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SELF-CONSISTENT, THREE-DIMENSIONAL EQUIL BRIUM EFFECTS ON TOKAMAK MAGNETIC FIELD RIP E

By

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SELF-CONSISTENT, THREE-DIMENSIONAL EQUILIBRIUM EFFECTS ON TOKAMAK MAGNETIC FIELD RIPPLE

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ABSTRACT

Self-consistent equilibrium effects on tokamak magnetic field ripple have been calculated using a three-dimensional equilibrium code. The effects are found to be large enough that they should be included in tokamak ignition experiment designs. Even the modification of the well depth associated with the flow of force-free plasma current along rippled field lines is substantial. An analysis of the results separates the contribution of the Shafranov shift to the ripple modification from the contributions of other finite-pressure effects.

I. INTRODUCTION

The axisymmetric, two-dimensional character of tokamak configurations has made them popular candidates for fusion devices. The constraints associated with this symmetry should make them particularly effective for plasma confinement. The situation in an actual device is not as favorable because of the discrete nature of the toroidal field coils. This introduces a magnetic field ripple that has a deleterious effect on plasma confinement. This loss mechanism is especially important for the high energy alpha particles which are produced in nuclear reactions. Magnetic ripple is, therefore, a particularly serious issue for ignition experiments.

Since the cost of a fusion device depends critically on both the number and size of the toroidal field coils, the determination of the magnitude of the magnetic field ripple is an important and integral part of machine design. The work that has been done up until now has been based on analysis of vacuum ripple fields. However, currents in the plasma can modify the ripple amplitude. Because of the existence of a threshold effect, a small increase in the ripple magnitude can lead to a dramatic increase in alpha particle loss [1]. It is therefore important to understand the changes in the nature and magnitude of the ripple field from the predictions of the simple vacuum field approximation.

Plasma currents can modify the ripple even at $\beta = 0\%$, due to the flow of the Ohmic current along rippled field lines. At finite β, the most obvious effect is the Shafranov shift, an outward displacement of the magnetic surfaces due to the magnetic induction and the Pfirsch-Schlüter current. This could provide significant enhancement of the ripple magnitude by moving the flux surfaces to regions where the ripple is large. The size of this effect can be estimated by using axisymmetric magnetohydrodynamic (MHD) calculations to determine the magnitude of these shifts as the pressure is increased. A more subtle effect of plasma pressure is the modification of the ripple field magnitude due to the flow of Pfirsch-Schlüter currents along rippled field lines. There is also an effect due to the Pfirsch-Schlüter currents which are generated by the ripple (i.e. by the change in curvature of the field lines due to the presence of the ripple field). All of these effects except for the Shafranov shift require a fully three-dimensional code for their calculation.

We describe our tools, model, and calculations in the next section and discuss our results in section III.

II. MODEL

Our principal tool for investigating the effect of plasma pressure on the magnitude of the magnetic field ripple is the PIES (Princeton Iterative Equilibrium Solver) Code [2-4]. It uses an iterative procedure where the magnetic field and pressure are assumed to be known at any given step and an improved current distribution is calculated. The new magnetic field, and thus the pressure, is calculated from this current, and the iteration process is continued until a self-consistent equilibrium is achieved.

In order to vary as few plasma parameters as possible in making a comparison, we treat equilibria with a fixed plasma surface; in this work our base configurations have circular cross sections with major radius $R_o = 1.75$ and minor radius $a = 0.6$. We work with a pressure distribution

$$p(\psi) = p_o(1 - \psi^{\alpha})^2, \tag{1}$$

and a safety factor

$$q(\psi) = q_o + (q_s - q_o)\psi^2, \tag{2}$$

with ψ a normalized poloidal flux, $\alpha = 1.6$, $q_o = 1.05$, and $q_s = 3.2$, values that correspond to typical tokamaks. The rotational transform, $\iota = 1/q$, is given in Fig. 1 as a function of ρ, a surface label which measures the maximum radius R_V of the surface and varies between 0 at the magnetic axis and 1 at the plasma surface. We use the PEST formalism [5] to obtain base configurations with different values of p_o. The good agreement that we get between these PEST calculations and the results of axisymmetric runs with the PIES code provides a validation of the PIES code. In this note we restrict consideration to cases where $\beta = 0\%$, a force-free equilibrium, and $\beta = 1.0\%$.

We model the discreteness of the toroidal field coils by treating them as if they were very thin, had very large ellipticity, and were shaped so as to spread the current sinusoidally in the toroidal angle ϕ. Then we have a current I at the axis $R = 0$, and

$$J_Z = I/(2\pi b\delta b)(1 + \cos N\phi), \qquad (b < R < b + \delta b), \tag{3}$$

where N is the number of toroidal field coils and b is the radius of the outer legs. The ripple field stream function varies as $R^N \sin N\phi$ inside the coil radius and as $R^{-N} \sin N\phi$ outside, so that the externally imposed magnetic field can be taken as

$$\mathbf{B} = 2I\nabla\{\phi + (R/b)^N/2N \sin N\phi\}, \qquad (R < b) \tag{4}$$

We typically fix the coil number $N = 16$ and radius $b = 3$, which gives us a field ripple of about 1% at the plasma surface.

We find it convenient to introduce this field ripple onto the equilibria that we obtain from the PEST code by changing the shape of the boundary surface, rather than by introducing the field itself. Thus we use Eq. (4) and the equation along the field lines $dR/B_R = Rd\phi/B_\phi$, together with a small ι_a approximation, to get

$$R = R_a\{1 + (R_a/b)^N/2N \cos N\phi\}, \tag{5}$$

$$Z = Z_a, \tag{6}$$

with R_a and Z_a defining the position of the axisymmetric boundary, and with ι_a the transform at the boundary.

