements at the various nodes *i*), V_f are the results of the kinematic interaction analysis for an infinitely flexible foundation. Expressing the condition that the tractions along the free surface of the excavation, and thus the resulting nodal forces F_f , must be zero

$$F_f = F_0 + S_f \left(V_f - V_0 \right) = 0 \tag{1}$$

leading to

$$V_f = V_0 - S_f^{-1} F_0 (2)$$

For an infinitely rigid (but massless) foundation, there could be nonzero tractions and forces F_f between the foundation and the excavation surface but their resultants should vanish. Letting c denote the point of reference for the motions (three translations and three rotations in the general case) and forces (three force components and three moments) acting on the foundation one can define a rigid body transformation matrix relating the displacements of any node *i* to those of point c.

$$L_{i} = \begin{bmatrix} 1 & 0 & 0 & 0 & (z_{i} - z_{c}) & -(y_{i} - y_{c}) \\ 0 & 1 & 0 & -(z_{i} - z_{c}) & 0 & (x_{i} - x_{c}) \\ 0 & 0 & 1 & (y_{i} - y_{c}) & -(x_{i} - x_{c}) & 0 \end{bmatrix}$$
(3)

Then,

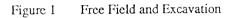
$$V_i = L_i V_c \tag{4}$$

and for the complete set of nodal displacements,

$$V = LV_c \tag{5}$$

where L is a $3n \times 6$ matrix if n is the number of nodes. L is the result of assembling the L_i matrices.





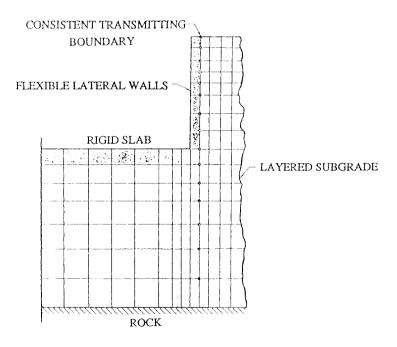


Figure 2 Finite Element Model

$$F_c = L^T F \tag{6}$$

where F results from the assembly of the nodal forces.

The condition is now $F_c = 0$ with

$$F = S_f (V - V_f) = F_0 + S_f (V - V_0)$$
⁽⁷⁾

Then,

$$L^{T}F = L^{T}F_{0} + L^{T}S_{f}(LV_{c} - V_{0}) = 0$$
(8)

or,

$$\left(L^{T}S_{f}L\right)V_{c} = L^{T}S_{f}V_{0} - L^{T}F_{0}$$
⁽⁹⁾

and calling

$$K_f = L^T S_f L \tag{10}$$

the dynamic stiffness matrix of the rigid (massless) foundation,

$$V_c = K_f^{-1} L^T S_f V_0 - K_f^{-1} L^T F_0$$
(11)

indicating that one can compute the motions of a rigid embedded foundation knowing the displacements V_0 and forces F_0 from free field wave propagation analyses and the dynamic stiffness matrix of the excavation S_f . The dynamic stiffness matrix of the rigid foundation K_f can be obtained from eq. (10).

In the case of a flexible foundation, one can either incorporate the model of the foundation with that of the structure letting the nodes *i* represent the contact points between the foundation and the soil, or consider the foundation by itself as a structure supported on the dynamic stiffness matrix S_f and subjected to the motions V_f . This latter alternative represents an intermediate step which offers no computational advantages and therefore the former is generally preferred.

An approximation to the above procedure to compute the consistent motions of a rigid embedded foundation was suggested by Iguchi [1] as

$$V_c = B^{-1} \int L^T u_0 - K_f^{-1} \int L^T t_0$$
(12)

where the integrals extend over the complete surface of the foundation-soil interface, u_0 and t_0 are the free field displacements and tractions as defined earlier, K_f is the dynamic stiffness matrix of the rigid foundation and the rigid body transformation matrix L is as L_i in eq. (3) replacing the coordinates of node i, $x_i y_i z_i$ by the coordinates of a generic point x y z. The superscript T denotes the transposed matrix. The main advantage of expression (12) is that it does not require the knowledge of the excavation stiffness matrix S_f . It can be seen that the second term representing the correction due to the condition of zero forces is essentially the same as in expression (11). The main difference is in the first term (averaging of the motions due to the rigid foundation) where the expression $K_f^{-1}L^TS_f$ has been replaced by $B^{-1}L^T$ where

