

Self-consistent equilibrium ocean tides

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Received 1978 March 14; in original form 1977 December 7

Summary. We compute the static response of the world ocean to an external zonal gravitational potential. The computation includes the effects of the self-attraction of the ocean, and the yielding of the Earth caused both by the external potential and the change in ocean load. We compare the computed tide with measurements of the fortnightly and monthly ocean tides. The short-wavelength departures from equilibrium found by Wunsch are still present. An average of observations at Pacific islands shows that the fortnightly tide departs significantly from equilibrium but the monthly may not. We have also calculated the effects of our computed tide on measurements of tidal gravity and tidal fluctuations in the length of day. Existing tidal gravity data are too imprecise to enable us to determine whether or not the spatial average of the ocean tides departs from equilibrium. The length of day data suggest that the monthly tide is farther from equilibrium than the fortnightly. We have not been able to resolve the apparent discrepancy between the length of day and ocean tide data.

1 Introduction

The equilibrium theory of the tides supposes that the surface of the sea at all times coincides with a gravitational equipotential, so that the tides simply reflect the change in shape of the equipotential caused by the variations of the attraction of the Sun and Moon. This theory thus ignores the dynamics of the water movement. This is a valid approximation provided that the oceans, regarded as a mechanical system, have no free oscillations at periods comparable to or less than those of the variations of the equipotential and have enough friction that the decay time is not much longer than these periods (Proudman 1960). Neither of these conditions holds at diurnal and semidiurnal periods; consequently any theory of these tides must include dynamical effects. It is these effects that make tidal theory so difficult.

There are slow variations of the tidal potential, with periods of a week and more, which produce small long-period tides. Because of frictional effects the ocean must have an equilibrium response at sufficiently long periods. The aim of this paper is to provide a self-consistent calculation of this static equilibrium response. By a self-consistent calculation we mean one in which the shift in sea level caused by an external potential is computed with the inclusion of the static response of the Earth as well as the oceans. We thus allow for the effects of the external potential on the Earth and oceans, and also for the changes caused by the movement of the water mass and its loading of the Earth.

This problem is not a new one, for equilibrium tidal theory originated with Newton and Daniel Bernoulli. The necessity of allowing for the yielding of the Earth was first pointed out by Thomson (1863). The first complete self-consistent treatment was worked out by Munk & McDonald (1960), and the first solutions using such a treatment were given by Merriam (1973) and Dahlen (1976). In all of these studies the applied potential and height of the tide were expressed in terms of spherical harmonics. Because of the irregular distribution of land and water, the equations for the different order harmonics do not decouple. Dahlen (1976, Section 5) discusses how one can sensibly truncate the resulting infinite system of equations to get useful results. This method is particularly suited to finding the low-order coefficients of a spherical harmonic expansion of an equilibrium tide. Our purpose here is to produce a contour map of the self-consistent equilibrium tide to compare with observations, and for this purpose we shall employ a very different method of solution.

2 Methods of computation

Our solution of the problem of the self-consistent equilibrium tide makes use of the Green function discussed by Farrell (1973), and resembles closely that which Farrell & Clark (1976) used in computing sea-level changes caused by the melting of glaciers.

As the tides are measured relative to the solid Earth, our computation must include both the change in the equipotential in some non-deformable frame and the vertical motion of the solid surface. We assume that the initial gravitational potential is $\phi(\theta, \psi)$. If we now impose an additional potential $V(\theta, \psi)$, which is a spherical harmonic of order 2, the new equilibrium potential will be

$$\phi + (1 + k_2)V + \phi'$$

where k_2 is the Love number expressing the solid Earth response to V , and $\phi'(\theta, \psi)$ the additional potential arising from the redistribution of the oceans and their altered loading of the Earth. Because of these additional potentials, the surface on which ϕ is constant will no longer be an equipotential surface. The distance we must go to to be on an equipotential surface is then $\epsilon(\theta, \psi)$, where

$$\epsilon(\theta, \psi) = g^{-1} [(1 + k_2)V(\theta, \psi) + \phi'(\theta, \psi)]$$

(see Farrell & Clark 1976); ϵ describes the motion of the potential surface relative to the centre of the Earth. The displacement of the surface of the solid Earth is

$$d(\theta, \phi) = h_2V + d'(\theta, \psi)$$

where d' is the contribution from loading and h_2V that from the external potential. The change in sea level is then

$$t(\theta, \psi) = \epsilon(\theta, \psi) - d(\theta, \psi) - c$$

where c is a constant introduced to ensure that sea level satisfies the conservation of mass. This requires that

$$\iint_0 t(\theta, \psi) d\Omega = 0$$

where the integral is taken over the oceans.

