Self-Dual Warped AdS_3 Black Holes

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AdS/CFT correspondence

Quantum gravity (string theory/M-theory) in (D + 1)-dimensional anti-de-Sitter spacetime is equivalent to D-dimensional conformal field theory at AdS boundary.

- A new way to study the strong coupling problems in field theory: AdS/QCD, AdS/CMT, ...
- A tool to study the problems in quantum gravity;
- Relies heavily on string theory technology.

AdS/CFT correspondence

A case not invoking string theory is AdS_3/CFT_2 correspondence: quantum gravity asymptotic to AdS_3 is holographically dual to a 2D CFT.

- For 3D pure gravity with a negative cosmological constant, the vacuum solution is AdS_3, with the isometry group $SL(2;\mathbb{R}) \times SL(2;\mathbb{R})$;
- The analysis of the symmetry of perturbations at asymptotic boundary give rise to a Virasoro algebra with a central term;
- The entropy of the BTZ black hole could be read from Cardy formula, which relates the asymptotic density of states in a 2D CFT and the symmetry algebra.

Topological Massive Gravity

- Pure 3D gravity: contains no local propagating degrees of freedom, thus hard to explain the microscopic origin of the BTZ black hole entropy.

- Topological Massive Gravity: with an additional gravitational Chern-Simons term, resulting a local, massive propagating degree of freedom.

$$I_{TMG} = \frac{1}{16\pi G}\left[\int d^3x\sqrt{-g}\,(R + 2/\ell^2) + \frac{1}{\mu}I_{CS}\right] \ , \quad (\mu > 0, G > 0) \tag{1}$$

$$I_{CS} = \frac{1}{2}\int d^3x\sqrt{-g}\varepsilon^{\lambda\mu\nu}\Gamma^{\rho}_{\lambda\sigma}\left(\partial_\mu\Gamma^{\sigma}_{\rho\nu} + \frac{2}{3}\Gamma^{\sigma}_{\mu\tau}\Gamma^{\tau}_{\nu\rho}\right) \tag{2}$$

The AdS_3 vacua in TMG are generally perturbatively unstable except at the chiral point $\mu\ell = 1$, where a consistent quantum theory of gravity is conjectured to exist and be dual to a chiral CFT.

Topological Massive Gravity

TMG possess other vacua for generic values of μ, namely warped AdS$_3$, admit isometry group $U(1) \times SL(2;\mathbb{R})$. Some of them are stable.

- Spacelike:

$$ds^2 = \frac{\ell^2}{(\nu^2+3)}\left[-\cosh^2\sigma d\tau^2 + d\sigma^2 + \frac{4\nu^2}{\nu^2+3}(du + \sinh\sigma d\tau)^2\right] \quad (3)$$

  stable for $\nu^2 > 1$ (stretched case), where $\nu = \mu\ell/3$.

- Timelike:

$$ds^2 = \frac{\ell^2}{(\nu^2+3)}\left[\cosh^2\sigma du^2 + d\sigma^2 - \frac{4\nu^2}{\nu^2+3}(d\tau + \sinh\sigma du)^2\right] \quad (4)$$

- Null: solution to TMG only for $\nu^2 = 1$

$$ds^2 = \ell^2\left[\frac{du^2}{u^2} + \frac{dx^+dx^-}{u^2} \pm \left(\frac{dx^-}{u^2}\right)^2\right] \quad (5)$$

Topological Massive Gravity

Quotients: Just as BTZ black holes are discrete quotients of ordinary AdS_3, there are black holes solutions as discrete quotients of warped AdS_3.

- Spacelike stretched black hole:

$$\frac{ds^2}{\ell^2} = dt^2 + \frac{dr^2}{(\nu^2+3)(r-r_+)(r-r_-)} + \left(2\nu r - \sqrt{r_+ r_-(\nu^2+3)}\right) dt d\theta$$
$$+ \frac{r}{4}\left(3(\nu^2-1)r + (\nu^2+3)(r_+ + r_-) - 4\nu\sqrt{r_+ r_-(\nu^2+3)}\right) d\theta^2 \quad (6)$$

  identifying points along isometry ∂_θ such that $\theta \sim \theta + 2\pi$.

