SEMANTIC PARAMODULATION FOR HORN SETS

William W. McCune
Lawrence J. Henschen

Dept. of Electrical Engineering and Computer Science
Northwestern University
Evanston. Illinois, U.S.A

ABSTRACT

We present a new strategy for semantic paramodulation for Horn sets and prove its completeness The strategy requires for each paramoduiation that either both parents be false positive units or that one parent and the paramodulant both be false relative to an interpretation We also discuss some of the issues involved in choosing an interpretation that has a chance of giving better performance that simple set-of-support paramoduiation.

1. Introduction

In [19] it is argued that paramoduiation has the following advantages over resolution with the equality axioms: 1. paramoduiation emphasizes the use of the functional representation as opposed to the relational representation; 2. in functional representation terms are not split up and so demodulation is more effective; and 3. paramoduiation works directly on deeply nested terms as opposed to resolution which uses function substitution to build up or tear down terms one level at a time. On the other hand, paramoduiation has proven difficult to control. Some restrictive strategies of a general nature have been developed [e.g., 13.18] but these are still not as effective as we would like. An often-times useful approach is to consider strategies for special classes of problems. In this paper we present a new semantic strategy for paramoduiation for Horn sets, extending the work of [4]. We prove the completeness of the strategy and discuss some issues relating to its use (in particular. some ideas about how interpretations should be chosen).

2. Preliminary Theoretical Results

Definition. A Horn semantic resolution with respect to an interpretation I is a resolution inference which satisfies one of the following two conditions: l) one of the parents is a positive unit which is false in I. or 2) one of the parents and the resolvent are both false in 1.

Lemma 1 ([4, Theorem l]). Horn semantic resolution is complete for unsatisnable Horn sets.

For the rest of this section we use slightly modified definitions of ground clause and ground resolution in order to simplify the analyses of ground deductions.

Definition A ground clause is a multiset of literals. We use the notation C: -Ll -12 ... LnD to represent n occurrences of literal -L in a clause C. D represents the remaining multiset of literals in C.

Definition Let Cl: -Ll ... Ln D and C2: Ll ... Lm E be two ground clauses where Cl contains at least n occurrences of literal L and clause C2 contains at least m occurrences of literal L. D and E are multisets of literals. The clause C3: D E is a ground resolvent of parent clauses Cl and C2. We say that the set of n occurrences of -L in Cl matches the set of m occurrences of L in C2 Duplicate literals in a resolvent are not merged.

Note that a Horn multiset clause still has only one occurrence of a positive literal and that resolution of Horn multiset clauses yields a Horn multiset clause.

Lemma 2. Let Dl be a Horn semantic resolution deduction of a clause Cl from a ground set S using interpretation I. Then there exists a Horn semantic resolution deduction D2 of a clause C2 where

1. C2 is logically equivalent to Cl, and
2. for each resolution in D2 exactly one literal of the false clause is matched.

Proof. The following algorithm constructs D2 from Dl

1. Choose a highest node in the tree representing Dl in which more than one literal from the false clause is matched. If there is no such node then stop, otherwise, let the parents of the resolution be P1. -Ll ... Ln D and P2: L E. The resolvent is R: D E.
2. Replace the above resolution by a sequence of n resolutions. The resolvents are Rl: -Ll... -Ln-1 D E, R2: -Ll ... -Ln-2 D El E2, ... , Rn: D El ... En.
3 Resolutions in Dl on descendents of literals in E are replaced (recursively) by resolutions on the corresponding multiple copies in El ... En. Go to 1

Neither Pl nor P2 can be a positive unit false in I. Therefore R must be false in I. Because -L is false in I, each of the clauses Rl,...,Rn is also false in I. Thus each resolution introduced in step 2 is Horn semantic. A highest node with the given property is selected at each iteration. The deduction is expanded, but only above the selected node; further, no new node above the selected node has the given property. Thus, the algorithm terminates. Clearly the resulting resolvent is logically equivalent to Cl.
QED

