Vol. 38 (2007) *ACTA PHYSICA POLONICA B* No 11

SELECTED TOPICS ON TAU PHYSICS[*]

A. Pich

Departament de Física Teòrica, IFIC, Universitat de València–CSIC,
Apt. de Correus 22085, E-46071 València, Spain

(Received October 22, 2007)

The B factories have generated a large amount of new results on the τ lepton. The present status of some selected topics on τ physics is presented: charged-current universality tests, bounds on lepton-flavour violation, the determination of α_{s} from the inclusive τ hadronic width, and the measurement of $|V_{us}|$ through the Cabibbo-suppressed decays of the τ lepton.

PACS numbers: 14.60.Fg, 13.35.–r, 12.15.–y, 12.38.–t

1. Introduction

The known leptons provide clean probes to perform very precise tests of the Standard Model and search for signals of new dynamics. The electroweak gauge structure has been successfully tested at the 0.1% to 1% level, confirming the Standard Model framework [1]. Moreover, the hadronic τ decays turn out to be a beautiful laboratory for studying strong interaction effects at low energies [2–5]. The τ is the only known lepton massive enough to decay into hadrons. Its semileptonic decays are then ideally suited for studying the hadronic weak currents in very clean conditions. Accurate determinations of the QCD coupling, $|V_{us}|$ and the strange quark mass have been obtained with τ decay data.

The huge statistics accumulated at the B factories allow to explore lepton-flavour-violating τ decay modes with increased sensitivities beyond 10^{-7}, which could be further pushed down to few 10^{-9} at future facilities. Moreover, BESIII will soon start taking data at the new Beijing Tau-Charm Factory. With the excellent experimental conditions of the threshold region, complementary information on the τ should be obtained, such as an improved mass measurement. Thus, τ physics is entering a new era, full of interesting possibilities and with a high potential for new discoveries.

---

[*] Presented at the XXXI International Conference of Theoretical Physics, "Matter to the Deepest", Ustroń, Poland, September 5–11, 2007.

2. Tests on charged-current universality

In the Standard Model all lepton doublets have identical couplings to the W boson. Comparing the measured decay widths of leptonic or semileptonic decays which only differ in the lepton flavour, one can test experimentally that the W interaction is indeed the same, *i.e.* that $g_e = g_\mu = g_\tau \equiv g$. As shown in Table I, the present data verify the universality of the leptonic charged-current couplings to the 0.2% level.

TABLE I

Experimental determinations of the ratios $g_l/g_{l'}$.

	$\Gamma_{\tau\to e}/\Gamma_{\mu\to e}$	$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$	$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$	$\Gamma_{W\to\tau}/\Gamma_{W\to\mu}$
$\vert g_\tau/g_\mu\vert$	1.0006 (22)	0.996 (5)	0.979 (17)	1.039 (13)
	$\Gamma_{\tau\to\mu}/\Gamma_{\tau\to e}$	$\Gamma_{\pi\to\mu}/\Gamma_{\pi\to e}$	$\Gamma_{K\to\mu}/\Gamma_{K\to e}$	$\Gamma_{K\to\pi\mu}/\Gamma_{K\to\pi e}$
$\vert g_\mu/g_e\vert$	1.0000 (20)	1.0021 (16)	1.004 (7)	1.0021 (25)
	$\Gamma_{W\to\mu}/\Gamma_{W\to e}$		$\Gamma_{\tau\to\mu}/\Gamma_{\mu\to e}$	$\Gamma_{W\to\tau}/\Gamma_{W\to e}$
$\vert g_\mu/g_e\vert$	0.997 (10)	$\vert g_\tau/g_e\vert$	1.0005 (23)	1.036 (14)

The τ leptonic branching fractions and the τ lifetime are known with a precision of 0.3% [4]. A slightly improved lifetime measurement could be expected from BaBar and Belle [6]. For comparison, the μ lifetime is already known with an accuracy of 10^{-5}, which should be further improved to 10^{-6} by the MuLan experiment at PSI [7]. The universality tests require also a good determination of m_τ^5, which is only known to the 0.06% level [8]. Two new measurements of the τ mass have been published recently:

$$m_\tau = \begin{cases} 1776.61 \pm 0.13 \pm 0.35 \text{ MeV} & \text{[Belle]}\,, \\ 1776.81\,^{+\,0.25}_{-\,0.23} \pm 0.15 \text{ MeV} & \text{[KEDR]}\,. \end{cases}$$