- Self-dual solution:

$$ds^2 = \frac{\ell^2}{\nu^2+3}\left(-\tilde{x}^2 d\tilde{\tau}^2 + \frac{d\tilde{x}^2}{\tilde{x}^2} + \frac{4\nu^2}{\nu^2+3}\left(\alpha d\tilde{\phi} + \tilde{x} d\tilde{\tau}\right)^2\right) \quad (7)$$

  with identification $\tilde{\phi} \sim \tilde{\phi} + 2\pi$.

Warped AdS/CFT correspondence

It is conjectured that the $\nu > 1$ quantum TMG with asymptotical spacelike stretched AdS_3 geometry is holographically dual to a 2D CFT with central charges

$$c_L = \frac{4\nu\ell}{G(\nu^2+3)}, \qquad c_R = \frac{(5\nu^2+3)\ell}{G\nu(\nu^2+3)} \tag{8}$$

- Entropy of spacelike stretched black hole could be reproduced through Cardy formula

$$S = \frac{\pi^2\ell}{3}(c_L T_L + c_R T_R) \tag{9}$$

- Analysis of the asymptotic symmetry lead to a central extended Virasoro algebra with above central charges.

Warped AdS/CFT correspondence is essential in Kerr/CFT correspondence, in the sense that the warped AdS_3 structure appears in the near-horizon geometry of Kerr black hole.

Kerr/CFT correspondence

- NHEK: near-horizon geometry of the extreme Kerr black hole

$$ds^2 = 2J\Gamma(\theta)\left(-r^2dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \Lambda(\theta)^2(d\phi + r dt)^2\right) \qquad (10)$$

with $\phi \sim \phi + 2\pi$. A slice of NHEK geometry at fixed polar angle is locally a self-dual warped AdS_3.
Conjecture: quantum gravity in NHEK is dual to a 2D CFT, with only left-moving temperature.

- Near-NHEK: near-horizon geometry of the near-extreme Kerr black hole

$$ds^2 = 2J\Gamma(\theta)\left(-r(r+4\pi T_R)dt^2 + \frac{dr^2}{r(r+4\pi T_R)} + d\theta^2 \right.$$
$$\left. + \Lambda(\theta)^2(d\phi + (r + 2\pi T_R)dt)^2\right) \qquad (11)$$

Conjecture: near-NHEK is dual to the same CFT with non-vanishing right temperature, since the right-moving sector gets excited.

Motivation

Is there near-NHEK like solutions in TMG?
If they exist, what's the holographic description of them?
Relationship with the warped AdS/CFT correspondence?

It turns out that such solutions do exist in TMG:
self-dual warped AdS_3 black holes.

Holographic dual to a chiral 2D CFT, with the same left central charge as in warped AdS/CFT, and non-vanishing left- and right-moving temperatures.

Provide another novel support to the conjectured AdS/CFT correspondence.

Motivation

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Self-dual warped AdS_3 black hole

Self-dual warped AdS_3 black hole: non-Einstein black hole solution of TMG, which is asymptotic to spacelike warped AdS_3

$$ds^2 = \frac{\ell^2}{\nu^2+3}\left(-(x-x_+)(x-x_-)\,d\tau^2 + \frac{1}{(x-x_+)(x-x_-)}\,dx^2 + \frac{4\nu^2}{\nu^2+3}(\alpha d\phi + (x - \frac{x_+ + x_-}{2})\,d\tau)^2\right), \quad (12)$$

where $\tau \in [-\infty, \infty]$, $x \in [-\infty, \infty]$ and $\phi \sim \phi + 2\pi$.

