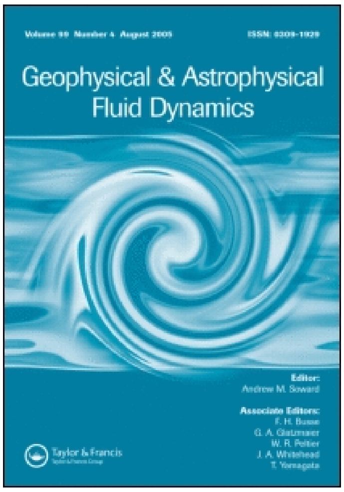

Geophysical & Astrophysical Fluid Dynamics
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713642804

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Online Publication Date: 01 April 2007

To cite this Article Vishwakarma, J. P., Maurya, Anil Kumar and Singh, K. K.(2007)'Self-similar adiabatic flow headed by a magnetogasdynamic cylindrical shock wave in a rotating non-ideal gas',Geophysical & Astrophysical Fluid Dynamics,101:2,155 — 168

To link to this Article: DOI: 10.1080/03091920701298112

URL: http://dx.doi.org/10.1080/03091920701298112

Geophysical and Astrophysical Fluid Dynamics,
Vol. 101, No. 2, April 2007, 155–168

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Self-similar adiabatic flow headed by a magnetogasdynamic cylindrical shock wave in a rotating non-ideal gas

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(*Received 6 September 2005; in final form 2 January 2007*)

Similarity solutions are obtained for one-dimensional adiabatic flow behind a magnetogasdynamic cylindrical shock wave propagating in a rotating non-ideal gas in presence of an azimuthal magnetic field. The density of the medium ahead of the shock is assumed to be constant. In order to obtain the similarity solutions the angular velocity of the ambient medium is assumed to be obeying a power law and to be decreasing as the distance from the axis increases. It is found that the similarity solutions exist, in both the cases, when the initial magnetic field is constant or obeying a power law. The effects of an increase in the value of the index for variation of angular velocity of the ambient medium, in the value of the parameter of the non-idealness of the gas and in the strength of the initial magnetic field are obtained.

Keywords: Shock wave; Magnetogasdynamics; Non-ideal gas; Rotating medium; Adiabatic flow; Similarity solutions

1. Introduction

The formulation of self-similar problems and examples describing the adiabatic motion of non-rotating gas models of stars, are considered by Sedov (1959), Zel'dovich and Raizer (1967), Lee and Chen (1968) and Summers (1975). Rotation of the stars significantly affects the process taking place in their outer layers. Therefore, question connected with the explosions in rotating gas atmospheres are of definite astrophysical interest. Chaturani (1970) studied the propagation of cylindrical shock waves through a gas having solid body rotation, and obtained the solutions by a similarity method adopted by Sakurai (1956). Nath *et al.* (1991) obtained the similarity solutions for the flow behind spherical shock waves propagating in a non-uniform rotating

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Geophysical and Astrophysical Fluid Dynamics
ISSN 0309-1929 print/ISSN 1029-0419 online
http://www.tandf.co.uk/journals
DOI: 10.1080/03091920701298112

interplanetary atmosphere with increasing energy. Ganguly and Jana (1998) studied a theoretical model of propagation of strong spherical shock waves in a self-gravitating atmosphere with radiation flux in presence of a magnetic field. They, also, considered the medium behind the shock to be rotating, but neglected the rotation of the undisturbed medium. In all of the works, mentioned above, the medium is taken to be a gas satisfying the equation of state of a perfect gas.

Because of high pressure and density that generally occur behind a shock wave, produced by an explosion, the assumption that the gas is ideal is no more valid. The popular alternative to the ideal gas is a simplified van der Waals model. Roberts and Wu (1996, 2003) adopted this model to discuss the shock wave theory of sonoluminescence. In the present work, we too adopt this as our model of a non-ideal gas to obtain the self-similar solutions for the flow behind a magnetogasdynamic cylindrical shock wave propagating in a rotating gas in presence of an azimuthal magnetic field. The non-ideal gas is assumed to have infinite electrical conductivity and constant specific heats. The initial density of the medium is assumed to be constant. In order to obtain similarity solutions, angular velocity of rotation of the ambient medium is assumed to be obeying a power law and to be decreasing as the distance from the axis increases. It is expected that such an angular velocity may occur in the atmospheres of rotating stars.

Effects of a change in the strength of ambient magnetic field, in the non-idealness of the gas, and in the index of variation of angular velocity of the ambient medium (or index of variation of ambient magnetic field) are investigated.

2. Basic equations and boundary conditions

The fundamental equations governing the unsteady adiabatic cylindrically symmetric motion of a non-ideal and perfectly conducting gas, which is rotating about the axis of symmetry and in which an azimuthal magnetic field is permeated and heat conduction and viscous stress are negligible (cf. Whitham 1958, Chaturani 1970), are

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\rho u}{r} = 0, \tag{1a}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\left(\frac{\partial p}{\partial r} + \mu h\frac{\partial h}{\partial r} + \frac{\mu h^2}{r}\right) - \frac{v^2}{r} = 0, \tag{1b}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} = 0, \tag{1c}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{uv}{r} = 0, \tag{1d}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} + p\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\rho}\right) = 0, \tag{1e}$$

where ρ, p, h are density, pressure and azimuthal magnetic field, respectively; u and v are radial and azimuthal components of the fluid velocity; μ is magnetic permeability; r and t are distance and time; and e is internal energy per unit mass. Also, we write

$$v = Ar, \tag{2}$$

where A is angular velocity of the medium at radial distance r from the axis of symmetry.