A variation of this model that we have also investigated was constructed by fixing a rippled surface outside the plasma boundary described in Eqs. (5) and (6), such that several computational surfaces exist in the vacuum region between the plasma surface and the wall. We then obtain a set of equilibria with similar pressure profiles and with the total toroidal current inside each magnetic surface fixed for different values of p_o. This provides a different measure of the effects of changing β in a free boundary system where the plasma boundary is displaced outward by the Pfirsch-Schlüter currents.

We first discuss the fixed boundary calculations with $p(\psi)$ and $q(\psi)$ prescribed. The magnetic surfaces at $\varphi = 0$ are given in Fig. 2a for the force free configuration and in Fig. 2b for the case where $\beta = 1.0\%$. The variation of the magnitude of the magnetic field along a line of force as it passes through several ripple field periods is given in Figs. 3 and 4 for these cases. The corresponding ripple field magnitude is given in Figs. 5 and 6. In all of these figures, we follow lines on magnetic surfaces that pass through their outermost positions at 2/5ths, 3/5ths, and 4/5ths of the distance between the magnetic axis and the plasma boundary and at the boundary itself. An alternate way of viewing the results is to compare the variations of the field magnitudes with β for the total field, the ripple field, and the ratio of the ripple field to the $N = 0$ (axisymmetric) component of the field. The results given in Figs. 7 through 9 are for surface 16, 4/5ths of the way out. The quantities in Fig. 9 have been normalized to their value at the plasma boundary.

Our results are summarized in Table I. The surfaces for which the data is given are the same as those used for the figures. The first column gives the values of the function $(B_v/B_s)/(R_v/R_s)^N$, with B_v the maximum ripple field on a magnetic surface, B_s that at the plasma surface, and R_v and R_s the corresponding radial positions. If the only change in the ripple magnitude were due to the displacement of the surface, these numbers would all be unity. Another measure of the ripple field magnitude, the function $(B_v/B_s)_{\beta=1\%}/(B_v/B_s)_{\beta=0\%}$ is given in the second column.

Results using the free boundary model, normalized to the field at the plasma boundary, are not much different from those where the plasma surface is kept fixed. For β-values up to 1% the rotational transform inside the plasma is not changed much at fixed current from what it was for the force free case. The scaled ripple field on surface 16 for this case is given in Fig. 10. The functions $(B_v/B_s)/(R_v/R_s)^N$ and $(B_v/B_s)_{\beta=1\%}/(B_v/B_s)_{\beta=0\%}$ for this case are also given in Table I.

III. DISCUSSION

Self-consistent equilibrium effects increase the severity of the ripple problem. We have not seen any case where the equilibrium effects shield the ripple and decrease its magnitude. The effect is always to decrease the rate at which the ripple strength decays in going towards the plasma center. Even at $\beta = 0\%$, the effect of the ripple field extends surprisingly far inward. As can be seen in Figs. 3 and 5, the ripple is noticeable even 3/5ths of the way out, even though only about a 1% ripple is imposed at the plasma surface. The introduction of plasma pressure increases this effect.

In the first column of Table I, we have normalized the fields to compensate for the R^N dependence of the vacuum magnetic field. (i.e. If the ripple field were strictly the externally imposed vacuum field, all these numbers would be unity.) At the same time, we have compensated for the Shafranov shift by using values of R_v corresponding to the shifted surfaces, as calculated by our code. The difference from unity at $\beta = 0\%$ is due to the flow of the Ohmic current along rippled field lines. Increased current densities associated with shaped cross sections would be expected to increase this effect. For example, for the shaping being contemplated for the Compact Ignition Tokamak (CIT) the cylindrical q would be decreased by a factor of about two, so that we could expect this effect to be increased by a similar factor.

The change in the first column in going from $\beta = 0\%$ to $\beta = 1\%$ is due to the flow of the Pfirsch-Schlüter currents along rippled field lines, as well as to the Pfirsch-Schlüter currents generated by the ripple. For this rather modest 1% value of β, the effect is relatively small. For the 5% β being considered for CIT, for example, we would expect effects at least five times as large. Such effects could be important.

The ripple field given in the second column in Table 1 has been normalized to the $\beta = 0\%$ field. Thus, these numbers do not include the zero β ripple effects included in column 1. They do include the effects of the Shafranov shift. Again, the effects would be larger for the values of β being contemplated for ignition experiments. The effect of shaping would be to decrease the Shafranov shift, and, therefore, lessen the severity of this contribution to the ripple.

The aim of this note has been to give a feel for the magnitude of the effects being neglected in present ripple models. On the basis of these results we feel that more extensive studies are warranted. In particular, we believe that these effects should be taken into account in the design of ignition experiments.

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TABLE I. Ripple field variations on different magnetic surfaces.

β	Surface	$(B_\psi/B_s)/(R_\psi/R_s)^N$	$(B_\psi/B_s)_\beta/(B_\psi/B_s)_{\beta=0\%}$
Constant ι, fixed boundary, normalized to plasma surface			
0.0%	20	1.00	
	16	1.09	
	12	1.21	
	8	1.37	
1.0%	20	1.00	1.00
	16	1.14	1.12
	12	1.29	1.24
	8	1.49	1.37
Constant I, free boundary, normalized to plasma surface			
1.0%	20	1.00	1.00
	16	1.11	1.11
	12	1.29	1.28
	8	1.50	1.46

FIGURE CAPTIONS

Fig. 1. Rotational transform, $\iota = 1/q$, as a function of ρ, the maximum major radius of a *magnetic surface scaled to be 0 at the magnetic axis and 1 at the plasma boundary.*

Fig. 2. Magnetic surfaces of model equilibrium at $\phi = 0$ for force free, $\beta = 0\%$ case (a) and for $\beta = 1.0\%$ (b).