Belle [9] has made a pseudomass analysis of $\tau \to \nu_\tau 3\pi$ decays, while KEDR [10] measures the $\tau^+\tau^-$ threshold production, taking advantage of a precise energy calibration through the resonance depolarization method. In both cases the achieved precision is getting close to the previous BES-dominated value, $m_\tau = 1776.99\,^{+\,0.29}_{-\,0.26}$ [8]. KEDR aims to obtain a final accuracy of 0.15 MeV. A precision better than 0.1 MeV should be easily achieved at BESIII [11], through a detailed analysis of $\sigma(e^+e^- \to \tau^+\tau^-)$ at threshold [12–14].

Table I shows also the constraints obtained from pion [8] and kaon decays [15], applying the recently calculated radiative corrections at NLO in chiral perturbation theory [16,17]. The accuracy achieved with K_{l3} data is already comparable to the one obtained from τ or π_{l2} decays.

Owing to the limited statistics available, the decays $W^- \to l^-\nu_l$ only test universality at the 1% level. At present, $\mathrm{BR}(W \to \nu_\tau\tau)$ is $2.1\,\sigma/2.7\,\sigma$ larger than $\mathrm{BR}(W \to \nu_e e/\nu_\mu\mu)$ [18]. The stringent limits on $|g_\tau/g_{e,\mu}|$ from W-mediated decays make unlikely that this is a real effect.

3. Lepton-flavour violating decays

TABLE II

Best limits (90% C.L.) on lepton-flavour-violating decays [8, 19, 20].

$\mu^- \to X^-$					
$e^-\gamma$	1.2×10^{-11}	$e^-2\gamma$	7.2×10^{-11}	$e^-e^-e^+$	1.0×10^{-12}
$\tau^- \to X^-$					
$e^-\gamma$	1.1×10^{-7}	$e^-e^+e^-$	3.6×10^{-8}	$e^-\mu^+\mu^-$	3.7×10^{-8}
$\mu^-\gamma$	4.5×10^{-8}	$\mu^-e^+e^-$	2.7×10^{-8}	$\mu^-\mu^+\mu^-$	3.2×10^{-8}
$e^-e^-\mu^+$	2.0×10^{-8}	$\mu^-\mu^-e^+$	2.3×10^{-8}	$\bar{\Lambda}\pi^-$	1.4×10^{-7}
$e^-\pi^0$	8.0×10^{-8}	$e^-\eta$	9.2×10^{-8}	$e^-\eta'$	1.6×10^{-7}
$\mu^-\pi^0$	1.1×10^{-7}	$\mu^-\eta$	6.5×10^{-8}	$\mu^-\eta'$	1.3×10^{-7}
$e^-\rho^0$	6.5×10^{-7}	$e^-\omega$	1.8×10^{-7}	$e^-\phi$	7.6×10^{-8}
$\mu^-\rho^0$	2.0×10^{-7}	$\mu^-\omega$	9.0×10^{-8}	$\mu^-\phi$	1.3×10^{-7}
e^-K_S	5.6×10^{-8}	e^-K^{*0}	8.0×10^{-8}	$e^-\bar{K}^{*0}$	7.7×10^{-8}
μ^-K_S	4.9×10^{-8}	μ^-K^{*0}	6.1×10^{-8}	$\mu^-\bar{K}^{*0}$	1.1×10^{-7}
$e^-K^+K^-$	1.4×10^{-7}	$e^-K^+\pi^-$	1.6×10^{-7}	$e^-\pi^+K^-$	3.2×10^{-7}
$\mu^-K^+K^-$	2.5×10^{-7}	$\mu^-K^+\pi^-$	3.2×10^{-7}	$\mu^-\pi^+K^-$	2.6×10^{-7}
$e^-\pi^+\pi^-$	1.2×10^{-7}	$\mu^-\pi^+\pi^-$	2.9×10^{-7}	$\Lambda\pi^-$	7.2×10^{-8}
$e^+\pi^-\pi^-$	2.0×10^{-7}	$e^+K^-K^-$	1.5×10^{-7}	$e^+\pi^-K^-$	1.8×10^{-7}
$\mu^+\pi^-\pi^-$	0.7×10^{-7}	$\mu^+K^-K^-$	4.4×10^{-7}	$\mu^+\pi^-K^-$	2.2×10^{-7}

We have now clear experimental evidence that neutrinos are massive particles and there is mixing in the lepton sector. The smallness of neutrino masses implies a strong suppression of neutrinoless lepton-flavour-violating processes, which can be avoided in models with other sources of lepton flavour violation, not related to m_{ν_i}. The scale of the flavour-violating new-physics interactions can be constrained imposing the requirement of a viable leptogenesis. Recent studies within different new-physics scenarios find interesting correlations between μ and τ lepton-flavour-violating decays, with $\mu \to e\gamma$ often expected to be close to the present exclusion limit [21–27].

