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Self-Consistent Off-Mass-Shell Amplitude Derived from the Generalized Veneziano Model

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Many authors[1] have tried to extend the Veneziano model to the N-particle amplitude satisfying the duality condition. Recently Namiki and the present author[2] and Sugawara[3] proposed possible methods to formulate the off-mass-shell amplitudes with the help of a "spurion" having the vacuum quantum numbers. In this letter we first point out an ambiguity in Sugawara's method to eliminate unwanted singularities and propose a possible prescription to formulate the off-mass-shell amplitude from the generalized Veneziano model.

Hereafter we follow Koba and Nielsen's notation.[1] The invariant mass squared and the corresponding trajectory of channel (i, j) are denoted respectively by s_{ij} and α_{ij}. The channel (i, j) consists of particles from the i-th particle to the j-th one. Sugawara[3] tried to obtain the four-point function corresponding to (current+hadron→current+hadron) from the six-point function including two spurions. This function comes to be a possible off-shell four-point function in the limit of zero spurion momenta, which has such unwanted singularities as double and/or triple poles. To eliminate them, Sugawara proposed the following rule: When some of the $s_{i,i+1}$ or $s_{j,j+2}$ become identical in the limit of zero spurion momenta, put all variables x equal to zero keeping only a certain one, say $x_{i,i+1}=-1-\alpha_{i,i+1}$ or $y_{j,j+2}=-2-\alpha_{j,j+2}$ $(=x_{j,j+2}-1)$. We can remark that there remains an ambiguity in choosing the nonzero variable mentioned above, and his method of making certain y's equal to zero seems to be unreasonable.

Here we are concerned only with the highest satellite terms. Our method is first to choose the nonzero variables in a unique way using the notion of the duality, and then to put the other variables $x_{i,j}=0$ (but never $y_{i,i+2}=0$), in order to eliminate the unwanted singularities:

(i) Let one of i-th and $(i+1)$-th particles be a spurion, then we can regard channel $(i, i+1)$ as an off-shell current.

(ii) Put all the variables $\tilde{x}_{i,i+1}=0$ corresponding to the channels dual to channel $(i, i+1)$. (It is to be noted that two off-shell currents are never constructed from two channels dual to each other.)

It will be seen later that the $(N-n)$-point function, obtained by our method from N-point function with n spurions, is identical with the $(N-n)$-generalized Veneziano function on the mass shell of $(N-n)$ particles. Sugawara's function never has such a property.

Now we apply our method to some simple examples. It is easy to show that three-point vertex function obtained from the four-point function becomes the simple pole term. We can get the four-point function including one off-shell current from the five-point Veneziano function in the limit p_2 (or p_3)$=0$ in Fig. 1. Following the rule (ii), we put $x_{12}=x_{34}=0$ and write the

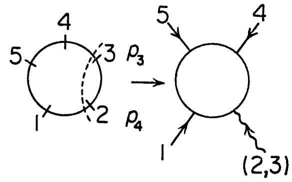

Fig. 1.

amplitude as

$$B(5\to4)=\iint_0^1 du_2du_3\left(\frac{w_{22}}{w_{23}}\right)^{x_{23}}\times u_3^{x_{13}}(w_{23})^{x_{24}}\frac{1}{w_{23}}$$

$$=\sum_{l=0}^{0}B(l+1, x_{23}+1)\frac{1}{x_{13}+1+l}\times\frac{\Gamma(x_{23}+1-x_{24}+l)}{\Gamma(x_{23}+1-x_{24})\Gamma(l+1)}, \qquad (1)$$

where $w_{ij}=1-\prod_{r=i}^{j}u_r$. Around the pole at $x_{23}=-1$ we get

$$B(5\to4)\sim\frac{1}{x_{23}+1}B(x_{13}+1, x_{24}+1), \qquad (2)$$

where the residue is nothing other than the four-point Veneziano function. We can generally show that the $(N-1)$-point function including one off-shell current can be derived from the N-point function in a similar way.