Fig. 3. Variation of the total magnitude of the magnetic field along a field line at surfaces 8, 12, 16, and 20 (the plasma boundary) for $\beta = 0\%$. The largest values of **B** are on the inner surfaces because of the $1/R$ dependence of the toroidal field.

Fig. 4. Variation of the magnitude of the total magnetic field along a field line for $\beta = 1.0\%$.

Fig. 5. Variation of the magnitude of the ripple magnetic field along a field line for $\beta = 0\%$.

Fig. 6. Variation of the magnitude of the ripple magnetic field along a field line for $\beta = 1.0\%$.

Fig. 7. Variation of the magnitude of the magnetic field along a field line on surface 16, 4/5ths of the way between the magnetic axis and the plasma boundary. The solid curve is for $\beta = 0\%$ and the dashed one is for $\beta = 1.0\%$.

Fig. 8. Variation of the magnitude of the ripple magnetic field along a field line on surface 16. The solid curve is for $\beta = 0\%$ and the dashed one is for $\beta = 1.0\%$.

Fig. 9. Variation of the ratio of the magnitude of the ripple magnetic field to that of the axisymmetric ($N = 0$) component of the total field along a field line on surface 16 for the calculations with a fixed plasma boundary. The ratio has been normalized to its value at the plasma boundary. The solid curve is for $\beta = 0\%$ and the dashed one is for $\beta = 1.0\%$.

Fig. 10. Variation of the ratio of the magnitude of the ripple magnetic field to that of the axisymmetric ($N = 0$) component of the total field along a field line on surface 16 for the calculations with a free plasma boundary. The ratio is again normalized to its value at the plasma boundary. The solid curve is for $\beta = 0\%$ and the dashed one is for $\beta = 1.0\%$.