In most of the cases the propagation of shock waves arises in extreme conditions under which the assumption that the gas is ideal is not a sufficiently accurate description. To discover how deviations from the ideal gas can affect the solutions, we adopt a simple model. We assume that the gas obeys a simplified van der Waals equation of state of the form (Roberts and Wu 1996, 2003)

$$p = \frac{\Gamma \rho T}{1 - b\rho}, \quad e = c_v T = \frac{p(1 - b\rho)}{\rho(\gamma - 1)}, \tag{3a, b}$$

where Γ is the gas constant, $c_v = \Gamma/\gamma(-1)$ is the specific heat at constant volume and γ is the ratio of specific heats. The constant b is the 'van der Waals excluded volume'; it places a limit, $\rho_{\max} = 1/b$, on the density of the gas.

We assume that a cylindrical shock is propagating outwards from the axis of symmetry in the non-ideal and perfectly conducting gas with constant initial density. Conditions across the magnetogasdynamic shock are

$$u_2 = (1 - \beta)V, \quad \rho_2 = \frac{\rho_1}{\beta}, \quad h_2 = \frac{h_1}{\beta}, \tag{4a–c}$$

$$p_2 = \left[\frac{1}{\gamma M^2} + \frac{2(1 - \beta)}{\beta(\gamma + 1) - (\gamma - 1) - 2b\rho_1}\left\{\frac{1}{M^2} + \frac{\gamma - 1}{4M_A^2}\left(\frac{1}{\beta} - 1\right)^2\right\}\right]\rho_1 V^2, \tag{4d}$$

$$v_2 = v_1, \tag{4e}$$

where

$$\beta^3 - \beta^2 L + \left\{\frac{\gamma + b\rho_1 - 2}{(\gamma + 1)M_A^2}\right\}\beta + \frac{b\rho_1}{(\gamma + 1)M_A^2} = 0, \tag{5a}$$

$$L = \frac{\gamma - 1}{\gamma + 1} + \frac{2b\rho_1}{(\gamma + 1)} + \frac{2}{(\gamma + 1)M^2} + \frac{\gamma}{(\gamma + 1)M_A^2}. \tag{5b}$$

Here V is the shock velocity, M is the shock-Mach number referred to the frozen speed of sound $(\gamma p_1/\rho_1)^{1/2}$, and M_A is the Alfven–Mach number. Quantities with suffices '1' and '2' correspond to their values just ahead and just behind the shock, respectively.

The shock-Mach number M_e referred to the speed of sound in non-ideal gas $[\gamma p_1/\rho_1(1 - b\rho_1)]^{1/2}$ and the Alfven–Mach number M_A are given by

$$M_e = M(1 - b\rho_1)^{1/2}, \quad M_A = \frac{V}{(\mu h_1^2/\rho_1)^{1/2}}, \tag{6a, b}$$

where

$$M = \frac{V}{(\gamma p_1/\rho_1)^{1/2}}. \tag{6c}$$

Ahead of the shock the azimuthal magnetic field varies as

$$h_1 = h_0 R^{\alpha}, \tag{7}$$

where h_0 and α are constants, and R is the shock radius.

In order to obtain the similarity solution, it is assumed that the initial angular velocity A_1 varies as

$$A_1 = A_0 R^{d}, \tag{8}$$

where A_0 and d are constants. The assumption of varying initial angular velocity is necessary as $d = 0$ implies from relation (13) $\alpha = 1$, which is inconsistent with the relation (19).

The momentum equation (1b) in the undisturbed state of the gas, gives

$$p_1 = \left[\rho_1 A_0^2 - (1+\alpha)\mu h_0^2\right]\frac{R^{2\alpha}}{2\alpha} + \text{constant}. \tag{9}$$

The total energy of the flow-field behind the shock is not constant, but assumed to be time dependent and varying as (Rogers 1958, Freeman 1968, Director and Dabora 1977)

$$E = E_0 t^{w}, \quad w \geq 0, \tag{10}$$

where E_0 and w are constants. The positive values of w correspond to the class in which the total energy increases with time. This increase can be achieved by the pressure exerted on the fluid by an expanding surface (a contact surface or a piston). Thus the flow is headed by a shock front and has an expanding surface as an inner boundary.

3. Similarity solutions

We introduce the following similarity transformations to reduce the equations of motion into ordinary differential equations:

$$u = VU(x), \quad \rho = \rho_1 \Omega(x), \quad p = \rho_1 V^2 P(x), \tag{11a–c}$$

$$v = VK(x), \quad \sqrt{\mu} h = \rho_1^{1/2} V H(x), \tag{11d, e}$$

where U, Ω, P, H, and K are the functions of the non-dimensional variable $x = r/R$. The shock front is represented by $x = 1$.

