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# Selected Preserver Problems on Algebraic Structures of Linear Operators and on Function Spaces



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# Contents

<b>Introduction</b> .....	1
0.1 Linear Preserver Problems .....	1
0.2 Survey of the Results of Chapter 1 .....	9
0.3 Some Structures of Linear Operators in Quantum Mechanics ..	11
0.4 Survey of the Results of Chapter 2 .....	16
0.5 Local Derivations, Local Automorphisms and Local Isometries of Operator Algebras and Function Algebras .....	21
0.6 Survey of the Results of Chapter 3 .....	25
0.7 Notation .....	27
<b>1 Some Linear and Multiplicative Preserver Problems on Operator Algebras and Function Algebras</b> .....	29
1.1 Some Linear Preserver Problems on $B(H)$ Concerning Rank and Corank .....	29
1.1.1 Summary .....	29
1.1.2 Formulation of the Results .....	29
1.1.3 Proofs .....	32
1.1.4 Remarks .....	38
1.2 Linear Maps on Factors Which Preserve the Extreme Points of the Unit Ball .....	39
1.2.1 Summary .....	39
1.2.2 Formulation of the Results .....	39
1.2.3 Proofs .....	41
1.2.4 Remarks .....	46
1.3 Diameter Preserving Linear Bijections of $C(X)$ .....	46
1.3.1 Summary .....	46
1.3.2 Formulation of the Result .....	47
1.3.3 Proof .....	48
1.3.4 Remarks .....	56
1.4 $*$ -Semigroup Endomorphisms of $B(H)$ .....	57
1.4.1 Summary .....	57

1.4.2	Formulation of the Results	58
1.4.3	Proof	59
1.4.4	Remarks	64
<b>2</b>	<b>Preservers on Quantum Structures</b>	<b>65</b>
2.1	Transformations on the Set of All $n$ -Dimensional Subspaces of a Hilbert Space Preserving Principal Angles	65
2.1.1	Summary	65
2.1.2	Formulation of the Main Result	65
2.1.3	Proof	67
2.1.4	Remarks	78
2.2	Orthogonality Preserving Transformations on Indefinite Inner Product Spaces: Generalization of Uhlhorn's Version of Wigner's Theorem	79
2.2.1	Summary	79
2.2.2	Formulation of the Results	79
2.2.3	Proofs	84
2.2.4	Remarks	92
2.3	Fidelity Preserving Maps on Quantum States	92
2.3.1	Summary	92
2.3.2	Formulation of the Results	93
2.3.3	Proofs	94
2.3.4	Remarks	97
2.4	Isometries of Quantum States	97
2.4.1	Summary	97
2.4.2	Formulation of the Results	97
2.4.3	Proofs	99
2.4.4	Remarks	104
2.5	Order Automorphisms of the Set of Bounded Observables	104
2.5.1	Summary	104
2.5.2	Formulation of the Results	104
2.5.3	Proofs	106
2.6	Linear Maps on the Set of Bounded Observables Preserving Maximal Deviation	112
2.6.1	Summary	112
2.6.2	Formulation of the Results	112
2.6.3	Proofs	116
2.6.4	Remarks	129
2.7	Some Preservers on Hilbert Space Effects	130
2.7.1	Summary	130
2.7.2	Formulation of the Results	130
2.7.3	Proofs	134
2.7.4	Remarks	141
2.8	Sequential Isomorphisms Between the Sets of von Neumann Algebra Effects	143

2.8.1	Summary	143
2.8.2	Formulation of the Results	143
2.8.3	Proof	145
2.8.4	Remarks	157
<b>3</b>	<b>Local Automorphisms and Local Isometries of Operator Algebras and Function Algebras</b>	<b>159</b>
3.1	The Automorphism and Isometry Groups of $B(H)$ Are Topologically Reflexive	159
3.1.1	Summary	159
3.1.2	Formulation of the Results	159
3.1.3	Proofs	160
3.1.4	Remarks	168
3.2	Reflexivity of the Automorphism and Isometry Groups of $C(X)$	169
3.2.1	Summary	169
3.2.2	Formulation of the Result	169
3.2.3	Proof	170
3.2.4	Remarks	171
3.3	Reflexivity of the Automorphism and Isometry Groups of the Suspension of $B(H)$	172
3.3.1	Summary	172
3.3.2	Formulation of the Results	172
3.3.3	Proofs	173
3.3.4	Remarks	185
3.4	2-Local Automorphisms of Operator Algebras on Banach Spaces	186
3.4.1	Summary	186
3.4.2	Formulation of the Results	187
3.4.3	Proofs	188
3.4.4	Remarks	195
3.5	2-Local Automorphisms of Some Quantum Structures	195
3.5.1	Summary	195
3.5.2	Formulation of the Results	196
3.5.3	Proofs	197
3.5.4	Remarks	203
	<b>Appendix</b>	<b>205</b>
	<b>Recent Results Added in Revision</b>	<b>211</b>
	<b>References</b>	<b>217</b>
	<b>Index</b>	<b>231</b>