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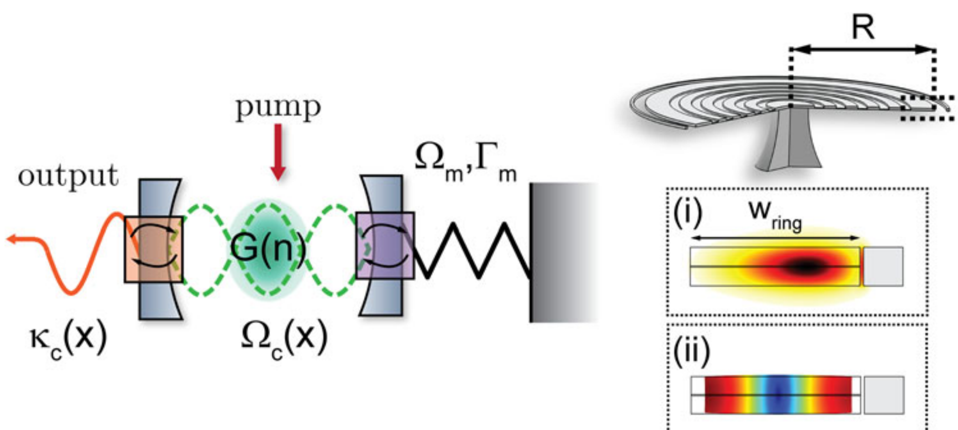

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Self-Sustained Laser Pulsation in Active Optomechanical Devices

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Abstract: We developed a model for an active optomechanical cavity embedding a semiconductor optical gain medium in the presence of dispersive and dissipative optomechanical couplings. Radiation pressure drives the mechanical oscillation and the back-action occurs due to the mechanical modulation of the cavity loss rate. Our numerical analysis utilizing this model shows that, even in a wideband gain material, such mechanism couples the mechanical vibration with the laser relaxation oscillation, enabling an effect of self-pulsed laser emission. In order to investigate this effect, we propose a bullseye-shaped device with high confinement of both the optical and the mechanical modes at the edge of a disk combined with a dissipative structure in its vicinity. The dispersive interaction is promoted by the strong photoelastic effect while the dissipative mechanism is governed by the boundary motion mechanism, enhanced by near-field interaction with the absorptive structure. This hybrid optomechanical device is shown to lead sufficient coupling for the experimental demonstration of the self-pulsed emission.

Index Terms: Semiconductors lasers, micro and nano opto-electro-mechanical systems (MOEMS).

1. Introduction

The interaction between light and mechanical vibrations in optical microcavities is usually described by the dispersive coupling between the optical and the mechanical modes. In such a scheme, the cavity resonance shifts due to the mechanical oscillation and the dynamical back-action allows amplification, cooling and interference of the mechanical modes [1]–[3]. A dissipative scheme of optomechanical coupling has been investigated more recently, where the mechanical motion modulates the decay rate of the optical cavity [4]–[6], leading to strong optomechanical coupling [7], [8] and high mechanical sensitivity [9]. An attractive aspect of these systems is the prospect of enhanced and less restrictive optomechanical cooling rates without requiring the good cavity condition: $\kappa \ll \Omega_m$, where κ is the optical decay rate and Ω_m the mechanical frequency [4].

In this sense, the coupling of a optomechanical resonator to a light emitter has also been an object of interest in the field of optomechanics due to its substantial potential to obtain laser cooling to the ground state of the mechanical motion outside the resolved sideband regime [10], [11]. The experimental demonstration of these hybrid systems has been pursued in a different number of devices, with increasing interest in observing the strong coupling regime [12]. Meanwhile, the development of laser micro-cavities with built-in mechanical degrees of freedom has enabled the investigation of the optomechanical interaction within active cavities. It was theoretically shown that active cavities with purely dispersive optomechanical coupling exhibit back-action either under an external coherent pump [13] or in the presence of resonant narrowband material gain [11]. Experimental demonstrations have been reported with semiconductor micro-lasers, exploring effects of tuning [14], [15] and interaction with strain waves [16], leading to modification of the emission properties beyond the optomechanical cooling.

In a previous work, we have shown that the modulation of the cavity loss rate by a mechanical degree of freedom can enable optomechanical feedback in an active microcavity fed by its own incoherent spontaneous emission. This was possible by exploiting the intrinsic geometric relation between the optical resonance frequency and the cavity photonic decay rate, given by the optical quality factor: $Q_o = \Omega_c/\kappa$ [17]. Such dissipative scheme is quite inefficient from this restriction and it is not obvious if one can find a design with realistic parameters that would allow such backaction. Nevertheless, in a more general scheme, Q_o may as well change with the mechanical vibration, allowing for the dispersive and dissipative optomechanical couplings to be independent. Therefore it is fundamental to improve both mechanisms in order to enable the observation of optomechanical feedback in such a cavity.