$B = \int L^T L$

(13)

An alternative to the use of equations (11) and (12) is the direct solution of the soil structure interaction problem for a rigid or flexible foundation, with or without mass, using any type of discretization. Figure 2 illustrates for example a possible finite element model for a circular foundation using finite elements under the base and sidewalls of the foundation and the consistent boundary matrix of Waas [2] and Kausel [3] to reproduce the far field. One could have used equally a hyperelement under the foundation as described by Tassoulas [4]. The base of the model (at any desired depth) can be considered fixed with the motion at the level in the free field specified as input or can be modeled with viscous dashpots to simulate a half space under the action of a plane train of waves with a specified angle of incidence. This viscous boundary can be selected to reproduce correctly the radiation of the waves in the freefield but will not absorb exactly the scattered (or diffracted) waves due to the geometry of the excavation. It must be placed therefore at a sufficient depth (depending on the amount of internal soil damping) to minimize the effects of possible reflections. When using this bottom boundary the input motions should be those expected at a hypothetical outcropping of the half space below the model.

3 Kinematic Interaction

Using this model, with a rigid base, Morray [5] conducted a number of parametric studies determining the motions (horizontal translation in the xdirection and rotation around the y axis) of embedded circular foundations subjected to trains of vertically propagating shear waves in the x - z plane (with the z axis vertical). He considered ratios of the total layer thickness H(depth from the free surface to base rock) to the radius of the foundation Rvarying from 1.5 to 2.5 and ratios of the embedment depth E to the radius Rfrom 0.5 to 1.5. Both rigid foundations and a foundation typical of a nuclear power plant's containment building with the properties of concrete were investigated. The results obtained were the horizontal translation and the rotation of the foundation as well as the horizontal motion at the free surface of the soil. From these results the transfer functions from the soil surface to the foundation were obtained. These transfer functions are expressed as the ratio of the horizontal translation of the center of the foundation to the horizontal translation of the free surface (top of the soil deposit) and the ratio of the vertical displacement of the edge of the foundation (rotation of the base multiplied by the radius in case of a rigid foundation) to the horizontal displacement of the free surface. The transfer functions will be complex functions of frequency indicating that there will be both a change in the amplitudes of the motions and in their phases. In this paper the results presented will be only the amplitudes of the transfer functions. Both amplitude and phase are needed, however, for SSI analyses.

Considering a train of vertically propagating shear waves in a homogeneous soil the free field motions would be given by

$$u = A[\exp(ipz) + \exp(-ipz)]\exp(i\Omega t)$$
(14)

where

$$p^2 = \rho \Omega^2 / G(1 + 2iD) \tag{15}$$

 ρ is the mass density of the soil, G its shear modulus and D the internal soil damping. A value of D of 0.05 (5%) was used. Ω is the frequency of the excitation in rad/sec. The term $\exp(i\Omega t)$ will be omitted from all the following expressions since it is the same for all.

The motion at the free surface would then be given by u = 2A and at the foundation level in the free field by $u = A[\exp(ipE) + \exp(-ipE)]$ or, if there is no damping $u = 2A\cos pE$. One could also define in the freefield a pseudo-rotation given by the difference in the horizontal motions divided into the distance E. Thus $\varphi = 2A(1 - \cos pE)/E$. The transfer functions for the 1D solution would be $\cos pE$ (for the horizontal translation) and $R(1 - \cos pE)/E$ for the rotation times the radius.