The B factories are pushing the experimental limits on neutrinoless lepton-flavour-violating τ decays beyond the 10^{-7} level [19,20], increasing in a drastic way the sensitivity to new physics scales. Future experiments could push further some limits to the 10^{-9} level [28], allowing to explore interesting and totally unknown phenomena. Complementary information will be provided by the MEG experiment, which will search for $\mu^+ \to e^+\gamma$ events with a sensitivity of 10^{-13} [29]. There are also ongoing projects at J-PARC aiming to study $\mu \to e$ conversions in muonic atoms, at the 10^{-16} [30] or even 10^{-18} [31] level.

4. The inclusive hadronic width of the τ lepton

The hadronic decays of the τ lepton provide a very clean laboratory to perform precise tests of the Standard Model [4]. The inclusive character of the total τ hadronic width renders possible an accurate calculation of the ratio [32–36]

$$R_\tau \;\equiv\; \frac{\Gamma[\tau^- \to \nu_\tau \,\mathrm{hadrons}\,(\gamma)]}{\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]} \;=\; R_{\tau,V} + R_{\tau,A} + R_{\tau,S}\,, \tag{1}$$

using analyticity constraints and the Operator Product Expansion. One can separately compute the contributions associated with specific quark currents: $R_{\tau,V}$ and $R_{\tau,A}$ correspond to the Cabibbo-allowed decays through the vector and axial-vector currents, while $R_{\tau,S}$ contains the remaining Cabibbo-suppressed contributions.

The theoretical prediction for $R_{\tau,V+A}$ can be expressed as [34]

$$R_{\tau,V+A} \;=\; N_C\, |V_{ud}|^2\, S_{\mathrm{EW}}\, \{1 + \delta_{\mathrm{P}} + \delta_{\mathrm{NP}}\}\,, \tag{2}$$

where $N_C = 3$ is the number of quark colours and $S_{\mathrm{EW}} = 1.0201 \pm 0.0003$ contains the electroweak radiative corrections [37–39]. The dominant correction ($\sim 20\%$) is the perturbative QCD contribution δ_{P}, which is fully known to $O(\alpha_{\mathrm{s}}^3)$ [34] and includes a resummation of the most important higher-order effects [35,40].

Non-perturbative contributions are suppressed by six powers of the τ mass [34] and, therefore, are very small. Their numerical size has been determined from the invariant-mass distribution of the final hadrons in τ decay, through the study of weighted integrals [41],

$$R_\tau^{kl} \;\equiv\; \int_0^{m_\tau^2} ds \left(1 - \frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_\tau}{ds}\,, \tag{3}$$

which can be calculated theoretically in the same way as R_τ. The predicted suppression [34] of the non-perturbative corrections has been confirmed by ALEPH [42], CLEO [43] and OPAL [44]. The most recent analysis [42] gives

