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Seismic response of two adjacent non-symmetric multistory buildings

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Abstract

The main aspect of seismic response of two adjacent multistory buildings is their possible pounding due to an earthquake. Both buildings are treated as non-symmetric with respect to their mass and/or stiffness characteristics, so, when subjected to earthquake excitation, both buildings perform the corresponding 3D motion. If the mechanical characteristics of buildings are different, buildings perform an out-of-phase motion so, if the initial separation of buildingss is not sufficient, occasional pounding between slabs of the same level is to be expected. The final result of collision between buildings may be a substantial damage, or even worse, a collapse of buildings. Possible occasional pounding of buildings due to earthquake excitation is analyzed by the combination of direct numerical integration of the corresponding differential equations of motion and the classical impact analysis of two rigid laminae in planar motion.

1 Introduction and basic assumptions

In the first place, it is not a normal situation to analyze the impact of adjacent buildings due to earthquake. If the buildings were built in accordance with the technical regulations, impact of adjacent buildings would have never occured. However, experiences from a lot of major earthquakes, starting with the 1971 San Fernando earthquake, show that impact of buildings due to earthquakes are really happening[1−2]. For instance, durnig the Mexico City earthquake in

1985, out of about 330 multistory buildings that were damaged or have collapsed during the earthquake, in more then 40% there was the evidence of pounding of adjacent buildings. Also, in 15% of all collapsed buildings, the primary reason was the collision with neighbouring buildings[1].

Numerical and analytical investigations of such problem are still relatively rare, but the interest in the problem is increasing[1,2]. There are basically two approaches in the analysis. The first is based on introduction of the special contact elements (spring and dashpot types) between adjacent buildings. Such impact elements become active when two masses are in contact and the main problem is how to assign the corresponding constants. Usually, the buildings are treated as the equivalent linear or non-linear s.d.o.f. systems, but also, there are more advanced approaches where m.d.o.f. systems with bilinear interstory resistance were considered. The other approaches are based on global balance of momentum and determination of internal impact impulses, sometimes combined with the Lagrange multipliers method to enforce the geometric compatibility conditions due to collision[3–6].

However, in the previous work only 2D problems were considered. It means that buildings were treated as symmetrical systems, without any torsional effects, so that each slab performs only translational motion with a single degree of freedom. As opposed to that, the present paper, which is based upon [5] and [6], is devoted to non-symmetrical buildings where each slab performs a planar motion with 3 d.o.f.

The basic assumptions in the seismic analysis of buildings are that slabs are treated as infinitely stiff in their planes, and that mass of the building is concentrated in slabs only (shear building assumptions). Therefore, all slabs are performing planar motion in parallel horizontal planes, while vertical elements (frames and/or shear walls) represent restraints to planar motion of slabs. Vertical elements are treated as planar structures, i.e. they have finite stiffness in their planes and negligible out-of-plane stiffness. Consequently, the mechanical model of each building is defined by 3N d.o.f., where N is the number of stories. Also, all the usual assumptions of linear elasticity are retained.

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2 Seismic analysis of non-symmetric multi-story building

Earthquake excitation of a building is treated in the usual way, i.e. as the forced translational ground motion in the horizontal plane imposed upon building's foundations. Such seismic translatory motion is uniquely defined by the ground acceleration function $\ddot{u}_g(t)$ and also by the dominant direction of ground motion. In the present analysis no soil-structure inteaction is considered, since the main objective of the analysis is the numerical implementation of the pounding situation during earthquake.

Differential equations of motion of a building due to an earthquake may be derived in the form:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\mathbf{b}\ddot{u}_g = \mathbf{g}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the mass, damping and stiffness matrices. Overdots denote differentiation with respect to time, while $\mathbf{x}$ and $\mathbf{g}$ represent the vector of generalized coordinates and the loading vector. Finally, the vector $\mathbf{b}$ represents the influence vector which defines the dominant direction of seismic ground motion in a horizontal plane.

Generally, there are two main ways to solve eqs.(1): the transformation methods (modal superposition, or superposition of Ritz or Lanczos vectors) and direct numerical integration methods. In the pounding response analysis it is more convenient to use direct numerical integration step by step. Among various possibilities, the implicit α-method is chosen. Therefore, the total duration of earthquake is divided into n_{max} equal time intervals Δt and within each time interval the equivalent static problem is obtained:

$$\mathbf{K}^*\mathbf{x}_{n+1} = \mathbf{g}^*_{n+\alpha} \qquad (n = 1, 2, \ldots, n_{max} - 1) \tag{2}$$

The equivalent stiffness matrix $\mathbf{K}^*$ and the vector of equivalent loading $\mathbf{g}^*_{n+\alpha}$ are given by the corresponding linear combination of $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ matrices and previously obtained solutions at the beginning of the considered time step: $\mathbf{x}_n$, $\dot{\mathbf{x}}_n$ and $\ddot{\mathbf{x}}_n$.