Here we propose and investigate a model for an active optomechanical cavity which includes both dispersive and dissipative couplings, in the absence of a coherent driving field. The uniqueness of such system is that dispersive and dissipative couplings interfere to enable optomechanical feedback through amplification of laser relaxation oscillations. While the dispersive interaction enables the back-action force, the dissipative coupling modulates the effective laser gain, therefore linking the motion and the laser relaxation oscillations. We thoroughly investigate this system and demonstrate that the optical spring effect and amplification of the mechanical oscillator can occur despite the absence of driving field. Also, we present a trade-off between the laser and the mechanical oscillator parameters which allows observing such effects in a realistic device.

Finally, we investigate the design of an active optomechanical resonator with enhanced dispersive and dissipative optomechanical couplings. This device brings together the recently demonstrated bullseye design [18], which allows high hybrid confinement of the optical and mechanical modes in a modified microdisk, with a highly dissipative structure in its vicinity, typically a metallic ring, spaced from the disk edge by a small air gap. With such design we predict a novel self-pulsation regime based on the coupling between the mechanical mode and the relaxation oscillations of the laser cavity.

2. Modified Laser Rate Equations With Optomechanical Coupling

We first recall the field dynamics of an optomechanical cavity in the presence of both dispersive and dissipative couplings [see the simplified scheme in Fig. 1(a)]. The material gain is added under a semi-classical approach, in order to modify the laser rate equations for a semiconductor laser with a mechanical degree of freedom. The mechanical motion for a certain normal mode is parameterized as a product of the normalized displacement $\mathbf{u}(\mathbf{r})$ multiplied by a time-dependent amplitude $x(t)$, i.e., $\mathbf{U}(\mathbf{r}, t) = \mathbf{u}(\mathbf{r})x(t)$. The evaluation of the motion equations for a passive cavity has been proposed previously, obtained from the Hamiltonian of the system coupled to an optical bath, such that the optical resonance Ω_c and the external coupling κ are both modulated by the mechanical motion [4], [6], [7]. The optical resonance is written as $\Omega_c(x) = \Omega_0 + g_\omega x(t)$, where Ω_0 is the bare optical resonance, and $g_\omega = \partial\Omega_c/\partial x\ |_{x=0}$ is the optomechanical frequency pull parameter. Analogously, the dissipative optomechanical coupling rate g_κ is defined such as $\kappa(x) = \kappa_0 + g_\kappa x(t)$,

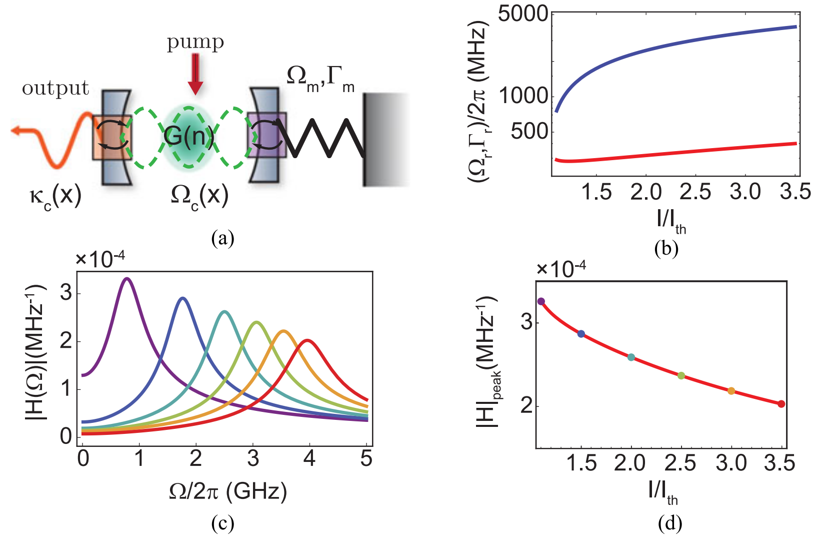

Fig. 1. The optomechanical laser fundamentals. (a) Scheme of an active optomechanical cavity presenting both dispersive and dissipative optomechanical couplings. The gain medium $G(n)$ is fed by optically or electrically generated carriers and the output is a lasing mode carrying the features of the mechanical system (Ω_m, Γ_m). (b) Relaxation Oscillation Frequency (in blue, Ω_r) and respective damping (in red, Γ_r) for the referred system in our analysis. (c) Small-signal intensity modulation of the system for varied injected current (d) Peak value of $H(\Omega)$ (see text), as function of the injected current ratio with the threshold current value I_{th} – each color point corresponds to the same color curve in (c).