The shock conditions (4) are transformed into

$$U(1) = 1 - \beta, \quad \Omega(1) = \frac{1}{\beta}, \quad H(1) = \frac{1}{\beta M_A}, \tag{12a–c}$$

$$P(1) = \frac{1}{\gamma M^2} + \frac{2(1-\beta)}{\beta(\gamma+1)-(\gamma-1)-2b\rho_1}\left\{\frac{1}{M^2} + \frac{(\gamma-1)}{4M_A^2}\left(\frac{1}{\beta}-1\right)^2\right\}, \tag{12d}$$

$$K(1) = \left[\frac{2\alpha}{\gamma M^2} + \frac{1+\alpha}{M_A^2}\right]^{1/2}, \tag{12e}$$

where

$$1 + d = \alpha. \tag{13}$$

The total energy behind the shock is given by

$$E = 2\pi\int_{r_p}^{R}\left\{\frac{1}{2}\rho\left(u^2+v^2\right) + \frac{p(1-b\rho)}{\gamma-1} + \frac{\mu h^2}{2}\right\}r\mathrm{d}r = E_0 t^w, \tag{14}$$

where r_p is the radius of inner expanding surface. Applying the similarity transformations (11) to the relation (14), we find that the motion of the shock front is given by the equation

$$R^2V^2 = \frac{E_0 t^w}{2\pi\rho_1 J}, \tag{15a}$$

where

$$J = \int_{xp}^{1}\left[\frac{1}{2}\Omega\left(U^2+K^2\right) + \frac{P(1-b\rho_1\Omega)}{\gamma-1} + \frac{H^2}{2}\right]x\mathrm{d}x, \tag{15b}$$

in which x_p is the value of x at the inner expending surface. Equation (15a) can be written as

$$R\frac{\mathrm{d}R}{\mathrm{d}t} = \left(\frac{E_0}{2\pi\rho_1 J}\right)^{1/2} t^{w/2}, \tag{16}$$

which on integration gives

$$R = \left(\frac{8E_0}{\pi\rho_1 J}\right)^{1/4}\frac{1}{\sqrt{w+2}}t^{(w+2)/4}. \tag{17}$$

From (34), we get the shock velocity

$$V = \frac{\mathrm{d}R}{\mathrm{d}t} = \frac{(w+2)}{4}\frac{R}{t} = \left(\frac{8E_0}{\pi\rho_1 J}\right)^{1/(w+2)}\frac{(w+2)^{w/(w+2)}}{4}R^{(w-2)/(w+2)}. \tag{18}$$

Since M and M_A are constants for similarity solutions, we have

$$\alpha = \frac{w-2}{w+2}. \tag{19}$$

Now, the following two cases arise:

Case 1 $\alpha = 0$
In this case, we have

$$d + 1 = 0 \tag{20}$$

and the shock velocity is constant. It follows that $p_1 =$ constant,

$$\rho_1 A_0^2 = \mu h_0^2 \quad \text{and} \quad M^2 = \frac{\mu h_0^2 M_A^2}{\gamma p_1}. \tag{21a, b}$$

Case 2 $\alpha \neq 0$
In this case, the constant in the right hand side of (9) must be zero, the shock velocity is variable and so

$$M^2 = \frac{\mu h_0^2 M_A^2}{\gamma\left[\rho_1 A_0^2 - (1+\alpha)\mu h_0^2\right]}. \tag{22}$$

Now, we have the following relations, valid in both the cases:

$$\frac{\mu h_0^2 M_A^2}{\rho_1} = \left[\left(\frac{8E_0}{\pi \rho_1 J}\right)^{1/(w+2)} \frac{(w+2)^{w/(w+2)}}{4}\right]^2 \tag{23a}$$

and

$$\frac{A_0^2 \rho_1}{\mu h_0^2} = \frac{2\alpha M_A^2}{\gamma M^2} + (1+\alpha), \quad 0 < M_A < \infty. \tag{23b}$$

To obtain the solution in a convenient form, we introduce the following transformations:

$$g = \frac{\rho}{\rho_2}, \quad y = \frac{p}{p_2}, \quad W = \frac{u}{V}, \quad Z = \frac{v}{V}, \quad s = \frac{h}{h_2}. \tag{24a–e}$$

Using the transformation (24), the equations of the motion (1) take the form

$$(W - x)\frac{\mathrm{d}g}{\mathrm{d}x} + g\frac{\mathrm{d}W}{\mathrm{d}x} + \frac{gW}{x} = 0, \tag{25a}$$

$$(W - x)\frac{\mathrm{d}W}{\mathrm{d}x} + \frac{\beta F}{g}\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{s}{\beta M_A^2 g}\frac{\mathrm{d}s}{\mathrm{d}x} + \frac{s^2}{\beta M_A^2 g x} - \frac{Z^2}{x} + \alpha W = 0, \tag{25b}$$

$$(W - x)\frac{\mathrm{d}s}{\mathrm{d}x} + s\frac{\mathrm{d}W}{\mathrm{d}x} + \alpha s = 0, \tag{25c}$$

$$(W - x)\frac{\mathrm{d}Z}{\mathrm{d}x} + \frac{ZW}{x} + \alpha Z = 0, \tag{25d}$$

$$2\alpha y+(W-x)\frac{\mathrm{d}y}{\mathrm{d}x}-\frac{\gamma y\beta(W-x)}{g(\beta-\overline{b}g)}\frac{\mathrm{d}g}{\mathrm{d}x}=0, \tag{25e}$$

where

$$F=\frac{1}{\gamma M^2}+\frac{2(1-\beta)}{\beta(\gamma+1)-(\gamma-1)-2\overline{b}}\left[\frac{1}{M^2}+\frac{\gamma-1}{4M_A^2}\left(\frac{1}{\beta}-1\right)^2\right] \tag{26}$$

and $\overline{b}=b\rho_1$ is the parameter of non-idealness of the gas.