Lemma 3. Let Dl be a ground Horn semantic resolution deduction of clause C from ground set S using interpretation 1. Suppose clause -L EO is false in I and occurs in Dl. Suppose also that -L does not occur in C. Then there exists a Horn semantic resolution deduction from S of clause C in which

1. the deduction of -L EO is the same as in Dl,
2. -L is the literal upon which clause -L EO is resolved, and

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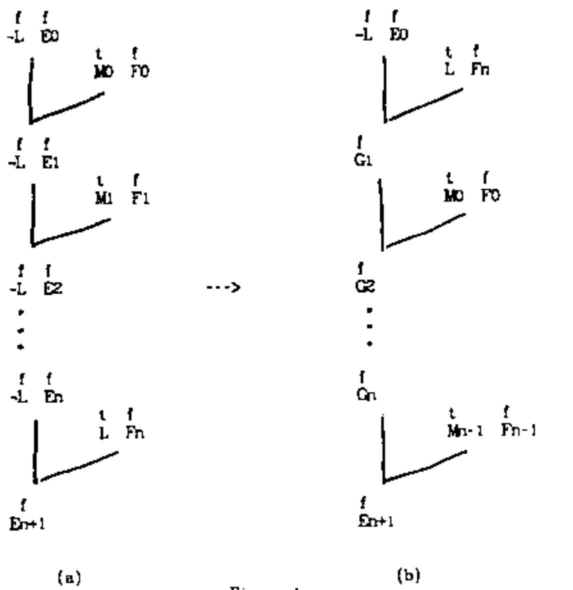

Figure 1

3. D2 has exactly the same number of nodes as DI.

Proof. Let the deduction be as shown in Figure la where Ei+1 = (Ei - |Mi) u Fi. L and Mi are literals, and Ei, Fi and Gi are multisets of literals. Assume that literal -L in clause -L En descends from literal -L in clause -L E0. (Recall that duplicate literals are never merged in a resolvent.) Clause -L E I is false in I because neither of its parents is a false positive unit. Therefore F0 is false. Similarly, each clause -L Ei, and each multiset Fi, 0<1<n, is false in I. Finally, En+1 is false in I. Now consider deduction D2 (Figure Ib) in which the clause L Fn has been moved up in the deduction and resolved with clause L E0. All other resolutions remain unchanged. Clearly Gi = Ei-1 + Fn, 1<1<n, is false in 1. Thus all resolutions in D2 are Horn semantic, It is clear that D2 has the same number of nodes as DI.
QED

Lemma 4. Similar results to those of Lemma 3 hold for the case in which L E0 is the start clause (false in I as before) and En is not empty.

Proof. L Ei , I<i<n, must be false because its parent L Ei-1 is a false non-unit clause. En+1 is false because En Is not empty by hypothesis. The rest of the proof is the same as in Lemma 3.
QED

Remark. Lemmas 2 and 3 allow us to assume without loss of generality that given a false clause in a ground Horn semantic resolution deduction, an arbitrary single occurrence of a negative iteral can be chosen as the literal matched for the next resolution.

Corollary 1. Let S be an unsatisflable set of clauses, and let 1 be an interpretation of the symbols of S. Then there exists a resolution refutation of S in which each resolution satisfies one of the following two conditions:

1. one of the parents is a positive unit false in I, or
2. the parent with the negative literal of resolution and the resolvent are both false in I.

Proof. Let D be a ground Horn semantic refutation of S which satisfies the conditions of Lemma 2. The following algorithm transforms D into a refutation satisfying the conditions of the corollary.

1. Let C be a highest non-unit clause in D in which the positive literal of the resolution is false in 1. If there is no such node then stop.
2. Let -L be a negative literal in C. By Lemma 3 there is a refutation with the same number of nodes in which the deduction of C is the same and -L is the literal resolved in clause C. Let D be such a refutation. Go to 1.

At step 1 the deduction of C satisfies the conditions of the corollary. After step 2 the deduction of the child of C satisfies the conditions. Thus the algorithm terminates because D is finite and the transformation does not increase the size of D.
QED

Notation. In order to simplify the notation we write Pa for P(...,a,...) and g(a) for g(...,a,...). Lower case letters a,b,c,... represent terms and upper case C,D,E,... represent (possibly empty) multisets of literals.