with $g_\kappa = \partial\kappa/\partial x \mid_{x=0}$. The Hamiltonian of the system can be written

$$\frac{H}{\hbar} = \Omega_c(x)a^\dagger a + \Omega_m b^\dagger b + \int d\omega\, \omega\, a^\dagger_\omega a_\omega - i\sqrt{\frac{\kappa(x)}{2\pi}} \int d\omega\, (a^\dagger_\omega a - a^\dagger a_\omega), \tag{1}$$

where a ($a^\dagger$) and a_ω ($a^\dagger_\omega$) are respectively the annihilation (creation) operators of the optical field and the annihilation (creation) of the optical bath at zero temperature; b ($b^\dagger$) is the annihilation (creation) operator of the mechanical mode, set at frequency Ω_m and normalized with respect to the zero-point fluctuation of the harmonic oscillator, x_{zpf}, as $x/x_{zpf} = b + b^\dagger$. The generalized optical force is derived from the gradient of H, $F = -\partial H/\partial x$, with first order approximation for x

$$F = -\hbar g_\omega a^\dagger a + i\hbar \frac{g_\kappa}{2\sqrt{\kappa_0}}(a^\dagger_{in} a - a^\dagger a_{in}), \tag{2}$$

In (2) the first term is related to the radiation pressure, proportional to the photon number $a^\dagger a$, while the second term is generated by the dissipative interaction. a_{in} is expressed in terms of the bath operator a_ω: $\frac{1}{\sqrt{2\pi}} \int d\omega a_\omega(t) = a_{in}(t) - \frac{\sqrt{\kappa(x)}}{2} a(t)$, from which follows the standard input-output relation $a_{out}(t) - a_{in}(t) = \sqrt{\kappa(x)} a(t)$ – the complete derivation is found in [19]. The time evolution for $a(t)$ is then described on time

$$\frac{da}{dt} = -i\Omega_c(x)a - \frac{\kappa(x)}{2} a + \sqrt{\kappa(x)} a_{in}(t), \tag{3}$$

In the correspondent semi-classical evaluation, the intracavity field amplitude α is the average value of a, such as a_{in} corresponds to an input field driving the cavity optical mode. For a semiconductor laser cavity, this coherent driving field is absent when the cavity photons are generated by spontaneous and stimulated emission due to the carrier recombination. Hence, the usual dissipation-induced optical force in (2) is zero, and the optical force is proportional to the total cavity-photon number, $P(t)$. The absence of the coherent drive allows for the usual semi-classical description of the laser dynamics using rate equations for the cavity-photon and carrier densities, $p(t)$ and $n(t)$

respectively, with $p(t) = P(t)/V$, where V is the optical cavity volume. In this approach, the field in the lasing mode is written $\alpha(t) = |\alpha(t)|\, e^{i\phi(t)}$, with $|\alpha(t)|^2 = P(t)$; the phase is decoupled and it can be treated separately, in contrast to usual optomechanics where the dynamics of the phase between a_{in} and α has an important role. Here we neglect the phase equation, since it is not involved in the back-action mechanism. Nevertheless its evaluation would be relevant if one would need to consider noise and the laser linewidth broadening. The advantage of using the photon number is the possibility of computing the volumetric spontaneous emission rate R_{sp} and the gain rate $Gp(t)$, where G is the net optical gain. From (3), we obtain the equation for photon density, and the gain is then added in the typical relation of gain minus losses, which rules the laser behavior. This optical gain originates from the stimulated emission due to the recombination of carriers, electrons and holes, assumed to be in equal density, $n(t)$. Considering a single lasing mode, the coupled rate equations are

$$\frac{dp}{dt} = [G - \kappa(x)]p + \beta R_{\text{sp}}, \tag{4}$$

$$\frac{dn}{dt} = \frac{I}{qV} - R_{\text{sp}} - Gp, \tag{5}$$

where I is the injected current obtained either by electrical or optical pumping, q is the elementary charge and β is the fraction of the spontaneous emission coupled to the lasing mode. The spontaneous emission rate is written proportional to the product of the electron and hole densities, $B_{\text{sp}}n^2$, neglecting non-radiative terms, where B_{sp} is the bimolecular recombination coefficient. Equations (4) and (5) describe the dynamics of a semiconductor laser with good accuracy, provided that the field oscillates slowly compared to Ω_c, and that the dielectric response of the material is fast compared with the carrier and photon lifetimes [20]. The presence of the dissipative optomechanical coupling appears clearly in (4), as the photon decay rate is a function of the mechanical degree of freedom x. We shall analyze the impact of this coupling on the laser dynamics.