In terms of the dimensionless variables x, W, y, g, s and Z the shock conditions take the form

$$x=1,\quad W=1-\beta,\quad g=1,\quad s=1,\quad y=1, \tag{27a–e}$$

$$Z=\left[\frac{(1+\alpha)}{M_A^2}+\frac{2\alpha}{\gamma M^2}\right]^{1/2}. \tag{27f}$$

Because of the dependence of the equations (25b), (25e) and (27) on $\overline{b}$, similarity solution exists only when $\overline{b}$ is constant i.e. only when the initial density ρ_1 is constant. The problem with the flow of a non-ideal gas is different from that of the perfect gas problem. In the latter case, similarity solution exists for initial density varying as some power of distance (Rogers 1958, Rosenau 1977), but it is not true for the problem with the flow of a non-ideal gas.

In addition to the shock conditions (27), the condition to be satisfied at the inner boundary surface is that the velocity of the fluid is equal to the velocity of inner boundary itself. This kinematic condition, from equations (11a) and (24), can be written as

$$W(x_p)=x_p. \tag{28}$$

From equations (25), we have

$$Bx\frac{\mathrm{d}W}{\mathrm{d}x}=\frac{\gamma y\beta^2FW}{\beta-\overline{b}g}+2\alpha\beta xyF-(W-x)\left[\frac{s^2}{\beta M_A^2}-Z^2g+\alpha Wgx\right]+\frac{\alpha s^2x}{\beta M_A^2}, \tag{29a}$$

$$\begin{aligned}Bx(W-x)\frac{\mathrm{d}s}{\mathrm{d}x}=-s\Bigg\{&\alpha x\left[(W-x)^2g-\frac{s^2}{\beta M_A^2}\right]+(1-\alpha x)\frac{\gamma y\beta^2FW}{\beta-\overline{b}g}\\&+2\alpha\beta xyF-(W-x)\left[\frac{s^2}{\beta M_A^2}-Z^2g+\alpha Wgx\right]+\frac{\alpha s^2x}{\beta M_A^2}\Bigg\},\end{aligned} \tag{29b}$$

$$\begin{aligned}Bx(W-x)\frac{\mathrm{d}g}{\mathrm{d}x}=g\Bigg\{&-2\alpha\beta xyF+(W-x)\left[\frac{s^2}{\beta M_A^2}-Z^2g+\alpha Wgx\right]\\&-\frac{\alpha s^2x}{\beta M_A^{-2}}-W\left[(W-x)^2g-\frac{s^2}{\beta M_A^2}\right]\Bigg\},\end{aligned} \tag{29c}$$

$$Bx(W-x)\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\gamma y\beta}{\beta-\overline{b}g}\left\{(W-x)\left[\frac{s^2}{\beta M_A^2}-Z^2g+\alpha Wgx\right]-\frac{\alpha s^2x}{\beta M_A^2}\right.$$
$$\left.-W\left[(W-x)^2g-\frac{s^2}{\beta M_A^{-2}}\right]\right\}-2\alpha xy\left[(W-x)^2g-\frac{s^2}{\beta M_A^2}\right], \tag{29d}$$

$$x(W-x)\frac{\mathrm{d}Z}{\mathrm{d}x}=-Z(\alpha x+W), \tag{29e}$$

where

$$B=(W-x)^2g-\frac{s^2}{\beta M_A^2}-\frac{\gamma yF\beta^2}{\beta-\overline{b}g}. \tag{30}$$

Now, the equations (29) may be integrated, numerically, with the boundary conditions (27) and the appropriate values of the constant parameters $\gamma, \alpha, \overline{b}, M$ and M_A, to obtain W, g, s, y and Z.

4. Results and discussion

Similarity considerations led to the following relations among the constants α, d and w:

$$1+d=\alpha, \quad \alpha=\frac{(w-2)}{(w+2)}. \tag{31a, b}$$

Then the following two cases may exist:

(i) The constant velocity shock ($\alpha=0$);
(ii) the decreasing velocity shock ($\alpha<0$).

Therefore, for the purpose of numerical calculations, we choose $\alpha=0, -0.5$ which correspond, respectively, to the following two sets of values of the constants:

(i) $\alpha=0,\ w=2,\ d=-1$; and
(ii) $\alpha=-1/2,\ w=2/3,\ d=-3/2$.

