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Self similarity of two point correlations in wall bounded turbulent flows

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Computational results of Moin & Moser (1987) and Spalart (1986) for two-point correlations of the normal v component of turbulence at two points y, y_1 $(y_1 > y)$, from the rigid walls bounding turbulent channel and boundary layer flows for Reynolds numbers 3200 and 7000 are shown to have an approximately self-similar form, when plotted in terms of y/y_1. It is found that

$$\frac{\overline{v(y)v(y_1)}}{\overline{v^2(y_1)}} = f\left(\frac{y}{y_1}\right)$$

where $0 \le y/y_1 \le 1$, $f(0) = 0$, $f(1) = 1$, and where f is approximately independent of y_1, for y_1 ranging from about 20 ν/u_τ to half the channel width;

$$f \approx 2(y/y_1)^2 - (y/y_1)^3 \pm .1.$$

The same kind of self similarity has been predicted for and measured in shear free boundary layers. But in that case, where $f \approx y/y_1$, the mechanism is one of 'blocking' or 'splatting' at the wall. In these sheared wall layers, the shear *also* has an important effect. There are important implications from this research for modeling wall bounded shear flow.

1. Introduction and Objective

The structure of turbulence at a height y from a wall is affected by the local mean shear at y, $(\frac{\partial U}{\partial y})$, by the *direct* effect of the wall on the eddies, and by the action of other eddies close to or far from the wall. Some researchers believe that a single one of these mechanisms is dominant, while others believe that these effects have to be considered together.

It is important to understand the relative importance of these effects in order to develop closure models, for example for the dissipation or for the Reynolds stress equation, and to understand the eddy structure of cross correlation functions and other measures. The specific objective of this research project was to examine the two point correlation R_{vv} of the normal velocity component v near the wall in a turbulent channel flow and in a turbulent boundary layer. This component of

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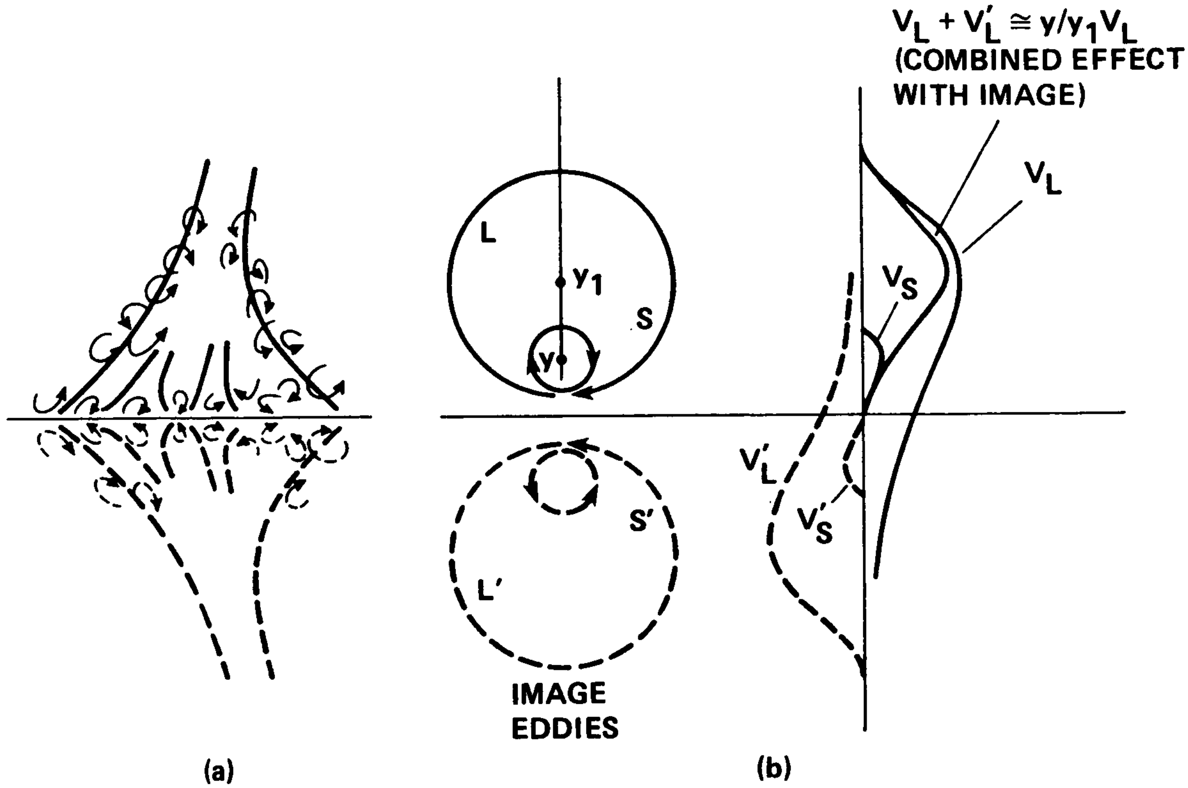

