# Self-trapping of polychromatic light in nonlinear periodic photonic structures

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**Abstract:** We study dynamical reshaping of polychromatic beams due to collective nonlinear self-action of multiple-frequency components in periodic and semi-infinite photonic lattices and predict the formation of *polychromatic gap solitons* and *polychromatic surface solitons* due to light localization in spectral gaps. We show that the self-trapping efficiency and structure of emerging polychromatic solitons depend on the input spectrum due to the lattice-enhanced dispersion, including the effect of crossover from localization to diffraction in media with defocusing nonlinearity.



**OCIS codes:** (190.4420) Nonlinear optics, transverse effects in; (190.5940) Self-action effects

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## 1. Introduction

The fundamental physics of periodic photonic structures is governed by scattering from modulations of the refractive index and subsequent interference. Such a resonant process is sensitive to a variation of the beam frequency and propagation angle [1]. Accordingly, refraction and diffraction of optical beams may depend strongly on the optical wavelength, allowing for construction of superprisms that realize a spatial separation of the frequency components.

Advances in the generation of light with broadband or supercontinuum spectrum in photonic-crystal fibers open new possibilities for characterization of photonic-crystal structures [2]. It was predicted that broadband excitations propagating through finite-size one- and two-dimensional nonlinear photonic crystals may generate multi-frequency gap solitons [3]. More recently, it was shown [4] that *polychromatic lattice solitons* may exist in extended periodic structures with noninstantaneous and broadband nonlinear response, such as optically-induced lattices created in photorefractive materials [5, 6, 7] or $LiNbO_3$ waveguide arrays [8, 9], where all frequency components can experience collective self-trapping and form a single localized beam, unlike the spatially separated localization at different wavelengths reported previously [3]. It was later shown that polychromatic lattice solitons can also be excited by white-light sources that are both spatially and temporally incoherent [10].

In this paper, we study the propagation of polychromatic light beams in nonlinear photonic lattices, and address an important question of how the periodicity-enhanced sensitivity of diffraction upon wavelength influences nonlinear beam self-action. We demonstrate that different frequency components can have the tendencies to experience either spatial self-focusing or self-defocusing in the same lattice with defocusing nonlinearity, resulting in a highly nontrivial dynamics of polychromatic light beams that *has no analogues* in the physics of white-light solitons in homogeneous configurations [11, 12] or lattices with focusing nonlinear response [10]. We also analyze nonlinear self-trapping at the interface between two different semi-infinite waveguide arrays and demonstrate the generation of *polychromatic surface solitons* which generalize the recently studied scalar [13, 14, 15] and multi-gap [16, 17] surface solitons.

We study the dynamics of polychromatic light in planar nonlinear photonic structures with a modulation of the refractive index along the transverse spatial dimension, such as optically-induced lattices or periodic waveguide arrays [5, 6, 7, 8, 9]. For an optical source with a high degree of spatial coherence, such as supercontinuum light generated in photonic-crystal fibers, the evolution of polychromatic beams in media with slow nonlinearity can be described by a set of normalized equations for the spatial beam envelopes $A_n(x,z)$ of the different frequency components at vacuum wavelengths $\lambda_n$,

$$i\frac{\partial A_n}{\partial z} + \frac{\lambda_n z_0}{4\pi n_0 x_0^2}\frac{\partial^2 A_n}{\partial x^2} + \frac{2\pi z_0}{\lambda_n}\left[\nu(x) + \gamma I\right]A_n = 0\,, \quad (1)$$

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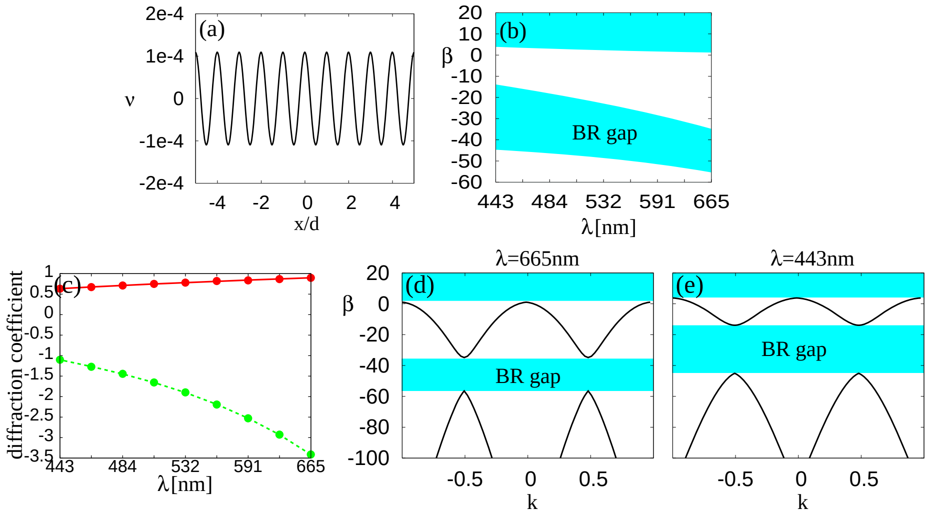


