

## SEISMIC WAVE EFFECTS ON SOIL-STRUCTURE INTERACTION

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## SUMMARY

One of the most commonly used hypotheses in the seismic analysis of structures is that the earthquake input motion is identical at all spatial points on a given level beneath the structure. This is clearly not generally true, and any actual spatial variation of this motion away from complete uniformity will give rise to effects not usually considered—in particular, self-cancelling effects by those displacement components which happen to oppose each other in direction, and torsional effects by those motions which have different lever arms with respect to the structural center of mass.

The present paper examines this situation analytically under the following circumstances: a rectangular structural mass or foundation block is assumed to rest upon distributed soil springs, and a travelling seismic wave is assumed to pass under the springs. The dynamics of this system are analyzed.

When soil motion is assumed to be in the direction of wave propagation (as with P-waves) it is found that the net forces on the structure which are initiated by soil motion are modified from those values that would be attributable to a spatially uniform quake. For example, each input harmonic of amplitude  $A_n$  of a uniformly distributed quake must be replaced by  $\tilde{A}_n$ , where

$$\tilde{A}_n = \left[ \frac{2(1 - \cos R_n)}{R_n^2} \right]^{1/2} A_n$$

$R_n$  being proportional to the ratio of structural dimension to the wavelength of the  $n$ th seismic wave harmonic.

The case of seismic motion transverse to the direction of wave propagation (as with shear waves) is shown to give analogous results, applicable to transverse, rather than longitudinal, effects. Furthermore, the origin of the torsional moment which is applied by this process to even symmetric structures is clearly delineated. Torsional effects have already been discussed by Newmark from a different vantage point.

The evolution of the above-mentioned effects with increasing wave number parameter  $\bar{R}_n$  is explored in the paper in two basic graphs. These latter permit the conclusion that the effects under consideration can become important for design, particularly since many practical cases can occur wherein seismic wavelengths are of the order of foundation dimensions. Because of the self-cancelling wave effects which enter into the calculation of the generalized forces of the problem, the study suggests that the commonly used hypothesis of a spatially uniform earthquake may be inherently highly conservative in some cases.

Though the present study is exploratory and based on an elementary soil-spring model, the issues it raises are of inevitable importance with other models—such as finite-element—as well.

The paper closes with remarks anticipating the analogous problem when spatial distribution of seismic input motion is alternatively postulated as random rather than simple wave passage.

One may infer a spatially distributed surface motion of the soil under a structure if an earthquake is simply assumed to consist of a complex of surface waves traversing the plan of the structural site.

Under these conditions, the effects of passing waves must be integrated over the structural area to obtain their net effects as exciting functions to the structure. One important result is the diminution or "self-cancelling" effect of some inputs. Another expected effect is the torsional excitation of the structure.

### 1. Introduction

A given earthquake time history or ground acceleration at a point represents the effects at a given site of a complex of seismic waves traveling past that point. In the case of structures such as nuclear power plants the length or width of the structure may be comparable in size to certain wavelengths of the seismic waves assumed to traverse the site. For such wavelengths there exist horizontal ground motions of different surface particles occurring simultaneously in opposite directions in the vicinity of the structure, and these actions may produce a net "self-cancelling" effect when integrated over the whole structural base area.

### 2. Structural Model

A very simple soil-structure interaction model will be assumed: namely, a rectangular, rigid block resting on top of the soil and connected to the soil by a continuously distributed set of springs. [See Fig. 1] These "soil springs" are one type of idealization which is commonly employed in soil-structure interaction analyses of important structures. The choice of these makes the present discussion simpler, but their exact form is not especially pertinent.

The foundation block may or may not have additional structure attached to it. This will be immaterial to the present discussion.

What will be focused on below is the type of soil motion and the average effect thereof to be expected at the "lower ends" of the theoretical soil springs.

### 3. Soil Motion Parallel to the Direction of Wave Propagation

For convenience here it will first be assumed that surface waves traveling with the "P-wave velocity"  $C_p$  traverse the structural site, particle motions being fore-and-aft with respect to the direction of  $C_p$ . This direction is called  $x$ , and, again for convenience, it is taken as normal to one of the plan dimensions of the structure. [See Fig. 2]

Let  $u$  be the absolute displacement of the foundation away from its rest position and  $\xi(x,t)$  be the absolute displacement of the soil at points  $x$  (at the "lower ends" of the soil springs) away from rest. Then the kinetic energy of  $x$ -translation of the base is

$$T = \frac{1}{2} M \dot{u}^2$$

where  $M$  is the total foundation mass, and its displacement relative to the soil is  $u - \xi$ . Hence the potential energy of the soil springs is

$$U = \frac{1}{2} \int_{fdn} k(x) (u - \xi)^2 dx$$

where  $k(x)$  is the distributed soil spring constant and the integral is over the foundation (i.e. 0,L).