FIGURE 1. Diagram to illustrate eddy motion near the ground, *(a)* typical structure of an updraft or "thermal" plus the "image" updraft, *(b)* the relation between the velocity at y_1 and y. The large eddy L, with velocity v_L, is centered at height y_1. The small eddy S, with velocity v_S is centered at height y. (Their "images" are L' and S') The profiles are shown of the vertical velocity of the large and small eddies, of their images and of the combined effect of both.

turbulence is the most sensitive to the relative effects of shear (which amplifies v) and the blocking effect of the surface (where $v = 0$, even in inviscid flow).

Recent research on shear free turbulent boundary layers, (such as occur in thermal convection between boundaries or in turbulence near a free surface or turbulence near a density inversion layer) has shown how the blocking effect leads to a self-similar form for R_{vv} when expressed as a ratio with $\overline{v^2}(y_1)$ (i.e., normalized at the upper point),

$$R_{vv} \equiv \frac{\overline{v(y)v(y_1)}}{\overline{v^2}(y_1)} = f\left(\frac{y}{y_1}\right) \approx \frac{y}{y_1} \qquad y < y_1 \tag{1.1}$$

The theory for the SFBL is valid when y_1 is much less than the turbulence scale far from the boundary. The explanation is given with the aid of figure 1. Let there be a large eddy (L) centered with maximum velocity at y_1 and a small eddy centered at y. Then the velocity at y_1, $v(y_1) \approx v_L$. The effect of the small eddy at y is small

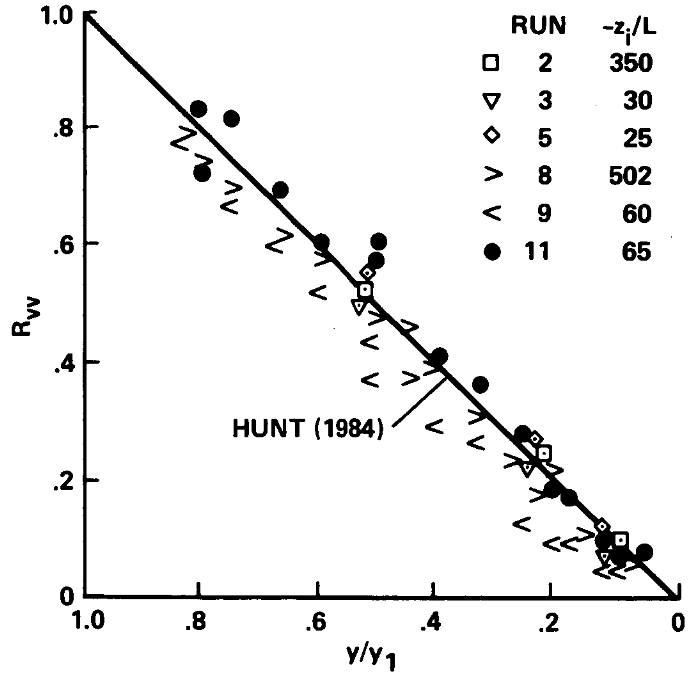

FIGURE 2. Measurements in the convective atmospheric boundary layer of the cross correlation of v at heights y and y_1, normalized by v^2 at y_1. The results are compared to the approximate form of the theoretical predictions of Hunt (1984), $\hat{R}_{vv} \approx y/y_1$. ($-z_i/L$ indicates the state of the convective boundary layer, the higher $-z_i/L$, the stronger the convection and the weaker the shear.) From Hunt, Kainal & Gaynor, (1987).

if y is less than about $y_1/2$. However, the velocity at y

$$v(y) = v_s + (y/y_1)v_L \tag{1.2}$$

has two components, from the small eddy and also from the large eddy. Because the vertical dependence of the large eddy is blocked by the surface, this component is reduced by a factor of about (y/y_1) for high Reynolds number turbulence. One can imagine an image vortex underneath the surface. Since the correlation between v_s and v_L is small if $y \leq y_1/2$ the correlation between $v(y)$ and $v(y_1)$ and thence the correlation, normalized at the upper point (N.U.P.) is

$$\hat{R}_{vv} \approx y/y_1. \tag{1.3}$$

It is interesting that the theory seems to agree with measurements for the atmospheric boundary layers during thermal convection. See figure 2 from Hunt, Kaimal & Gaynor 1987. The same general idea might be appropriate for a wall bounded shear flow at moderate Reynolds number, but now the velocity at height y is not a simple function of y/y_1. In general we expect the component of v_L to be given by a function $g[(y - y_1)/L^v(y_1)]$, depending on the distance between y and y_1 and the scale of turbulence $L^v(y_1)$ at y_1. Therefore, we might predict:

$$\hat{R}_{vv} \approx g((y - y_1)/L^v(y_1)) \tag{1.4}$$

But in a wall-bounded flows $L^{(v)}(y_1)$ is proportional to y_1, (at least for unstable and neutral, but not for stably stratified flows). Therefore, we might expect,

$$g\left(\frac{y - y_1}{L^v(y_1)}\right) = g\left(\frac{y - y_1}{\alpha y_1}\right) \tag{1.5}$$

where $L^v(y_1) \approx \alpha y_1$, which can be written as another function, i.e.

