Selected Mapping without Side Information for PAPR Reduction in OFDM

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***Abstract*—Selected mapping (SLM) is a technique used to reduce the peak-to-average power ratio (PAPR) in orthogonal frequency-division multiplexing (OFDM) systems. SLM requires the transmission of several side information bits for each data block, which results in some data rate loss. These bits must generally be channel-encoded because they are particularly critical to the error performance of the system. This increases the system complexity and transmission delay, and decreases the data rate even further. In this paper, we propose a novel SLM method for which no side information needs to be sent. By considering the example of several OFDM systems using either QPSK or 16-QAM modulation, we show that the proposed method performs very well both in terms of PAPR reduction and bit error rate at the receiver output provided that the number of subcarriers is large enough.**

***Index Terms*—Orthogonal frequency-division multiplexing (OFDM), peak-to-average power ratio (PAPR), selected mapping, side information.**

I. Introduction

HIGH peak-to-average power ratio (PAPR) is a well-known drawback of orthogonal frequency-division multiplexing (OFDM) systems. Among all the techniques that have been proposed to reduce the PAPR (see, e.g., [1] - [7]), selected mapping (SLM) is one of the most promising ones because it is simple to implement, introduces no distortion in the transmitted signal, and can achieve significant PAPR reduction [2]. The idea in SLM consists of converting the original data block into several independent signals, and then transmitting the signal that has the lowest PAPR. The selected signal index, called *side information index* (SI index), must also be transmitted to allow for the recovery of the data block at the receiver side, which leads to a reduction in data rate. This index is traditionally transmitted as a set of bits (the SI bits). The probability of erroneous SI detection has a significant influence on the error performance of the system since the whole data block is lost every time the receiver does not detect the correct SI index. In practice, a channel code must thus be used to protect the SI bits. This further reduces the data rate, makes the system more complex, and increases the transmission delay.

It may therefore be worth trying to implement the SLM method without having to explicitly send any SI bit. A few techniques for doing so have already been proposed such as, for example, the scrambling method described in [8] or the maximum-likelihood decoding scheme introduced in [9]. More recently, another method was proposed in [10] that combines channel estimation using *high-power* pilot tones and PAPR reduction via SLM. One of the key ideas in [10] consists of choosing the location of the pilot tone used inside each data sub-block depending on the SI index, and then exploit the power disparity between this pilot tone and the data symbols in the same sub-block in order to recover this index at the receiver side. The technique in [10] assumes the use of several pilot tones in each data block, which is not a suitable option for fading channels that change slowly. The practical limitations of the work in [10] can however be suppressed by leaving out the pilot tones and replacing them with data symbols. By doing so, we remove the restrictions regarding the possible locations of the high-power subcarriers inside the data block, and are therefore left with a problem which is much more general than that addressed in [10]. In this paper, we propose to address this problem by describing a novel SLM method without side information and studying its performance in terms of probability of erroneous SI detection, bit error rate (BER), and PAPR reduction. In our SLM method, each SI index is associated with a particular set of locations inside the data block at which the modulation symbols have been *extended*. In the receiver, a SI detection block attempts to determine the locations of the extended symbols.

The paper is organized as follows: In Section II, the proposed SLM technique is presented. Then, in Section III, we consider an example for which we provide various computer simulation results. Finally, conclusions are drawn in Section IV.

II. The Proposed SLM Technique without Side Information

In this section, a notation in the form $V = (v_q)_Q$ shall be used to denote a vector V composed of Q scalar quantities v_q, $q \in \{0, 1, ..., Q-1\}$.

A. The proposed SLM transmitter

Consider an OFDM system using N orthogonal subcarriers. A data block is a vector $X = (x_n)_N$ composed of N complex symbols x_n, each of them representing a modulation symbol transmitted over a subcarrier. In the classical SLM technique, X is multiplied element by element with U vectors $B_u = (b_{u,n})_N$ composed of N complex numbers $b_{u,n}$, $u \in \{0, 1, ..., U-1\}$, defined so that $|b_{u,n}| = 1$, where $|\cdot|$ denotes the modulus operator. Each resulting vector $X_u = (x_{u,n})_N$, where $x_{u,n} = b_{u,n} \cdot x_n$, produces, after inverse discrete Fourier transform, a corresponding vector X'_u composed of N symbols

Manuscript received May 3, 2007; revised August 22, 2007; accepted November 29, 2007. The associate editor coordinating the review of this paper and approving it for publication was X. Wang.