- There seems to be two horizons at x_+ and x_-.
- The vacuum is chosen to be given by $x_+ = x_- = 0$ and $\alpha = 1$, which is the self-dual warped AdS_3.
- For $\nu^2 > 1$, the solution is free of naked CTCs.

Self-dual warped AdS_3 black hole

Self-dual warped AdS_3 black hole is related to the self-dual warped AdS_3 through coordinate transformation

$$\begin{aligned} \tilde{\tau}^{\pm} &\equiv \tilde{\tau} \pm \frac{1}{\tilde{x}} = \tanh\left[\frac{1}{4}\left((x_+ - x_-)\tau \pm \ln\frac{x - x_+}{x - x_-}\right)\right], \\ \tilde{\phi} &= \phi + \frac{1}{2}\ln\left[\frac{1 - (\tilde{\tau}^+)^2}{1 - (\tilde{\tau}^-)^2}\right]. \end{aligned} \tag{13}$$

- Globally, the maximal analytic extension of the self-dual warped black hole is diffeomorphic to the self-dual warped AdS_3.
- However, the above coordinate transformations are singular at the boundary $x \to \infty$, indicating different physics.
- The situation here is very similar to the relation between NHEK and near-NHEK, or between AdS_2 and AdS_2 black hole.
- Observer at fixed x measure a Hawking temperature proportional to $x_+ - x_-$. The entropy does not depend on $x_+ - x_-$, but the scattering amplitudes do depend on $x_+ - x_-$.

Self-dual warped AdS_3 black hole

Self-dual warped black hole is locally equivalent to spacelike warped AdS_3

$$ds^2 = \frac{\ell^2}{\nu^2+3}\left(-\cosh^2\sigma dv^2 + d\sigma^2 + \frac{4\nu^2}{\nu^2+3}(du + \sinh\sigma dv)^2\right) \qquad (14)$$

through coordinate transformation

$$\begin{aligned} v &= \tan^{-1}\left[\frac{2\sqrt{(x-x_+)(x-x_-)}}{2x-x_+-x_-}\sinh\left(\frac{x_+-x_-}{2}\tau\right)\right], \\ \sigma &= \sinh^{-1}\left[\frac{2\sqrt{(x-x_+)(x-x_-)}}{x_+-x_-}\cosh\left(\frac{x_+-x_-}{2}\tau\right)\right], \\ u &= \alpha\phi + \tanh^{-1}\left[\frac{2x-x_+-x_-}{x_+-x_-}\coth\left(\frac{x_+-x_-}{2}\tau\right)\right]. \end{aligned} \qquad (15)$$

Self-dual warped AdS_3 black hole

Killing vectors of spacelike warped AdS_3:

$$
\begin{aligned}
J_2 &= 2\partial_u \\
\tilde{J}_1 &= 2\sin\upsilon\tanh\sigma\partial_\upsilon - 2\cos\upsilon\partial_\sigma + \frac{2\sin\upsilon}{\cosh\sigma}\partial_u \\
\tilde{J}_2 &= -2\cos\upsilon\tanh\sigma\partial_\upsilon - 2\sin\upsilon\partial_\sigma - \frac{2\cos\upsilon}{\cosh\sigma}\partial_u \\
\tilde{J}_0 &= 2\partial_\upsilon
\end{aligned}
$$

Self-dual warped AdS_3 black hole solution is free of curvature singularity and regular everywhere.

- Strictly speaking, it does not belong to the category of regular black holes, which require the existence of a geometric or causal singularity shielded by an event horizon.
- On the other hand, similar to black holes, this solution have regular thermodynamic behavior, satisfying the first law of thermodynamics.