The equation of motion is then

$$M \ddot{u} + \left( \int_{fdn} k(x) dx \right) u = \int_{fdn} k(x) \xi(x, t) dx \quad (1)$$

Ordinarily an earthquake time history (artificial or natural) is specified at a single soil surface point (say  $x=0$ ) by its acceleration  $\ddot{\xi} = \ddot{\xi}(t)$ . However, here it will be convenient to focus upon the displacement, which will be supposed expressed by the Fourier series

$$\xi(t) = \sum_{n=1}^N A_n \cos(\omega_n t + \phi_n) \quad (2)$$

The coefficients  $A_n$  will here be assumed known for any given case; they are directly related to the Fourier amplitude spectrum of the quake. For an artificial earthquake the phase angles  $\phi_n$  are arbitrarily established, for example through a choice of  $N$  random numbers on  $(0, 2\pi)$ . (See Ref. [1]).

Introducing now the viewpoint that seismic excitations are actually due to traveling waves (here assumed to move at some velocity  $C_p$ ), the eq. (2) generalizes to

$$\xi(x, t) = \sum_{n=1}^N A_n \cos\left(\omega_n t + \phi_n - \frac{\omega_n x}{C_p}\right) \quad (3)$$

Seismic waves (see for example Ref. [2]) may well be dispersive, but  $C_p$  may be considered here merely a constant local, or site velocity, or one associated with the frequency  $\omega_n$ .

The right-hand side of eq. (1) describes a generalized force now requiring the integration of displacements given by eq. (3), weighted by the distributed spring-constant factor  $k(x)$ . Thus it may be observed that longitudinal variation of earthquake displacement can cause varying amounts of input to the structure, depending both on distributed local soil properties and the  $x$ -phasing of the wave displacement.

A simple result is obtained if it is assumed that  $k(x) = k$  is constant with  $x$ . Then eq. (1) requires an input force

$$F(t) = \int_0^L k \left( \sum_{n=1}^N A_n \cos\left(\omega_n t + \phi_n - \frac{\omega_n x}{C_p}\right) \right) dx \quad (4)$$

Use of the relation

$$C_p = \frac{\omega_n}{2\pi} \lambda_n$$

among velocity, circular frequency  $\omega_n$  and wavelength  $\lambda_n$  then results in the following expression for the  $n$ th component  $F_n$  of eq. (4):

$$F_n = k L \tilde{\xi}_n(t) \quad (5)$$

where  $\tilde{\xi}_n$  is an average displacement expressed by

$$\tilde{\xi}_n(t) = -A_n \frac{\sqrt{2} \sqrt{1 - \cos R_n}}{R_n} \cos(\omega_n t + \phi_n - \psi_n) \quad (6)$$

In this expression

$$\tan \psi_n = \frac{1 - \cos R_n}{\sin R_n}$$

where

$$R_n = \frac{2\pi L}{\lambda_n} \quad (7)$$

is a parameter proportional to the ratio of structural x-dimension to the particular wavelength associated with the nth term.

It may be seen that the averaging of the seismic wave under the structure results in a different effective "one-point" earthquake which may be considered to shake the structure in the classical fashion usually assumed in earthquake structural analyses. To define this new effective input the original earthquake displacement coefficient  $A_n$  is now replaced by  $\tilde{A}_n$ , with

$$\frac{\tilde{A}_n}{A_n} = \left[ \frac{2(1 - \cos R_n)}{R_n^2} \right]^{\frac{1}{2}} \quad (8)$$

and the phase of the nth term is shifted backward by  $\psi_n$ .

If one is creating an artificial earthquake for use in analysis, as in Ref. [1], through the use of randomly distributed phase angles  $\phi_n$ , the phase modifications  $\psi_n$  appearing here in eq. (6) can probably be disregarded, since the value  $\phi_n - \psi_n$  remains random,  $\phi_n$  being initially arbitrary.

The function  $\tilde{A}_n/A_n$  given by eq. (8) is plotted in Fig. 3 and, as might be expected, resembles a Fourier transform.