$$\hat{R}_{vv} \approx f(y/y_1).$$

This is the first hypothesis to be tested.

We have argued that the wall "blocks" the normal component of the large-scale eddies centered above the wall at $y = y_1$. In a shear flow the streamwise u and normal v components of the turbulence are correlated at the height y_1, i. e., $R_{uv} \equiv \overline{uv}(y_1) \neq 0$. Therefore the cross-correlation between the normal velocity $v(y)$ at y ($< y_1$) and the horizontal velocity at y_1, $u(y_1)$, should steadily decrease near the wall as $y/y_1 \to 0$. If the scaling argument of (1.5) is valid one should expect that, in the log layer, where $y << y_1$, the $u - v$ correlation normalized by $\overline{uv}$ at y_1 has the form:

$$\hat{R}_{uv} \equiv \frac{R_{uv}(y_1, y)}{R_{uv}(y_1, y_1)} \approx f_{12}\left(\frac{y}{y_1}\right) \tag{1.6}$$

This is the second hypothesis to be tested.

In general, correlations involving the horizontal component are affected by the inactive or irrotational motions. Consequently, the presence of the wall exerts a weaker influence on these correlations.

The computations of the structure of homogeneous turbulence in a uniform, mean velocity gradient have shown that the main effect of the mean shear is to reduce the scale of the turbulence in the spanwise or z direction (Townsend 1976). Unlike many statistical effects this one is so strong that this channeling of turbulent eddies can be seen in the instantaneous pattern of streamlines, as derived from flow-visualization studies and direct simulations (e. g., Lee, Kim and Moin 1987).

The picture of these eddies in shear flow, indicated in figure 3, looks very different from the conventional circular vortex-like eddy of homogeneous isotropic turbulence; such as illustrated in figure 1. One suggestion is to represent the eddies as vertically-elongated structures with a defined spanwise scale a_3.

This suggests that the structure of the large eddy is approximately described by

$$v \approx f_v\left(\frac{y_2}{y_1}\right)g\left(\frac{r_3}{a_3}\right).v_L \tag{1.7}$$

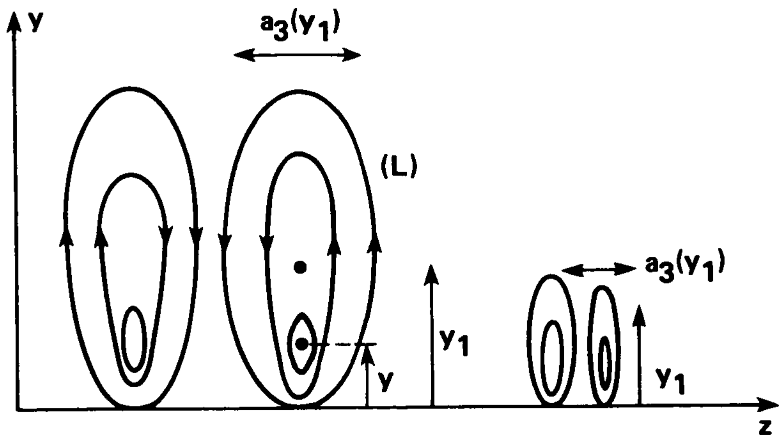

FIGURE 3. Schematic of spanwise structure of eddies in the boundary layer, showing how the spanwise scale (a_3) of the large eddy (L) at y_1 determines the spanwise scale closer to the wall (at y). Also, if y_1 is smaller, a_3 is also smaller.

