Self-similarity of complex networks and hidden metric spaces.

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Self-similarity and scale invariance are traditionally known as characteristics of certain geometric objects, such as fractals, or of field theories describing system dynamics near critical points of phase transitions. In these cases, objects or physical systems are intrinsically embedded in metric spaces and distance scales in these spaces are natural scaling factors. In complex networks, it is difficult to define self-similarity and scale invariance in a proper geometric sense because many networks are not explicitly embedded in any physical space and they lack any metric structure except the topological one given by the collection of lengths of shortest paths between nodes, which is a poor source of length-based scaling factors since it does not have large lengths as it usually exhibits the small-world property. We demonstrate [1] that hidden metric spaces underlying the observed topologies of some complex networks –in particular the Border Gateway Protocol map of the Internet at the Autonomous System level and the Pretty Good Privacy social web of trust– appear to provide a plausible explanation of the observed self-similarity of their main topological characteristics -degree distributions, degree-degree correlations, and clustering– with respect to a simple degree-thresholding renormalization scheme which produces a hierarchy of nested subgraphs. Nodes are assumed to reside in an underlying hidden metric space, meaning that for all pairs there are defined hidden distances satisfying the triangle inequality. Clustering –cycles of length three– in the topology becomes then a natural consequence of the triangle inequality in this metric space underneath. If we take the most generic interpretation of hidden distances as measures of either structural or functional similarity between nodes, and admit that more similar nodes are more likely to be connected, then the hidden and observable forms of transitivity become clearly related. In future work, hidden metric spaces may find far-reaching applications such as the design of efficient routing and searching algorithms for communication and social networks.

[1] M.A. Serrano, D. Krioukov and M. Boguñá, Phys. Rev. Lett. **100**, 078701 (2008).