We define a function $G_t(\Delta)$ such that at an angular distance Δ from a load of unit mass/area,

$$G_t(\Delta) = g^{-1} \phi' - d'.$$

This is the Green function for changes in gravitational potential (see Farrell 1972 for a discussion of related Green functions). This function may be written as a sum of Legendre functions weighted by the appropriate combination of load Love numbers k'_n, h'_n :

$$G_t(\Delta) = \frac{ag}{M_E} \sum_{n=0}^{\infty} (1 + k'_n - h'_n) P_n(\cos \Delta) \tag{2.1}$$

where M_E is the mass, and a the radius, of the Earth (Farrell & Clark 1976).

The equation for the change in sea level caused by an external potential V is then

$$t(\theta, \psi) = g^{-1} (1 + k_2 - h_2) V(\theta, \psi) + g^{-1} \phi'(\theta, \psi) - d(\theta, \psi) - c = (1 + k_2 - h_2) \frac{V}{g}(\theta, \psi) + \iint_0 G_t(\Delta) \rho t(\theta', \psi') d\Omega' - \frac{1}{A} \iint t(\theta', \psi') d\Omega' \tag{2.2}$$

where Δ is the angular distance between (θ, ψ) and (θ', ψ') , ρ the density of seawater (1030 kg m^{-3}), and A the area of the oceans.

We have solved equation (2.2) for $t(\theta, \psi)$ by iteration using an input potential of the form

$$V(\theta, \psi) = V_0 Y_{20}(\theta, \psi) \tag{2.3}$$

which is appropriate for the long-period tides. (We use the spherical harmonic normalization of Munk & Cartwright 1966.) The result of each stage of the iteration was the sum of the first two terms on the right-hand side of (2.2), with a constant subtracted to make the result conserve mass. Starting with $t = 0$, in four iterations the result converges to the point that the rms difference between sea level on the third and fourth iteration is 0.5 per cent of V_0/g .

To show the effects of the irregular distribution of land, we have compared our solution with that expected for an ocean-covered earth. In this case a spherical harmonic treatment is most convenient; we use the addition theorem to rewrite (2.1) as

$$G_t(\Delta) = 4\pi \frac{ag}{M_E} \sum_{n=0}^{\infty} \frac{(1 + k'_n - h'_n)}{2n + 1} \sum_{m=-n}^n Y_{nm}(\theta, \psi) Y_{nm}^*(\theta', \psi').$$

If we then take

$$t(\theta', \psi') = \sum_{l,k} t_{lk} Y_{lk}(\theta', \psi') \tag{2.4}$$

we see that (2.2) becomes

$$t(\theta, \psi) = (1 + k_2 - h_2)g^{-1}V_0Y_{20}(\theta, \psi) + \frac{4\pi a^3 g \rho}{M_E} \sum_{n=0}^{\infty} \frac{1 + k'_n - h'_n}{2n + 1} \sum_{m=-n}^n t_{nm} Y_{nm}(\theta, \psi) - \frac{t_{00}}{4\pi a^2}.$$

By the orthogonality of the spherical harmonics, all the coefficients t_{lk} are zero except for

$$t_{20} = \frac{V_0}{g} (1 + k_2 - h_2) \left[1 - \frac{3\rho}{5\rho_E} (1 + k'_2 - h'_2) \right]^{-1} \tag{2.5}$$

where ρ_E is the mean density of the solid Earth. This is the result of Dahlen (1976, p. 385).

The expression (2.5) is worth a further look. If the term in square brackets is ignored we have the classical result for the alteration of the equilibrium tide by the yielding of the Earth. The bracketed term expresses the effects of ocean loading and the self attraction of the water. For a typical earth model $1 + k'_2 - h'_2$ is about 1.7, and evaluation of (2.5) shows that these effects thus increase the equilibrium tide height of a global tide over the classical result by about 25 per cent. It is clear that for smaller bodies of water, loading and self attraction will be less, and this augmentation smaller. In the limit of very small bodies of water, such as a fluid tiltmeter (Michelson & Gale 1919) the size of the tide-raising potential would be $1 + k_2 - h_2$ times V/g .