$$\delta_{\rm NP} \; = \; -0.0043 \pm 0.0019 \, . \tag{4}$$

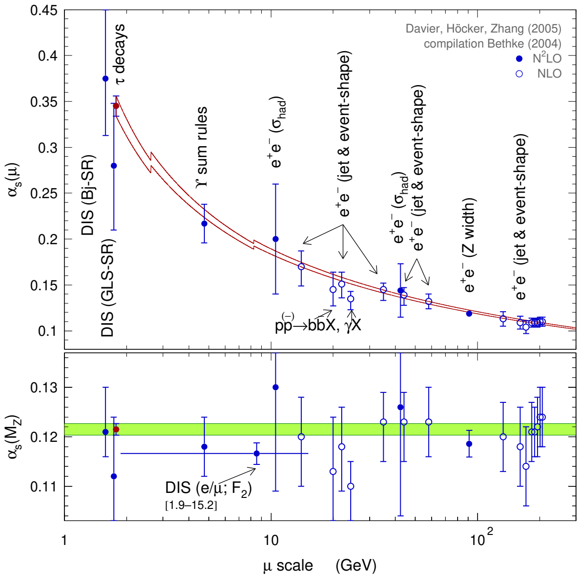

Fig. 1. Measured values of $\alpha_{\rm s}$ at different scales. The curves show the energy dependence predicted by QCD, using $\alpha_{\rm s}(m_\tau^2)$ as input. The corresponding extrapolated $\alpha_{\rm s}(M_Z^2)$ values are shown at the bottom, where the shaded band displays the τ decay result within errors [45].

The QCD prediction for $R_{\tau,V+A}$ is then completely dominated by the perturbative contribution; non-perturbative effects being smaller than the perturbative uncertainties from uncalculated higher-order corrections. The result turns out to be very sensitive to the value of $\alpha_{\rm s}(m_\tau)$, allowing for an accurate determination of the fundamental QCD coupling [33, 34]. The experimental measurement $R_{\tau,V+A} = 3.471 \pm 0.011$ implies [45]

$$\alpha_{\rm s}(m_\tau) \; = \; 0.345 \pm 0.004_{\rm exp} \pm 0.009_{\rm th} \, . \tag{5}$$

The strong coupling measured at the τ mass scale is significantly larger than the values obtained at higher energies. From the hadronic decays of the Z, one gets $\alpha_{\rm s}(M_Z) = 0.1186 \pm 0.0027$ [18], which differs from $\alpha_{\rm s}(m_\tau)$ by more than twenty standard deviations. After evolution up to the scale M_Z [46], the strong coupling constant in (5) decreases to [45]

$$\alpha_{\rm s}(M_Z) \; = \; 0.1215 \pm 0.0012 \, , \tag{6}$$

in excellent agreement with the direct measurements at the Z peak and with a similar accuracy. The comparison of these two determinations of $\alpha_{\rm s}$ in two very different energy regimes, m_τ and M_Z, provides a beautiful test of the predicted running of the QCD coupling; *i.e.*, a very significant experimental verification of *asymptotic freedom.*

5. $|V_{us}|$ determination from Cabibbo-suppressed tau decays

The separate measurement of the $|\Delta S| = 0$ and $|\Delta S| = 1$ tau decay widths provides a very clean determination of V_{us} [47, 48]. To a first approximation the Cabibbo mixing can be directly obtained from experimental measurements, without any theoretical input. Neglecting the small SU(3)-breaking corrections from the $m_s - m_d$ quark-mass difference, one gets:

$$|V_{us}|^{\rm SU(3)} \; = \; |V_{ud}| \, \left(\frac{R_{\tau,S}}{R_{\tau,V+A}}\right)^{1/2} \; = \; 0.210 \pm 0.003 \qquad [0.215 \pm 0.003] \, . \quad (7)$$

We have used $|V_{ud}| = 0.97377 \pm 0.00027$ [8], $R_\tau = 3.640 \pm 0.010$ and the value $R_{\tau,S} = 0.1617 \pm 0.0040$ [48], which results from the most recent BaBar [49] and Belle [50] measurements of Cabibbo-suppressed tau decays. The new branching ratios measured by BaBar and Belle are all smaller than the previous world averages, which translates into a smaller value of R_S and $|V_{us}|$. For comparison, we give in brackets the result obtained with the previous value $R_{\tau,S} = 0.1686 \pm 0.0047$ [45].

This rather remarkable determination is only slightly shifted by the small SU(3)-breaking contributions induced by the strange quark mass. These corrections can be theoretically estimated through a QCD analysis of the differences [47, 48, 51–59]

$$\delta R_\tau^{kl} \; \equiv \; \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} \, . \quad (8)$$