The solution of the differential equations (29) with boundary conditions (27) depends on five constant parameters γ, M, M_A, $\overline{b}$ and α. Numerical integration of these differential equations is performed to obtain the reduced variables W, Z, g, y, s, starting from the shock surface to the inner expanding surface for $\gamma=5/3$; $M=10$; $M_A^{-2}=0.02, 0.1$; $\overline{b}=0, 0.05, 0.1$; $\alpha=0, -0.5$ (Rosenau and Frankenthal 1976, Rosenau 1977, Roberts and Wu 1996, 2003, Vishwakarma and Yadav 2003). For a fully ionized gas $\gamma=5/3$, and therefore it is applicable to stellar medium. Rosenau and Frankenthal (1976) have shown the effects of magnetic field on the flow-field behind the shock are significant when $M_A^{-2}\geq 0.01$; therefore the above values of M_A^{-2} are taken for calculations in the present problem. The value $\overline{b}=0$ corresponds to the perfect gas case.

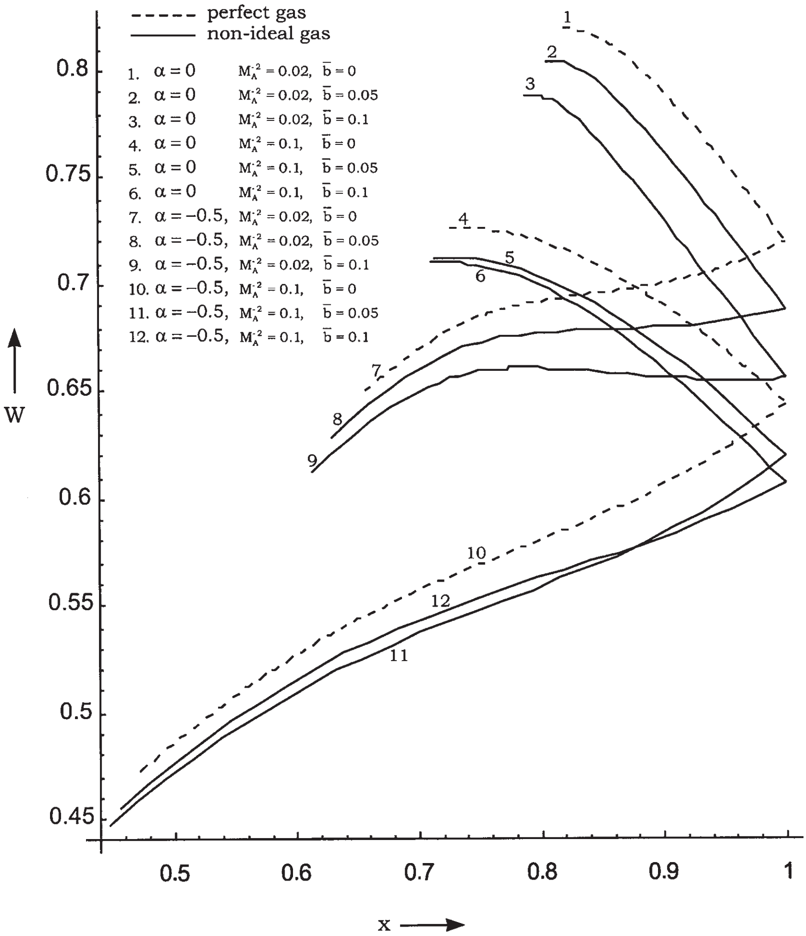

Figure 1. Variation of the reduced radial velocity W in the flow-field behind the shock front.

The results are shown in figures 1–5. Values of x_p (the reduced position of the inner expanding surface) and the density ratio across the shock front $\beta = \rho_1/\rho_2$ are shown in tables 1 and 2 for different cases.

Figure 1 shows that the radial velocity W increases from the shock front to the inner expanding surface when $\alpha = 0$; whereas it decreases when $\alpha = -0.5$. Figures 2 and 4 show that the density g and the pressure y decrease rapidly behind the shock front. Also, the azimuthal velocity Z decreases rapidly behind the shock front when $\alpha = 0$, and it decreases slowly when $\alpha = -0.5$ (figure 5). Figure 3 shows that the azimuthal magnetic field S increases rapidly from shock front to inner expanding surface, and this increase becomes slower when α is decreased or when M_A^{-2} is increased.

From tables 1 and 2 and figures 1–5, it is found that the effects of an increase in the value of M_A^{-2} (i.e. the effects of an increase in the strength of ambient