(as opposed to the isotropic eddy structure where $v \approx f[(r_2^2 + r_3^2)/L^2]v_L)$. Here $r_2 \equiv (y_1 - y)$ and r_3 are the distances from the center of the eddy. Thus at a height y, and spanwise displacement z,

$$v(y, r_3) = v_s + v_L f_v(\frac{y}{y_1})\ g(\frac{r_3}{a_3})$$

and therefore

$$\overline{v(y, r_3)v(y_1, 0)} \approx v_L^2 f_v(\frac{y}{y_1}) g(\frac{r_3}{a_3}).$$

This model also implies that

$$\hat{\hat{R}}_{vv}(y, r_3) \equiv \frac{\hat{R}_{vv}(y, r_3)}{\hat{R}_{vv}(y, 0)} = \frac{\overline{v(y, r_3)v(y_1, 0)}}{\overline{v(y, 0)v(y_1, 0)}} \approx g(\frac{r_3}{a_3}) \tag{1.8}$$

where the function g is independent of z. If the hypothesis (1.6) is valid we would also expect that

$$\hat{\hat{R}}_{vu}(y, r_3) \equiv \frac{\hat{R}_{vu}(y, r_3)}{\hat{R}_{vu}(y, 0)} = \frac{\overline{v(y, r_3)u(y_1, 0)}}{\overline{v(y, 0)u(y_1, 0)}} \approx g(\frac{r_3}{a_3})$$

Since the eddy structure (1.7) occurs only on strong shear flows, it is natural to suppose that a_3 depends on the *mean velocity gradient* dU/dy and the vertical turbulence intensity v' in the log layer. Near the wall the lateral structure is likely to be determined by instabilities within the wall layer. So we postulate that

$$a_3(y_1) \approx \alpha_s \frac{v'}{dU/dy(y_1)} + \alpha_w \frac{v'}{u_\tau}. \tag{1.9}$$

This is the third hypothesis to be tested here.

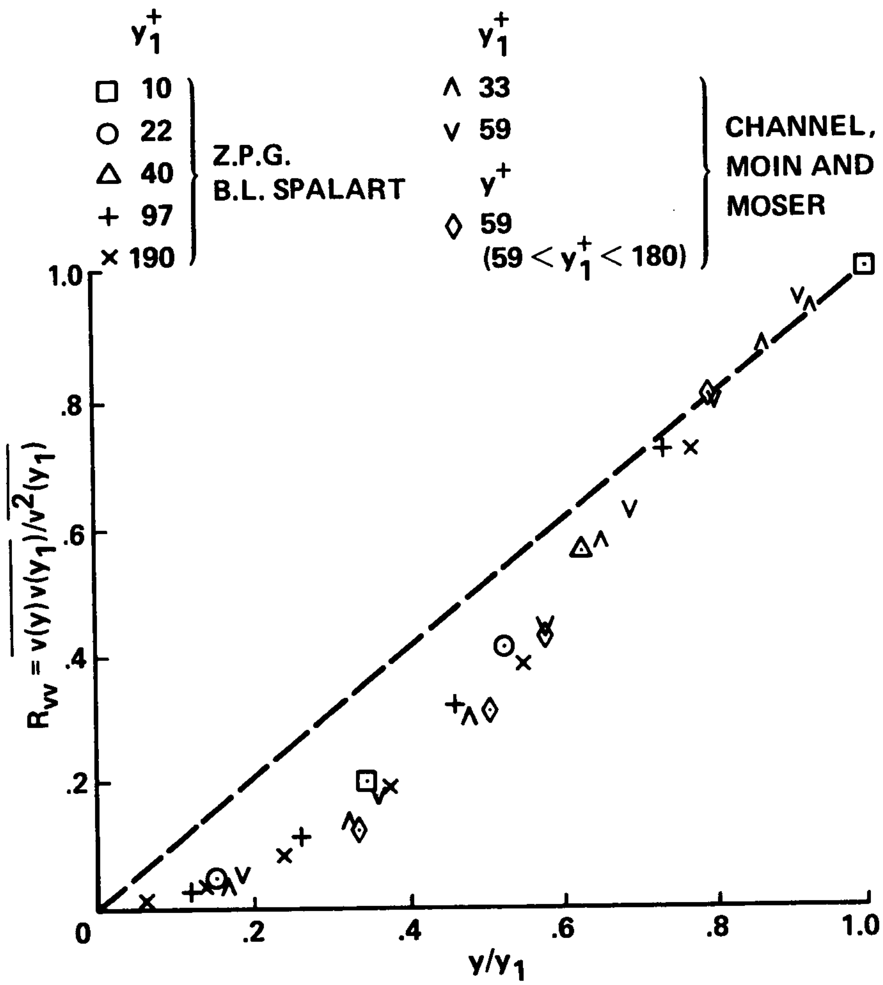

FIGURE 4. Cross correlation of v at heights y and y_1, normalized by v^2 at y_1 computed from direct numerical simulations of the zero pressure gradient boundary layer (Spalart 1987), and the plane channel (Moin & Moser 1987). Also shown are the theoretical predictions of Hunt (1984) ($\hat{R}_{vv} \approx y/y_1$).