Fig. 1. (a) Refractive index profile for the photonic lattice. (b) Dependence of the bandgap spectrum on the wavelength and (c) diffraction coefficients at the top (red) and bottom (green) of the first band. (d,e) Spatial Bloch-wave dispersion for two wavelengths of 665nm and 443nm, respectively. Transverse Bloch wavevector component $k$ is normalized to $K = 2\pi/d$. Grey shading in (b,d,e) marks the spectral gaps: semi-infinite gap at the top (for large values of $\beta$) and Bragg-reflection gaps (for smaller values of $\beta$).

where $x$ and $z$ are the transverse and longitudinal coordinates normalized to $x_0 = 10\mu m$ and $z_0 = 1mm$, respectively, $I = \sum_{n=1}^{N} |A_n|^2$ is the total intensity, $N$ is the number of components, $n_0$ is the average refractive index, $\nu(x)$ is the refractive index modulation in the transverse spatial dimension, and $\gamma$ is the nonlinear coefficient. We consider the case of a Kerr-type medium response, where the induced change of the refractive index is proportional to the light intensity. We neglect higher-order nonlinear effects such as saturation in order to identify the fundamental phenomena independent on particular nonlinearity. We note that Eq. (1) with $\lambda_n = \lambda$ describe one-color multigap solitons [18, 19, 20, 21], yet we find that the physics of multi-color interaction may give rise to completely new effects.

## 2. Generation of polychromatic gap solitons in periodic lattices

First, we discuss the generation of polychromatic solitons in periodic structures, where the distant boundaries do not influence the beam propagation. In this case, linear dynamics of optical beams is defined through the properties of extended eigenmodes called Bloch waves [1]. We consider an example of the lattice with $\cos^2$ refractive index modulation [see Fig. 1(a)] with the period $d = 10\mu$m, corresponding to characteristic shape of optically induced or fabricated lattices [6, 7, 8, 9]. We calculate dependencies between the longitudinal ($\beta$, along $z$) and transverse ($k$, along $x$) wave-numbers for Bloch waves, as depicted in Figs. 1(b-e). The top spectral gap is semi-infinite (it extends to large $\beta$), and it appears due to the effect of the total internal reflection. The effective diffraction of Bloch waves becomes anomalous at the upper edges of Bragg-reflection gaps, where $D_{\rm eff} = -\partial^2\beta/\partial k^2 < 0$.

The presence of Bragg-reflection gaps and associated anomalous diffraction allows for the formation of monochromatic spatial gap solitons even for defocusing nonlinearity [22, 6, 8, 9]. Figures 1(b-e) show that the spatial bandgap spectrum depends on the optical wavelength [4, 10]. We note that the *anomalous diffraction regime is strongly frequency dependent* and we de-

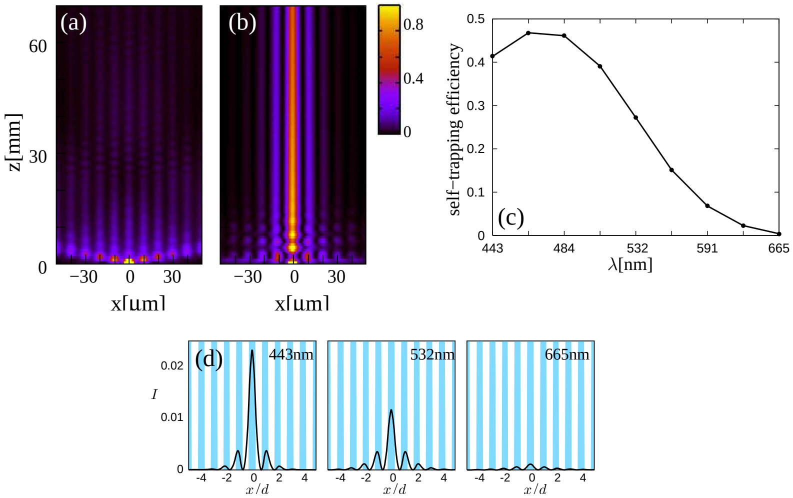


Fig. 2. (a) Linear diffraction of a polychromatic beam; darker shading marks lower intensity; (b-d) Nonlinear self-focusing and generation of a polychromatic gap soliton for $I = 1.11$: (b) total intensity, (c) output power spectrum of self-trapped soliton, and (d) output intensity profiles of individual components for $\lambda = 443$nm, 532nm, and 665nm.

rive an asymptotic expression $D_{\text{eff}} \sim -\lambda^3$ at large wavelengths, whereas the normal diffraction coefficient at the top of the first band and in the bulk is proportional to $\lambda$. Different frequency sensitivity of normal and anomalous diffraction coefficients is shown in Fig. 1(c). Additionally, the Bragg-reflection gap becomes much narrower at larger wavelengths, limiting the maximum degree of spatial localization that is inversely proportional to the gap width.

