LETTERS

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Self-induced oscillations in the shock wave flow pattern formed in a stationary supersonic flow over a double wedge

G. Ben-Dor
Pearlstone Center for Aeronautical Engineering Studies, Department of Mechanical Engineering, Ben-Gurion University of the Negev, Beer Sheva 84105, Israel

E. I. Vasilev
Department of Computational Mechanics, Volgograd State University, Volgograd, Russia

T. Elperin
Pearlstone Center for Aeronautical Engineering Studies, Department of Mechanical Engineering, Ben-Gurion University of the Negev, Beer Sheva 84105, Israel

A. V. Zenovich
Department of Computational Mechanics, Volgograd State University, Volgograd, Russia

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Numerical simulations of a two-dimensional supersonic flow of an inviscid perfect gas over a double wedge in the Mach numbers range $5 \leqslant M \leqslant 9$, revealed the existence of self-induced oscillations in the shock wave flow pattern in a narrow range of geometrical parameters. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1625646]

We consider a supersonic flow of an inviscid perfect gas over a double wedge (Fig. 1) in the range of high Mach numbers for which the shock waves are attached to the leading edges of both the first and the second wedges. The interaction of these two waves results in the formation of a complex shock wave flow pattern. Such flow fields can occur during flight of supersonic/hypersonic aircrafts or in the course of the re-entry of space shuttles. A numerical solution of this problem, in a stationary formulation, was conducted elsewhere.[1]

Our recent success in discovering the existence of hysteresis processes in numerous cases of the interaction of supersonic flows with various geometries[2] motivated us to thoroughly complement these studies and check whether a hysteresis phenomenon exists in the case of a supersonic flow over a double wedge.

Not only did our study reveal that there is a hysteresis, we also found out that there are self-induced oscillations in the shock wave flow pattern for various angles of inclination of the second wedge.

The flow is described by the nonstationary Euler equations for an inviscid perfect diatomic gas ($\gamma=1.4$). The parameters in the problem are the free stream flow Mach number, M, the ratio of the lengths of the surfaces of the double wedge, L_1/L_2, the angle of the first wedge, θ_1, and the angle of the second wedge, θ_2, or alternatively the difference between the two wedge angles, $\Delta\theta=\theta_2-\theta_1$.

A W-modification of Godunov's scheme[3] that has second-order accuracies both in space and time was used in the calculations. The stationary solution was determined by settling of the nonstationary solution with time. Since the inclination of the first shock wave and the induced flow field behind it could be determined analytically, the size of the computational domain was reduced to include only the region of the interaction of the two shock waves (dashed line in Fig. 1). In order to damp the numerical oscillations that occur behind strong shock waves in stationary flows the following technique was employed. At each time step two independent calculations were performed using standard and diagonal stencils (Fig. 2), and their average was used at the following time step. This extension of the stencil enabled us to damp the numerical fluctuations and to avoid the need to use artificial viscosity.

The investigation was performed with the following parameters: $L_1/L_2=2$, $M=9$, $\theta_1=15°$, and $\Delta\theta$ was changed in the course $20°\rightarrow35°\rightarrow20°$. The goal was to check whether there is a hysteresis phenomenon. This was done by changing the value of $\Delta\theta$ while keeping all the other parameters fixed. $\Delta\theta$ was continuously changed during one dimensionless unit of time by 0.2°, and then five units of dimensionless time were spent to enable the solution to settle. The time was non-dimensionalized by L_1/a (the ratio of the length of the first wedge surface, L_1, to the speed of sound of the free stream flow, a).

As a criterion for attaining a stationary solution the dis-

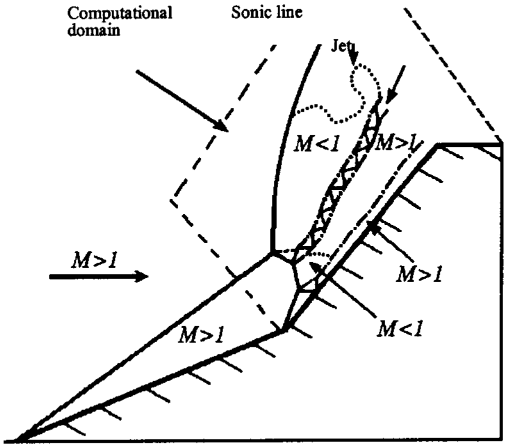

FIG. 1. The computational domain.

crete analog of the following non-stationary residual was used:

$$R_n = \frac{1}{S(\mathbf{\Omega})} \int_{\Omega} \frac{1}{\rho} \left| \frac{\partial \rho}{\partial t} \right| ds, \tag{1}$$

where $S(\mathbf{\Omega})$ is the area of the domain Ω where R_n is calculated. We used one third (from the left) of the computational domain as Ω, i.e., the region inside a control volume with interacting shock waves (Fig. 1).