2. Preliminary results

In figure 4 we present a graph of $R_{vv}(y/y_1)$ combining the results of the computations of the zero pressure gradient (Z. P. G.) boundary layers (Spalart 1986) and of the channel flow (C.F.) (Moin & Moser 1987). The range of data is as follows: ZPG: $\delta u_\tau/\nu = 300$ (where δ is the boundary layer thickness) and

$$y^+ = 10,\ 22,\ 40,\ 97,\ 190$$

Channel Flow: $\delta u_\tau/\nu = 180$ (where δ is the channel half width) and

$$y_1 u_\tau/\nu = 33,\ 59,\ \ldots\ 180$$

In figure 4 the results for the channel alone are plotted including the exceptionally small value of $y_1 = 5\nu/u_\tau$.

It appears that for these two wall bounded shear flows, the self similar plot of $\hat{R}_{vv}$ is a good description of the measurements. Comparing figures 2 and 4 indicates as good a 'collapse' as observed for shear free boundary layer. (But remember that figure 2 is a plot of experimental points in the atmosphere!). Note that the similarity hypothesis is more accurate for smaller values of y, as expected since the assumption that the small eddies at y and the large eddies as y_1 are uncorrelated, (i.e. $\overline{v_s v_L} \approx 0$) is more valid when y/y_1 is small.

It is particularly surprising that approximately the same curve describes the distribution of $\hat{R}_{vv}$ for points both within and well above the viscous sublayer. However very close to the surface within the sublayer we must expect that, since $v \propto y^2$ as $yu_\tau/\nu \to 0$, $R_{vv} \to (y/y_1)^2$ as $yu_\tau/\nu \to 0$.

However the computation of Kim, Moin or Moser (1987) have shown that some vertical eddying motion exists on a scale even smaller than $5\nu/u_\tau$ (because v is not exactly proportional to y^2). This is quite consistent with the fact that the two point correlation $\hat{R}_{vv}$ is greater than $(y/y_1)^2$ when $y_1 u_\tau/\nu = 5$.

These results show that there is a significant difference in the measured value of $\hat{R}_{vv}$ between these shear boundary layers (figures 4 and 5*a*) and the shear-free boundary layers in figure 2. They show that the effect of shear is to reduce the correlation length of the normal velocity in the normal direction. (But it is important to note that the smallest scale of v in a shear flow is in the spanwise or z-direction, Townsend (1976). So these curves of R_{vv} do *not* give a basis for estimating dissipation or the dissipation length scale.)

Figure 5*b* is added to show that if the conventional two point correlation is plotted against y/y_1, the points do not tend to zero as $y/y_1 \to 0$ and the curves do not have any general pattern.

In figure 5*c* we present the cross correlation of the Reynolds stress R_{uv} as defined by (1.6). These curves for different values of (y_1/δ) show that R_{uv} is not a universal function of (y/y_1) over the whole channel width. However, the three sets of curves for which y_1 is in the log layer, i.e., $30 \leq y+ \leq 100$ do exhibit a strong degree of similarity- (recall that the abscissa is being stretched by a factor of 3, and yet the curves are self similar).

It is interesting to note that the shape of the self similar curves of R_{uv} is markedly different to the curves for the normal velocity correlation, R_{vv}. The correlation is higher.

In figure 6, we present the cross correlation for the vertical velocity separated by a normal and spanwise spacing normalized by the correlation at the same height, but zero spanwise spacing i.e.,

$$\hat{\hat{R}}_{vv}(y, r_3; y_1) = \frac{\hat{R}_{vv}(y, r_3, y_1)}{\hat{R}_{vv}(y, 0, y_1)} = \frac{\overline{v(y, r_3)v(y_1, 0)}}{\overline{v(y, 0)v(y_1, 0)}}$$

We also plotted the correlation of u and v defined as

$$\hat{\hat{R}}_{vu}(y, r_3; y_1) = \hat{R}_{vu}(y, r_3; y_1)/\hat{R}_{vv}(y, 0, y_1)$$