The variation of the gap width can have a dramatic effect on self-action of an input beam focused at a single site of a defocusing nonlinear lattice [9], where a sharp crossover from self-trapping to defocusing occurs as the gap becomes narrower. We find that, most remarkably, these distinct phenomena can be observed in the same photonic structure but for different wavelength components. We note that these effects do not occur in lattices with positive or self-focusing nonlinearity [10]. In our numerical simulations, we put $\gamma = -10^{-4}$ and choose the lattice parameters such that the critical wavelength corresponding to the crossover is around 591nm. We confirm that the monochromatic beam with $\lambda = 443nm$ experiences strong self-trapping, whereas the largest fraction of input beam power becomes delocalized at a longer wavelength $\lambda = 665nm$. We then address a key question of how an interplay between these opposite effects changes the nonlinear propagation of polychromatic beams.

We model the self-action of polychromatic light beams by simulating the propagation of nine components with the wavelengths ranging between 443nm and 665nm. The input corresponds to a narrow Gaussian beam that has the width of one lattice site, i.e. in our case 5$\mu$m. Figure 2 shows our numerical results for the propagation of polychromatic light over 70mm. The input spectrum of light is ‘white’, i.e. the light beams of different wavelength all have the same input profile and intensity. In the linear regime (small input intensity), all components of the beam strongly diffract, and the beam broadens significantly at the output, as shown in Fig. 2(a). As the input power is increased, we find that the spatial spreading can be compensated in a broad spectral region by self-defocusing nonlinearity. We observe a spatially localized total intensity profile at the output, indicating the formation of *a polychromatic gap soliton* [Fig. 2(b)].

We note that the spatial localization of the soliton components depends strongly on the wavelength [Figs. 2(d)], so that the long-wavelength component has a much larger spatial extent

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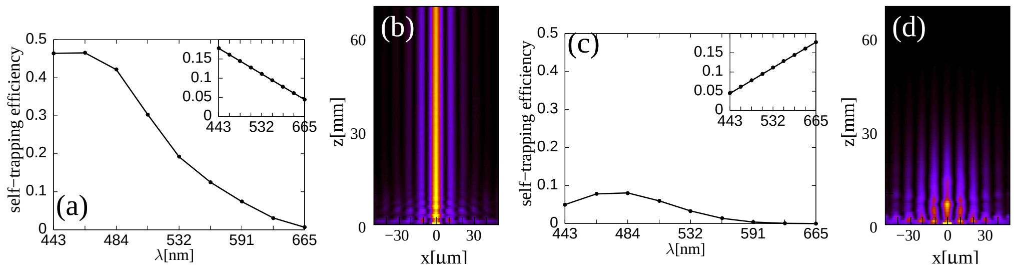


Fig. 3. (a,c) Fraction of the light trapped by the lattice as a function of wavelength; (b,d) the corresponding evolution of the total intensity profiles for incident light beams with the same input profiles, but different frequency spectra shown in the insets. The peak input intensity is (a,c) the same or (b,d) twice as large compared to Figs. 2(b-d).

than the short-wavelength component. Hence, the soliton has a blue center and red tails, and this effect is more pronounced than for the solitons with the same spectra in bulk. Additionally, the power spectrum of the soliton becomes blue-shifted at the output. Figure 2(c) shows the self-trapping efficiency defined here as the percentage of light that remains in the three central waveguides of the optical lattice after the propagation for each wavelength. This value is essentially identical to the trapped fraction of light, as even for longer propagation distances the light would remain localized in these waveguides. We see that more that 40% of the light with the wavelengths between 443nm and 484nm is trapped, whereas for the longer wavelengths that percentage drastically decreases due to the narrowing of the Bragg-reflection gap. However, due to coupling of different wavelengths, there still is a noticeable amount of light from the red side of the spectrum that is trapped (roughly 8% of the light at 591nm). This is in a sharp contrast to the case of monochromatic red light, for which the self-trapping efficiency vanishes.