When considering a 3D, cylindrical, rigid foundation embedded in a soil stratum (or a half space) the vertically propagating shear waves will produce both a horizontal translation of the base and a rotation. Figure 3 shows the transfer function of the translation for the case with E = R and H = 2R. The results from the one dimensional solution (motion at the level of the foundation in the free field) are also shown. The 3D solution follows very closely the 1D motion up to roughly 0.75 of the first natural frequency in shear of the embedment layer f_1 . After that the 3D solution oscillates around a mean value with only moderate amplitudes while the 1D motion exhibits much more significant oscillations. Because of this the 1D solution would severely underestimate the amplitudes of the motions around the natural frequencies of the embedment layer, while overestimating them at the midpoints between these frequencies. From inspection of the results for the various cases considered it appeared (Elsabee and Morray [6]) that a reasonable approximation could be obtained defining the transfer function for the horizontal translation (amplitude) by

$$F_{\mu}(\Omega) = \begin{cases} \cos \pi f/2f_1 & \text{for } f \le 0.7f_1 \\ 0.453 & \text{for } f \ge 0.7f_1 \end{cases}$$
(16)

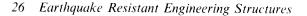
with $f_1 = c_s/4E$ the fundamental frequency of the embedment layer and c_s the shear wave velocity of the soil.

Figure 4 shows the amplitude of the transfer function for the rotation multiplied by the radius of the same foundation (E = R, H = 2R). Shown in the same figure is the transfer function for the 1D pseudo-rotation multiplied by the scaling factor 0.257. The agreement is very good in the low frequency range but it deteriorates again for higher frequencies with the 1D solution exhibiting much larger oscillations than the true 3D rotation. From inspection of the figures for the various cases studied it appeared that a reasonable approximation could be provided by the expression

$$RF_{\varphi}(\Omega) = \begin{cases} 0.257(1 - \cos \pi f/2f_1) & \text{for } f \le f_1 \\ 0.257 & \text{for } f \ge f_1 \end{cases}$$
(17)

with f_1 as above defined. In these studies, the rotation was considered positive in the clockwise direction.

Figure 5 shows for the same foundation the effect of assuming elastic sidewalls with the properties of concrete while still maintaining a rigid base.



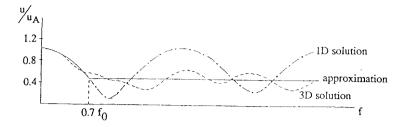


Figure 3 Transfer Functions for Horizontal Motion

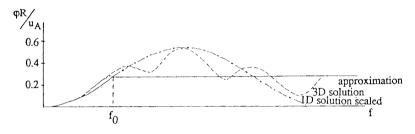
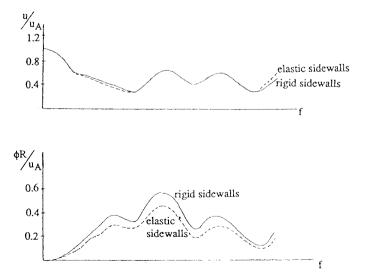
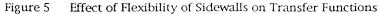


Figure 4 Transfer Functions for Rotation





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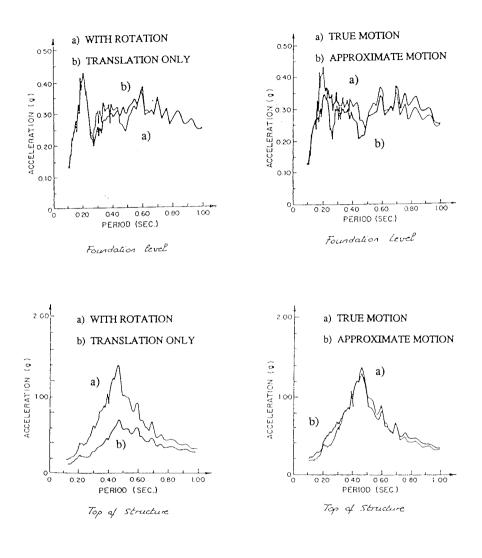


Figure 6 Effect of Base Rotation on Figure 7 Effect of Approximate Rules for Amplified Response Spectra Motion on Amplified Response Spectra

It can be seen that the effect on the horizontal translation is negligible. The amplitude of the base rotation is reduced on the other hand by about 20%. In the limiting case, if there were no sidewalls the foundation would still have a rotation but of opposite sign. The actual conditions of the backfill would also influence the magnitude of the rotation as well as the fact that some slippage could take place between the sidewalls and the soil during vibration. Thus while the approximate expressions would yield results consistent with those obtained performing a direct (one step) solution of the combined soil-structure system, in practice the rotations might be expected to be somewhat smaller.