We now consider equality (E-) interpretations and equality axioms. Note that an E-interpretation will never make the following assignments to ground instances of the equality axioms.

1. reflexivity	x=x	<f>
2. symmetry	x≠y y=x	<ff> <tt>
3. transitivity	x≠y y≠z x=z	<fff> <ftt> <tft>
4. predicate substitution	x≠y -Px Py	<fff> <ftt>
5. function substitution	x≠y g(x)=g(y)	<ff>

Lemma 5. Let C be a clause with the following properties.

1. C occurs in a ground Horn semantic resolution deduction with respect to E- interpretation I.
2. C contains a positive literal L that is true in I, L descends from an equality axiom, and any remaining literals of C are false in 1.

Then C also has one of the following properties.

3. C is itself the equality axiom.
4. C is a unit clause obtained by resolving two false positive units with either transitivity <ttt> or predicate substitution <ttt>.
5. C is an immediate resolvent of function substitution <tt> with a false positive unit.
6. C is an immediate resolvent of predicate substitution <tft> with a false positive equality unit.

Proof. Trivial by case analysis.

Lemma 6. Let C be a clause with properties 1 and 2 above but with L a negative equality literal, anot=b.

Then C also has one of the following properties.

3. C is itself the equality axiom.
4. C is an immediate resolvent of predicate substitution <ttf> with a false positive non-equality unit.
5. C is an immediate resolvent of transitivity <ttf> with a false positive equality unit.

Proof. Trivial by case analysis.

Theorem 1. Let S be an E-unsatisriable set of ground Horn clauses, and let 1 be an E-interprc. ition for the symbols of S. Then there exists a resolution and paramodulation refutation of S *u* fa=a| a is a ground termj in which each resolution is Horn semantic and each paramodulation satisfies one of the following two conditions:

1. both parents are positive units which are false in 1, or

2. one parent and the paramodulant are both false in I.

(Paramodulation is defined so that either the left or right argument of an equality literal can match the term to be replaced.)

Remark. We assume symmetric matching for equality. That is, we allow the literals a=b and bnot =a to match for a resolution. This simplifies the proof of the theorem and does not weaken the theorem.

Proof There exists a finite set E" of ground instances of equality axioms such that S u E' is unsatisfiable. We know from Lemma 1 that there exists a resolution refutation, say DI, which satisfies the resolution restrictions of the theorem. We will construct from DI an acceptable resolution and paramodulation refutation D2.

Because we are assuming symmetric matching for equality and paramodulatic *i* is allowed from both sides of the equality, we may assume without loss of generality that DI contains no instances of symmetry

Let the initial deduction DI be written as a tree with the false parent of each node on the left. We transform DI by repeatedly replacing the leftmost occurrence of an equality axiom by paramodulation. Because the equality axiom is true, it must occur as the right parent where it enters DI. Because equality axioms are eliminated left to right, no literal to the left of the axiom being eliminated can descend from another equality axiom Special cases (AI, A.2) are introduced to handle the replacement when an earlier replacement has caused paramodulation into or from a descendant of an equality axiom occurring further to the right. Each of the cases below is labeled with the type of equality axiom (P for predicate substitution. T for transitivity, etc) as well as with the truth value assignments for its literals. Due to space limitations, we describe in prose only a few of the cases. The remaining ones are similar and are described by the Figures 2-20.

Case P.ttt (Figure 2). The two negative literals must resolve immediately with false positive units a=b and Pa. The order is not important. The resulting paramodulation introduces no new cases because both parents of paramodulation were to the left of P.ttt.

Case P.tff (Figure 3). The true Literal a not =b must resclve first. Lemma 3 allows us to assume that -Pa resolves next. The resolution is replaced by a paramodulation as shown in Figure 3. If Pa descends from another equality axiom, that axiom is eliminated by special case A.I.

Case P.ftf (Figure 4). The true literal -Pa must resolve first. Lemma 3 allows us to assume that anot =b resolves next. If a=b descends from another equality axiom, that axiom is eliminated by special case A.2.