The results of the numerical calculations of two wave configurations for $\Delta\theta = 26.4°$ and $\Delta\theta = 28°$ are shown in Fig. 3. The important difference between these wave configurations is the occurrence of a Mach stem with a subsonic flow patch (Fig. 1) behind it in Fig. 3(b).

The main result of the present study was the finding that there is a wedge angle range, $27° \leqslant \Delta\theta < 28°$, inside which a stationary flow was not established. Instead, our numerical study revealed the excitation of self oscillations inside this range of wedge angles. These oscillations are demonstrated in Fig. 4 where the time variation of the non-stationary residual, R_n, is shown. In the time interval $0 < t < 6$, where $\Delta\theta$ is augmented $26.6° \rightarrow 26.8°$ at $0 < t < 1$, and $\Delta\theta = 26.8°$ at $1 < t < 6$, the residual is sharply decreased. This in turn assured that a stationary solution was reached.

A further increase of the magnitude of $\Delta\theta$ resulted in an almost periodic fluctuations in R_n, i.e., excitation of the oscillations. During one period of oscillations the wave pattern alternated between the wave patterns shown in Figs. 3(a) and 3(b). For $\Delta\theta \geqslant 28°$ the oscillations disappeared and a stationary solution was obtained. The evolution of the wave configuration during one period of oscillations is shown in Fig. 5 where successive frames with constant density contours (isopycnics) with a time step equal to 1/20 of the period are showed. Frames 1, 3, 5, 9, 11 and 13 are not shown since the changes of the flow in these frames are insignificant.

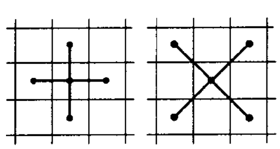

FIG. 2. Standard and diagonal stencils.

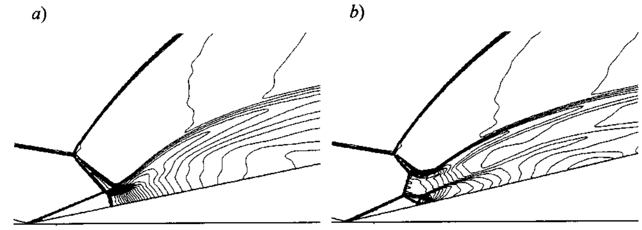

FIG. 3. Constant density contours (isopycnics) for stable wave configurations (mesh 300×125). (a) $\Delta\theta = 26.4°$ and (b) $\Delta\theta = 28°$.

Frame 0 shows a regular interaction between the shock wave emanating from the leading edge of the second wedge and the one emanating from the triple point that was formed along the shock wave emanating from the leading edge of the first wedge (Fig. 1). The refracted shock wave of the latter one is seen to reflect as a Mach reflection from the surface of the second wedge (the Mach stem of this Mach reflection is very short). Frames 0 to 4 indicate that this Mach stem grows and moves upstream along the second wedge surface. A remarkable change in the shock wave flow pattern is evident in frame 6. As can be seen a triple point develops along the shock wave emanating from the leading edge of the second wedge too. The Mach stems of this triple point (the lower one) and the one that develops along the shock wave emanating from the leading edge of the first wedge (the upper one) are seen to interact in a regular manner to result in a jet as is shown schematically in Fig. 1. In addition, the reflected shock wave of the lower triple point is seen to reflect as a regular reflection from the surface of the second wedge. The Mach stem of the lower triple point is seen to attain its maximum length in frame 7. The subsonic flow patch behind the Mach stem is bounded by two contact surfaces, one that emanates from the lower triple point and one that bounds the above mentioned jet (Fig. 1). After reaching its maximal length the Mach stem is seen to decrease in its length until it reaches its minimal length at frame 10. The fast shift in the position of the triple points generates an instability of the contact surfaces that is clearly visible in frame 8. Following frame 12 the Mach stem starts to grow slowly again until it attains its second maximum in frame 16. The second process of the decrease of the Mach stem length (frames 16–20) is accompanied with a transition to a regular interaction, and the wave pattern returns to the initial one that was shown in frame 0 (note that frames 20

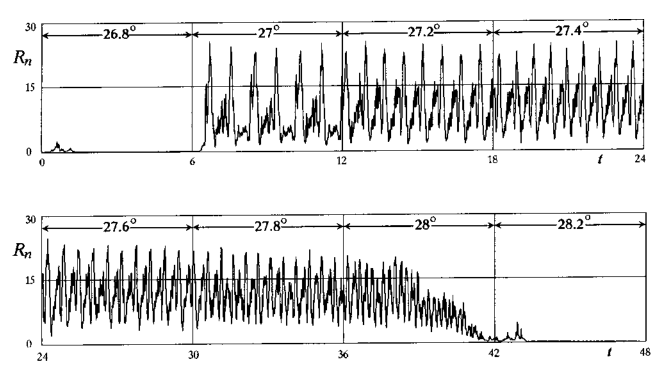

FIG. 4. Time variation of the non-stationary residual R_n during the change in the angle $\Delta\theta$.