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Digital Object Identifier 10.1109/TWC.2009.070463

$x'_{u,q}$, $q \in \{0, 1, ..., N-1\}$, given by

$$x'_{u,q} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_{u,n} \cdot \exp\left(j\frac{2\pi nq}{N}\right). \quad (1)$$

Among the U vectors X'_u, the one leading to the lowest PAPR, denoted as X'_v, is selected for transmission. To allow for the recovery of the original vector X at the receiver side, one needs to transmit the index v of the selected vector. This index, hereafter called *SI index*, is generally transmitted as a set of $\log_2(U)$ bits.

In this paper, we propose a novel SLM technique that allows for the SI index v to be reliably embedded in the transmitted vector X'_v so that no additional bits need to be sent to the receiver. In this technique, the vectors $B_u = (b_{u,n})_N$, $u \in \{0, 1, ..., U-1\}$, are such that, for each B_u, the moduli of K elements $b_{u,n}$ are set to a constant $C > 1$, called *extension factor*, whereas the moduli of the other $(N-K)$ elements $b_{u,n}$ remain equal to the unit. Note that the phases of these elements can be set to any random values as in classical SLM. For a given vector B_u, the locations of the elements $b_{u,n}$ for which $|b_{u,n}| = C$ form a set S_u composed of K integers. For instance, if a vector B_u is associated with the set $\mathrm{S}_u = \{0, 7, 25, 37\}$, it means that only the complex elements $b_{u,n}$ in positions $n = 0, 7, 25$, and 37 in B_u have a modulus equal to C. The set S_u is actually representative of the index u. To allow for SI recovery in the receiver, there must be a one-to-one correspondence between an index u and a set S_u. In other words, two distinct vectors B_u must NOT be associated with identical sets. The number of distinct sets S_u that can be generated is given by the binomial coefficient $\binom{N}{K} = \frac{N!}{K!\cdot(N-K)!}$. Since the SI index can take U values, we only need to use U sets S_u among the $\binom{N}{K}$ available sets. In the next section, we will describe a simple method to select the U sets S_u.

Once all vectors B_u have been generated, our SLM method works exactly like the classical one, i.e. the data block X is multiplied element by element with each B_u so as to produce U vectors $X_u = (x_{u,n})_N$, with $x_{u,n} = b_{u,n} \cdot x_n$, as well as U corresponding vectors X'_u. In a given vector X_u, the symbols $x_{u,n}$ with $n \in \mathrm{S}_u$ have an average energy C^2 times greater than that of the other symbols, and are hereafter referred to as *extended symbols*. This disparity in average energies between extended and non-extended symbols is what allows the receiver to recover the SI index after transmission. Finally, the vector X'_u with the lowest PAPR is transmitted. We recall that, throughout this paper, this particular vector is denoted as X'_v and is associated with the vectors $X_v = (x_{v,n})_N$ and $B_v = (x_{v,n})_N$. The index v represents the SI index to be transmitted.

In the proposed SLM technique, the average energy per transmitted symbol is increased when the data block X is multiplied by the vectors B_u because the fact that $|x_{u,n}| = |b_{u,n}| \cdot |x_n|$, with $|b_{u,n}| = 1$ or C, implies that $E[|x_{u,n}|^2] > E[|x_n|^2]$, where $E[\cdot]$ designates the expectation operator. Note that any energy increase in X_u translates into an identical energy increase in X'_u (Parseval's theorem). This energy increase G, expressed in decibels (dB), is given by

$$G = 10 \cdot \log_{10}\left[1 + \delta \cdot (C^2 - 1)\right], \quad (2)$$

where $\delta = \frac{K}{N}$. In practice, the energy increase is compensated for by decreasing the average energy per complex symbol x_n, i.e. reducing the minimum Euclidean distance between signal points in the constellation from which the symbols x_n are drawn. Remarkably, this does not necessarily result in some significant error performance degradation at the receiver output, as will be seen later.