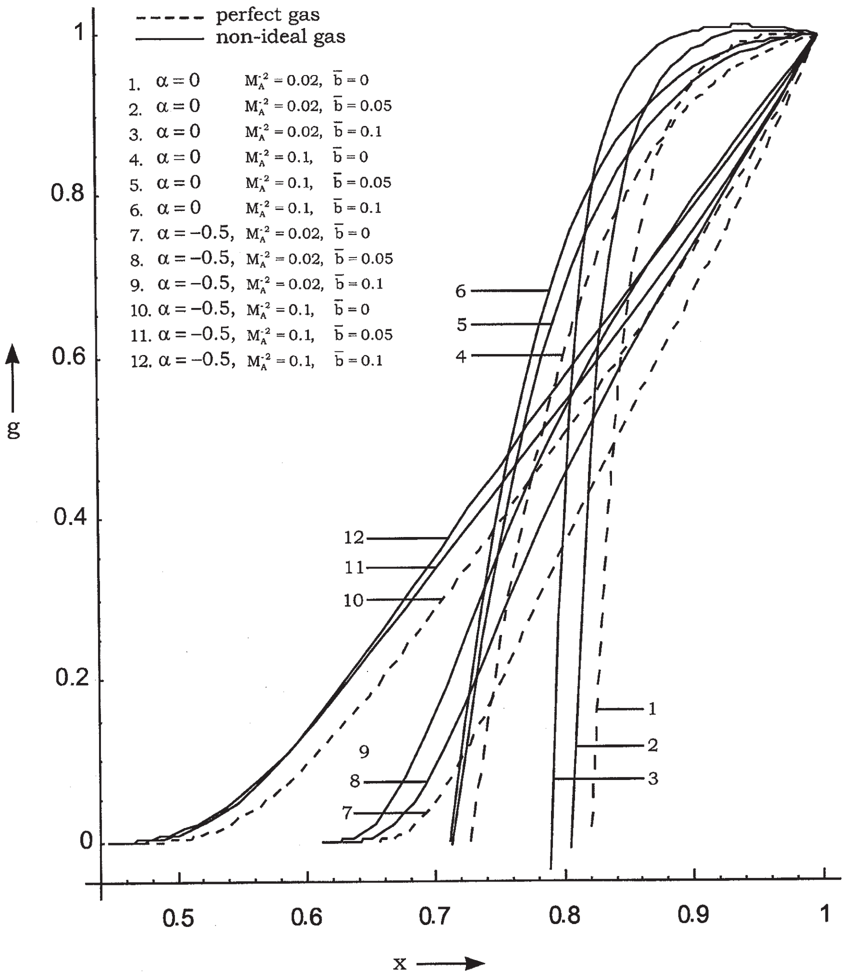

Figure 2. Variation of the reduced density g in the flow-field behind the shock front.

magnetic field) are

(i) to decrease x_p, i.e. to increase the distance of inner expanding surface from the shock front. Physically it means that the gas behind the shock is less compressed, i.e. the shock strength is reduced;
(ii) to increase the value of β, i.e. to decrease the shock strength, which is the same as given in (i) above;
(iii) to decrease the radial velocity and to increase the azimuthal velocity at any point in the flow-field behind the shock;
(iv) to decrease the slopes of the profiles of density, pressure and azimuthal magnetic field, i.e. to reduce the tendency of abrupt fall of the density and the pressure and abrupt increase of the azimuthal magnetic field as we move inwards from the shock front.

Thus the presence of magnetic field has decaying effect on the shock wave.

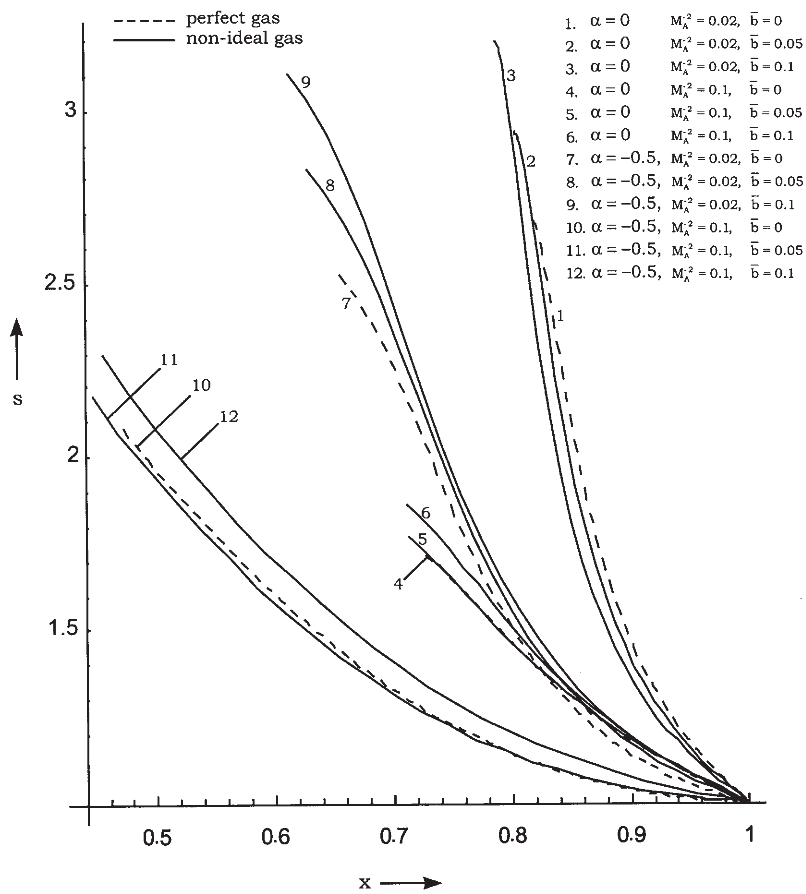

Figure 3. Variation of the reduced azimuthal magnetic field s in the flow-field behind the shock front.

The effects of an increase in the value of the parameter of the non-idealness of the gas $\bar{b}$ are

(i) to increase the distance of the inner expanding surface from the shock front (table 1);
(ii) to increase the value of β (table 2), i.e. to decrease the shock strength. Therefore the non-idealness of the gas has decaying effect on the shock wave;
(iii) to decrease the radial velocity, in general; and to increase the azimuthal velocity slightly, at any point in the flow-field behind the shock (figures 1 and 5);
(iv) to decrease the slope of density profiles and to increase the slope of profiles of azimuthal magnetic field (figures 2 and 3).