3. Small Signal Analysis

In order to study the impact of mechanically induced fluctuations on the laser dynamics we have assumed a small amplitude oscillation for $x(t)$, $x(t) = x_0 + \delta x e^{-i\Omega t}$, where x_0 is the static displacement, and we sought for the carrier population and photon number response using the ansatz $n(t) = n_0 + \delta n e^{-i\Omega t}$ and $p(t) = p_0 + \delta p\, e^{-i\Omega t}$, where n_0 and p_0 are the steady state solutions with fluctuations δn and δp – it is equivalent to solve the system in the frequency space. The fluctuations in the photon number, δp, are of most importance as they induce a corresponding optical force that may back-act on the mechanical system. To quantify the impact of the laser properties on this fluctuation, we define a laser modulation sensitivity to the mechanical motion as $H(\Omega) = \frac{\delta p/p_0}{g_\kappa \delta x}$. An analytical expression for $H(\Omega)$ can be obtained by writing the stimulated emission rate for a frequency flat gain, $G(n)$, written then explicitly in the logarithm approximation (suitable for semiconductor gain medium based on quantum wells, without loss of generality): $G(n) = G_n \ln[(n + n_s)/(n_{tr} + n_s)]$, where G_n is a gain coefficient, n_{tr} is the carrier density for transparency and n_s is a fitting parameter. Under the small signal approximation, $H(\Omega)$ is given by

$$H(\Omega) = \frac{\delta p/p_0}{g_\kappa \delta x} = -\frac{\Gamma_n - i\Omega}{\Omega_r^2 - (\Omega + i\Gamma_r)^2}, \tag{6}$$

where $\Gamma_n = 2B_{\text{sp}}n_0 + G_n p_0/n_0$ is the carrier fluctuation damping rate, $\Omega_r^2 = G_0 G_n p_0/n_0 + 2G_0 B_{\text{sp}} n_0 - (\Gamma_n - \Gamma_p)^2/4$ is the usual laser Relaxation Oscillation Frequency (ROF), with a decay rate $\Gamma_r = (\Gamma_n + \Gamma_p)/2$, and $\Gamma_p = \beta B_{\text{sp}} n_0^2/p_0$ is the photon fluctuation decay rate, with G_0 equal to the gain at the steady-state [20]. Equation (6) is similar to laser loss modulation efficiency and reveals that the resonant nature of relaxation oscillation plays a fundamental role in the dissipative optomechanical interaction. For semiconductor lasers, the ROF can range from hundreds of MHz [21] up to tens of GHz [22], while the associated damping rates Γ_n, Γ_p and Γ_r are in the order of tens to hundreds of MHz. Due to the resonant response of laser cavity loss modulation, the mechanical modes must match the laser relaxation oscillation frequency (Ω_r) in order to have appreciable dissipative optomechanical effects. For a simple microring optical cavity, such as the design we will investigate below, both Ω_r and Γ_r can be (roughly) linearly tuned through the injected current above the laser threshold (I_{th}), as shown Fig. 1(b). Under high injection current, $\Gamma_r \approx \Gamma_n$, and the photonic decay Γ_p decreases with the current. Despite the ROF tunability through the injected current, increasing it well-above laser threshold leads to a steady reduction of the peak response in $H(\Omega)$, as can be readily noted in Fig. 1(c) and (d). Even with the tunability, the ROF lies in the few GHz range for a typical semiconductor microring cavity like investigated below. Therefore, it is necessary that this type of cavity support dissipatively coupled mechanical modes in this frequency range.

From the previous analysis, we see that the dissipative coupling g_κ is essential for the photon number response to mechanical oscillations. Now we can demonstrate that the dispersive optical force – either electrosctriction or radiation pressure in a dielectric cavity – accompanying these photon number oscillations will back-act and drive the mechanical degree of freedom. The generalized optical force was calculated previously in (2), and it was shown that the dissipative term is zero for our incoherently driven system, leaving only the the dispersive term, $-\hbar g_\omega P$. Thus the equation for the driven harmonic mechanical oscillator is

$$\frac{d^2x}{dt^2} + \Gamma_m \frac{dx}{dt} + \Omega_m^2 x = -\frac{\hbar P g_\omega}{m_{eff}}, \tag{7}$$

where Ω_m is the mechanical resonance frequency, the mechanical damping rate is $\Gamma_m = \Omega_m/Q_m$, and m_{eff} is the effective motional mass [1]. Using the ansatz for $x(t)$ in (7), together with (6) for the photon modulation, we arrive at an harmonic oscillator equation with modified frequency and damping terms,

$$\delta\Omega_m(\Omega = \Omega_m) = \mathcal{G}^2 \frac{\Gamma_n(\Omega_m^2 - \Omega_r^2 - \Gamma_r^2) - 2\Gamma_r\Omega_m^2}{(\Omega_m^2 + \Gamma_r^2)^2 - 2\Omega_r^2(\Omega_m^2 - \Gamma_r^2) + \Omega_r^4}, \tag{8}$$

$$\Gamma_{om}(\Omega = \Omega_m) = \mathcal{G}^2 \frac{2\Omega_m(\Omega_m^2 - \Omega_r^2 + 2\Gamma_n\Gamma_r - \Gamma_r^2)}{(\Omega_m^2 + \Gamma_r^2)^2 - 2\Omega_r^2(\Omega_m^2 - \Gamma_r^2) + \Omega_r^4}, \tag{9}$$