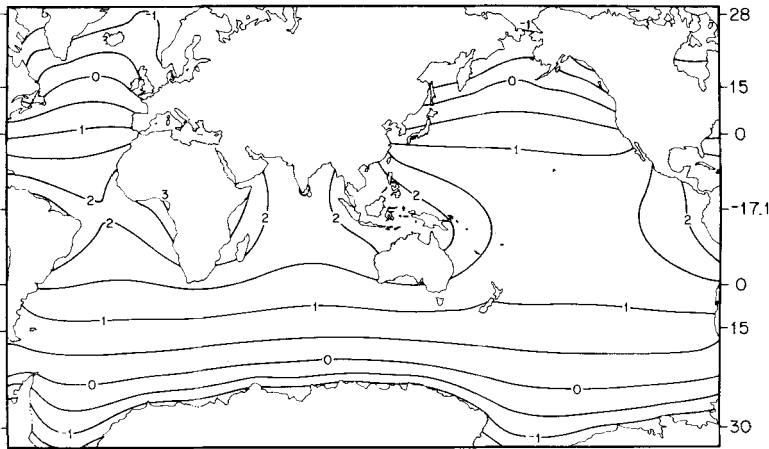


Figure 1. Contour map showing the departure of the self-consistent equilibrium zonal tide from the global equilibrium tide whose value is given by (2.5), and shown in the margins of the figure. The height of the ocean tide is given by the sum of the contoured value and 17.1 $(3 \cos^2 \theta - 1)$; some values of the latter are shown at the edge of the map. The applied potential is such that $V/g = 20(3 \cos^2 \theta - 1)$.

In Fig. 1 we show the departure of the self-consistent equilibrium zonal tide from the global equilibrium tide whose value is given by (2.5), and shown in the margins of the figure. The main features of this map are that the departure is itself predominantly dependent on latitude, indicating that the true tide is very nearly a spherical harmonic Y_{20} , but with a smaller coefficient than given by (2.5). The predominantly positive value of the departure is a result of the conservation of mass requirement, which means that the true tide must look like a spherical harmonic minus a constant. The irregularities introduced by the uneven distribution of water (and hence water load) are also plainly visible.

3 Comparison with observed tides

Having computed what the equilibrium tide should be, we may ask whether any of the observed long-period tides are in fact of this form. The largest long-period tides have periods of about a fortnight, a month, six months and a year. The last two are unobservable in sea-level records because of the presence of much larger movements caused by weather, so that our comparison can be carried out only for the first two tides. As we noted in the introduction, we may expect the tides to be equilibrium only if the ocean has no free oscillations at or below the frequency of forcing. Because of the rotation of the Earth, the ocean seems able to support a wide range of oscillations, which become more densely spaced in frequency at larger and longer periods (some examples, for various special cases, are illustrated in Longuet-Higgins 1966 and Wunsch 1967). In a system which has free modes at zero frequency, the response at low frequencies will depart from the equilibrium response unless the period of forcing is less than the decay time due to friction (Proudman 1960). The effect of friction in keeping the low-frequency modes of the ocean from distorting the tidal

Table 1. Long-period ocean-tide admittances (relative to self-consistent equilibrium tide).