Since the strong interactions are flavour blind, these quantities vanish in the SU(3) limit. The only non-zero contributions are proportional to powers of the quark mass-squared difference $m_s^2(m_\tau) - m_d^2(m_\tau)$ or to vacuum expectation values of SU(3)-breaking operators such as $\delta O_4 \equiv \langle 0| m_s \bar s s - m_d \bar d d |0\rangle \approx (-1.4 \pm 0.4) \times 10^{-3}$ GeV^4 [47, 52]. The dimensions of these operators are compensated by corresponding powers of m_τ^2, which implies a strong suppression of δR_τ^{kl} [52]:

$$\delta R_\tau^{kl} \; \approx \; 24 \, S_{\rm EW} \left\{ \frac{m_s^2(m_\tau)}{m_\tau^2} \left(1 - \varepsilon_d^2\right) \Delta_{kl}(\alpha_{\rm s}) - 2\pi^2 \, \frac{\delta O_4}{m_\tau^4} \, Q_{kl}(\alpha_{\rm s}) \right\} , \quad (9)$$

where $\varepsilon_d \equiv m_d/m_s = 0.053 \pm 0.002$ [60]. The perturbative QCD corrections $\Delta_{kl}(\alpha_{\rm s})$ and $Q_{kl}(\alpha_{\rm s})$ are known to $O(\alpha_{\rm s}^3)$ and $O(\alpha_{\rm s}^2)$, respectively [52, 59].

The theoretical analysis of $\delta R_\tau \equiv \delta R_\tau^{00}$ involves the two-point correlation functions of vector and axial-vector quark currents, which can be separated into their transverse ($J = 1$) and longitudinal ($J = 0$) components. The longitudinal contribution to $\Delta_{00}(\alpha_{\rm s})$ shows a rather pathological behaviour, with clear signs of being a non-convergent perturbative series. Fortunately, the corresponding longitudinal contribution to δR_τ can be estimated phenomenologically with a much better accuracy, $\delta R_\tau|^{\rm L} = 0.1544 \pm 0.0037$ [47, 61], because it is dominated by far by the well-known $\tau \to \nu_\tau \pi$ and $\tau \to \nu_\tau K$ contributions. To estimate the remaining transverse component, one needs an input value for the strange quark mass. Taking the range $m_s(m_\tau) = (100 \pm 10)\,\text{MeV}$ $[m_s(2\,\text{GeV}) = (96 \pm 10)\,\text{MeV}]$, which includes the most recent determinations of m_s from QCD sum rules and lattice QCD [61], one gets finally $\delta R_{\tau,\rm th} = 0.216 \pm 0.016$, which implies [48]

$$|V_{us}| = \left(\frac{R_{\tau,S}}{\frac{R_{\tau,V+A}}{|V_{ud}|^2} - \delta R_{\tau,\rm th}} \right)^{1/2} = \left\{ \begin{array}{c} 0.2165 \pm 0.0026_{\,\rm exp} \pm 0.0005_{\,\rm th} \\ \\ [0.2212 \pm 0.0031_{\,\rm exp} \pm 0.0005_{\,\rm th}] \end{array} \right. . \tag{10}$$

Again, the first number is the updated value, including the recent BaBar and Belle data, while the number inside brackets gives the previous result.

Sizeable changes on the experimental determination of $R_{\tau,S}$ are to be expected from the full analysis of the huge BaBar and Belle data samples. In particular, the high-multiplicity decay modes are not well known at present and their effect has been just roughly estimated or simply ignored. Thus, the result (10) could easily fluctuate in the near future. However, it is important to realize that the final error of the V_{us} determination from τ decay is completely dominated by the experimental uncertainties. If $R_{\tau,S}$ is measured with a 1% precision, the resulting V_{us} uncertainty will get reduced to around 0.6%, *i.e.* ±0.0013, making τ decay the best source of information about V_{us}.

An accurate experimental measurement of the invariant-mass distribution of the final hadrons in Cabibbo-suppressed τ decays could make possible a simultaneous determination of V_{us} and the strange quark mass, through a correlated analysis of several weighted differences δR_τ^{kl}. The extraction of m_s suffers from theoretical uncertainties related to the convergence of the perturbative series $\Delta_{kl}^{\rm L+T}(\alpha_{\rm s})$, which makes necessary a better understanding of these corrections. Further work in this direction is in progress [62].

I want to thank Michal Czakon, Henryk Czyż and Janusz Gluza for organizing an enjoyable conference. This work has been supported by the Spanish Ministry of Education (Grant FPA2004-00996), by Generalitat Valenciana (GVACOMP2007-156), by EU funds for regional development and by the EU MRTN-CT-2006-035482 (FLAVIA*net*).

REFERENCES

[1] A. Pich, *The Standard Model of Electroweak Interactions*, in Proc. 2006 European School of High-Energy Physics (Aronsborg, Sweden), ed. R. Fleischer, CERN-2007-005 p. 1, `0705.4264 [hep-ph]`.