It is important to notice that the rotation is an integral and important part of the foundation motion. Ignoring it, while deamplifying the translational component, may lead to important errors on the unconservative side. To illustrate this point, Figure 6 shows the results of a SSI analysis performed on a structure with characteristics similar to those of typical containment buildings in nuclear power plants using both components of motion and only the translation. The characteristics of the motions at the base and at the top of the structure (including inertial interaction effects) are depicted in terms of their response spectra. It can be seen that the results of both analyses are very similar at the base of the structure, where the rotation has very little effect (the small differences are due to the coupling terms in the stiffness matrix). At the top of the structure, however, the results accounting for the base rotation are almost twice those considering only the translation. Figure 7 shows the results using the estimates of the translation and the rotation provided by the approximate rules suggested above. The agreement with the more rigorous solution also shown in the figure, is remarkably good, particularly at the top of the structure.

In these analyses the dynamic stiffness matrix of the foundation, needed for the dynamic solution of the complete system, was computed using the same finite element model with consistent transmitting boundaries employed for the kinematic interaction studies and shown schematically in Figure 2. The solution was carried out in the frequency domain, then converted to time domain using the Fast Fourier Transform.

The effect of embedment on the foundation motions was further investigated by Dominguez [7] who considered square and rectangular foundations embedded in a homogenous half space and used the boundary element method for the formulation. Dominguez considered trains of waves at arbitrary angles of incidence. Typical results for a square foundation with side 2B and angles of incidence of 0, 45 and 90 degrees with respect to the horizontal axis are shown in Figure 8. An angle of incidence of 90 degrees corresponds to vertically propagating waves in this case. f_1 is again the fundamental natural frequency of the embedment layer in shear. The results were obtained in this case over a smaller range of frequencies. For vertically propagating waves the results are similar to those reported by Morray over this reduced frequency range. It can be seen, however, that for other angles of incidence the rotational component decreases significantly while a torsional component of motion appears.

Similar studies to those performed by Morray were carried out using a wider range of embedment ratios. As a result of these studies, it was recommended to extend the original formulas as

$$F_{\mu}(\Omega) = \begin{cases} \cos \pi f/2f_1 & \text{for } f \le \alpha f_1 \\ \cos \pi \alpha/2 & \text{for } f \ge \alpha f_1 \end{cases}$$
(18)

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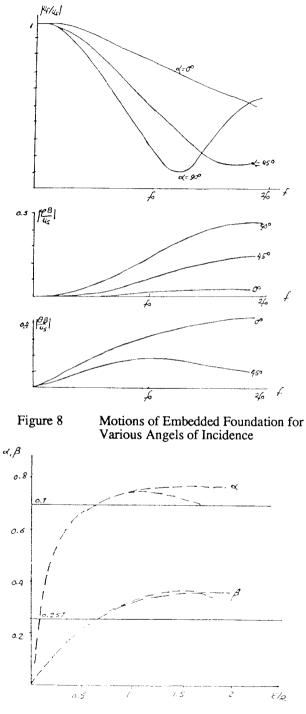


Figure 9 α, β Coefficients

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$$RF_{\varphi}(\Omega) = \begin{cases} \beta(1 - \cos \pi f/2f_1) & \text{for } f \le f_1 \\ \beta & \text{for } f \ge f_1 \end{cases}$$
(19)

with α and β functions of the embedment ratio E/R as shown in Figure 9.

4 Foundation Stiffnesses

Using the same finite element model with consistent lateral boundaries shown schematically in Figure 2, Elsabee [6, 8] conducted parametric studies determining the dynamic stiffness matrix of embedded rigid circular foundations with different ratios of the embedment depth to the radius and of the layer thickness to the radius. For each foundation, results were obtained using three different finite element meshes with elements of square cross section and sides equal to one-quarter, one-eighth and one-sixteenth of the radius. The results from these three meshes were then extrapolated (a linear extrapolation with respect to element size) to obtain an improved estimate.