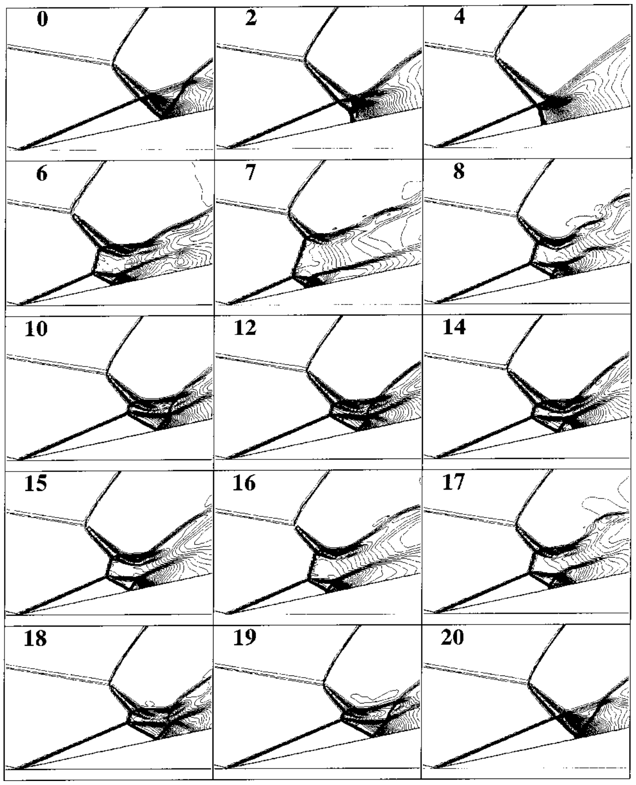

FIG. 5. Successive frames with constant density contours of the time evolution of the wave pattern in the regime with self oscillations for $\Delta\theta = 27°$.

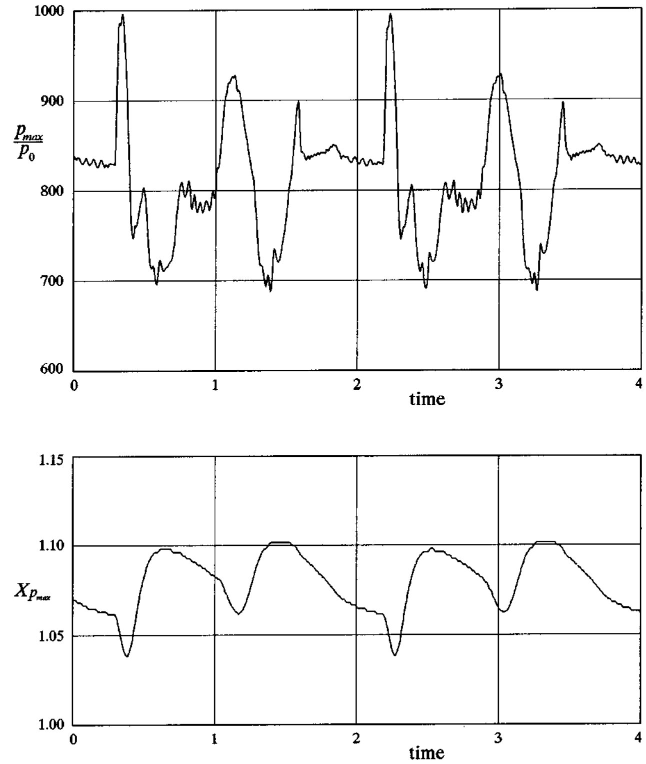

FIG. 6. Time variation of the maximum pressure along the wedge surface (top plot) and its location (bottom plot).

and 0 are identical). Then the entire process repeats itself. The computational time step in these calculations was $7\cdot10^{-5}$–$8\cdot10^{-5}$.