B. The proposed SLM receiver

In this paper, we assume transmission over a quasi-static frequency-selective Rayleigh fading channel with Z equal-power taps. For each transmitted symbol $x'_{v,q}$, $q \in \{0, 1, ..., N-1\}$, the corresponding received sample y'_q is thus given by

$$y'_q = \sum_{z=0}^{Z-1} h'_z \cdot x'_{v,q-z} + n'_q, \quad (3)$$

where h'_z is a complex zero-mean Gaussian sample representing the fading experienced by the zth tap. In (3), n'_q denotes a complex Gaussian noise sample with zero mean and variance $\sigma^2 = N_0$, where N_0 denotes the one-sided power spectral density of the additive white Gaussian noise (AWGN). We also assume that the Z fading samples h'_z are independent and perfectly known at the receiver side, i.e. perfect channel state information (CSI) is considered. Under these assumptions, we can show that, after discrete Fourier transform, the received sample y_n corresponding to the nth subcarrier, $n \in \{0, 1, ..., N-1\}$, is given by [11]

$$y_n = h_n \cdot x_{v,n} + n_n, \quad (4)$$

where

$$n_n = \frac{1}{\sqrt{N}} \sum_{q=0}^{N-1} n'_q \cdot \exp\left(-j\frac{2\pi nq}{N}\right) \quad (5)$$

is a complex zero-mean Gaussian noise sample with variance $\sigma^2 = N_0$ and

$$h_n = \sum_{z=0}^{Z-1} h'_z \cdot \exp\left(-j\frac{2\pi nz}{N}\right) \quad (6)$$

is a complex Gaussian noise sample with zero mean and unit variance.

The role of the SI detection block is to recover the index v by processing both vectors $Y = (y_n)_N$ and $H = (h_n)_N$, the latter being computed using (6). This block has the knowledge of the U possible vectors B_u and the one-to-one correspondence between an index u and a set S_u. Under these conditions, the optimal performance in terms of SI detection is obtained by applying the well-known maximum-likelihood (ML) detection algorithm: the receiver selects as SI index the value of u that minimizes the term

$$\sum_{n=0}^{N-1} |y_n - h_n b_{u,n} x_n|^2, \qquad u \in \{0, 1, ..., U-1\}. \quad (7)$$

When 2^p-ary complex symbols x_n are employed, the number of terms to compute is equal to $U \cdot 2^{pN}$, which leads to a prohibitive computational complexity for practical values of N. In fact, for each vector B_u to be considered, there are 2^{pN} possible sequences of symbols x_n, i.e. 2^{pN} possible vectors X_u (or X). In practice, it is thus necessary to use a suboptimal algorithm instead of ML detection. In this sub-section, we propose an algorithm that exploits the disparity in average energy between the K extended symbols and the $(N-K)$ non-extended symbols in vector $X_v = (x_{v,n})_N$ so as to recover the SI index v. Assume a particular location $n \in \{0, 1, ..., N-1\}$ in the received vector and its corresponding fading sample h_n. If the SI index was $u \in \{0, 1, ..., U-1\}$, the average received energy at location n in the absence of additive noise would be given by

$$E[|h_n|^2|x_{u,n}|^2] = |h_n|^2|b_{u,n}|^2\gamma, \tag{8}$$

where γ denotes the average energy per complex symbol x_n. At the same time, we can show that the average energy of the received sample y_n is given by

$$E[|y_n|^2] = |h_n|^2|b_{v,n}|^2\gamma + \sigma^2. \tag{9}$$

At this stage, we can introduce a metric $\alpha_{u,n}$ defined as

$$\alpha_{u,n} = \left| E[|y_n|^2] - \sigma^2 - |h_n|^2|b_{u,n}|^2\gamma \right|. \tag{10}$$

By combining (9) and (10), we can show that

$$\alpha_{u,n} = |h_n|^2\gamma \left| |b_{v,n}|^2 - |b_{u,n}|^2 \right|. \tag{11}$$

The minimal value of the metric $\alpha_{u,n}$ is equal to 0 and obtained for $|b_{v,n}| = |b_{u,n}|$. Extending this reasoning to the whole received vector rather than focusing on a particular location n leads us to introduce another metric β_u defined as

$$\beta_u = \sum_{n=0}^{N-1} \alpha_{u,n}. \tag{12}$$

The minimal value of this metric β_u is also equal to 0 and obtained when the condition $|b_{v,n}| = |b_{u,n}|$ is satisfied for all values of n, i.e. $B_v = B_u$ or equivalently $u = v$. This shows that the SI index can be recovered by determining, using (10) and (12), the value of u that minimizes the metric β_u. It is important to mention that, in the computation of (10), the receiver uses the sample y_n to approximate the term $E[|y_n|^2]$ as follows:

$$E[|y_n|^2] \approx |y_n|^2. \tag{13}$$

Evaluating an average by only considering a single sample may obviously lead to a gross estimate of this average. In case we use too many unreliable estimates of the terms $E[|y_n|^2]$, $n \in \{0, 1, ..., N-1\}$, in the computation of (10), the minimal value of the metric β_u may not be obtained for $u = v$, thus leading to an erroneous detection of the SI index. In order to minimize the probability of occurence of such an error event, we must ensure that the U vectors B_u are *as different as possible*, i.e. the number of locations n in two distinct vectors B_u and $B_{u'}$ for which $|b_{u,n}| \neq |b_{u',n}|$ is maximised. This should, in most cases, prevent a metric β_u, $u \in \{0, ..., v-1, v+1, ..., U-1\}$, from satisfying the condition $\beta_u < \beta_v$, even if we have $\alpha_{u,n} < \alpha_{v,n}$ for several locations $n \in \{0, 1, ..., N-1\}$.

III. Example

To illustrate the proposed SLM technique, we consider in this section the example of several OFDM systems based on either QPSK or 16-QAM modulation with Gray mapping. Our goal is to achieve PAPR reduction using $U = 10$ vectors B_u. All results given in this section are obtained by computer simulations considering the frequency-selective fading channel model previously described with $Z = 4$ taps. We also assume the use, at the transmitter output, of a nonlinear solid-state power amplifier (SSPA) simulated using Rapp's model [12] with a smoothness parameter $p = 3$ and an input backoff (IBO) of 7 dB.

A. Construction of the vectors B_u

In this sub-section, we present a technique to construct the set of $U = 10$ vectors $B_u = (b_{u,n})_N$, $u \in \{0, 1, ..., U-1\}$. First, for a given vector B_u, the phases of the complex elements $b_{u,n}$ can be chosen randomly, as already suggested in previous works (see, e.g., [13]). The first step of the procedure used for defining the moduli $|b_{u,n}|$ consists of dividing the vectors B_u into subvectors of length M, where M is the smallest possible integer satifying the condition $\binom{M}{k} \geq U$, k being any integer smaller than M. In our example, $U = 10$, and therefore $M = 5$ because $\binom{5}{k} = 10$ when $k = 2$ or 3. If U was equal to 11 instead of 10, we should have taken $M = 6$ because there is no integer k for which $\binom{5}{k} \geq 11$. Each vector B_u is thus viewed as a succession of $L = N/M$ subvectors, each of them composed of M complex elements. Before moving to the next step, we also have to select a particular value of k among all those satisfying the condition $\binom{M}{k} \geq U$. For reasons explained later, we keep the smallest value of the integer k for further considerations. In our example, we thus find $M = 5$ and $k = 2$.

Hereafter, we denote by $b_{m,l}$ the $(m+1)$th complex element in the $(l+1)$th subvector, $m \in \{0, 1, ..., M-1\}$, $l \in \{0, 1, ..., L-1\}$. In each subvector, the moduli of k elements are set to a constant $C > 1$, whereas the moduli of the other $(M-k)$ elements remain equal to the unit. The locations in each subvector of the elements $b_{m,l}$ for which $|b_{m,l}| = C$ are identical for each subvector, i.e. the moduli $|b_{m,l}|$ do not depend on the index $l \in \{0, 1, ..., L-1\}$. This implies that, once the M moduli have been defined for the first subvector (corresponding to $l = 0$), this pattern of moduli is simply repeated $(L-1)$ times in order to obtain the moduli for all locations $n \in \{M, M+1, ..., N-1\}$ in the corresponding vector B_u. The fact that $\binom{M}{k} \geq U$ implies that there are enough possible permutations of k *extended* elements in a M-element subvector in order to generate the U distinct sets S_u that are needed. As an example, Fig. 1 shows the $U = 10$ subvectors that can be generated when $M = 5$ and $k = 2$. Note that Fig. 1 only shows the moduli of the elements $b_{m,l}$ since their phases can basically take any random values.

The set of U vectors B_u, $u \in \{0, 1, ..., U-1\}$, obtained using the simple procedure described above is such that, when

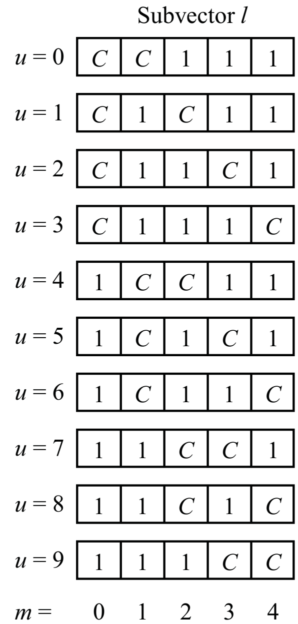

Fig. 1. List of the 10 possible subvectors that can be used to apply our SLM technique, when $M = 5$ and $k = 2$. We only show the moduli of the elements $b_{m,l}$ since their phases can take any random values.