where the effective optomechanical coupling was defined as $\mathcal{G}^2 = g_\omega g_\kappa x_{zpf}^2 P_0$, with $x_{zpf}^2 = \hbar/(2m_{eff}\Omega_m)$ the mechanical zero-point fluctuation. Such a dissipative-dispersive effective optomechanical coupling is completely analogous to the light-enhanced optomechanical coupling in passive optomechanics [1]. In analogy with passive optomechanical cavities, the origin of the optical spring effect (8) and optomechanical damping (9) lies on the phase lag between the mechanical motion and the optical forces induced by the fluctuation of the optical intensity – here though, enhanced by the relaxation oscillation phenomena. In an active optomechanical cavity, therefore, the feedback is caused by a combination of dispersive and dissipative processes. Also, we notice that the phase response between mechanical motion and relaxation oscillation, obtained from the imaginary part of $H(\Omega)$, depends on the sign of g_κ, which will define if δx precedes or lags δp in the blue/red side of the ROF resonance. Another important difference regarding passive optomechanical cavities is the absence of the standard detuning parameter of optomechanics, since there is no driving external field. Using $\Omega_m \approx \Omega_r$, $\Gamma_n \approx \Gamma_r$ and $\Gamma_r^2 \ll \Omega_r^2$, we obtain simple expressions for the optically-induced spring-effect and damping, $\delta\Omega_m = -\mathcal{G}^2/(2\Gamma_r)$ and $\Gamma_{om} = \mathcal{G}^2/(2\Omega_r)$. Interestingly, the spring effect is related to the relaxation oscillation damping and the optomechanical damping is related to its frequency, i.e., the real and imaginary parts of the involved mechanical and ROF frequencies are mixed, an inverted behavior when compared to the usual dynamical backaction, where $\delta\Omega_m$ is related primarily to the detuning and Γ_{om} relates to κ [1]. Such character comes from the dual nature of the optomechanical coupling here: while the force originates from the dispersive mechanism, mechanical modulation of optical intensity occurs through the loss. We are specially interested in the self-sustained oscillation regime of the mechanical oscillator, where $\Gamma_m + \Gamma_{om} = \Gamma_{eff} \leq 0$,

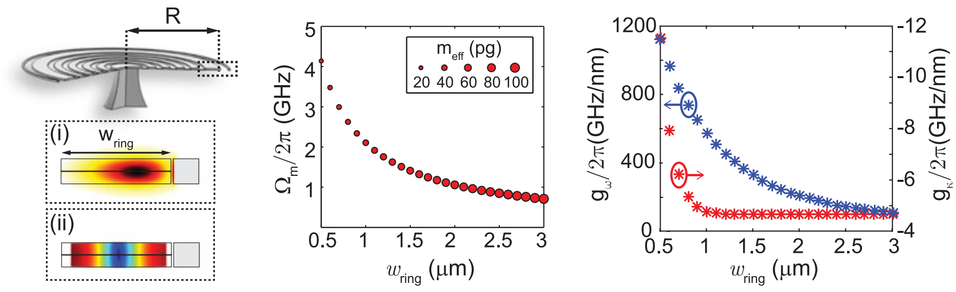

Fig. 2. Active bullseye with single quantum well and metallic ring in the vicinity. (a) The chosen device confines both the optical and the mechanical mode in the edge of a disk of radius R and it is surrounded by an absorptive metallic ring separated by a small gap. A limited region of the disk with length w_{ring} is shielded by a mechanical grating in the disk top surface. We took $R = 12\ \mu$m and gap $= 20$ nm for our simulations in a disk that is 230 nm thick. The metallic ring, based on Chromium, has the same thickness of the disk and it is anchored to the substrate – such fixation structure is not shown in the schematic. In the inset (i), which corresponds to the indicated disk edge cross section, we show the optical field density for $w_{\text{ring}} = 1\ \mu$m, a typical WGM with azimuthal number $m = 244$ and with evanescent field confined in the air spacing. The quantum well 7 nm thick is localized in the middle of the disk. The normalized displacement modulus $|\mathbf{u}(\mathbf{r})|$ for the chosen mechanical mode (fundamental Fabry-Perot-like) is shown in (ii). (b) Dependency of the mechanical frequency and effective mass with respect to the length w_{ring}. (c) Calculated optomechanical dispersive (g_ω) and dissipative (g_κ) coupling strengths for the structure, considering moving boundary and photoelastic contributions.

when the mechanical amplitude is steady in time, leading to a steady behavior of the photon and carrier fluctuation, as shown in [17], where laser self-pulsed emission is obtained. Additionally, the discussion of optomechanical cooling capability in this system requires proper treatment of the spontaneous emission noise. This calculation is yet to be performed and lies beyond the scope of this work.