We now study the effect of input spectrum on the nonlinear self-action of polychromatic light. We perform numerical simulations for the same profiles of the input beam as in Fig. 2(b), but considering different power distribution between the frequency components. Figures 3(a-d) show the characteristic propagation results for beams with blue- and red-shifted input spectra. For the blue-shifted input spectrum [Figs. 3(a,b)], we observe self-trapping of the polychromatic light beam, and a small percentage of the red light is trapped by the nonlinear index change caused by the blue parts of the spectrum. In fact, the self-trapping efficiency for the red part of the spectrum is almost identical to the case of a white spectrum shown in Fig. 2(c). Fundamentally different behavior is observed for a polychromatic beam with red-shifted spectrum [Figs. 3(c,d)]. In this case, the beam strongly diffracts and self-trapping does not occur even when the total input intensity is increased several times compared to the case of white spectrum. This happens due to the tendency of red components to experience enhanced diffraction as the effect of defocusing nonlinearity is increased at higher intensities. We note that, according to Fig. 3(c), the blue part of the spectrum is also diffracting.

### 3. Generation of polychromatic surface solitons

Next, we study the influence of boundaries on the soliton generation, and consider an interface between two different periodic lattices as shown in Fig. 4(a). Such interfaces may support localized *linear* modes that generalize the so-called *surface Tamm states* known to exist in other similar structures. Nonlinear media allows for localization even in the cases when linear surface states are absent [13, 14, 15, 16, 17]. Light can only be localized when the spectral gaps of *both* lattices overlap. We find that by selecting different waveguide widths and refractive index contrasts of lattices, it becomes possible to tailor the wavelength dispersion of individ-

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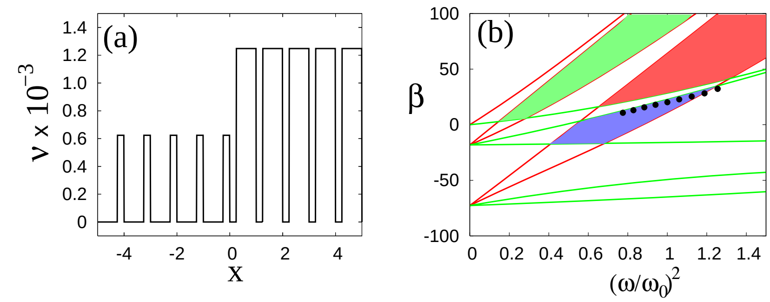


Fig. 4. (a) Structure of the waveguide interface, (b) edges of the spectral bands for the narrow waveguide array (green lines) and the wide waveguide array (red lines), and the overlap regions (red, green, and blue) vs. the light frequency scaled to $\omega_0 = 2\pi/532$nm.

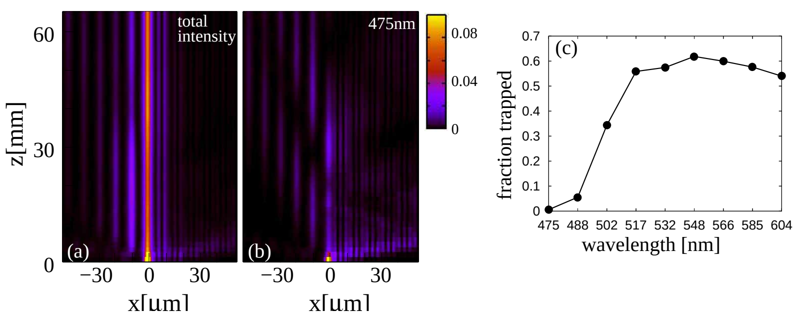


Fig. 5. Generation of a polychromatic interface soliton composed of nine input components. (a,b) Evolution of the (a) total beam intensity and (b) the blue-most component during a 10cm propagation, (c) fraction of the input light trapped at the two waveguides closest to the interface after the propagation.

ual lattice spectra and thereby control the localization and diffraction of polychromatic beams. Figure 4(b) shows the overlaps of the bandgap-spectra of the lattices as a function of the light frequency, and the overlap regions where the localized surface states may exist. When a polychromatic beam is launched into the narrow waveguide closest to the interface (the input beam width at FWHM is 5.3$\mu$m), a large fraction of the light localizes at the interface in the form of an interface gap soliton, as shown in Fig. 5(a).

We have estimated numerically the propagation constants of each of the nine components of the generated polychromatic soliton and marked them with dots in Fig. 4(b). The propagation constants are shifted due to defocusing nonlinearity into the first band gap of the narrow lattice, however for the chosen frequency spectrum the localization is only possible in the overlap region with the the second gap of the wide lattice, shown with blue shading in Fig. 4(b). We see that the overlap vanishes for light of higher frequencies, and accordingly blue components are not localized at the interface which acts as an optical low-pass filter, see Figs. 5(b,c). This effect is in a sharp contrast to the beam dynamics in bulk nonlinear media or periodic photonic lattices with cos-type modulation, where blue light does generally focus into solitons more easily than red light as discussed above in Sec. 2.

In conclusion, we have studied the generation of polychromatic gap solitons in periodic and semi-infinite photonic lattices, and demonstrated that self-action of light can be employed to reshape multiple frequency components of propagating beams in media with noninstantaneous and broadband nonlinear response.

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