Case A.I. Paramodulation into a true positive nonequality literal that descends from an equality axiom. Let the clause be Pb C, where Pb is true and C is false. By Lemma 5 there are possible 4 subcases.

Subcase A.I.1 (Figure 16). Pb C is predicate substitution with paramodulation into Pb. Lemma 3 allows us to assume that the two negative literals resolve next. The order is not important. This section of the deduction is replaced by two paramodulation inferences. Paramodulation from a=b is covered in case A.2, and paramodulation into Pa is covered in case A. 1.

The remaining 17 cases are similar.
QED

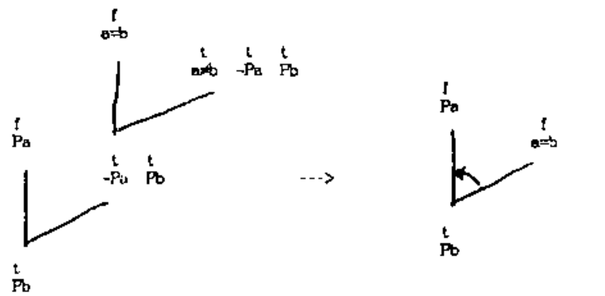

Figure 2 -- P ttt

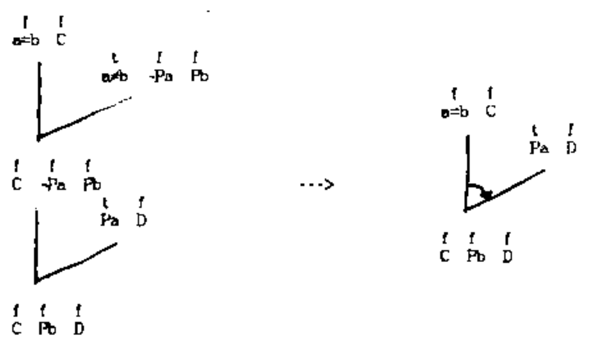

Figure 3 -- P tff

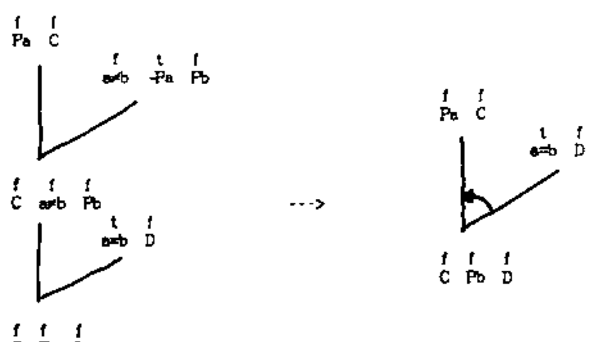

Figure 4 -- P.ftf