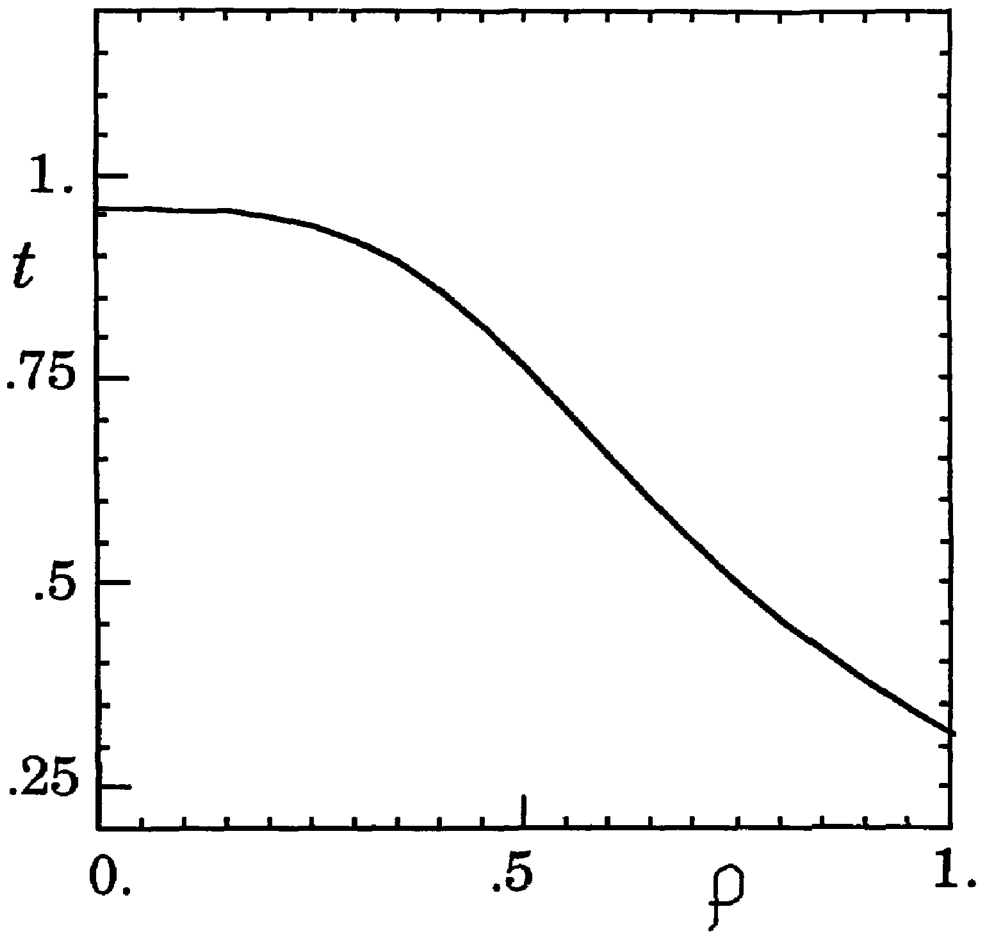

Fig. 1

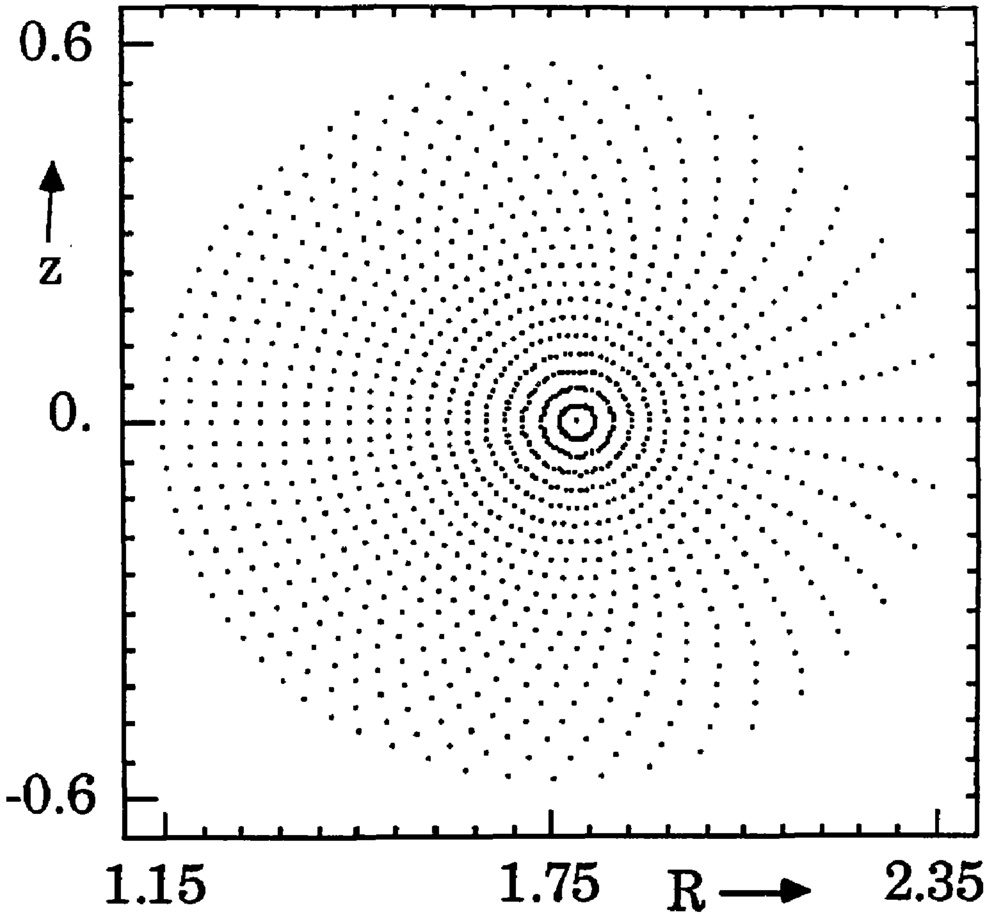

Fig. 2(a)

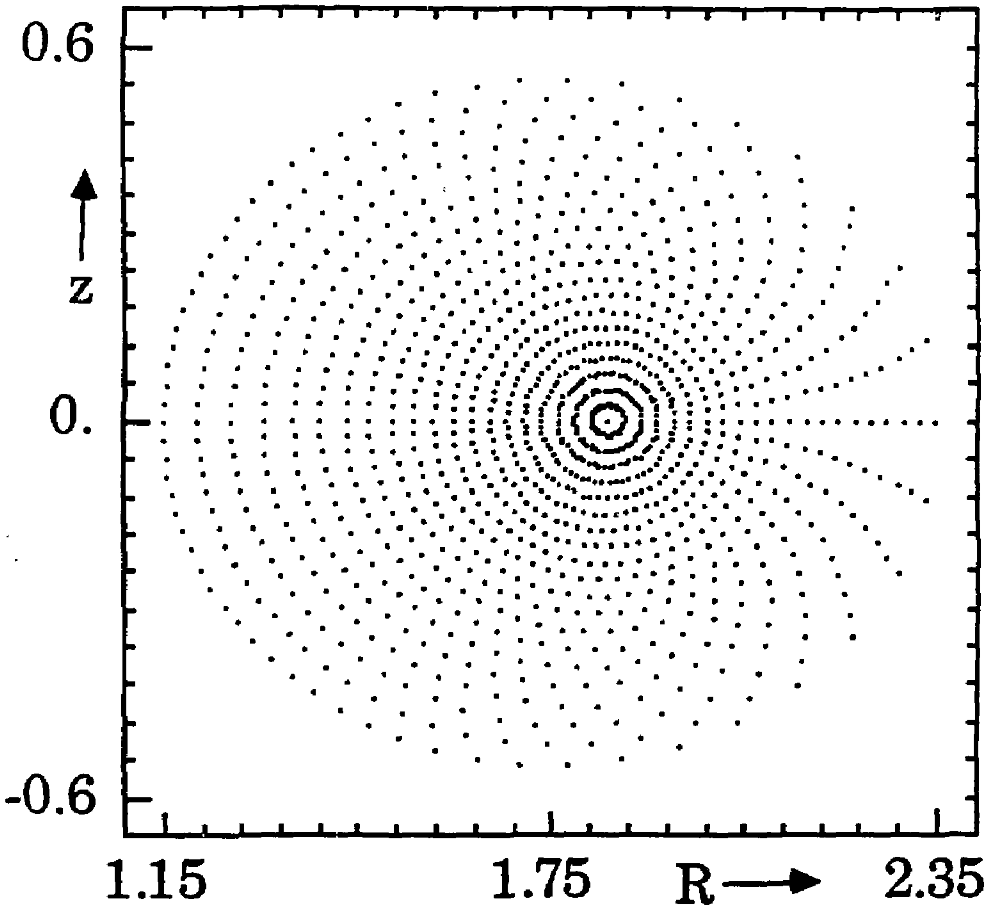

Fig. 2(b)