4. Design of an Active Bullseye Resonator for Enhanced g_ω and g_κ

We present in the following a realistic optomechanical laser design and show a path towards experimental observation of the self-sustained optomechanical oscillations in an active cavity. Since the effective optomechanical coupling rate in our system, $\mathcal{G}$, depends on the combination of dispersive (g_ω) and dissipative coupling (g_κ), the challenge is to find a suitable cavity design that maximize both type of coupling simultaneously. Previously reported works often implemented the dissipative optomechanical coupling factor through a boundary shift that changed the external coupling rate κ, which depended exponentially on the distance between a loss structure, for instance a bus waveguide, and the main cavity. In these works the coupling led to values of $\partial\kappa/\partial x$ in the order of 1-10 MHz/nm [5], [23] and in some cases, strong disturbance of the optical mode and a consequent increase of the radiation losses [9].

Our design is inspired by a recently demonstrated device, the bullseye optomechanical resonator [18] [Fig. 2(a)] which relies on a strong photo-elastic dispersive coupling. By evanescently coupling the bullseye cavity to a metallic rim, we show below that a significant dissipative coupling can be achieved [9], [24]. In such a design, the optical mode is confined within the edge of the disk, as a typical whispering gallery mode (WGM) [Fig. 2(a)(i)], while a dielectric nanostructured grating built in at the top surface of the disk allows the mechanical mode to be confined within the optical mode region. This optimized overlap of the optical and the mechanical modes ensures the strong photo-elastic contribution to the dispersive optomechanical coupling while retaining high mechanical quality factors. Also, relatively high optical quality factor, Q_o, can be achieved without compromising the mechanical resonator properties. By adding a strongly dissipative structure in the vicinity of the disk – a metallic ring – a large dissipative coupling is reachable as well, without compromise of the laser operation. As shown in Fig. 2(a), the dissipative bullseye device has a metallic ring separated by a small air-gap, which can readily be anchored to a surrounding substrate (not shown in the scheme). Such loss modulation scheme is completely compatible with the current micro and nanofabrication techniques, and has been effective in the demonstration of PT-cavity lasers [25], where a metallic tip is brought to proximity with the laser cavity. Our proposal relies on optical pumping of the gain medium, but it also could be implemented with an electrically injected pump scheme [26]. Since this outer metallic ring is attached to the substrate, all the modulation is due to the bullseye boundary displacement alone. We have chosen the mechanical breathing mode of the outer edge region for the analysis, which is represented in Fig. 2(a)(ii).

III-V alloys represent a natural choice of materials and we picked up a GaAs based structure, since this platform is already well established in the field of cavity optomechanics. Specially, the photoelastic effect has been well explored [27], [28] and recent work shows good results for surface passivation of GaAs based devices [29]. The gain medium is based on a single quantum well of InGaAs with 13% of In and 7 nm thick. The disk is 230 nm thick in total and 12 μm radius. For this structure, the laser active transition is at 933 nm, but we will work slightly red shifted, at 950 nm (correspondent azimuthal number $m = 244$), in order to have smaller differential gain. Indeed there is a trade-off between laser and optomechanical parameters: a high photon density is required for large effective optomechanical coupling, but it is desirable to not increase the ROF significantly to not diminish Γ_{om}. Approximating the laser relaxation oscillation frequency by $\Omega_r^2 \approx G_0 G_n p_0 / n_0$, it is clear that a differential gain reduction compensates an increase in the photon density and one may be able to reach lasing without changing significantly the photon number. In this sense, the logarithmic nature of the gain dependence on carrier density for quantum wells helps avoiding large changes in the laser relaxation oscillation frequency, since the differential gain decreases with the carrier density. In this design, we chose a lasing mode red-shifted from the transition edge – a small but sufficient gain is still possible in this region due to the intraband relaxation process that broadens the spontaneous emission spectrum. At this wavelength, the calculated gain provides $G_n = 1.1 \times 10^{11}$ s^{-1}, $n_{\text{tr}} = 1.4 \times 10^{24}$ m^{-3} and $n_s = -1.0 \times 10^{24}$ m^{-3}. Other laser parameters used in the numerical evaluation are $\beta = 10^{-4}$ and $B_{\text{sp}} = 10^{-16}$ m^{3}s^{-1}. The intrinsic loss was taken $\kappa_i / 2\pi = 7.9$ GHz (equivalent to a spatial absorption rate of roughly 500 m^{-1}) and $\kappa_0 = \kappa_i + \Omega_0 / Q_o$, with Q_o calculated for each specific w_{ring} and gap distance to the metal.