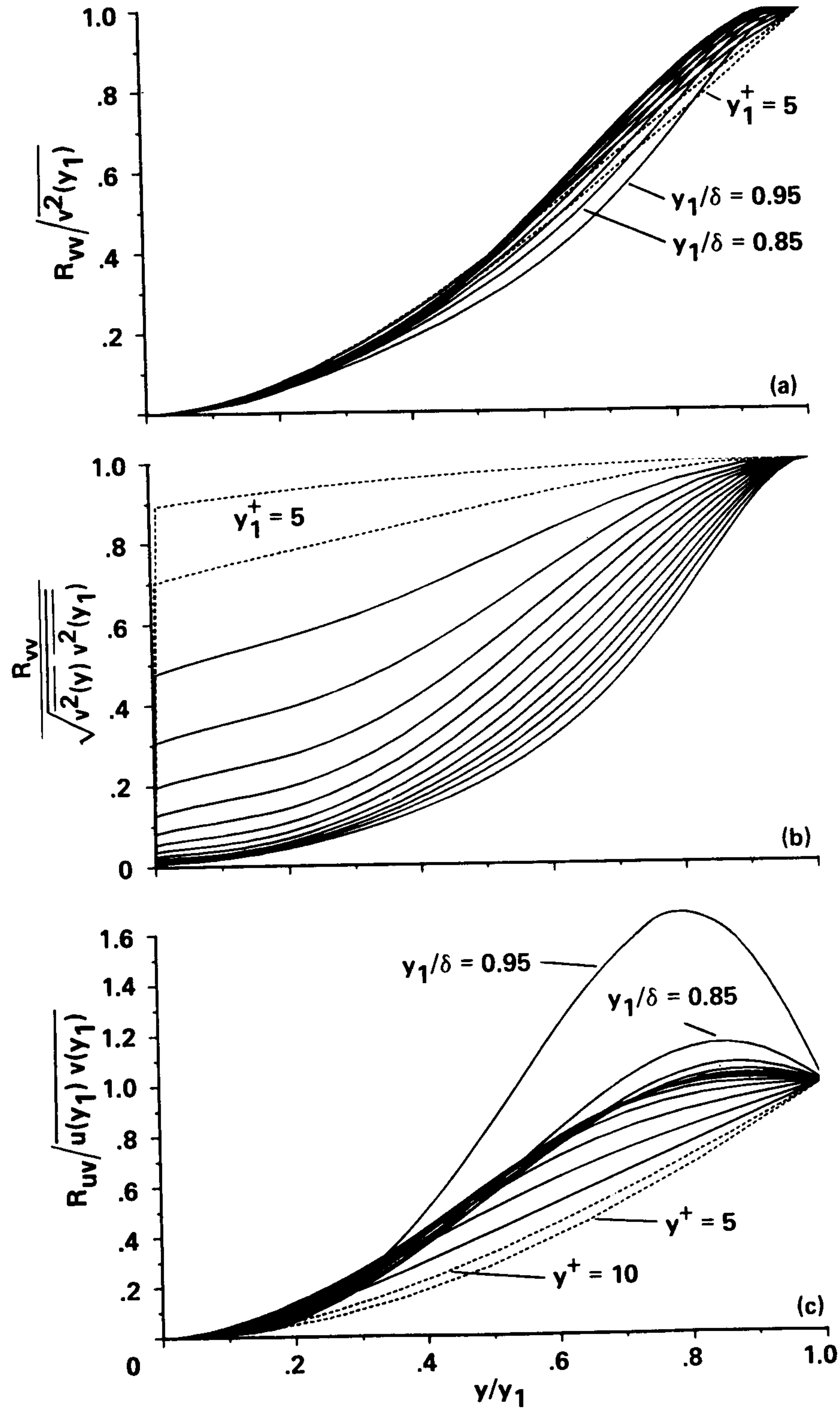

FIGURE 5. Normalized cross correlations of velocities at y and y_1 from turbulent plane channel flow (Moin & Moser, 1987) for various values of y_1. *(a)* $\overline{v(y)v(y_1)}/\overline{v^2(y_1)}$, *(b)* $\overline{v(y), v(y_1)}/\sqrt{\overline{v^2(y)v^2(y_1)}}$, *(c)* $\overline{u(y)v(y_1)}/\overline{u(y_1)v(y_1)}$.