The stiffness matrix for an embedded, rigid, circular foundation considering only two degrees of freedom (horizontal translation and rotation) will be of the form

$$K_f = \begin{bmatrix} K_{xx} & K_{x\varphi} \\ K_{\varphi x} & K_{\varphi \varphi} \end{bmatrix}$$
(20)

The terms $K_{ij}(K_{xx}, K_{x\varphi} = K_{\varphi x} \text{ and } K_{\varphi \varphi})$ will be complex functions of frequency. It is common to write them in the form

$$K = K_0 (1 + 2iD) [k + ia_0 c]$$
(21)

where K_0 is the static value (corresponding to zero frequency), D is the internal soil damping (assumed to be of a linear hysteretic nature), k and c are the dynamic stiffness coefficients (functions of frequency) and a_0 is a dimensionless frequency equal to pR with p as defined in eq. (15). It should be noticed that when there is internal damping, a_0 will be complex with this notation. Alternatively, one can use only the real part of a_0 , modifying accordingly the values of k and c, which would become functions also of D.

Based on his studies Elsabee extended the formulas derived by Kausel [3] for the static stiffness of circular foundations on the surface of a soil layer of finite depth, to account for embedment. The expressions he suggested are

$$K_{xx0} = \frac{8GR}{2 - \nu} \left(1 + \frac{1}{2} \frac{R}{H} \right) \left(1 + \frac{2}{3} \frac{E}{R} \right) \left(1 + \frac{5}{4} \frac{E}{H} \right)$$

$$K_{\varphi\varphi0} = \frac{8GR^3}{3(1 - \nu)} \left(1 + \frac{1}{6} \frac{R}{H} \right) \left(1 + 2\frac{E}{R} \right) \left(1 + 0.7\frac{E}{H} \right)$$

$$K_{x\varphi0} = \left(0.4\frac{E}{R} - 0.03 \right) RK_{xx0}$$
(22)

Figure 10 shows the variation of the stiffness coefficients k and c versus frequency for the K_{xx} and the $K_{\varphi\varphi}$ terms and a foundation with E = R and H = 3R. The results are compared to those of a surface foundation on a layer

of the same thickness H and those of a circular mat on the surface of a homogenous half space. When dealing with a layer of finite depth the coefficient c which represents the loss of energy by radiation of the waves away from the foundation will be zero below a threshold frequency which is the fundamental natural frequency in shear of the stratum for horizontal translation and the vertical natural frequency in rocking if Poisson's ratio $v \leq 0.3$. For larger values of Poisson's ratio it is a frequency intermediate between the two. For the half space the term c starts with a nonzero value from the beginning for the horizontal stiffness. It starts however at 0 for the rocking term and a surface foundation. The real stiffness coefficient k has a number of oscillations associated with the natural frequencies when dealing with a soil layer of finite depth and is much smoother for a deep layer (if there is some internal soil damping) or for a homogenous half space. It is also smoother in rocking than for horizontal translation.

For a surface foundation on a half space the dynamic stiffness coefficients k and c corresponding to horizontal translation are essentially constant with values of 1 and 0.6 approximately (the value of c is a function of Poisson's ratio). The same variations could be used for an embedded foundation in a half space but the value of c would increase, depending on the embedment ratio E/R. For E = R it can be seen that $c \cong 0.9$. For a finite layer the value of k would have oscillations as mentioned above and the value of c would have to be truncated below the threshold frequency. The values of k and c are functions of frequency for the rocking case. Approximate expressions for a surface foundation on a half space are

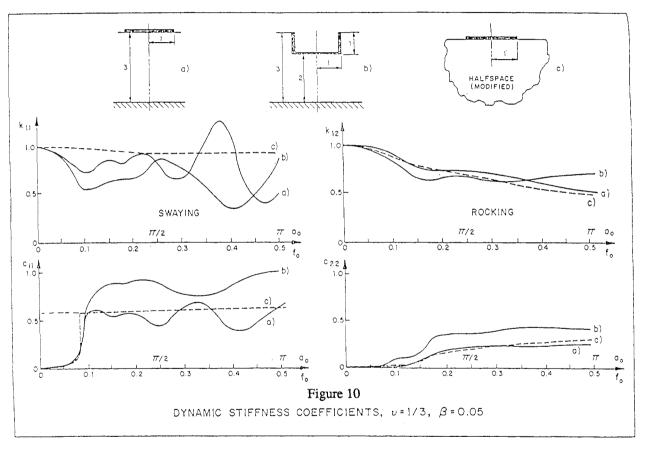
$$k = \begin{cases} 1 - 0.2a_0 & \text{for } a_0 \le 2.5 \\ 0.5 & \text{for } a_0 \ge 2.5 \end{cases}$$
(23)

and

$$c = \frac{0.35a_0^2}{1+a_0^2} \tag{24}$$

More accurate expressions for these coefficients, function of Poisson's ration were proposed by Veletsos and Verbic (9). One could again use the same expressions for k in case of an embedded foundation, but the coefficient 0.35 in eq. (24) would increase with embedment (to 0.45 or so for E = R).