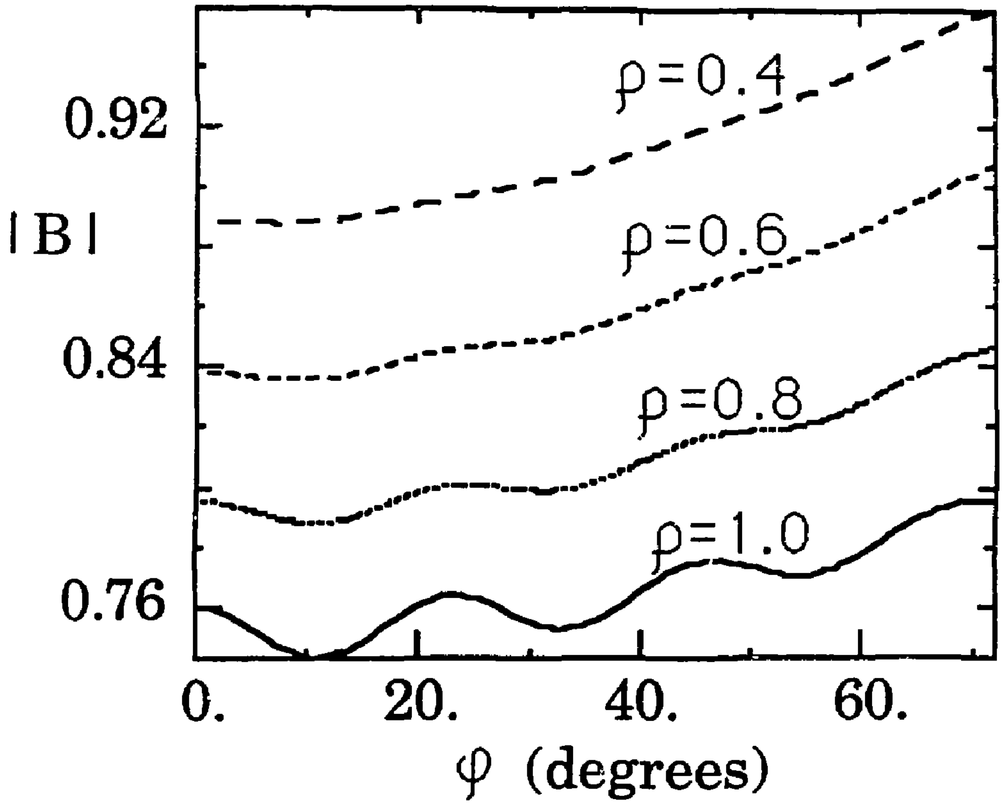

Fig. 3

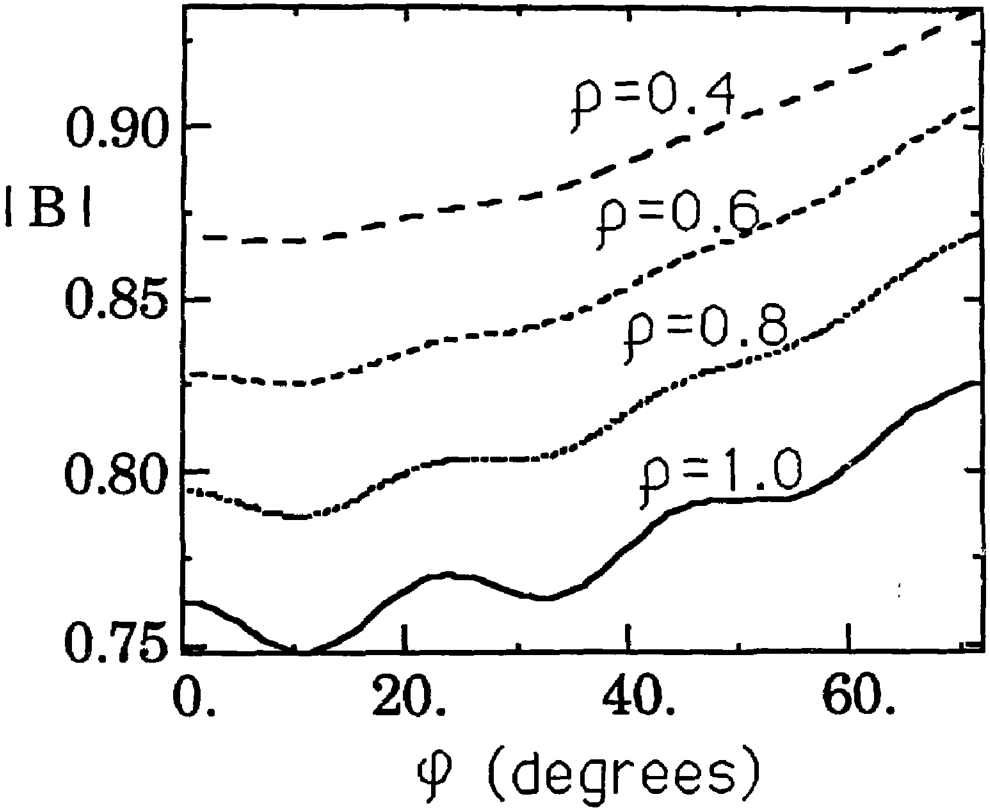

Fig. 4