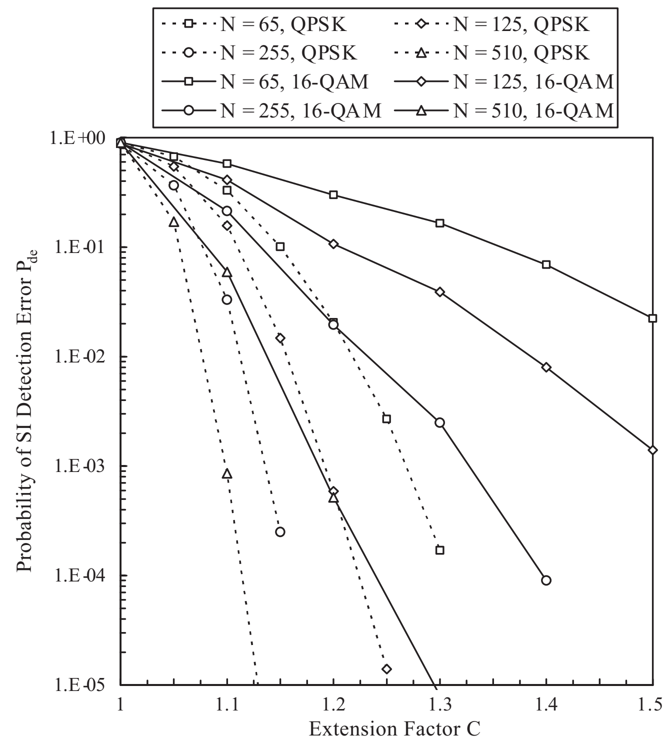

Fig. 2. Probability of side information detection error, P_{de}, obtained with the proposed SLM technique as a function of the extension factor C, for $N = 65$, 125, 255, and 510 subcarriers. The OFDM systems use either QPSK or 16-QAM modulation, with Gray mapping in both cases. The system parameters are: $E_b/N_0 = 10$ dB, quasi-static frequency-selective Rayleigh fading channel with $Z = 4$ equal-power taps and perfect CSI, suboptimal SI detection algorithm, $M = 5$, $k = 2$, SSPA with $p = 3$ and IBO = 7 dB.

considering any pair of vectors B_u and $B_{u'}$, $u \neq u'$, the number of locations n for which $|b_{u,n}| \neq |b_{u',n}|$ is *at least* equal to $\frac{2N}{M}$. In other words, the probability of erroneous SI detection can be lowered by increasing the ratio $\frac{N}{M}$. This is why it is crucial to minimize, for given values of N and U, the length M of the subvectors. With the construction procedure proposed in this sub-section, the total number of extended symbols in the transmitted vector X_v is given by $K = \frac{Nk}{M}$. Note that $\delta = \frac{K}{N} = \frac{k}{M}$. Hence, in order to minimize the energy increase given by (2), we must ensure to use the smallest possible value of k among all those satisfying the condition $\binom{M}{k} \geq U$.

B. Probability of SI detection error

Fig. 2 shows the probability of SI detection error, P_{de}, as a function of the extension factor C, for four different numbers of subcarriers, N = 65, 125, 255, and 510. We consider two OFDM systems using either QPSK or 16-QAM modulation. All results are obtained for $E_b/N_0 = 10$ dB, where E_b designates the average energy per bit and N_0 is the one-sided power spectral density of white Gaussian noise. The parameter P_{de} represents the probability that the receiver cannot recover the SI index v, i.e. a complete OFDM frame (vector X) is lost. Note that, for simplicity sake, the numbers of subcarriers are chosen to be multiples of $M = 5$ close to the usual powers of two used in practice. Finally, the suboptimal algorithm introduced in the previous section is used for recovering the SI index.