Place	Tide	Amplitude	Phase (deg)	Reference
Guam (13.5° N, 144.7° E)	M _f	0.65 ± 0.11	-6 ± 14	Wunsch (1967)
	M _m	0.25 ± 0.15	-2 ± 41	
Eniwetok (11.5° N, 162.3° E)	M _f	0.66 ± 0.08	34 ± 10	Wunsch (1967)
	M _m	0.76 ± 0.14	-9 ± 15	
Kwajalein (9.3° N, 167.5° E)	M _f	0.72 ± 0.10	6 ± 11	Wunsch (1967)
	M _m	0.68 ± 0.21	10 ± 22	
Wake Island (19.3° N, 166.6° E)	M _f	0.54 ± 0.11	31 ± 14	Wunsch (1967)
Ocean Island (0.9° S, 169.6° E)	M _f	1.16 ± 0.32	18 ± 21	Wunsch (1967)
Arorae Island (2.6° S, 176.8° E)	M _f	1.20 ± 0.28	-1 ± 18	Wunsch (1967)
	M _m	0.68 ± 0.30	-16 ± 32	
Hull Island (4.5° S, 172.2° W)	M _f	0.57 ± 0.27	28 ± 34	Wunsch (1967)
	M _m	0.90 ± 0.20	62 ± 18	
Canton Island (2.8° S, 171.7° W)	M _f	0.77 ± 0.12	47 ± 12	Wunsch (1967)
	M _m	1.26 ± 0.23	24 ± 15	
Johnston Island (16.7° N, 169.5° W)	M _f	0.62 ± 0.45	25 ± 46	Wunsch (1967)
Christmas Island (2.0° N, 157.5° W)	M _f	0.84 ± 0.10	18 ± 10	Wunsch (1967)
	M _m	1.48 ± 0.29	-2 ± 16	
Honolulu (21.3° N, 157.8° W)	M _f	0.66 ± 0.09	38 ± 6	Munk & Cartwright (1966)
	M _m	1.05 ± 0.26	13 ± 20	
Kahului (20.9° N, 156.5° W)	M _f	0.59 ± 0.16	51 ± 19	Wunsch (1967)
	M _m	1.29 ± 0.47	-34 ± 27	
Hilo (19.7° N, 155.1° W)	M _f	0.65 ± 0.10	52 ± 13	Wunsch (1967)
	M _m	0.95 ± 0.58	-17 ± 41	
San Francisco (37.8° N, 122.42° W)	M _f	2.03 ± 0.71	-19 ± 26	Wunsch (1967)
	M _m	1.92 ± 2.24	36 ± 59	
La Jolla (32.7° N, 117.1° W)	M _f	0.39 ± 0.29	-88 ± 46	Wunsch (1967)
Malin (55.4° N, 7.4° W)	M _f	1.40 ± 0.31	52 ± 16	Cartwright (1968)
	M _m	1.06 ± 0.46	-13 ± 22	
Aberdeen (57.2° N, 2.1° W)	M _f	1.11 ± 0.13	29 ± 6	Cartwright (1968)
	M _m	1.13 ± 0.42	18 ± 20	
Stornoway (58.2° N, 6.4° W)	M _f	1.12 ± 0.18	22 ± 9	Cartwright (1968)
	M _m	1.24 ± 0.28	-2 ± 13	
Lerwick (60.2° N, 1.2° W)	M _f	1.23 ± 0.11	6 ± 5	Cartwright (1968)
	M _m	1.06 ± 0.32	1 ± 15	
Average of Pacific islands (weighted mean)	M _f	0.69 ± 0.02	26 ± 2	
	M _m	0.90 ± 0.05	7 ± 4	

response at long periods is unclear (Wunsch 1967, pp. 469–470). In any case, it is not known what decay time is to be expected from reasonable models of frictional dissipation in the ocean. Proudman (1960) estimated a decay time of about a month, which would allow the monthly and fortnightly tides to depart from equilibrium.

Wunsch (1967) argued that these tides are in fact not equilibrium. Using tidal observations from a number of Pacific islands, he found that there appeared to be short-wavelength fluctuations in the tidal admittances (which in general fell somewhat below 1 in amplitude), which he explained as the effect of Rossby wave dynamics. His admittances have very large error bars, but seem to differ significantly, though not substantially, from unity. These admittances were computed relative to the input potential. We have used our results to recompute them as admittances relative to the true equilibrium tide. The results are given in Table 1 (we have also included the admittances found by Munk & Cartwright 1966 for Honolulu and Cartwright 1968 for Scotland). Since the equilibrium zonal tide is in phase with the driving potential, the phases of the admittances do not change.

With the exception of La Jolla, for which the movement of the node owing to mass conservation has a large effect, none of the admittances is much changed. The island admittances are moved closer to 1 but still differ from it, and the short wavelength fluctuations from island to island are not significantly reduced. We also give the weighted averages of the island observations in Table 1. These indicate that the spatial average of the monthly tides does not differ significantly from equilibrium, but that the average of the fortnightly tides probably does. In view of the large errors in the admittances, we are reluctant to draw any stronger conclusions.