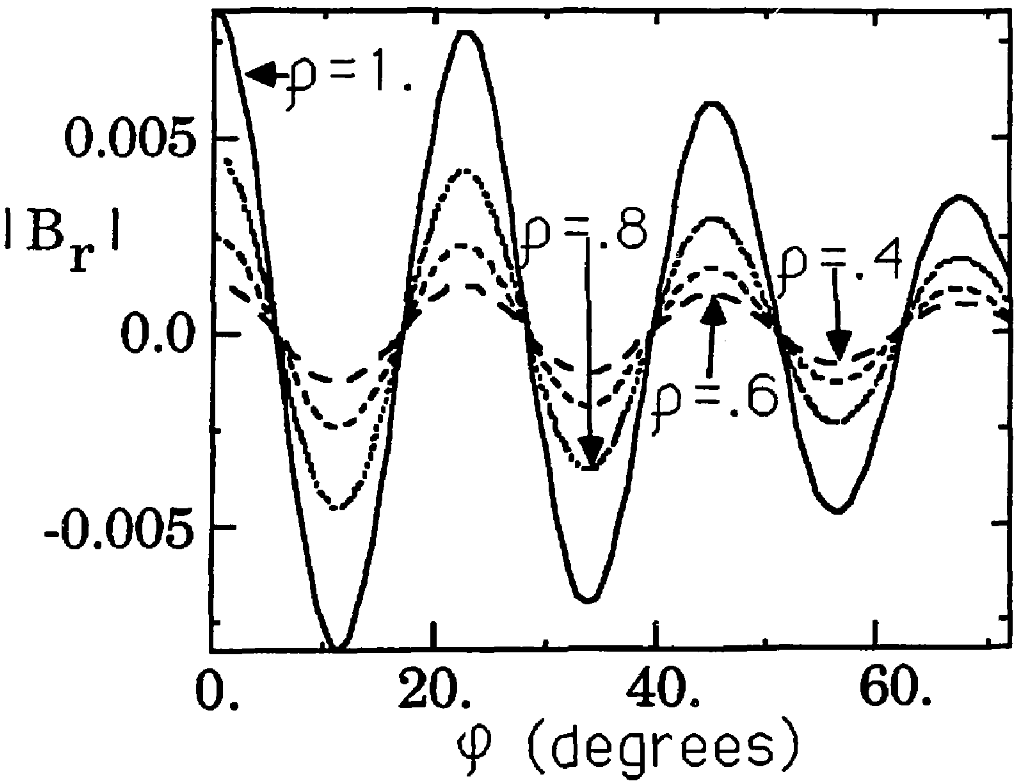

Fig. 5

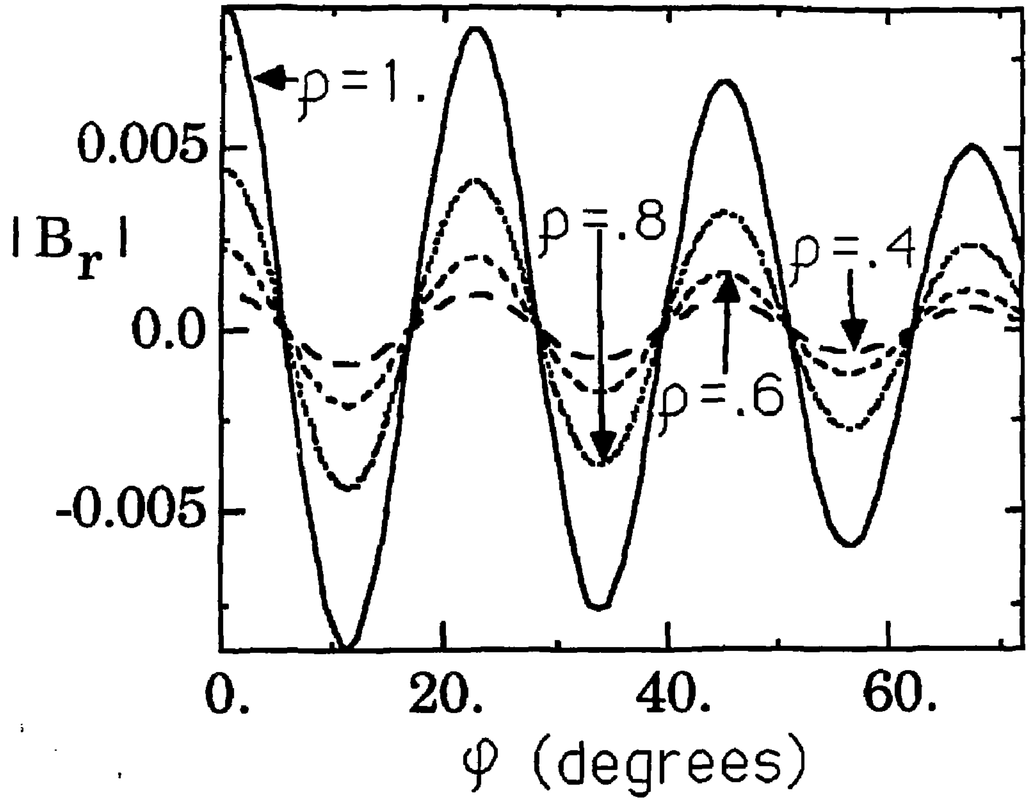

Fig. 6

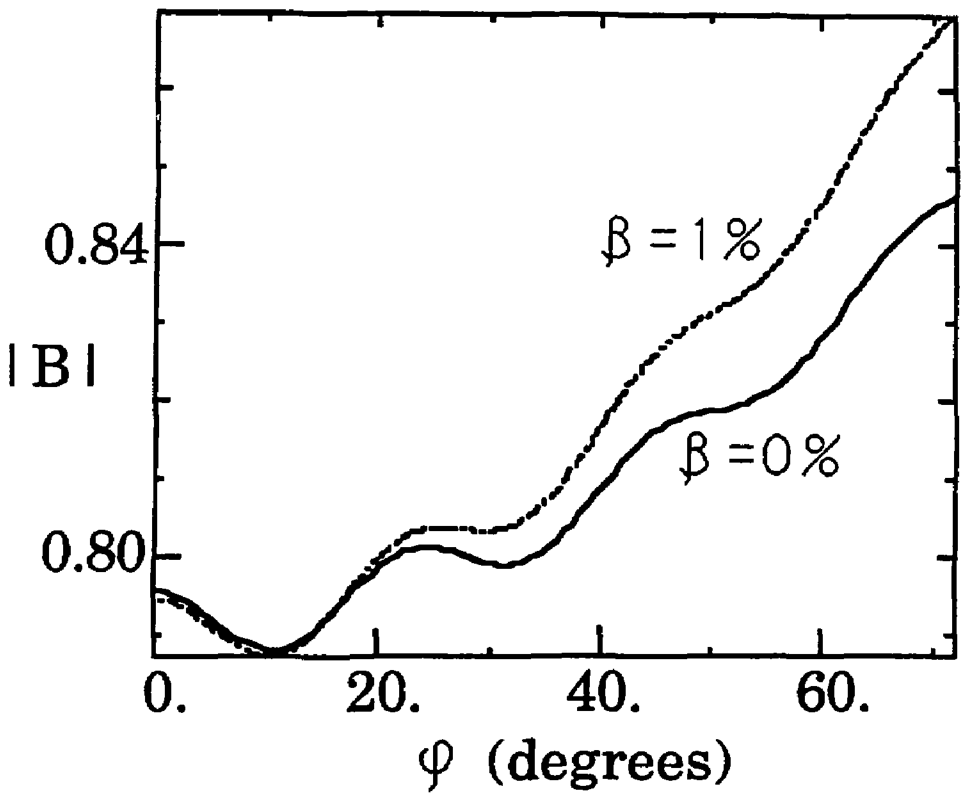