From Fig. 2, it is observed that the value of P_{de} depends on the extension factor C, the number N of subcarriers, and the modulation scheme that is employed. As C is increased, the performance of our suboptimal algorithm improves simply because a higher value of C allows for a better distinction between extended and non-extended symbols after transmission through the channel, which makes the occurence of an erroneous detection event less likely. Increasing the number N of subcarriers also results in a lower probability of detection error. We recall that, if we consider two distinct vectors B_u and $B_{u'}$, the minimum number of locations n for which $|b_{u,n}| \neq |b_{u',n}|$ is equal to $\frac{2N}{M}$. Hence, any increase in the number of subcarriers results in an increase in this number of locations, which then reduces the probability of an erroneous detection event.

The results in Fig. 2 also indicate that our SI detection algorithm performs better with QPSK than with 16-QAM. This can be explained by the fact that, in QPSK, the energy per symbol x_n is constant, i.e. we always have $|x_n|^2 = \gamma$. This is not the case for 16-QAM where the term $|x_n|^2$ can take three different values: $|x_n|^2 = \frac{\gamma}{5}$, γ, or $\frac{9\gamma}{5}$, meaning that the term $|x_n|^2$ may significantly differ from its average value γ. Hence, the corresponding received sample y_n, given by (4), has an energy $|y_n|^2$ that tends to deviate more from its average value $E[|y_n|^2]$ in 16-QAM than it does in QPSK. As a consequence, the approximation $E[|y_n|^2] \approx |y_n|^2$ used in the computation of (10) tends to be more accurate in QPSK than in 16-QAM, which makes the occurence of an erroneous detection event less likely with QPSK than with 16-QAM.

C. Bit error rate performance

It is also important to study the error performance degradation caused by the application of the technique proposed in this

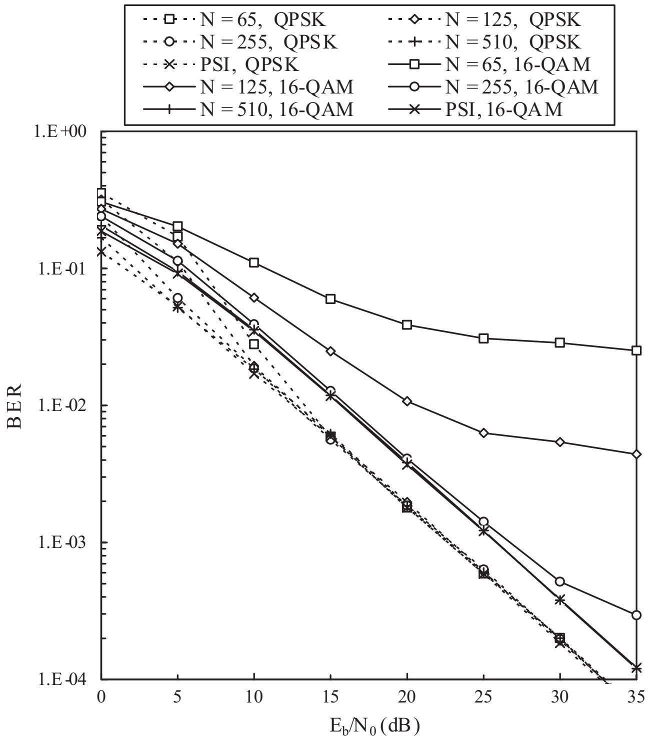

Fig. 3. BER performance of several OFDM systems using the proposed SLM technique, for $N = 65$, 125, 255, and 510 subcarriers. The OFDM systems use either QPSK or 16-QAM modulation, with Gray mapping in both cases. The system parameters are: $C = 1.2$, quasi-static frequency-selective Rayleigh fading channel with $Z = 4$ equal-power taps and perfect CSI, suboptimal SI detection algorithm, $M = 5$, $k = 2$, SSPA with $p = 3$ and IBO = 7 dB. For comparison purposes, the BER plots obtained with equivalent OFDM systems based on classical SLM with perfect side information (PSI) are also displayed.

paper. Such degradation is potentially due to both the energy increase G and the occasional SI detection error events. In Fig. 3, we have plotted the BER versus E_b/N_0 curves obtained for the OFDM systems based on QPSK and 16-QAM, for N = 65, 125, 255, and 510 subcarriers. For all simulations, the extension factor value is $C = 1.2$. For comparison purposes, we have also plotted the BER curves obtained with equivalent OFDM systems using classical SLM with perfect, i.e. error-free, side information (SLM-PSI). The latter scenario typically implies the use of a channel code entirely dedicated to the protection of the SI bits during transmission, which is probably more an ideal situation than a practical one.