Finite elements method (FEM) simulations were performed for the mechanical properties of the device. Chromium was chosen as the material for the ring due to its high absorption coefficient and high refractive index, favoring the mode spreading within the air gap, likewise a slot waveguide. For this design/simulation, this gap was set to 20 nm. In Fig. 2(b) we show the mechanical frequency and the effective mass as a function of w_{ring}. We calculated the optomechanical dispersive and the dissipative coupling factors separately, as shown in Fig. 2(c). The dispersive part is given by perturbation theory [30],

$$g_\omega = -\frac{\Omega_c}{2} \frac{\langle \mathbf{E}^* | \partial \epsilon / \partial x | \mathbf{E} \rangle}{\langle \mathbf{E}^* | \epsilon | \mathbf{E} \rangle}, \tag{10}$$

including the contribution from photoelastic and moving boundary terms in the dielectric perturbation $\partial \epsilon / \partial x$, with the first term being dominant. In the other hand, the dissipative coupling is obtained from the full calculation of the complex optical frequency for a standing dielectric ring with its edge moving with respect to the metallic ring position – assuming that the radial moving boundary is the most important component for the dissipative modulation, being directly related to the gap between the disk edge and the metallic ring. g_κ is then calculated from the spatial derivative of the imaginary part of the frequency. Component analysis shows that g_ω and g_κ have opposite signs [30] – in brief, the approximation of the metal induces a reduction of the optical frequency and an increase of the decay rate. As shown in Fig. 2(c), we highlight the very high g_κ obtained with this strategy, of the order of GHz/nm, while g_ω is kept with high values as well, allowing for very high overall optomechanical coupling. This is a result of the intrinsic separation of the coupling mechanism, where the dispersive interaction is generated mainly by photoelastic effect and the dissipative interaction arises from a boundary motion process.

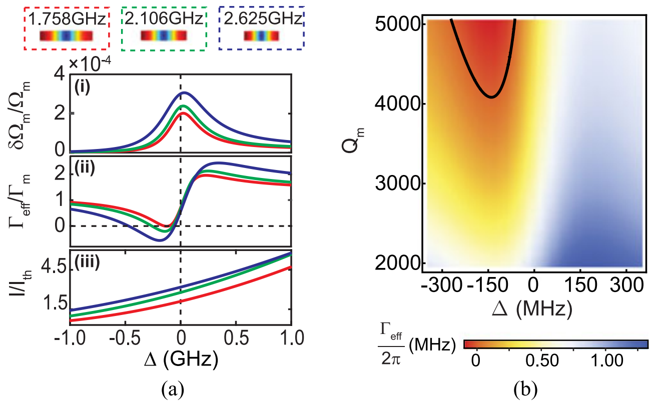

Fig. 3. Optical spring effect and limit for the self-oscillation regime. (a) We have calculated the optomechanical feedback for three values of w_{ring}: 1200 nm (red), 1000 nm (green) and 800 nm (blue), with respective mechanical frequencies of 1.758 GHz, 2.106 GHz and 2.625 GHz for the indicated mechanical mode – displacements profiles shown as insets on the top. We evaluate the relative shift in the mechanical frequency (i) and the relative optomechanical damping (ii) for a fixed value of $Q_m = 5000$ as function of the detuning $\Delta = (\Omega_r - \Omega_m)/2\pi$, with interest in the amplification region, where $\Gamma_{eff} \leq 0$. The injected current tunes Ω_r, as explicit in (iii). (b) The density plot shows the value of the effective damping Γ_{eff} for the case of $w_{ring} = 1000$ nm, a fixed current and variable Q_m, in order to evaluate the impact of small variations on the mechanical frequency from the design. The current is fixed in $I/I_{th} = 2.5$, where $\Omega_r/2\pi = 1.95$ GHz, and we calculate the minimum Q_m needed to achieve $\Gamma_{eff} = 0$, highlighted in black. Therefore all the points above this curve will present self-pulsed emission, supporting a mechanical frequency variation of 12% from the design without need of higher Q_m.