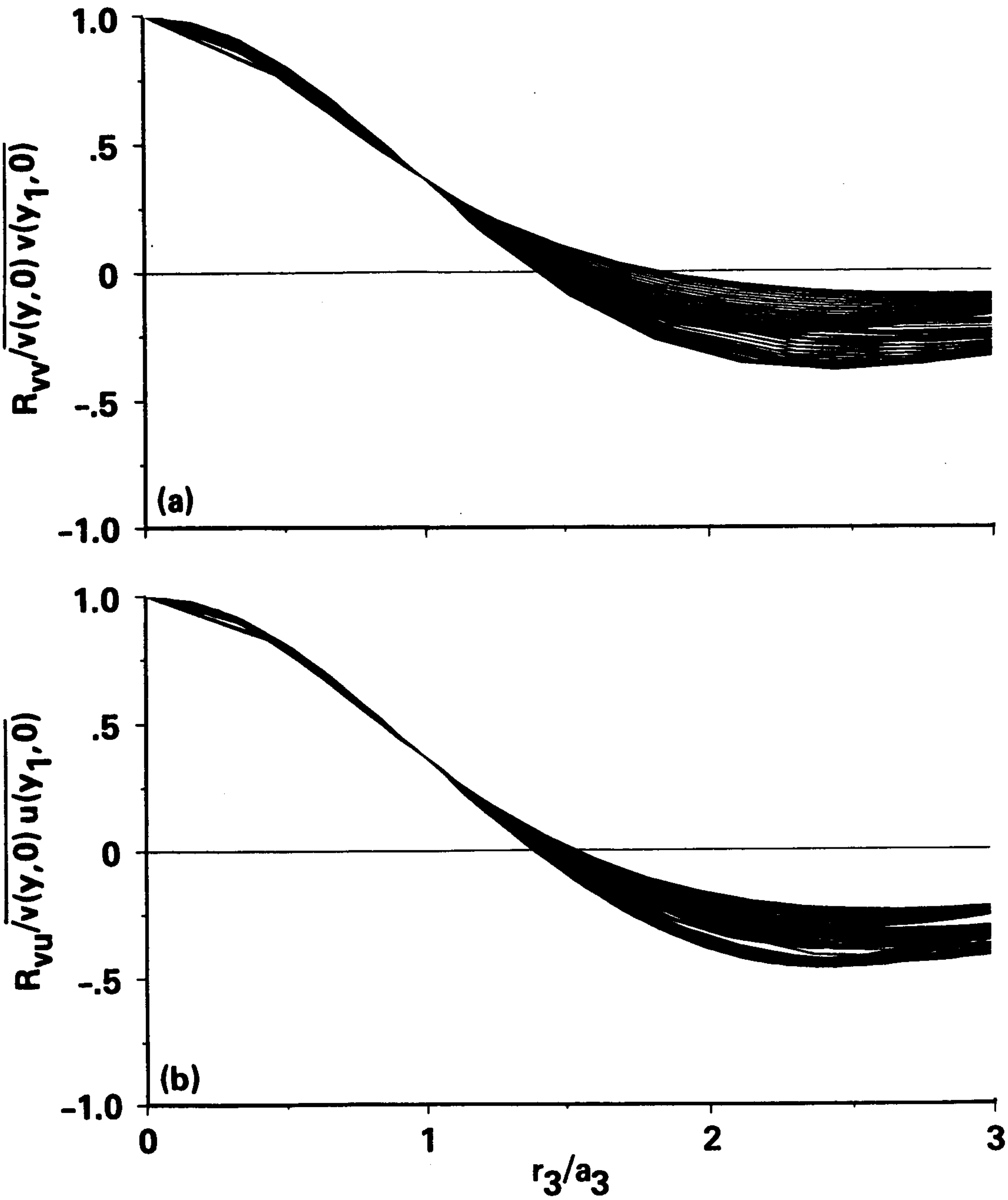

FIGURE 6. Normalized cross correlations of velocities at y and y_1 with separation in z (r_3) from turbulent plane channel flow (Moin & Moser, 1987) for various values of y_1. *(a)* $\overline{v(y,r_3),v(y_1,0)}/\overline{v(y,0)v(y_1,0)}$, *(b)* $\overline{v(y,r_3),u(y_1,0)}/\overline{v(y,0)u(y_1,0)}$. The value of a_3 is chosen for each curve to be the r_3 location at which the curve passes through $1/e$.