[2] A. Pich, Tau Physics, in *Heavy Flavours II*, eds. A.J. Buras, M. Lindner, Advanced Series on Directions in High Energy Physics 15, World Scientific, Singapore 1998, p. 453 [`hep-ph/9704453`].

[3] A. Pich, *Int. J. Mod. Phys.* **A21**, 5652 (2006).

[4] A. Pich, *Nucl. Phys. B (Proc. Suppl.)* **169**, 393 (2007) [`hep-ph/0702074`].

[5] A. Stahl, *Physics with Tau Leptons*, Springer Tracts in Modern Physics 160, Springer, Berlin 2000.

[6] A. Lusiani, *Nucl. Phys. B (Proc. Suppl.)* **144**, 105 (2005).

[7] MuLan Collaboration, *Phys. Rev. Lett.* **99**, 032001 (2007).

[8] W.-M. Yao *et al.*, *Review of Particle Physics*, *J. Phys. G* **33**, 1 (2006) and 2007 partial update for edition 2008 `[http://pdg.lbl.gov/]`.

[9] Belle Collaboration, *Phys. Rev. Lett.* **99**, 011801 (2007).

[10] V.V. Anashin *et al.*, *JETP Lett.* **85**, 347 (2007).

[11] X.H. Mo, *Nucl. Phys. B (Proc. Suppl.)* **169**, 132 (2007).

[12] P. Ruiz-Femenía, A. Pich, *Phys. Rev.* **D64**, 053001 (2001); *Nucl. Phys. B (Proc. Suppl.)* **121**, 205 (2003).

[13] M.B. Voloshin, *Phys. Lett.* **B556**, 153 (2003).

[14] B.H. Smith, M.B. Voloshin, *Phys. Lett.* **B324**, 117 (1994); **B333**, 564 (1994).

[15] R. Wanke, `0707.2289 [hep-ex]`; FLAVIA*net* Kaon Working Group, `http://ific.uv.es/flavianet/`

[16] I. Rosell, V. Cirigliano, `0707.3439 [hep-ph]`, `0707.4464 [hep-ph]`.

[17] V. Cirigliano *et al.*, *Eur. Phys. J.* **C23**, 121 (2002).

[18] The LEP Collaborations ALEPH, DELPHI, L3 and OPAL and the LEP Electroweak Working Group, `hep-ex/0612034`; `http://www.cern.ch/LEPEWWG/`

[19] BaBar Collaboration, `0708.3650 [hep-ex]`; *Phys. Rev. Lett.* **98**, 061803 (2007); **96**, 041801 (2006); **95**, 191801, 041802 (2005); **92**, 121801 (2004).

[20] Belle Collaboration, `0708.3272 [hep-ex]; 0708.3276 [hep-ex]` `0705.0650 [hep-ex]`; *Phys. Lett.* **B648**, 341 (2007); **B640**, 138 (2006); **B639**, 159 (2006); **B632**, 51 (2006); **B622**, 218 (2005); **B589**, 103 (2004); *Phys. Rev. Lett.* **93**, 081803 (2004); **92**, 171802 (2004).

[21] V. Cirigliano, G. Isidori, V. Porretti, *Nucl. Phys.* **B763**, 2287 (2007).

[22] B. Grinstein *et al.*, *Nucl. Phys.* **B763**, 35 (2007); **B728**, 121 (2005).

[23] S. Antusch *et al.*, *J. High Energy Phys.* **0611**, 090 (2006).

[24] E. Arganda, M.J. Herrero, *Phys. Rev.* **D73**, 055003 (2006).

[25] M. Blanke *et al.*, *J. High Energy Phys.* **0705**, 013 (2007).

[26] M. Ciuchini *et al.*, *Nucl. Phys.* **B783**, 112 (2007).

[27] P. Paradisi, *J. High Energy Phys.* **0602**, 050 (2006); **0510**, 006 (2005).

[28] M. Bona *et al.*, *SuperB: A High-Luminosity Asymmetric e^+e^- Super Flavor Factory. Conceptual Design Report*, `0709.0451 [hep-ex]`.

[29] T. Mori, *Nucl. Phys. B (Proc. Suppl.)* **169**, 166 (2007).