Another kind of observation which contains information about the ocean tides is measurements of Earth tides, which are partly influenced by ocean loading. Since these provide data on the ocean tide averaged over a considerable area, they would seem to be ideal for testing whether the spatial average of the long-period tides is approximately equilibrium. According to the results of Wunsch (1967) this should be the case, but this is hard to test using direct measurements of the tide because of short-wavelength variations in the admittances.

The problem of removing environmental effects makes measurement of the long-period Earth tides very difficult. Only a few results have been published, all for the gravity tides. In Table 2 we compare these results with those predicted using our equilibrium ocean-tide model. The load tide is only one or two per cent of the total gravity tide except near the nodal latitudes. The existing measurements are not precise enough to allow us to say whether or not the true load tide is significantly different from that expected from an equilibrium ocean tide.

Table 2. Fortnightly gravity tide admittances (including the effects of a self-consistent equilibrium ocean tide).

Place	Amplitude (δ) observed	Computed*	Phase (deg) observed†	Reference
Talgar (43.2° N, 77.1° E)	1.169 ± 0.076	1.1551	5.1 ± 4.0	Barsenkov (1972)
Strasbourg (48.4° N, 7.4° E)	1.16 ± 0.09	1.1352	1.0 ± 4.0	Lecolazet & Steinmetz (1966)
South Pole (90.0° S)		1.1358		
Piñon Flat (33.6° N, 116.5° W)		1.0429		

* For an oceanless earth $\delta = 1.1554$.

† Computed phase is 0 in all cases.

4 Effect of ocean tides on changes in the length of day

There is one geophysical observation which is significantly dependent on the global form of the long-period tides: the tidal changes in the length of day. At the moment the observations are too inaccurate to be of great oceanographic value. Their main interest lies in the fact that they seem to conflict with the ocean tide observations.

That the long-period Earth tides create periodic fluctuations in the speed of rotation of the Earth was first noted in 1928 by Jeffreys, who thought the effect too small to measure (Jeffreys 1976, Section 7.02). The first computation to include the effect of the ocean tides is due to Pertzov & Ivanova (1975) who did not allow for ocean loading and self attraction in computing their equilibrium tide. We have computed the expected response for a self-consistent equilibrium tide.

We first review the derivation of the tidal fluctuations of the Earth's rate of rotation, which is most conveniently done using MacCullagh's formula (Jeffreys 1976, Section 4.027). In a coordinate system chosen so that the Earth's moment of inertia tensor is diagonal, this formula gives for the external potential (to order $1/r^3$)

$$\begin{aligned} \phi(r, \theta, \psi) = & \frac{GM_E}{r} - \frac{G}{r^3} \left\{ 2\sqrt{\frac{\pi}{5}} [C_{33} - \frac{1}{2}(C_{11} + C_{22})] Y_{20}(\theta, \psi) + \frac{3}{2}\sqrt{\frac{2\pi}{15}} [C_{11} - C_{22}] \right. \\ & \left. \times [Y_{22}(\theta, \psi) + Y_{2-2}(\theta, \psi)] \right\}. \end{aligned} \quad (4.1)$$

We assume that changes in the inertia tensor of the Earth and oceans caused by tidal deformations are such that we can write

$$C_{11} = A + c_{11} \quad C_{22} = A + c_{22} \quad C_{33} = C + c_{33}$$

with the off-diagonal components remaining zero. Substituting these expressions into (4.1), we find that the potential due to deformation is

$$\phi_d(r, \theta, \psi) = -\frac{G}{r^3} \left\{ 3\sqrt{\frac{\pi}{5}} c_{33} Y_{20}(\theta, \psi) + \sqrt{\frac{3\pi}{10}} [c_{11} - c_{22}] [Y_{22}(\theta, \psi) + Y_{2-2}(\theta, \psi)] \right\}. \quad (4.2)$$

We have here made use of the fact that for tidal deformations of the Earth-ocean system $\text{Tr}\vec{C}$ is constant (Dahlen 1976, p. 391; Rochester & Smylie 1974) so that $c_{11} + c_{22} = -c_{33}$.