Fig. 7

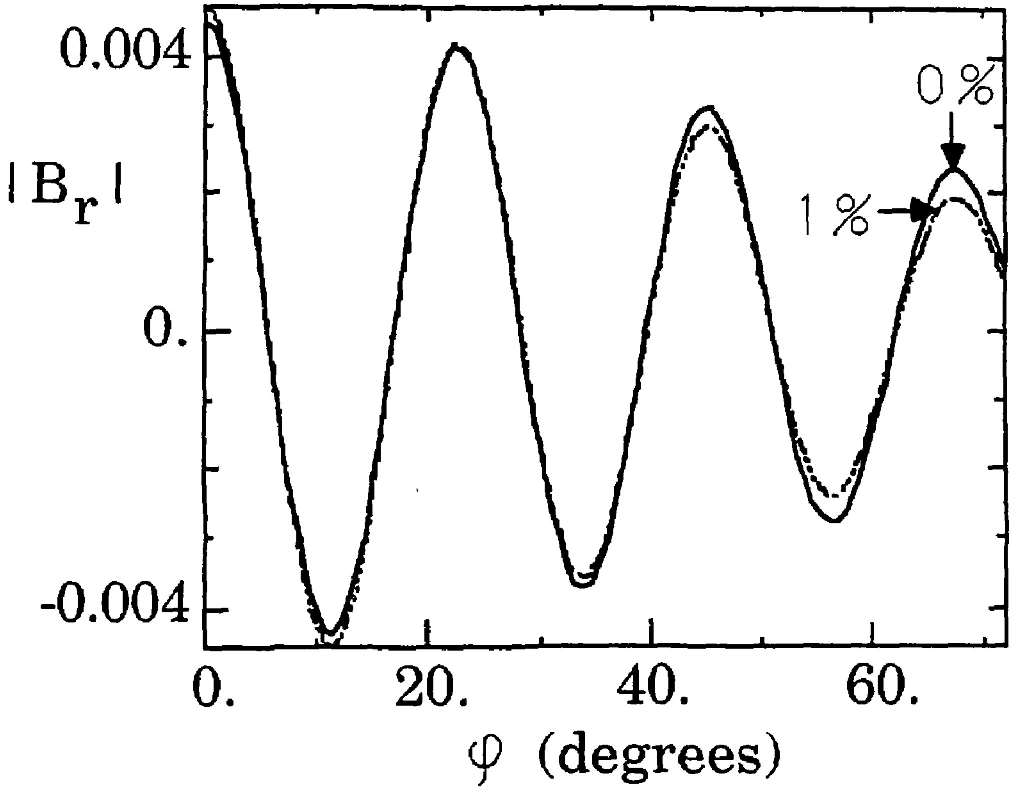

Fig. 8

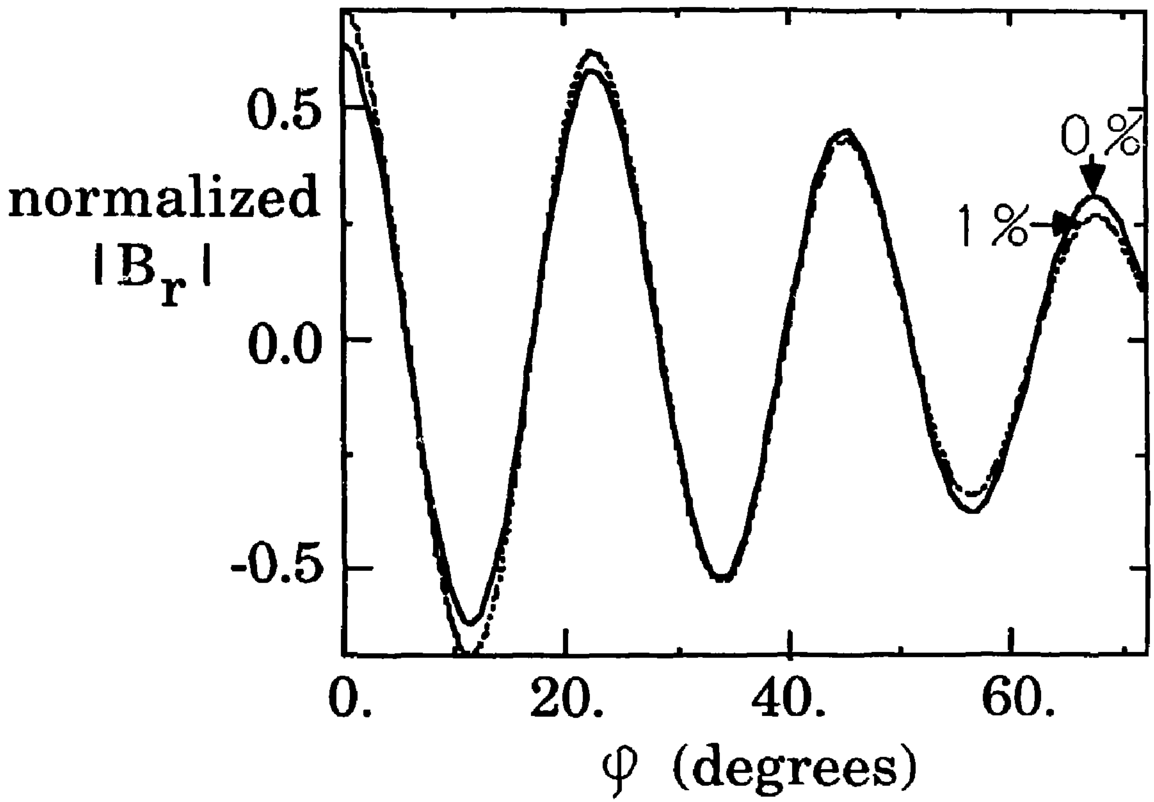