The solution of eqs. (2) gives the vector of generalized coordinates at the end of the considered time step, $\mathbf{x}_{n+1}$, while the generalized velocities and accelerations at the end of time step are then given by

$$\dot{\mathbf{x}}_{n+1} = \frac{\gamma}{\beta\Delta t}(\mathbf{x}_{n+1} - \mathbf{x}_n) - (\frac{\gamma}{\beta} - 1)\dot{\mathbf{x}}_n - \Delta t(\frac{\gamma}{2\beta} - 1)\ddot{\mathbf{x}}_n \tag{3}$$

$$\ddot{\mathbf{x}}_{n+1} = \frac{1}{\beta\Delta t^2}(\mathbf{x}_{n+1} - \mathbf{x}_n) - \frac{1}{\beta\Delta t}\dot{\mathbf{x}}_n - (\frac{1}{2\beta} - 1)\ddot{\mathbf{x}}_n \qquad (4)$$

3 Impact analysis of two adjacent buildings

Two neighbouring buildings are treated as having the same storey heights and as being exposed to the same seismic excitation $\ddot{u}_g(t)$ in the same principal direction. Therefore, the behaviour of each building is described by the corresponding differential equations of motion given by (1). Utilizing the same α method of numerical integration, the solution of eqs. of motion is reduced to two systems of algebraic equations for each time step Δt:

$$\mathbf{K}_i^*\mathbf{x}_{i,n+1} = \mathbf{g}_{i,n+\alpha}^* \qquad (n = 1, 2, \ldots, n_{max} - 1; \quad i = 1, 2) \quad (5)$$

The solution of each system (5) gives the corresponding generalized coordinates at the end of the current time step, while generalized velocities and accelerations are then obtained by eqs. (3)-(4).

3.1 Conditions of impact

Under the adopted assumptions related to the horizontal load analysis of the building, each slab is treated as the rigid lamina undergoing the planar motion in a horizontal plane. The corresponding generalized coordinates defining the motion of each slab are the two coordinates of displacement vector of the center of mass of the slab, and one coordinate that defines the vector of elementary rotation of the slab. Of course, the shape of each slab is known: generally speaking, it may be considered as a polygonal one. Consequently, it is easy to obtain the exact position of each slab with reference to the inertial coordinate system.

Since neighbouring buildings are treated as having the same storey heights, there are two slabs in each horizontal plane. Since the shear building assumtion is adopted, it is not possible to analyse the worst case of an impact of a slab at a midstorey of neighbouring building. If the regions that both slabs of the same level are occupying at any instant of time, expressed with reference to the inertial system, are denoted respectively by $\mathcal{A}$ and $\mathcal{B}$, the possible relationship between

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them may be expressed as:

$$\mathcal{A} \cap \mathcal{B} = \begin{cases} \oslash & \text{empty set} \\ Q & \text{point} \\ \mathcal{C} & \text{non-empty set} \end{cases} \quad (6)$$

Relationship (6) means that, respectively,

- regions are not in contact, ie. there is no collision;
- regions are having a contact in a point, ie. possible collision;
- regions are overlapping, ie. the collision has already happened within the time step.

In the first case, there is no collision between slabs, so the generalized velocities and accelerations are determined from eqs. (3)-(4) and then the equations of motion (5) for each building are solved for the next time step.

The second case represents the situation when slabs are having a contact in a point and it will be called *the position condition* for impact of slabs. However, contact in a point is the necessary but not the sufficient condition of impact. Namely, in order to really have the impact at the point of contact, the velocities of the points of contact of both slabs should be such as to imply the tendency for overlaping of slabs. It will be called *the velocity condition* of impact[5]. Therefore, the position and velocity conditions of impact of two slabs, occurring at point Q, may be expressed as

$$\vec{r}_Q^{(A)} - \vec{r}_Q^{(B)} = 0 \quad (\vec{v}_Q^{(A)} - \vec{v}_Q^{(B)})\, \vec{n} > 0 \quad \Rightarrow \quad v_{n1} - v_{n2} > 0 \quad (7)$$

where $\vec{r}$ and $\vec{v}$ are the position and velocity vectors of point Q, while $\vec{n}$ is the outward normal defined for region $\mathcal{A}$. If the outward normal is defined just for one of the regions in the point of contact, then v_{n1} is related to the region for which the outward normal $\vec{n}$ is defined.

Finally, the third case expressed by (6) represents the situation when the impact *has already happened* within the current time step. In such a case equations of motion for both buildings are solved again for the same time step, ie. for the same *beginning* of the step, but the value of the time step Δt is reduced to one half of the previous one. This is done iteratively in order to capture the situation when *at the end* of the time step slabs are in the state of collision.