Novak [10] had proposed an alternative simplified procedure to compute approximately the dynamic stiffnesses of circular foundations embedded in a half space. In this approach the stiffness terms are assumed to consist of the contribution of the base (with a stiffness equal to that of a surface foundation) and the sidewalls (reproduced by a series of frequency dependent springs and dashpots obtained from the study of a rigid disk vibrating in a plane). The values of the springs and dashpots can be expressed in closed form in terms of modified Bessel functions. Studies by Chen [11] have shown that the real part of the stiffnesses (the terms K_0k) predicted by Novak's approach tend to be smaller than the results of finite element analyses especially in rocking. On the other hand the values of the imaginary parts of the stiffnesses (the terms K_0a_0) predicted by Novak's approximation are in excellent agreement with the finite element results for a half space. They are also a very good approximation for a foundation embedded in a layer of finite depth above the threshold frequency. Below this frequency Novak's solution 迹



would predict nonzero values of the imaginary terms, which should be zero.

Once the dynamic stiffness matrix of the foundation is known, the third and final step of the SSI analysis obtaining the response of the structure on a flexible foundation to the motions computed accounting for kinematic interaction is straightforward particularly when performed in the frequency domain. Some complications arise, however, when attempting to use more traditional methods of structural dynamics in the time domain, such as modal analysis or especially modal spectral analysis. It is particularly important that the structural model be able to account for both translational and rotational motions for an embedded rigid foundation or for independent, different motions of the various contact points for flexible foundations.

The main effects of the inertial interaction, which have been extensively discussed in the literature, are the modification in the natural period of the structure (an elongation of the period due to the added flexibility of the foundation) and a change in the effective damping of the system which is in most cases an increase (when there is radiation damping). The importance of these effects depends on the relative stiffness of the structure with respect to the soil. As embedment increases so do the foundation stiffnesses and therefore the inertial interaction effects tend to decrease in importance at least in relation to the change in the effective natural period. The effective damping decreases because of the reduced interaction but increases due to the larger values of the coefficient c.

For any given earthquake record the effect of the change in period can be beneficial or detrimental depending on the value of the initial period (on a rigid base) and the characteristics of the seismic motion (as represented for instance by the response spectra). For a smoothed design spectrum the effect tends to be beneficial at least for nuclear power plants. The increase in effective damping, if there is an increase, will always be beneficial.

5 Final Considerations

Soil structure interaction effects are often associated only with the inertial interaction phenomena (elongation of the effective natural period and change in the effective damping). These are indeed the only effects when considering the dynamic response of a structure subjected to external loads applied directly on the structure (wind loads, wave loads, or machine vibrations). For the seismic case, however, one must also consider kinematic interaction effects. When dealing with structures on surface foundations the effects of the kinematic interaction tend to be small unless one has very long and rigid foundations. Inertial interaction effects tend to be then the predominant ones. For embedded foundations, on the other hand, kinematic interaction effects can become important and much more so than inertial interaction.

The approximate methods discussed in this paper, in combination with the formulas available in the literature to compute the inertial interaction effects for an equivalent single degree of freedom system, allow to conduct preliminary analyses and estimate the relative importance and significance of the various effects. Because kinematic interaction will result in a seduction in the amplitudes of the translational motion, which can be significant for deeply embedded foundations, there has been a controversy about accepting it. Two mistakes which have complicated further the issue are the confusion between the transfer functions for the motions of an embedded foundation (3D solution) and the 1D solution for the motion at the foundation level in the free

field (which has much larger oscillations), and ignoring the rotational component of motion. It is true on the other hand that the values of the translational and rotational components of motion are a function of the angle of incidence of the waves but for soft soils where the effects are important the waves in the embedment region should be traveling almost vertically.

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