Thermodynamics

- Conserved charges:

$$\mathcal{M}^{ADT} = 0\,, \qquad \mathcal{J}^{ADT} = \frac{(\alpha^2 - 1)\nu\ell}{6G(\nu^2 + 3)} \tag{16}$$

- Entropy: with the Chern-Simons contribution

$$S = S_E + S_{CS} = \frac{2\pi\alpha\nu\ell}{3G(\nu^2 + 3)} \tag{17}$$

- Hawking temperature T_H and the angular velocity of the horizon Ω_h ,

$$T_H = \frac{x_+ - x_-}{4\pi\ell}\,, \qquad \Omega_h = -\frac{x_+ - x_-}{2\alpha\ell} \tag{18}$$

- The first law

$$d\mathcal{M}^{ADT} = T_H\, dS + \Omega_h\, d\mathcal{J}^{ADT} \tag{19}$$

is satisfied for a variation of the black hole parameter α.

Thermodynamics

Remarks:

- ADT mass and angular momentum do not depend on x_+ and x_-.
  Entropy S does not depend on x_+ and x_- either.
  Observers at fixed x measure a Hawking temperature proportional to $x_+ - x_-$.

  Similar to the case of near-NHEK.

- Even with a different choice of vacuum, the first law of thermodynamics still holds.

Temperatures in dual CFT

Since the self-dual warped black hole metric looks much similar to the near-NHEK geometry, it is sensible to define a quantum vacuum in analogy to the Frolov-Thorne vacuum of Kerr.

The construction of the vacuum begins by expanding the quantum fields in eigenmodes of the asymptotic energy ω and angular momentum k. For a scalar field,

$$\Phi = \sum_{\omega,k,l} \phi_{\omega k l} e^{(-i\omega\tau + ik\phi)} \mathcal{R}_l(x) \tag{20}$$

After tracing over the region inside the horizon, the vacuum is a diagonal density matrix in the energy-angular momentum eigenbasis with a Boltzmann weighting factor

$$\exp\left(-\hbar \frac{\omega - k\Omega_H}{T_H}\right) \tag{21}$$

Temperatures in dual CFT

In AdS/CFT dictionary, black hole in AdS is dual to finite temperature CFT. Taking black hole as a thermodynamical system, the thermal equilibrium in black hole system could be compared to thermal equilibrium of finite temperature CFT.

2D CFT contains two independent sectors: left-moving one and right-moving one, possibly with different central charges and temperatures.

Temperatures in dual CFT

The Boltzman factor should be identified with that in CFT

$$\exp\left(-\hbar\frac{\omega - k\Omega_H}{T_H}\right) = \exp\left(-\frac{n_L}{T_L} - \frac{n_R}{T_R}\right) \tag{22}$$

The left and right charges n_L , n_R associated to ∂_ϕ and ∂_τ are

$$n_L \equiv k\,, \qquad n_R \equiv \omega \tag{23}$$

This defines the left and right temperatures:

$$T_L = \frac{\alpha}{2\pi\ell}\,, \qquad T_R = \frac{x_+ - x_-}{4\pi\ell} \tag{24}$$

Temperatures in dual CFT

- The right temperature denotes the deviation from extremality, originates essentially similar to the case that Rindler observers detect radiations in the Minkowski vacuum (as discussed for AdS_2 black hole).
- The left temperature arises from the periodical identification of points in spacelike warped AdS_3 along ∂_ϕ. Expressing ∂_ϕ in terms of the spacelike warped AdS_3 coordinates,

$$\partial_\phi = \frac{\alpha}{2} J_2 = \pi \ell T_L J_2 \tag{25}$$

  the temperature of the dual 2D CFT can be read from the coefficient of the shift. This periodical identification makes no contribution to the right temperature.

Asymptotic behavior

To acquire the central charge of the dual CFT through the asymptotic symmetry analysis, we impose the following consistent boundary conditions

$$\begin{pmatrix} h_{\tau\tau} = O(1) & h_{\tau x} = O(1/x^3) & h_{\tau\phi} = O(x) \\ h_{x\tau} = h_{\tau x} & h_{xx} = O(1/x^3) & h_{x\phi} = O(1/x) \\ h_{\phi\tau} = h_{\tau\phi} & h_{\phi x} = h_{x\phi} & h_{\phi\phi} = O(1) \end{pmatrix} \tag{26}$$

where $h_{\mu\nu}$ is the deviation of the full metric from the vacuum.