A close inspection of frames 6 and 7 reveals a difference in the motion of the upper and the lower triple points. The upper triple point reaches its extreme left position in frame 6, while the lower triple point reaches its extreme left position in frame 7 when the upper triple point already moves in the right direction. Thus the motion of the lower triple point occurs with a small delay relatively to that of the upper triple point. This delay allows elucidating the mechanism of the observed oscillations. When the upper and the lower triple points move they affect each other. The motion of the lower triple point changes the angles of inclination of the contact surfaces (frames 7 and 12). The latter results in a shift of the main reflected shock wave front (see Fig. 5), and, consequently, a shift of the upper triple point. The shift of the upper triple point changes the location of the intermediate triple point and the size of the subsonic domain behind the Mach stem. The latter results in a shift in the position of the Mach stem since its size is determined by the size (area) of the critical cross section. The lower triple point moves together with the Mach stem. In the case when these interactions are balanced, the stationary flow shown in Fig. 3(b) is formed.

Such a situation occurs when $\Delta\theta \geqslant 28°$. However, if the delay in these interactions exceeds some critical value, the stable equilibrium is violated and the flow becomes oscillatory. The transition to the oscillatory flow regime occurs in the range $27° \leqslant \Delta\theta < 28°$.

The foregoing presented oscillations in the shock wave flow pattern result in a time variation of the pressure along the second wedge surface. The time variation of the maximum non-dimensional pressure and its non-dimensional location along the second wedge surface for two periods of oscillations in the flow regime are shown in Fig. 6. The pressure is normalized by the free stream flow pressure and the location along the second wedge surface is normalized by the first wedge length. The range of the variation in the maximum pressure is quite large, from 700 to 1000. Note that the pressure maximum is located immediately behind the re-

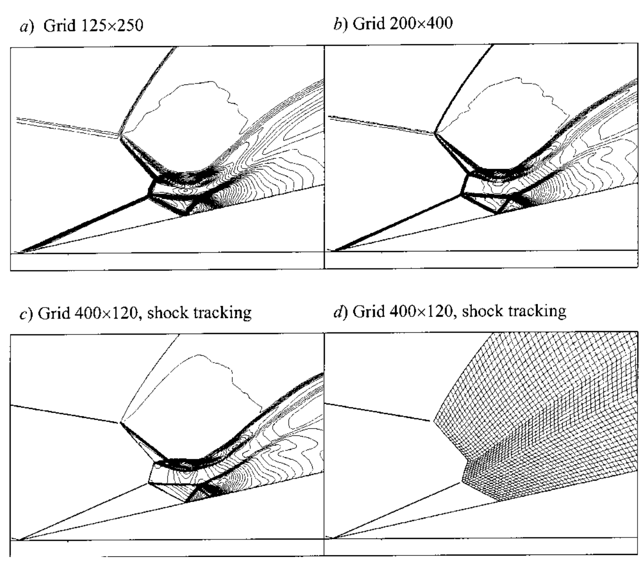

FIG. 7. Constant density contours with mesh refinement [(a)–(c)], and the grid used for shock tracking (d).

flected shock wave at the second wedge surface, and that the small oscillations in the figure are caused by the fluctuations of the numerical solution behind the strong shock waves. These fluctuations are local and their influence on entire flow pattern is small. An inspection of these plots shows that the largest maximum pressure is associated with the minimum of $X_{p_{\max}}$. At these moments the length of the Mach stem is maximal (frames 7 and 16 in Fig. 5).

Additional investigations showed that mesh refinement did not change the above-described oscillations of the shock waves pattern and only slightly changed the period of the oscillations. The results of the calculations for $\Delta\theta = 27.4°$, using three different grids at the time when the Mach stem height is close to its minimal value, are shown in Figs. 7(a), 7(b) and 7(c). In the case shown in Fig. 7(c) we employed a completely different movinggrid with capturing of the main shock waves.[4,5] This mesh is shown in Fig. 7(d) where each third grid line is plotted. Notably, in all these calculations we observed self-oscillations of the shock waves pattern with periods 0.64, 0.62 and 0.61, correspondingly. The oscillations observed for this value of $\Delta\theta$ are not accompanied by the transition to regular reflection, i.e., the Mach stem always existed and as a consequence, the amplitude of the maximum pressure at the wall was lower than that on Fig. 6 by a factor of two.

It should be noted that the above-presented results are based on an ideal fluid flow model. Viscous and thermal conduction effects could modify the flow topology, mainly inside the boundary layers. However, since the observed phenomenon is a result of the shock wave interactions outside the boundary layers and the shock wave reflections from the wedge surface, the findings of the present study will most likely prevail in real fluids.

Finally, it is possible that the large fluctuations and fast shifts of the location of the maximal pressure along the wedge surface can cause high-frequency vibrations and mechanical damage to supersonic aircraft/re-entry vehicles that have double-wedge like geometries.

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