Typical values of  $C_p$  might be: for alluvium 570 m/sec and, for shale bedrock 2200 m/sec. (Ref [2]). As an illustration, Table 1 is calculated for a wave speed of 1000 m/s and typical frequencies in the seismic range of interest to nuclear power plants. ( $L = 65$  m.)

These considerations suggest as a main conclusion that appreciable reduction in the effective higher-frequency components of an earthquake may be attributed to the "self-cancellation" phenomenon under study. Whether the exact phenomenon actually occurs in the form postulated could well be a subject for further discussion. Spatial distributions of the actual seismic wave displacement or acceleration time histories of earthquakes have rarely, if ever, been collected in the field. To date, mainly single-point time histories are the rule. If horizontal spatial distributions should prove to be more random and less deterministic than postulated above, the phenomenon of wave effect self-cancellation could still occur. One might in fact expect different time histories, differently cross-correlated, at different soil points "under" the foundation. Only simpler cases will be mentioned here, however.

#### 4. Other Cases

Soil motion transverse to the direction of wave propagation can be analyzed in a manner analogous to that done above. Details will not be presented here, but the main conclusions will be stated. Both transverse and torsional effects may generally be expected from passage of a transverse wave. Torsional forcing occurs even in the absence of inertial and elastic coupling of the structure about the center of gravity.

The net transverse shears exhibit an effect very analogous to that indicated in Fig. 3, and the net effective earthquake effect when integrated over a rigid foundation, can be considerably less than that anticipated from a uniform earthquake definition.

The net torsional moment developed has the following characteristics: for very long wavelengths it is negligible. It rises to a maximum and falls again in decreasing waves as the inverse wavelength parameter  $R_n$  increases.

When arbitrary waves traverse the structural site, a complex phenomenon occurs, but the tendency toward the decreasing of nominal "point earthquake" effects remains.

Table I

$f_n = \omega_n/2$	$\lambda_n$	$R_n$	$\bar{A}_n/A_n$
0	$\infty$	0	1
1 Hz	1000 m	0.408	0.99
3	333	1.226	0.94
5	200	2.042	0.84
10	100	4.084	0.44
20	50	8.168	0.20
30	33	13.614	0.07

References

- [1] Scanlan, R. H. and Sachs, K., "Earthquake Time Histories and Response Spectra", Jnl. Engrg. Mech. Div., ASCE, Vol. 100, Aug., 1974, pp. 635-655.
- [2] Bolt, B. A., "Elastic Waves in the Vicinity of the Earthquake Source", Ch. 1 of Earthquake Engineering, R. L. Wiegel, Ed., Prentice-Hall, Englewood Cliffs, N.J., 1970.



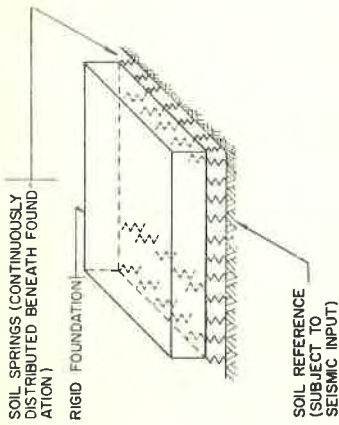


Fig 1 FOUNDATION MODEL ON SOIL SPRINGS

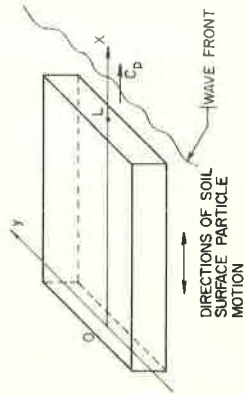


FIG. 2 P-WAVE MODEL

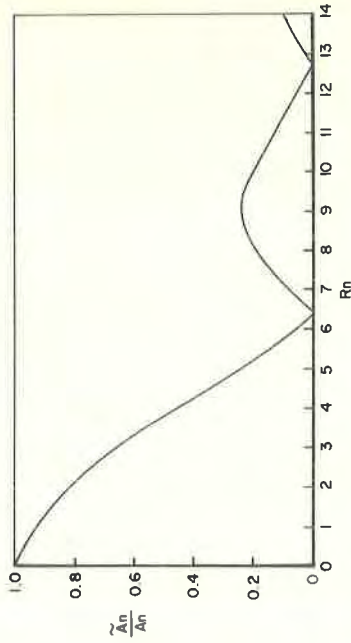


Fig. 3 WAVE AMPLITUDE REDUCTIONS vs WAVE NUMBER