These boundary conditions differ from both the ones in spacelike warped AdS_3 and the Kerr black holes, since the allowed deviations $h_{\tau\phi}$ and $h_{\phi\phi}$ are of the same order as the leading terms.

Asymptotic behavior

- The asymptotic symmetry group, preserving the above boundary conditions, contains one copy of the conformal group of the circle generated by

$$\xi_\epsilon = \epsilon\left(\phi\right)\partial_\phi \tag{27}$$

Since $\phi \sim \phi + 2\pi$, it is convenient to define $\epsilon_n(\phi) = e^{in\phi}$ and $\xi_n = \xi(\epsilon_n)$. They admit the following commutators

$$i\left[\xi_m, \, \xi_n\right] \; = \; (m - n)\, \xi_{m+n} \tag{28}$$

and ξ_0 generates the $U(1)$ rotational isometry. That is, the $U(1)$ isometry is enhanced to a Virasoro algebra.

- The conserved charge associated with an asymptotic Killing vector ξ represent the asymptotic symmetries algebra via a covariant Poisson bracket, up to a central term

$$i\{Q_m, Q_n\} \;\; = \;\; (m-n)Q_{m+n} + \frac{c_L}{12} m(m^2 - 2)\delta_{m+n,0} \, , \tag{29}$$

where the central charge

$$c_L \; = \; \frac{4\nu\ell}{G(\nu^2+3)} \tag{30}$$

Asymptotic behavior

- The c_L is exactly the value of the left central charge conjectured in warped AdS/CFT correspondence for spacelike warped AdS_3.

- The entropy of the self-dual warped black hole can be reproduced from the Cardy formula

$$S = \frac{2\pi\alpha\nu\ell}{3G(\nu^2+3)} = \frac{\pi^2\ell}{3}\frac{4\nu\ell}{G(\nu^2+3)}\frac{\alpha}{2\pi\ell} \equiv \frac{\pi^2\ell}{3}c_L T_L \qquad (31)$$

- Conjecture: our self-dual warped black hole solution is dual to a chiral 2D CFT with the temperatures and central charges

$$T_L = \frac{\alpha}{2\pi\ell}, \quad T_R = \frac{x_+ - x_-}{4\pi\ell}; \quad c_L = \frac{4\nu\ell}{G(\nu^2+3)} \qquad (32)$$

Further supported by the real-time correlator of scalar perturbations.

Scalar perturbation

Consider a scalar field Φ with mass m in black hole background

$$\Phi = e^{-i\omega\tau + ik\phi}\mathcal{R}(x) \tag{33}$$

The radial wave function $\mathcal{R}(x)$ satisfies the equation

$$\frac{d}{dx}\left((x-x_+)(x-x_-)\frac{d}{dx}\right)\mathcal{R}(x) - \left(\frac{\nu^2+3}{4\nu^2}\frac{k^2}{\alpha^2} + \frac{\ell^2}{\nu^2+3}m^2 - \frac{\left(\omega + \frac{k}{\alpha}\left(x - \frac{x_+ + x_-}{2}\right)\right)^2}{(x-x_+)(x-x_-)}\right)\mathcal{R}(x) = 0 \tag{34}$$

Choose ingoing boundary condition at the horizon for calculating the retarded Green's function, the solution is

$$\mathcal{R}(x) = N\left(\frac{x-x_+}{x-x_-}\right)^{-\frac{i}{2}\left(\frac{k}{\alpha}+\frac{2\omega}{x_+-x_-}\right)}\left(\frac{x_+-x_-}{x-x_-}\right)^{\frac{1}{2}-\beta} \cdot F\left(\frac{1}{2}-\beta-i\frac{k}{\alpha}, \frac{1}{2}-\beta-i\frac{2\omega}{x_+-x_-}, 1-i\left(\frac{k}{\alpha}+\frac{2\omega}{x_+-x_-}\right); \frac{x-x_+}{x-x_-}\right) \tag{35}$$