We may of course also expand the external potential caused by deformation in spherical harmonics. If the applied potential that causes the deformation is of the form (2.3), the external potential is

$$\phi_d = \sum_{l=2}^{\infty} \sum_{m=-l}^l V_0 \kappa_{lm} \left(\frac{a}{r}\right)^{l+1} Y_{lm}(\theta, \psi). \quad (4.3)$$

We define $\kappa = \kappa_{20}$, and call this the zonal response coefficient of the Earth-ocean system. In reality the external potential will depend on the frequency of the applied potential, and the response coefficients will therefore be functions of frequency. Since we are using the static response, the coefficients (including κ) are simply numbers. The reason for singling κ_{20} out of the expansion (4.3) is that it is relatively easily measurable. Comparing (4.2) and (4.3) we see that

$$c_{33} = -\kappa \frac{a^3}{3G} \sqrt{\frac{5}{\pi}} V_0$$

and c_{33} is measurable through its influence on the speed of rotation of the Earth, which is given by (Munk & McDonald 1960, Chapter 6)

$$\Omega = \omega(1 + c_{33}/C).$$

Here Ω is the instantaneous, and ω the steady, angular velocity of the Earth.

It has been conventional in the past to regard the coefficient κ as being equivalent to the Love number k_2 , and to compare those estimates of it from length of day measurements with others derived from Earth tide measurements or seismic models. This is not correct. The use of Love numbers assumes spherical symmetry, in which case the infinite matrix in (4.3) above becomes diagonal, with the diagonal element κ_{ii} being the Love number k_i . Though the solid Earth is nearly spherically symmetric, when the effects of the oceans are to be included this approximation is no longer adequate. The other geophysical parameter which appears to depend on k_2 , the period of the Chandler wobble, is also substantially affected by the Oceans (Dahlen 1976) and by the core (Smith 1977).

The effect of an equilibrium ocean tide is to make κ greater than k_2 . If we expand the tide height as in (2.4), we see that the external potential from the ocean tide is

$$4\pi G\rho a \sum_{n=0}^{\infty} \sum_{m=-n}^n \frac{(1+k'_n)}{(2n+1)} \left(\frac{a}{r}\right)^{n+1} t_{nm} Y_{nm}(\theta, \psi).$$

This expression includes the potential due to loading of the Earth. Adding this potential to that due to the deformation of the solid Earth caused by the external potential, we find that

$$\kappa = k_2 + \frac{3\rho}{5\rho_E} (1+k'_2) t_{20} \left(\frac{V_0}{g}\right)^{-1}.$$

For an ocean-covered earth we may use (2.5) to find κ directly. For the Gutenberg–Bullen model (Farrell 1972) it turns out that in this case $\kappa = 0.3040 + 0.0663 = 0.3703$.

We have evaluated κ for an equilibrium tide on a realistic earth by integrating over the computed ocean tide to get the coefficient t_{20} . (As a check, we performed the computation of the self-consistent tide and integration to find κ on an ocean-covered earth; the computation gave $\kappa = k_2 + 0.0666$, only 0.5 per cent different from the result given above.) The result that we found for a realistic ocean–continent distribution was

$$\kappa = 0.3040 + 0.0381 = 0.3421.$$

In a private communication, F. A. Dahlen has informed us that κ may be found using the results in Dahlen (1976); he has used them to obtain $\kappa = 0.3010 + 0.0403 = 0.3413$. The value of k_2 is about 1 per cent smaller because a different earth model was used; the difference in the computed ocean correction arises from differences in the representation of the distribution of land. The value of κ is probably uncertain by about 1 per cent.

Any departure of the observed κ from this value must be due to a departure either of the long-period ocean tide from equilibrium or of the effective elastic parameters of the Earth at fortnightly and monthly periods from those at seismic periods. Dahlen (1976, 1977) and Smith (1977) have shown that elastic parameters inferred from seismic data and corrected for attenuative dispersion give a value for the Chandler wobble period that does not differ significantly from that observed. Drastic changes in the elastic properties at intermediate frequencies seem unlikely. We thus expect that determinations of the tidal change in the length of day are valuable primarily as a measure of the globally averaged behaviour of the long-period ocean tides. We note that κ is about 10 per cent dependent on the ocean tides. It is therefore about an order of magnitude more sensitive than tidal gravity measurements to variations in the long-period ocean tides.