[30] *An Experimental Search for a μ^-– e^- Conversion at Sensitivity of* 10^{-16} *with a Slow-Extracted Bunched Beam*, J-PARC P21-LoI (15 December 2006), `http://j-parc.jp/NuclPart/pac_0701/pdf/P21-LOI.pdf`

[31] *An Experimental Search for a μ^-–e^- Conversion at Sensitivity of the Order of* 10^{-18} *with a Highly Intense Muon Source: PRISM*, J-PARC P20-LoI (28 April 2006), `http://j-parc.jp/NuclPart/pac_0606/pdf/p20-Kuno.pdf`

[32] E. Braaten, *Phys. Rev. Lett.* **60**, 1606 (1988); *Phys. Rev.* **D39**, 1458 (1989).

[33] S. Narison, A. Pich, *Phys. Lett.* **B211**, 183 (1988).

[34] E. Braaten, S. Narison, A. Pich, *Nucl. Phys.* **B373**, 581 (1992).

[35] F. Le Diberder, A. Pich, *Phys. Lett.* **B286**, 147 (1992).

[36] A. Pich, *Nucl. Phys. B (Proc. Suppl.)* **39B,C**, 326 (1995) [`hep-ph/9412273`].

[37] W.J. Marciano, A. Sirlin, *Phys. Rev. Lett.* **61**, 1815 (1988).

[38] E. Braaten, C.S. Li, *Phys. Rev.* **D42**, 3888 (1990).

[39] J. Erler, *Rev. Mex. Phys.* **50**, 200 (2004) [`hep-ph/0211345`].

[40] A.A. Pivovarov, *Z. Phys.* **C53**, 461 (1992).

[41] F. Le Diberder, A. Pich, *Phys. Lett.* **B289**, 165 (1992).

[42] ALEPH Collaboration, *Phys. Rep.* **421**, 191 (2005); *Eur. Phys. J.* **C4**, 409 (1998); *Phys. Lett.* **B307**, 209 (1993).

[43] CLEO Collaboration, *Phys. Lett.* **B356**, 580 (1995).

[44] OPAL Collaboration, *Eur. Phys. J.* **C7**, 571 (1999).

[45] M. Davier, A. Höcker, Z. Zhang, *Rev. Mod. Phys.* **78**, 1043 (2006).

[46] G. Rodrigo, A. Pich, A. Santamaria, *Phys. Lett.* **B424**, 367 (1998).

[47] E. Gámiz *et al.*, *Phys. Rev. Lett.* **94**, 011803 (2005); *J. High Energy Phys.* **0301**, 060 (2003).

[48] E. Gámiz *et al.*, *Theoretical Progress on the V_{us} Determination from τ Decays*, `0709.0282 [hep-ph]`.

[49] BaBar Collaboration, `0707.2981 [hep-ex]`; *Phys. Rev.* **D76**, 051104 (2007).

[50] Belle Collaboration, *Phys. Lett.* **B654**, 65 (2007); **B643**, 5 (2006).

[51] M. Davier, *Nucl. Phys. B (Proc. Suppl.)* **55C**, 395 (1997).

[52] A. Pich, J. Prades, *J. High Energy Phys.* **9910**, 004 (1999); **9806**, 013 (1998).

[53] S. Chen *et al.*, *Eur. Phys. J.* **C22**, 31 (2001). M. Davier *et al.*, *Nucl. Phys. B (Proc. Suppl.)* **98**, 319 (2001).

[54] K.G. Chetyrkin, J.H. Kühn, A.A. Pivovarov, *Nucl. Phys.* **B533**, 473 (1998).

[55] J.G. Körner, F. Krajewski, A.A. Pivovarov, *Eur. Phys. J.* **C20**, 259 (2001).

[56] K. Maltman, C.E. Wolfe, *Phys. Lett.* **B639**, 283 (2006).

[57] J. Kambor, K. Maltman, *Phys. Rev.* **D62**, 093023 (2000); **D64**, 093014 (2001).

[58] K. Maltman, *Phys. Rev.* **D58**, 093015 (1998).

[59] P.A. Baikov, K.G. Chetyrkin, J.H. Kühn, *Phys. Rev. Lett.* **95**, 012003 (2005).

[60] H. Leutwyler, *Phys. Lett.* **B378**, 313 (1996).

[61] M. Jamin, J.A. Oller, A. Pich, *Phys. Rev.* **D74**, 074009 (2006).

[62] E. Gámiz *et al.*, work in progress.