Scalar perturbation

At asymptotical infinity, the radial eigenfunction has the behavior

$$\mathcal{R}(x) \sim A x^{-\frac{1}{2}-\beta} + B x^{-\frac{1}{2}+\beta} \tag{36}$$

where

$$A = N\left(x_+ - x_-\right)^{\frac{1}{2}+\beta} \frac{\Gamma\left(-2\beta\right)\Gamma\left(1 - i\left(\frac{k}{\alpha} + \frac{2\omega}{x_+ - x_-}\right)\right)}{\Gamma\left(\frac{1}{2} - \beta - i\frac{k}{\alpha}\right)\Gamma\left(\frac{1}{2} - \beta - i\frac{2\omega}{x_+ - x_-}\right)} \tag{37}$$

$$B = A\left(\beta \to -\beta\right) \tag{38}$$

The real-time retarded Green's function could be computed using a prescription in terms of the boundary values of the bulk fields. Consider a real $\beta > 0$ without loss of generality, the retarded correlator is given by

$$G_R \sim \frac{A}{B} = \left(x_+ - x_-\right)^{2\beta} \frac{\Gamma(-2\beta)}{\Gamma(2\beta)} \frac{\Gamma\left(\frac{1}{2} + \beta - i\frac{k}{\alpha}\right)\Gamma\left(\frac{1}{2} + \beta - i\frac{2\omega}{x_+ - x_-}\right)}{\Gamma\left(\frac{1}{2} - \beta - i\frac{k}{\alpha}\right)\Gamma\left(\frac{1}{2} - \beta - i\frac{2\omega}{x_+ - x_-}\right)} \tag{39}$$

Scalar perturbation

Throwing a scalar Φ at the black hole is dual to exciting the CFT by acting with an operator $\mathcal{O}_\Phi$. For an operator of dimensions (h_L, h_R) at temperature (T_L, T_R), the momentum-space Euclidean Green's function is determined by conformal invariance

$$G_E(\omega_{L,E}, \omega_{R,E}) \sim T_L^{2h_L-1} T_R^{2h_R-1} e^{i\frac{\omega_{L,E}}{2T_L}} e^{i\frac{\omega_{L,E}}{2T_R}} \Gamma(h_L + \frac{\omega_{L,E}}{2\pi T_L})\Gamma(h_L - \frac{\omega_{L,E}}{2\pi T_L}) \cdot \Gamma(h_R + \frac{\omega_{R,E}}{2\pi T_R})\Gamma(h_R - \frac{\omega_{R,E}}{2\pi T_R}) \quad (40)$$

the Euclidean correlator $G_E(\omega_{L,E}, \omega_{R,E})$ is related to the value of the retarded correlator $G_R(\omega_L, \omega_R)$ by

$$G_E(\omega_{L,E}, \omega_{R,E}) = G_R(i\omega_{L,E}, i\omega_{R,E}) \quad (41)$$

Comparing the arguments of the Gamma functions among (39) and (41), we find precise agreement under the following identification

$$h_L = h_R = \frac{1}{2} + \beta, \quad \omega_L = k/\ell, \quad \omega_R = \omega/\ell, \quad T_L = \frac{\alpha}{2\pi\ell}, \quad T_R = \frac{x_+ - x_-}{4\pi\ell}$$

Summary

- A novel example of warped AdS/CFT correspondence: the self-dual warped AdS_3 black hole is dual to a chiral CFT with non-vanishing left central charge.
- The quantum topological massive gravity asymptotic to the same spacelike warped AdS_3 in different consistent ways may be dual to different 2D CFTs.

Thank you !