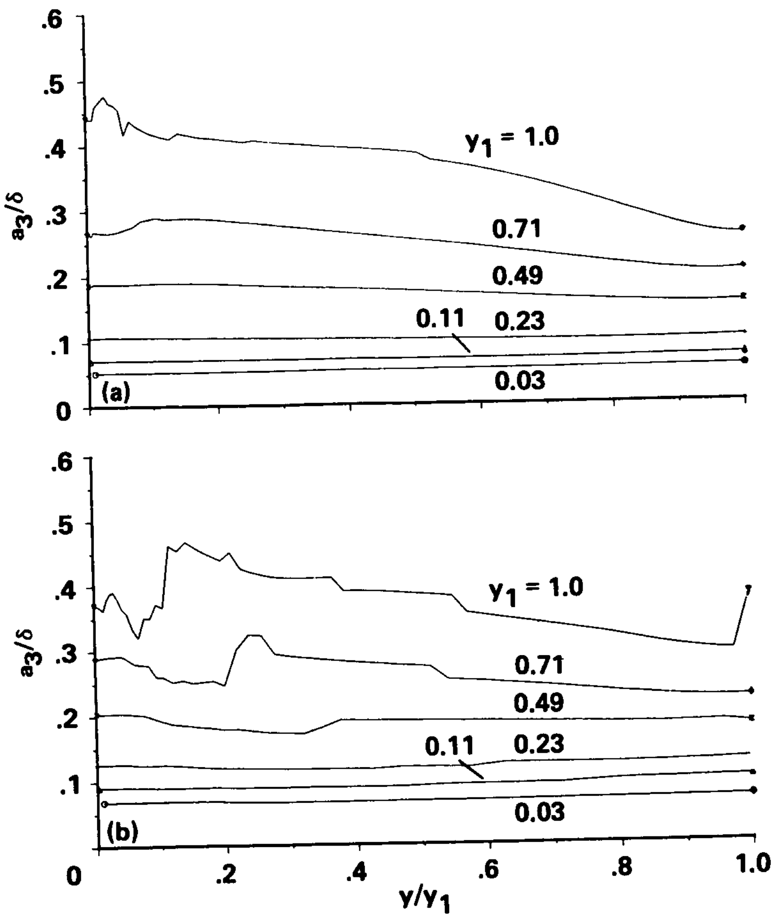

FIGURE 7. Values of a_3 as a function of y/y_1 and y_1, *(a)* as used to scale R_{vv} (figure 6*a*), *(b)*) as used to scale R_{vu} (figure 6*b*).

The curves are plotted as functions of r_3/a_3 for different y and y_1, where a_3 is defined as the value of r_3 where $\hat{R}_{vv} = 1/e = 0.36$ The results show firstly, the small variation of the form of the spanwise structure and the negligible variation of the spanwise scale of the eddies in these boundary layers. Using this particular correlation emphasizes this point quite nicely. This approximate invariance is found for values of $y_1/\delta \leq 0.8$. It is not true for y_1 at the centerline, $y_1/\delta \approx 1.0$.

Secondly, these results show how the scale a_3 increases with y_1. In Figure 7, we have plotted a_3^+ against y^+. It appears that this scale is of the order of 9 wall units near the wall and then begins to increase when $y_1^+ \geq 10$. This would be consistent with the ideas suggested in the introduction. A satisfactory curve fit could be (figure 8)

$$a_3^+ \approx (1.4\frac{\partial U^+}{\partial y^+} + 7)$$
$$\approx 0.3y^+ \quad \text{in the log layer.}$$

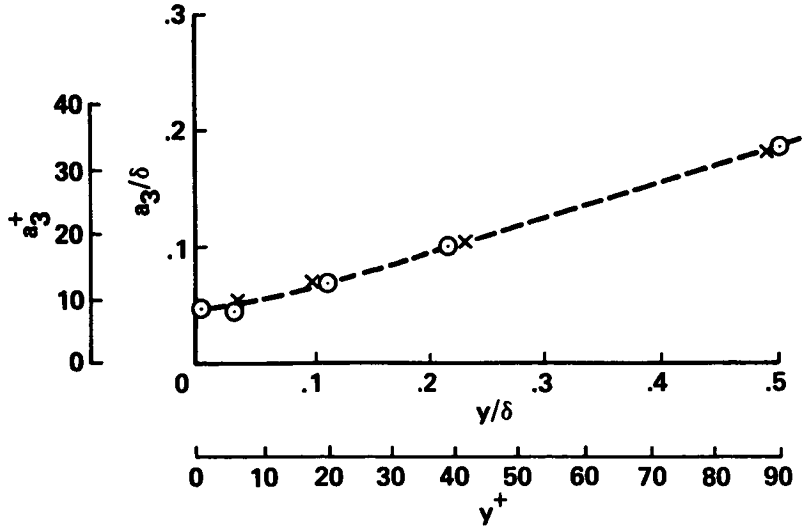

FIGURE 8. Values of a_3 as used to scale R_{vv} (figures 6*a* and 7*a*), as a function of the reference height y_1. × – simulation results, ○ – the model $a_3 = 7 + (1.4\frac{du^+}{dy^+})^{-1}$.

This value of a_3^+ is of the same order but a little *greater* than the dissipation length scale L_ϵ (based on $\overline{v^2}$ in the log layer) where

$$L_\epsilon = \frac{\epsilon}{\overline{v^2}^{3/2}} \approx 0.18y$$

Other results have been computed for R_{uv}, R_{uu}. They also show equally strong channeling of the spanwise structure; though the value of a_3 for R_{uu} is about twice as great as for R_{vv}.

3. Implication and further work

The preliminary results show that even in the inhomogeneous turbulent boundary layer, the two-point correlation function may have self similar forms. The nature of these self similar functions can be inferred by using rapid-distortion theory. The results shows that the effects of shear and of blocking are equally important in the form of correlation functions for spacing normal to the wall. But for spanwise spacing, we have found that the eddy structure is quite different in these shear flows; this aspect of eddy structure is largely controlled by the shear and perhaps by small scale structures very close to the wall. So any theory for the turbulent structure must take both these effects into account.

The results suggest further study

a. Comparison with laboratory and atmospheric measurement.
b. The effects of curvature and pressure gradient should be investigated. We would still expect to see these self similar two point correlations.
c. Further theoretical calculations should be done using RDT Theory for uniform shear near a wall.

d. The two point correlation functions are likely to be self similar for other components and in other direction (eg. for the spanwise, z, spacing).

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