The effects of an increase in the value of the index for variation of ambient azimuthal magnetic field α, i.e. the effects of an increase in the value of the index for variation of

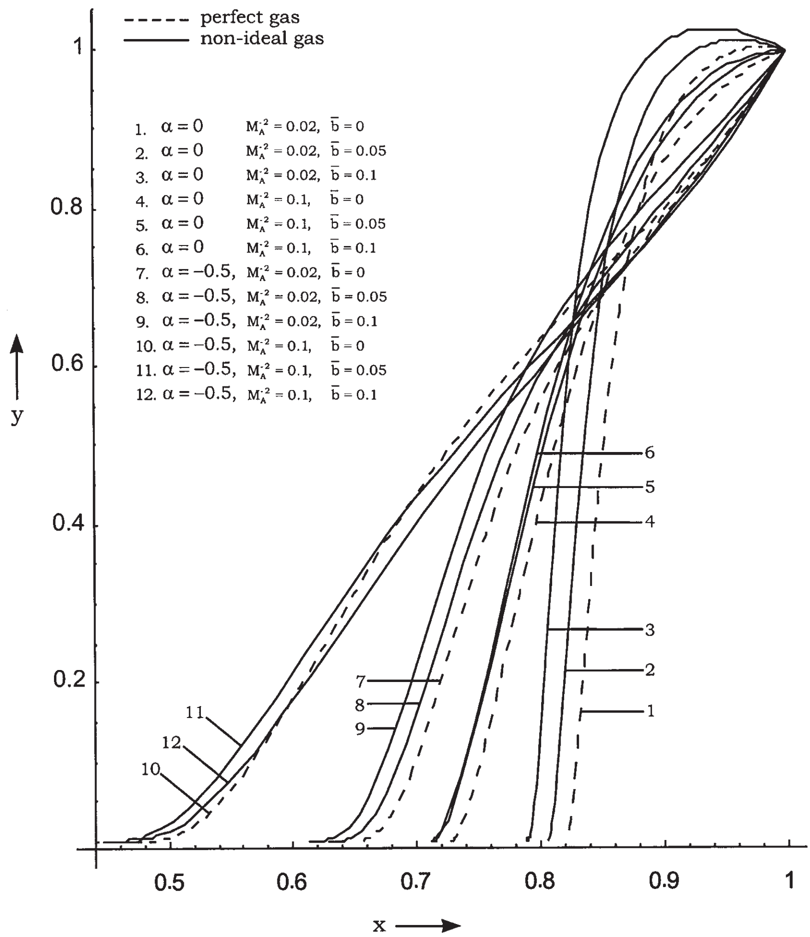

Figure 4. Variation of the reduced pressure y in the flow-field behind the shock front.

the angular velocity of the ambient medium d are

(i) to decrease the distance of inner expanding surface from the shock front. It means that the shock is stronger when the ambient magnetic field is uniform ($\alpha = 0$) in comparison with that when it is decreasing ($\alpha = -0.5$). It also means that the shock is stronger when the angular velocity of the ambient medium is slowly decreasing (see the relation (31a));

(ii) to increase the radial velocity and azimuthal magnetic field at any point in the flow-field behind the shock (figures 1 and 3);

(iii) to increase the tendency of rapid increase in azimuthal magnetic field and rapid decrease in azimuthal velocity, density and pressure.

Present self-similar model may be used to describe some of the overall features of a "driven" shock wave produced by a flare energy release $E(=E_0 t^w)$ that is time dependent. For $w > 0$ the energy E increases with time and the solutions then