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3.2 Impact analysis of two slabs

If both impact conditions (7) are met, the classical impact analysis of two rigid bodies in planar motion is performed. Now the generalized velocities obtained at the end of the time step, when slabs are in the state of impact, represent the known velocities immediately before the impact. They are denoted with the upper prime index. The impact of two bodies is treated as sudden (discontinuous) change in velocity fields of two bodies. Therefore, immediately after the impact, slabs occupy the same space position, but the velocity fields are changed. Unknown generalized velocities immediately after the impact are denoted by $(\ldots)''$.

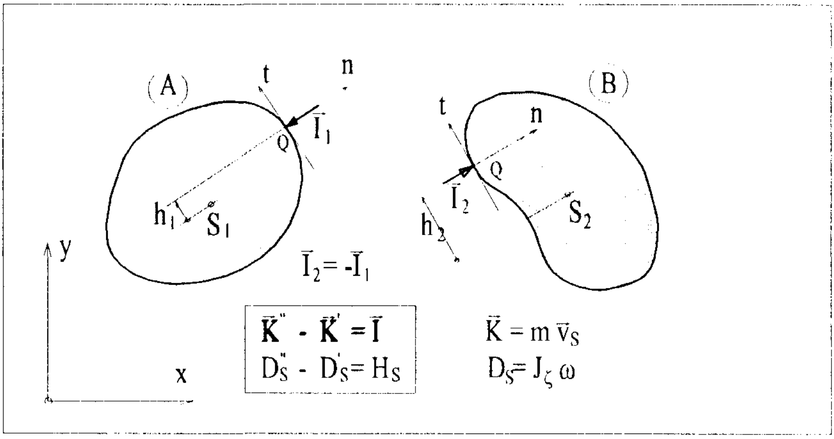

Figure 1: Separated slabs with internal impact impulses

Impact analysis of two slabs is performed in the classical way based on direct evaluation of internal impact impulse developed at the point of contact during the impact. The internal impulse $I_1 = I_2 = I$ is acting in direction along the normal axis n which is perpendicular to the contour of one of the slabs. Namely, if both slabs are considered as separated, the following balances of momentum $\mathbf{K} = m\vec{v}_S$ and moment of momentum $D^S = J_\zeta \omega$ may be written for each slab:

- Slab $\mathcal{A}$

$$\begin{aligned} m_1 \dot{u}_1'' - m_1 \dot{u}_1' &= -I\cos\theta \\ m_1 \dot{v}_1'' - m_1 \dot{v}_1' &= -I\sin\theta \\ J_{\zeta_1}\dot{\varphi}_1'' - J_{\zeta_1}\dot{\varphi}_1' &= -I\sin\theta \bar{x}_{Q_1} + I\cos\theta \bar{y}_{Q_1} = I h_1 \end{aligned} \tag{8}$$

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- Slab $\mathcal{B}$

$$\begin{aligned} m_2\dot{u}_2'' - m_2\dot{u}_2' &= I\cos\theta \\ m_2\dot{v}_2'' - m_2\dot{v}_2' &= I\sin\theta \\ J_{\zeta_2}\dot{\varphi}_2'' - J_{\zeta_2}\dot{\varphi}_2' &= -I\sin\theta\bar{x}_{Q_2} - I\cos\theta\bar{y}_{Q_2} = -Ih_2 \end{aligned} \tag{9}$$

Besides the six unknown generalized velocities of both slabs immediately after the impact, the internal impact impuls I is the additional seventh unknown. The closure equation is the definition of the coefficient of restitution (or impact), which may be given as

$$k = \frac{|v_{n2}'' - v_{n1}''|}{|v_{n1}' - v_{n2}'|} \qquad k \in [0,\ 1] \tag{10}$$

where v_{ni}, $(i = 1, 2)$ represent the components of the velocity of points Q of both regions in direction of the outward normal n. Of course, if the value of coefficient of impact k is adopted to be equal to one, $k = 1$, it represents the case of ideally elastic impact where the total kinetic energy is conserved during the impact. The other extreme case, when k is equal to zero, $k = 0$, represents the case of ideally plastic impact, while all other, more or less realistic cases, correspond to some intermediate value of $k \in [0,\ 1]$.

It is easy to express the generalized velocities of both regions as a function of unknown impulse I from eqs. (8)-(9). Introducing obtained relations into (10), the internal impact impulse I may be obtained as

$$I = (1 + k)\frac{b}{a} \tag{11}$$

where

$$a = \sum_{i=1}^{2}\left(\frac{1}{m_i} + \frac{h_i^2}{J_{\zeta_i}}\right) > 0 \qquad b = v_{n1}' - v_{n2}' > 0 \tag{12}$$

As mentioned before, obtained generalized velocities of both slabs immediately after the impact, as calculated from eqs. (8)-(12), are then imposed as the initial velocities for the next time step. Initial generalized accelerations at the beginning of the next time step are unchanged as obtained from eqs. (4) from the previous time step solution.