Fig. 9

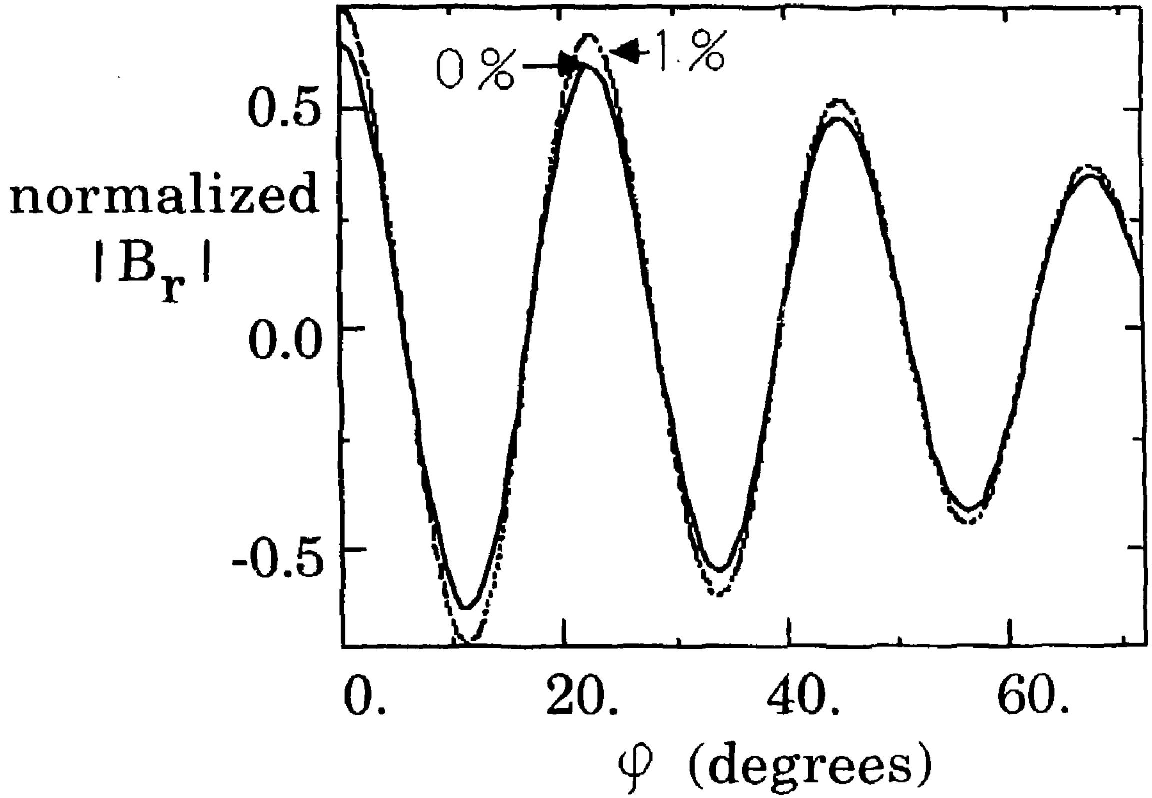

Fig. 10

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