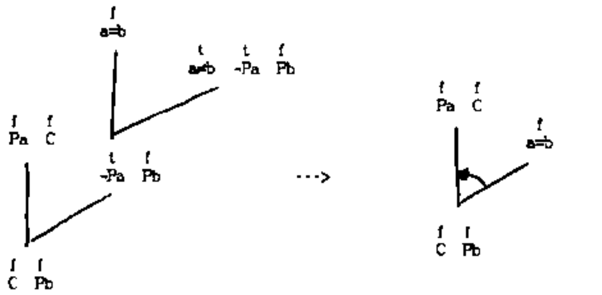

Figure 5a -- P.ttf

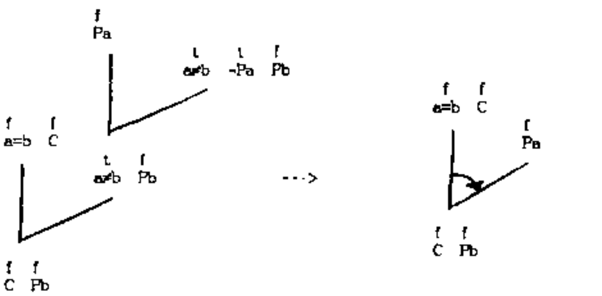

Figure 5b -- P.ttf

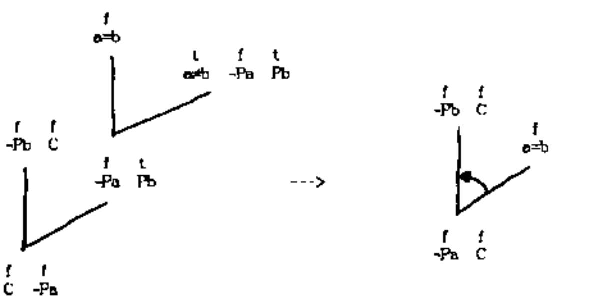

Figure 6 -- P.tft

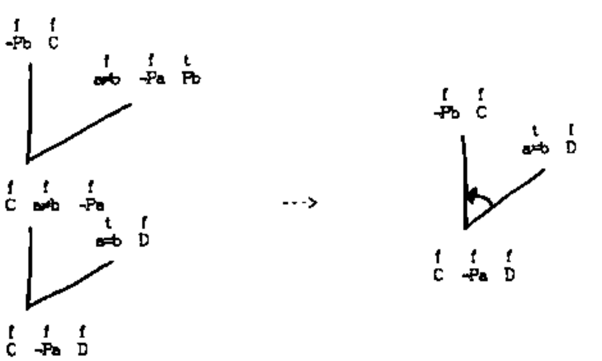

Figure 7 -- P.fft

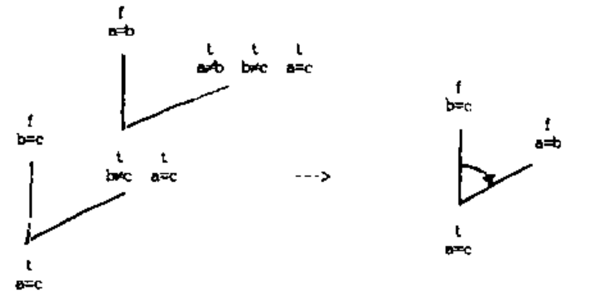

Figure 8 -- T.ttt

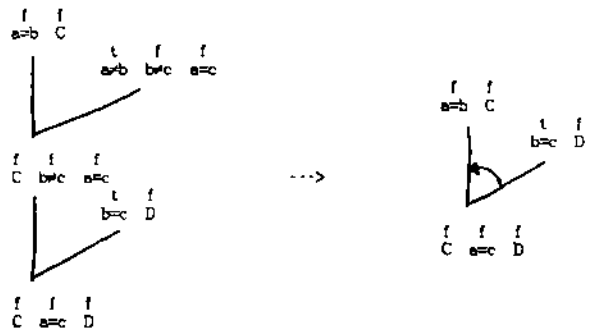

Figure 9 -- T.tff

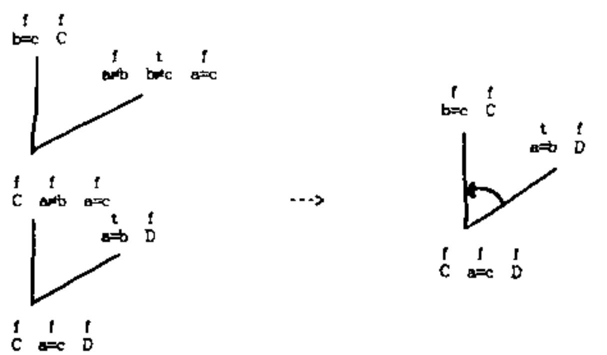

Figure 10 -- T.ftf

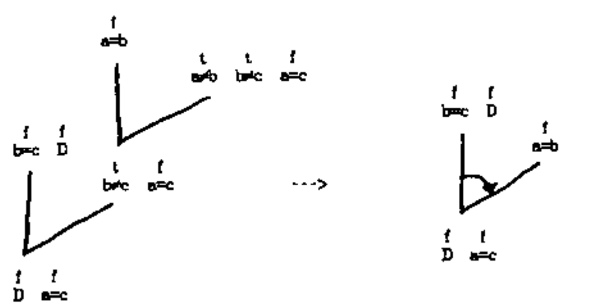

Figure 11a -- T.ttf

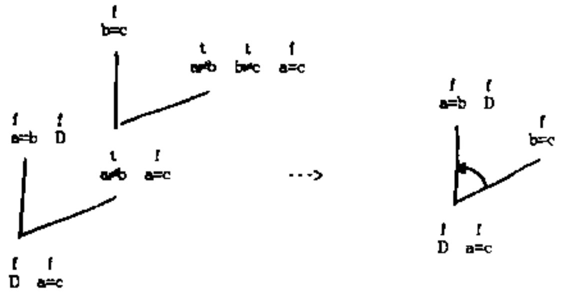

Figure 11b -- T ttf

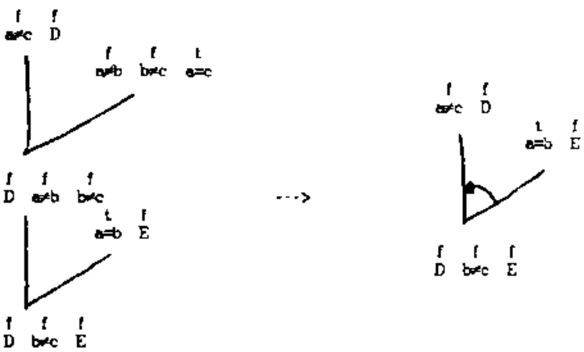

Figure 12 -- T fft

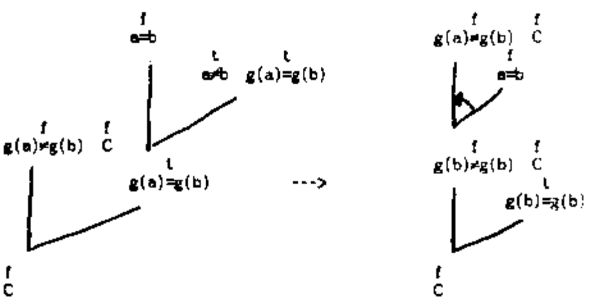

Figure 13 -- F.tt

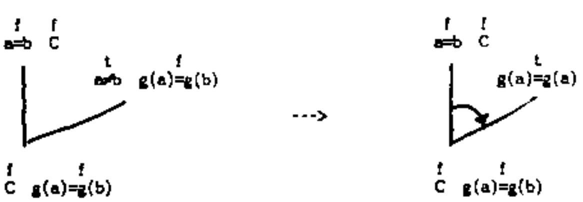

Figure 14 -- F.tf

Figure 15 -- F ft

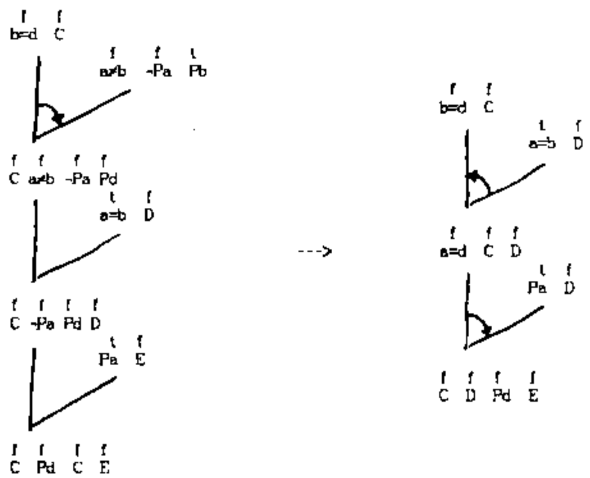

Figure 16 -- A 1 1