We can also derive the four-point function with two off-shell currents from the six-point Veneziano function. There are two cases as shown in Fig. 2. Since the channels (5, 6), (2, 4) and (3, 4) are dual to channel (4, 5) in case (a), we get

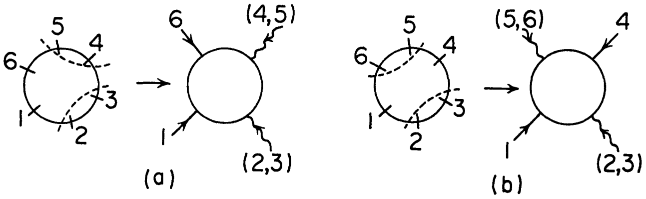

Fig. 2.

$$B^{(a)}(6\to 4)=\iiint_0^1 du_2du_3du_4\left(\frac{w_{22}}{w_{23}}\right)^{x_{23}}\left(\frac{w_{44}}{w_{34}}\right)^{x_{45}}(w_{24})^{x_{61}}u_3{}^{x_{13}}\frac{1}{w_{23}w_{34}}$$
$$=\sum_{l,m,n=0}^{\infty}B(x_{23}+1,l+n+1)B(x_{45}+1,m+n+1)\frac{1}{x_{13}+1+l+m+n}$$
$$\times\frac{\Gamma(x_{23}+1+l)\Gamma(x_{45}+1+m)\Gamma(-x_{61}+n)}{\Gamma(x_{23}+1)\Gamma(l+1)\Gamma(x_{45}+1)\Gamma(m+1)\Gamma(-x_{61})\Gamma(n+1)}. \quad (3)$$

Around poles at $x_{23}=-1$ and $x_{45}=-1$, we have

$$B^{(a)}(6\to 4)\sim\frac{1}{(x_{23}+1)(x_{45}+1)}B(x_{13}+1,x_{61}+1). \quad (4)$$

Similarly in case (b), we can get the amplitude

$$B^{(b)}(6\to 4)=\iiint_0^1 du_2du_3du_4\left(\frac{w_{22}}{w_{23}}\right)^{x_{23}}u_4{}^{x_{56}}u_3{}^{x_{13}}\left(\frac{w_{23}}{w_{24}}\right)^{x_{24}}\frac{1}{w_{23}w_{34}}$$
$$=\sum_{l,m,n=0}^{\infty}\frac{B(x_{23}+1,l+m+1)\Gamma(x_{23}+1-x_{24}+l)\Gamma(x_{24}+m)}{(x_{56}+1+m+n)(x_{13}+1+l+m+n)\Gamma(x_{23}+1-x_{24})\Gamma(l+1)\Gamma(x_{24})\Gamma(m+1)}, \quad (5)$$

which yields the pole term at $x_{23}=-1$ and $x_{56}=-1$,

$$B^{(b)}(6\to 4)\sim\frac{1}{(x_{23}+1)(x_{56}+1)}B(x_{13}+1,x_{24}+1). \quad (6)$$

In Eqs. (4) and (6) the residues are just the four-point Veneziano amplitude.

Sugawara[3] suggested that the procedure like the rule (ii) may yield fixed poles. It is easy to see that Eq. (3) has the fixed pole at nonsense point in the t-channel angular momentum plane. The residue at $J=1$ is obtained by substituting Eq. (3) into Gribov-Froissart formula[4] as follows:

$$\sum_{l,m=0}^{\infty}\{{}_3F_2(-x_{61},l+1,m+1;x_{23}+l+2,$$
$$x_{45}+m+2;1)\}/\{(x_{23}+1+l)$$
$$\times(x_{45}+1+m)\}. \quad (7)$$

For $x_{23}=x_{45}=0$ Eq. (7) becomes

$$\sum_{l=0}^{\infty}\frac{\Gamma(x_{61}+1)}{\Gamma(x_{61}+2+l)}[\psi(x_{61}+2+l)-\Gamma'(l+1)]$$
$$+\sum_{l\neq m=0}^{\infty}\frac{1}{m-l}[B(x_{61}+1,l+1)$$
$$-B(x_{61}+1,m+1)], \quad (8)$$

which has certainly simple poles at x_{61}

$=-1, -2, \cdots$. The pole-approximated form factor is obtained by making use of the current conservation and the s-channel pion-pole dominance.[4]

In this letter we have tried to formulate the off-shell current from the two-particle correlation. Our method will be extended to the case including more than three-particle correlations.

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