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3.3 Overall seismic analysis with possible impact

The overall seismic analysis of two adjacent buildings is now self-evident. Differential eqs. of motion of both buildings (5) are being simultaneously solved using direct numerical integration step by step. After each time step the current configuration of both buildings is being checked. It means that each pair of neighbouring slabs of the same level, storey by storey, is being analysed in order to establish the current relationship between them at the end of the time step. Namely, the considered pair of neighbouring slabs could be without contact, could be in a collision at the end of the current time step, or the collision has already hapened previously sometimes within the considered time step.

If there is no collision between any pair of slabs, the simultaneous time integration of equations of motions of both buildings is continued. In the case of impact between one or more pairs of slabs of neighbouring buildings, the impact analysis given by (8)-(12) is then performed in order to establish internal impact impulses between slabs and particularly the new velocities immediately after the impact. Obtained new velocity field is used as the new initial velocity conditions for the next time step integration.

On the other hand, if there is overlaping of any pair of slabs of the same level, it means that collision between them has already occured sometimes during the just considered time step. In such a case, the previously considered time step is reduced by half and the time integration, with reduced time step, is performed again in order to capture the situation of impact at the end of new time step. Having in mind the physical nature of the problem, the convergency in the engineering sense is to be expected.

4 Computer implementation and numerical examples

The corresponding computer program has been developed[6], which enabled various numerical experiments. As the illustrative example, seismic response of two buildings presented in Fig. 2 is considered. Buildings are 3 and 5 stories high, both are rectangular in plan and have orthogonal, but non-symmetic arrangement of frames. Seismic excitation is adopted as El Centro accelerogram scaled to max ground

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acceleration of 0.32 g.

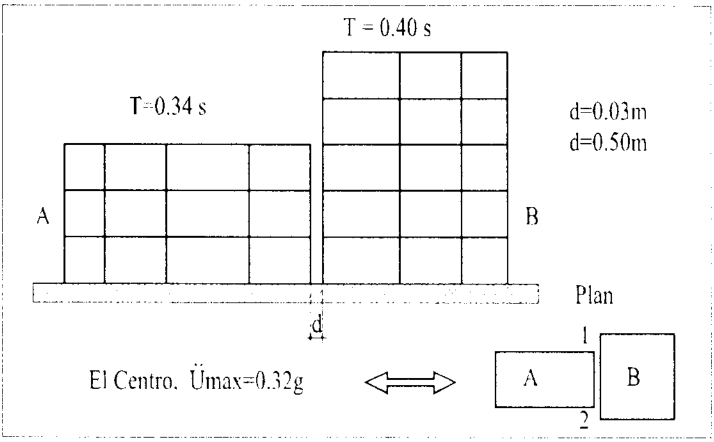

Figure 2: Layout of two adjacent buildings

In order to obtain the effect of pounding compared to the situation without pounding, the response of both buildings for the same excitation is obtained twice. The first run was for the case of separation between buildings being equal to $d = 0.5\ m$, which obviously means that buildings are more than sufficently separated, so there is no pounding, and the second run was for the separation gap of $d = 0.03\ m$, which means that pounding is possible.

Obtained results revealed that within the duration of the accelerogram of about 12 seconds, pounding between slabs of the third floor happened 19 times. Out of that, 16 times the corner point of the lower bulding denoted as "2" hit into the slab of the higher building and the remaining 3 times the other corner denoted as "1" hit into the other slab. The figure 3 presents the time history of displacement component u in direction of x axis of the corner point "2" of the third floor of the lower bilding.

As may be concluded from Fig.3, the displacement amplitudes of point "2" in direction of the accelergram are reduced when compared to the no-collision case. The overall conclusion of the whole analysis is that the analysis may be used for estimation of the separation gap which is necessary to prevent the possible pounding during the earthquake.

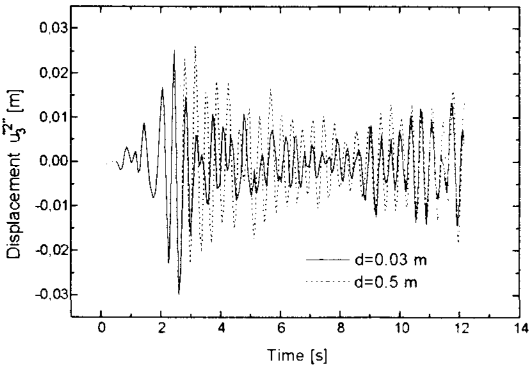

Figure 3: Time history with pounding between slabs

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