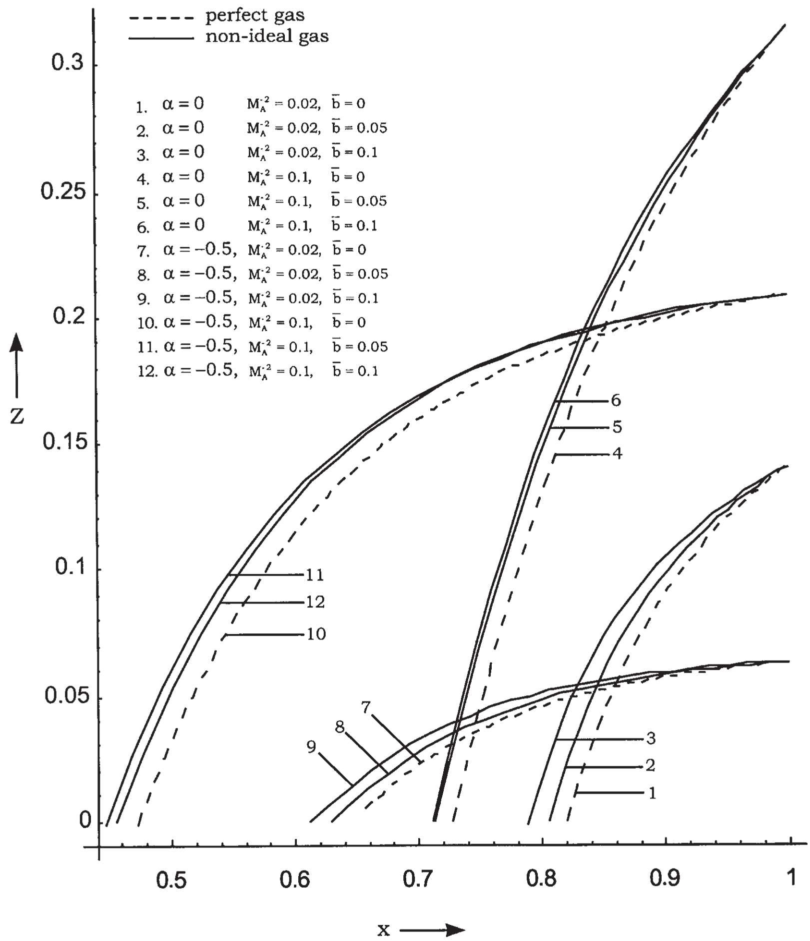

Figure 5. Variation of the reduced azimuthal velocity Z in the flow-field behind the shock front.

Table 1. Position of inner expanding surface x_p for $\gamma = 5/3$, $M = 10$ and various values of M_A^{-2}, $\overline{b}$ and α.

M_A^{-2}	$\overline{b}$	α	x_p
0.02	0	0	0.820
		−0.5	0.656
	0.05	0	0.806
		−0.5	0.628
	0.1	0	0.788
		−0.5	0.612
0.1	0	0	0.727
		−0.5	0.473
	0.05	0	0.714
		−0.5	0.446
	0.1	0	0.711
		−0.5	0.456

Table 2. Density ratio β across the shock front for $M_A^{-2} = 0.02, 0.1$; $\overline{b} = 0, 0.05, 0.1$; $M = 10$ and $\gamma = 5/3$.

	$\beta = \rho_1/\rho_2$	
$\overline{b}$	$M_A^{-2} = 0.02$	$M_A^{-2} = 0.1$
0	0.278962	0.355192
0.05	0.310485	0.379733
0.1	0.343672	0.393009

correspond to a blast wave produced by intense, prolonged flare activity in a rotating star when the wave is driven by fresh erupting plasma for some time and its energy tends to increase as it propagates from the star.

References

Chaturani, P., Strong cylindrical shocks in a rotating gas. *Appl. Sci. Res.*, 1970, **23**, 197–211.

Director, M.N. and Dabora, E.K., An experimental investigation of variable energy blast waves. *Acta Astronaut.*, 1977, **4**, 391–407.

Freeman, R.A., Variable energy blast wave. *J. Phys. D*, 1968, **1**, 1697–1710.

Ganguly, A. and Jana, M., Propagation of shock wave in a self-gravitating radiative magnetohydrodynamic non-uniform rotating atmosphere. *Bull. Cal. Math. Soc.*, 1998, **90**, 77–82.

Lee, T.S. and Chen, T., Hydromagnetic interplanetary shock waves. *Planet. Space Sci.*, 1968, **16**, 1483–1502.

Nath, O., Ojha, S.N. and Takhar, H.S., A study of stellar point explosion in a radiative magneto-hydrodynamic medium. *Astrophys. Space Sci.*, 1991, **183**, 135–145.

Ray, G., Deb Similarity solutions for cylindrical blast waves in magnetogasdynamics. *Phys. Fluids*, 1973, **16**, 559–560.

Roberts, P.H. and Wu, C.C., Structure and stability of a spherical implosion. *Phys. Lett. A*, 1996, **213**, 59–64.

Roberts, P.H. and Wu, C.C., The shock-wave theory of sonoluminescene. In *Shock Focussing effect in Medical Science and Sonoluminescene*, edited by R.C. Srivastava, D. Leutloff, K. Takayama and H. Groning, 2003 (Springer-Verlag: Heidelberg).

Rogers, M.H., Similarity flows behind strong shock waves. *Quart. J. Mech. Appl. Math.*, 1958, **XI**, 411–422.

Rosenau, P., Equatorial propagation of axisymmetric magnetohydrodynamic shocks. II. *Phys. Fluids*, 1977, **20**, 1097–1103.

Rosenau, P. and Frankenthal, S., Shock disturbances in a thermally conducting solar wind. *Astrophys. J.*, 1976, **208**, 633–637.

Sakurai, A., Propagation of spherical shock waves in stars. *J. Fluid Mech.*, 1956, **1**, 436–453.

Sedov, L.I., *Similarity and Dimensional Methods in Mechanics*, 1959 (Academic Press: London).

Summers, D., An idealised model of a magnetohydrodynamic spherical blast wave applied to a flare produced shock in the solar wind. *Astron. Astrophys.*, 1975, **45**, 151–158.

Vishwakarma, J.P. and Yadav, A.K., Self-similar analytical solutions for blast waves in inhomogeneous atmospheres with frozen-in-magnetic field. *Eur. Phys. J.B.*, 2003, **34**, 247–253.

Whitham, G.B., On the propagation of shock waves through regions of non-uniform area or flow. *J. Fluid Mech.*, 1958, **4**, 337–360.

Zel'dovich, Y.B. and Raizer, Yu.P., *Physics of Shock Waves and High Temperature Hydrodynamic Phenomena*, Vol. II, 1967 (Academic Press: New York).