Fig. 3 shows that the performance gap between our SLM technique and SLM-PSI can be made very small. When QPSK is employed, this gap is actually only significant for low SNRs and small numbers N of subcarriers, i.e. in situations where the probability of erroneous SI detection is sufficiently high to have an impact on the overall BER. Some unpublished simulation results indicated that, when using QPSK and a small number of subcarriers, the performance gap at low SNRs can be reduced by increasing the value of the extension factor C. When QPSK is replaced by 16-QAM, the performance difference between our method and SLM-PSI becomes significantly larger due to the increased probability of erroneous SI detection. However, as N is increased, the gap diminishes to the extent of actually becoming negligible for $N = 510$ subcarriers.

As a conclusion, if the probability of erroneous SI detection can be made small enough (e.g., by increasing the SNR and/or the number of subcarriers), the performance difference between our SLM method and SLM-PSI is actually marginal. This is *a priori* rather surprising given the fact that the energy increase per symbol obtained with our SLM method is $G = 0.70$ dB in this example (for which $C = 1.2$ and $\delta = \frac{k}{M} = \frac{2}{5}$). At first glance, we would therefore expect to have a performance difference of $G = 0.70$ dB in the absence of any erroneous SI detection event.

Hereafter, we propose to clarify this result by considering, for simplicity sake and without loss of generality, the example of BPSK and ignoring the presence of a nonlinear SSPA at the transmitter output. As previously mentioned, throughout our work, we actually compensate for the increase in average energy per transmitted symbol $x_{v,n}$ (or equivalently $x'_{v,q}$) by decreasing the average energy per symbol x_n, i.e. making the constellation *more compact*. This results in a reduced minimum Euclidean distance between signal points, and thus some error performance degradation at the receiver output. In the case of BPSK, the symbols $x_n \in \{\pm\sqrt{E_b}\}$ are simply replaced by the symbols $x_n \in \{\pm\sqrt{E_b/\omega}\}$, where $\omega = 1 + \delta \cdot (C^2 - 1)$. It is easy to show that extending K symbols by a factor C and leaving $(N-K)$ of them unchanged in a frame of N BPSK symbols $x_n \in \{\pm\sqrt{E_b/\omega}\}$ leads to an average energy per symbol equal to E_b, and thus identical to the energy per symbol obtained using classical SLM with $x_n \in \{\pm\sqrt{E_b}\}$.

If the transmission channel over each BPSK subcarrier can be modeled as AWGN, the bit error probability p_{eb} obtained with the proposed SLM method in the absence of any SI detection error can then be expressed as

$$p_{eb} = \frac{1-\delta}{2} \cdot \mathrm{erfc}\left(\sqrt{\frac{1}{\omega} \cdot \frac{E_b}{N_0}}\right) + \frac{\delta}{2} \cdot \mathrm{erfc}\left(\sqrt{\frac{C^2}{\omega} \cdot \frac{E_b}{N_0}}\right). \quad (14)$$

This equation indicates that, at high SNR, the performance gap over AWGN channel between our SLM method and SLM-PSI is equal to $10 \cdot \log_{10}[\omega]$ dB, which corresponds exactly to the energy increase G given by (2). However, as shown by (4), the transmission channel over each subcarrier can actually be modeled more accurately as flat Rayleigh fading rather than AWGN. Based on (14), we can demonstrate that the bit error probability for BPSK over flat Rayleigh fading channel is given by [14]

$$p_{eb} \approx \frac{1-\delta}{2\omega'} \cdot \int_0^{+\infty} \mathrm{erfc}\left(\sqrt{x}\right) \cdot \exp\left(-\frac{x}{\omega'}\right) \cdot dx + \frac{\delta}{2\omega' \cdot C^2} \cdot \int_0^{+\infty} \mathrm{erfc}\left(\sqrt{x}\right) \cdot \exp\left(-\frac{x}{\omega' \cdot C^2}\right) \cdot dx, \quad (15)$$

where $\omega' = \frac{1}{\omega} \cdot \frac{E_b}{N_0}$. This equation has the simple closed-form expression

$$p_{eb} = \frac{1}{2} \cdot \left[1 - (1-\delta) \cdot \left(1 + \frac{1}{\omega'}\right)^{-\frac{1}{2}} - \delta \cdot \left(1 + \frac{1}{C^2 \cdot \omega'}\right)^{-\frac{1}{2}}\right], \quad (16)$$