Three recent determinations of κ for the fortnightly and monthly tides are shown in Fig. 2. (The phase of the response does not seem to have been measured.) The error bars are large, but even so the fortnightly tide results seem to show that it is somewhat less than

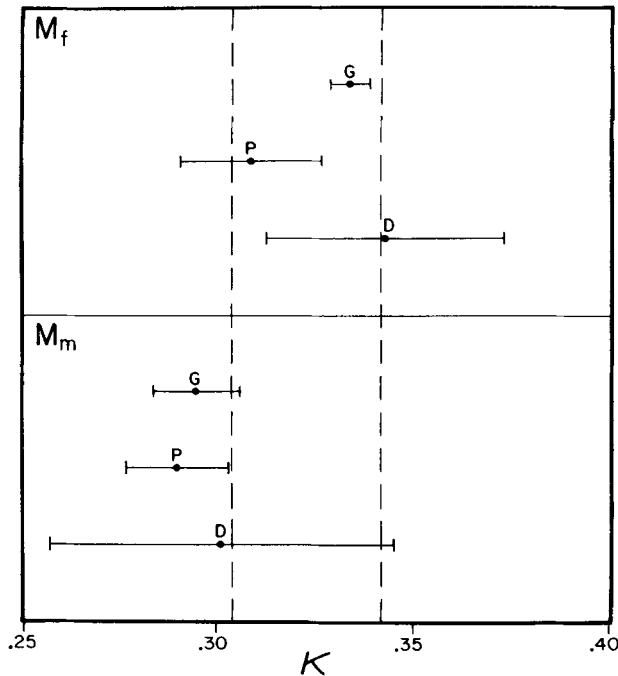


Figure 2. Measurements of the zonal response coefficient, κ , made from length of day observations. The left-hand dashed line shows the value expected for an oceanless earth; the right-hand one, that expected for equilibrium ocean tides. The letters indicate the sources of the measurements, which are: G – Guinot (1974); P – Pil’nik (1974); D – Djurovic (1976).

equilibrium, just as is suggested by the island observations summarized in Table 1. Unfortunately this agreeable concordance is lost for the monthly tides. The length of day data suggest that the monthly ocean tide is either almost absent or nearly in quadrature to the driving potential, while the island data suggest that it is closer to being equilibrium than the fortnightly tide. (We may note that the island observations, being from equatorial latitudes, are particularly relevant to changes in the moment of inertia.) The source of this disagreement is unclear. One possible source could be the effects of the non-linear interaction tides $M_2 - N_2$ and $M_2 - K_2$. Unfortunately the effects of the stronger $M_2 - S_2$ tide on length of day variations at the period of the MS_f tide cannot be evaluated because the M_2 tide aliases into this tide in astronomical observations (Munk & McDonald 1960, p. 70). Lambeck & Cazenave (1974) have suggested that noise due to meteorological fluctuations in the length of day might bias estimates of κ at monthly periods. Unless the meteorology is coherent with the tidal potential (which seems unlikely) it should be possible to remove its effects by proper statistical treatment. The response method of Munk & Cartwright (1966) would seem to be a promising way to analyse length of day data. Noise arising from observational errors should be less for future observations because of the use of better techniques such as very long baseline interferometry. Certainly there seems to be a discrepancy to resolve, and further study is warranted.

5 Conclusions

We have computed the self-consistent equilibrium ocean-tide response to an applied zonal potential, with the purpose of providing a reference against which observations of long-

period Earth and ocean tides might be checked. The ocean-tide observations, though not very precise, suggest that the monthly and fortnightly tides depart from equilibrium, though more over short wavelengths than on a spatially averaged basis. The available observations of long-period Earth tides are too imprecise to provide any evidence.

We have also computed the effect of an equilibrium ocean tide on tidal changes in the length of day. Observations of these changes are in conflict with the available ocean-tide data, for the astronomical observations indicate that the monthly tide is farther from equilibrium than the fortnightly, which is the opposite of what the oceanographic data show. If this discrepancy can be cleared up, and the precision of the astronomical measurements improved, they would seem to offer the best measurements of whether the long-period ocean tides are, on a global basis, equilibrium.

Acknowledgments

We should like to thank F. A. Dahlen for helpful discussions, and for allowing us to publish his result for κ . This research has been supported by grants NSF DES 75-14566 and NSF EAR 77-00897, and is a contribution of the Scripps Institution of Oceanography.

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