5. Self-Sustained Laser Pulsation

In the following, we present the calculation of the optical spring effect and optomechanical damping for our design in order to evaluate the possibility of obtaining self-sustained laser pulsation. There is an aimed optimal range of mechanical frequencies, near few GHz, where $H(\Omega)$ is high enough, i.e efficient transduction between the mechanical motion and the optical loss modulation is allowed [see Fig. 1(c) and (d)], and the dissipative optomechanical coupling is still sufficiently high, based on Fig. 2(b) and (c). Therefore, it is important to evaluate the device for different values of w_{ring}. We have chosen to investigate three different w_{ring} lengths: 1200 nm, 1000 nm and 800 nm in order to find the optimum size. The calculated relative spring effect (i) and optomechanical damping (ii) are shown in Fig. 3(a) (curves shown in red, green and blue, respectively). The profiles of the modes are shown as insets at the top of the figure, with respective mechanical frequencies $\Omega_m/2\pi$: 1.758 GHz, 2.106 GHz, and 2.625 GHz. The mechanical quality factor was fixed with a value of $Q_m = 5000$, which is about two times the experimental value obtained for this design of cavity, but expected to be achievable in reference [18]. We defined a detuning, in analogy with the term in usual optomechanics, as $\Delta = (\Omega_r - \Omega_m)/2\pi$. In Fig. 3(a)(i) we notice a maximum spring modification occurs near the zero detuning, where Ω_m is slightly blue shifted from Ω_r – from (8), we see that this occurs when $\Omega_m^2 \approx \Omega_r^2 - \Gamma_r^2$. Also, $\delta\Omega_m/\Omega_m$ is always positive, and it is not symmetric due to the non symmetric behavior of Ω_r and Γ_r with the current. Fig. 3(a)(ii) shows the relative optomechanical damping, defining two regions with respect to zero detuning, increasing or decreasing Γ_m. Finally, Fig. 3(a)(iii) shows the dependence between detuning and the injection current which shows that one may position the system under different conditions of the optomechanical interaction. In order to have self-sustained pulsation, optomechanical amplification is necessary. This is achieved under the condition where the damping coefficient is negative, i.e.,, $\Gamma_{eff} \leq 0$, and this only occurs when the mechanical frequency is blue shifted with respect to the relaxation oscillation frequency – in the figure, this is readily achieved by the green and blue curves. Although the current allows to sweep the relaxation oscillation frequency, the amplification region is limited to a small bandwidth, as seen in Fig. 3(a)(ii). The self-sustained oscillation regime was discussed in our previous work [17]. Essentially, when the laser is turned on, with an injected current above the laser threshold, $I > I_{\mathrm{th}}$, the mechanical oscillator receives an initial kick and starts oscillating with an amplitude decay rate related to its own dissipation. The oscillation lifetime is of the order of $\tau_{\mathrm{m}} = Q_{\mathrm{m}}/\Omega_{\mathrm{m}}$. However, if the mechanical damping is modified, such that $\Gamma_{\mathrm{eff}} \leq 0$, after the initial kick the mechanical oscillation amplitude would grow with time, until it reaches a dynamical steady-state, shifted from the rest position and with a non-zero small amplitude oscillation. Such oscillation is transferred to the photon and carrier population, leading to a self-pulsation state of the laser emission.

We have checked the system robustness regarding this bandwidth for the observation of the self-pulsed emission. This concerns essentially small changes of the mechanical frequency and/or quality factor of the real device. An interesting map of the oscillation condition can be obtained by plotting the damping rate Γ_{eff} for different detuning and different mechanical quality factor. Fig. 3(b) presents this map where we have fixed the $w_{\mathrm{ring}} = 1000$ nm, and operate the laser with an injection current 2.5 times the lasing threshold value, $I/I_{\mathrm{th}} = 2.5$. Under this condition, the relaxation oscillation frequency is $\Omega_{\mathrm{r}}/2\pi = 1.95$ GHz. The black curve denotes $\Gamma_{\mathrm{eff}} = 0$, i.e., the onset of the sustained oscillations. Points above this curve are therefore in the amplification regime, showing the robustness of the device – a variation of 12% of the mechanical frequency from the design is supported without necessary increase of the mechanical quality factor. These predictions are consistent with the full numerical integration of the optomechanical laser rate equations, as shown previously [17]. Thus we expect this model to provide good analytical predictions about the threshold for the self-sustained optomechanical laser oscillation.

An obvious concern about the feasibility of this device is the very small gap which is necessary to achieve a reasonable value of g_{κ}. We observe that these simulations are based on a mode resonant aroung 950 nm and with the first radial order of the optical mode. There are two possible ways of improving this scenario: by using a higher radial order mode, which is supposed to be less confined and have a more spread evanescent field, naturally leading to higher dissipative coupling; and/or by using different semiconductor alloys and then work with emission between 1300 nm and 1600 nm, since the mode is less confined in these cases as well. These are ongoing subjects beyond the scope of this paper. Nevertheless, given the well established current nanofabrication techniques, this value for the gap between the dielectric disk and the metallic ring, although challenging, should not be an impediment for an experimental implementation of the device. Furthermore, the model presented is general of a semiconductor laser cavity and can be applied to other geometries and gain media, leading to similar dynamics. The most relevant limitation is the absence of thermal effects. Heating of the device may occur due to non-radiative recombination or excess free carriers and shift the optical resonances and the gain envelope. It may also cause degradation of the optical and mechanical quality factors and diminish the efficiency of the laser. Nevertheless, our proposed design has robust working points and acceptable parameters. Besides this, thermal effects can be mitigated at low temperature due to the reduced thermal expansion and reduced thermo-optic effect. Therefore, the active bullseye represents a realistic device for the pursuit of the optomechanical laser and the predicted self-sustained pulsed emission.

6. Conclusion

We have shown the existence of coupling between the laser relaxation oscillation and the mechanical motion in an optomechanical laser with both dispersive and dissipative optomechanical coupling under a semi-classical approach. Small signal analysis reveals the dependence of the optomechanical damping and optical spring effect with the laser parameters, controlled by the injected current. Even in the absence of a coherent external pump, it is possible to achieve an optomechanical amplification, when the laser relaxation oscillations couple to the mechanical oscillation, leading to self-oscillating light emission. A proposed design based on high confinement of both the optical and the mechanical mode combined with a near-field dissipative structure results in high dispersive and dissipative optomechanical coupling factors, compatible with experimental demonstration. Incorporation of optical gain in optomechanical cavities may now enable the development of a new class of active devices.

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