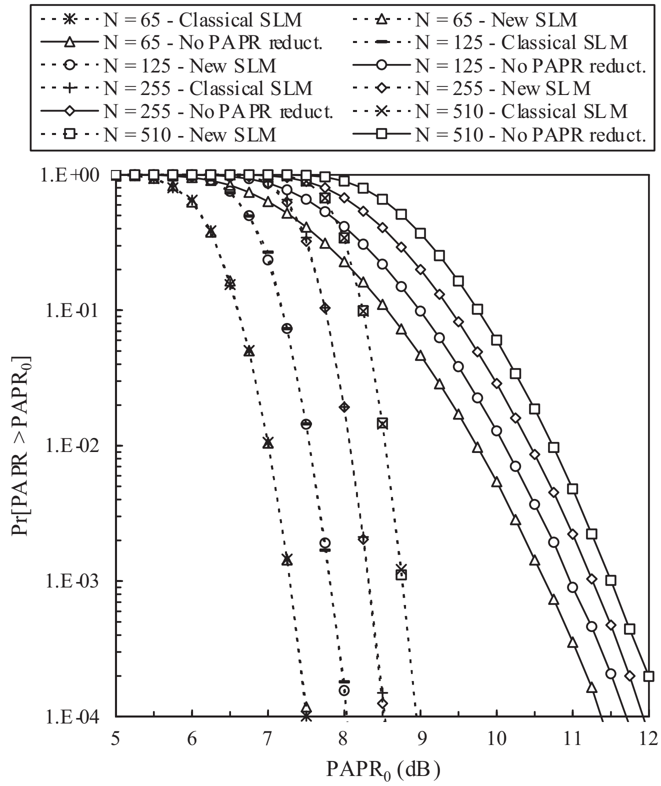

Fig. 4. CCDF of the PAPR obtained with the proposed SLM technique (*New SLM*), for $N = 65$, 125, 255, and 510 subcarriers. The OFDM systems use 16-QAM modulation with Gray mapping. The system parameters are: $C = 1.2$, $M = 5$, $k = 2$, oversampling factor of 4. For comparison purposes, we also display for each value of N the plots obtained with classical SLM as well as those associated with an equivalent OFDM system without any PAPR reduction.

which can be approximated at high SNR ($\omega' \to +\infty$) using

$$p_{eb} \approx \frac{\left[1+\delta \cdot (C^2-1)\right] \cdot \left[C^2-\delta \cdot (C^2-1)\right]}{4C^2} \cdot \left(\frac{E_b}{N_0}\right)^{-1}. \quad (17)$$

Using (17), it is possible to evaluate the expected error performance gap at high SNR between our SLM technique and SLM-PSI in the absence of any erroneous SI detection event. Note that the bit error probability expression for SLM-PSI is obtained by computing (17) with $C = 1$ and $\delta = 0$. When $C = 1.2$ and $\delta = \frac{2}{5}$, we find a gap of approximately 0.14 dB, which is indeed significantly smaller than the energy increase $G = 0.70$ dB. This theoretical result is therefore coherent with the observations made from the BER plots in Fig. 3.

D. PAPR reduction performance

Finally, Fig. 4 shows the complementary cumulative distribution function (CCDF) of the PAPR obtained with the proposed SLM technique when using 16-QAM modulation, for $C = 1.2$ and $N = 65$, 125, 255, and 510 subcarriers. The plots corresponding to the ordinary OFDM without PAPR reduction and the classical SLM method are also depicted for comparison purposes. These results are obtained by using an oversampling factor equal to 4 [15]. We observe that, for all configurations, the performance of our SLM method in terms of PAPR reduction is identical to that of classical SLM. Note that the results obtained with the proposed SLM method take into account the increase in average energy given by (2).

IV. CONCLUSION

We have proposed a simple SLM technique for PAPR reduction in OFDM that does not require the explicit transmission of SI bits. Our investigations, performed by considering OFDM schemes based on QPSK and 16-QAM modulations, have shown that this technique is particularly attractive for systems using a large number of subcarriers. In fact, the probability of SI detection error can be made very small by increasing the extension factor and/or the number of subcarriers. In cases where this probability becomes sufficiently low, the BER performance difference between the proposed technique and classical SLM using error-free side information has been shown to be negligible. In addition, the application of our SLM technique leads to a reduction in PAPR which is identical to that obtained with classical SLM, for any number of subcarriers. For OFDM systems with a large number of subcarriers, the only significant price to pay for employing the proposed technique instead of classical SLM is a slight complexity increase at the receiver side due to the use of a SI detection block.

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