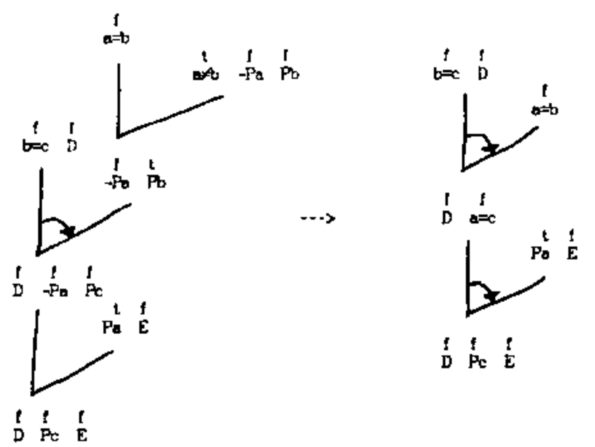

Figure 17 -- A.1.4

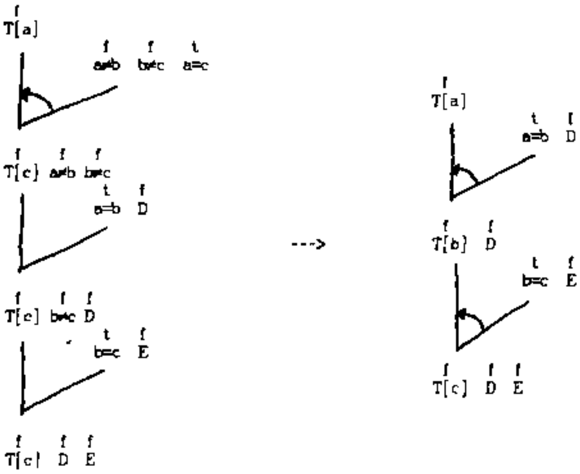

Figure 18 -- A 2.1

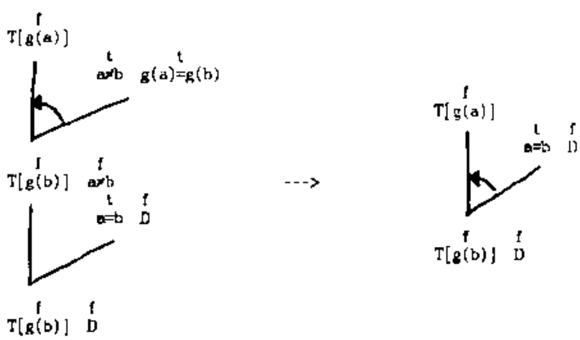

Figure 19 -- A 2 1

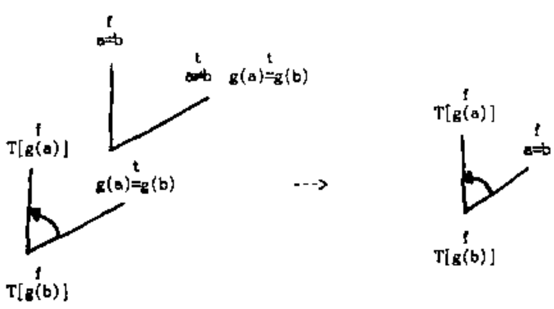

Figure 20 -- A.2.3

Theorem 2. Let S be an E-unsatisftabie set of (general) Horn clauses and let I be an E-interpretation for the symbols of S. Then there is a resolution and paramoauiatioa refutation of S u (x=x} u (functional reflexive axioms) satisfying the semantic conditions of Theorem 1.

Proof, Let D' be a proof as in Theorem 1 for an appropriate set S' of ground instances of S. Resolutions in D' are lifted in the normal way. Now let T'[b'] be the ground paramodulant of T'[a'] and a*=b' C, and let T and a=b C be the corresponding clauses of S. If the position in T corresponding to the position of a' in T'[a] exists, then paramodulation lifts directly. If not, functional reflexive axioms are used.

Case 1. T'[a'] is false. The appropriate set of functional reflexive axioms are paramodulated into T to instantiate it so that it gets a subterm corresponding to a". Each paramodulant is false because each has T'[a'] as an instance. The paramodulation now lifts directly.

Case 2. T'[a'] is true and a'=b' C" is false. In this case, a=b C is paramodulated into the appropriate functional reflexive axioms to obtain a clause g(..(a)...)=g(...(b)...) C which can match the existing term structure in T'[a'J. g(...(a)...)=g(..(b)...) is false because T'[a"] is true and T'[b'] is false. Thus each paramodulant is false The paramodulation now lifts directly.
QED

In case F.tf (Figure 14) of Theorem 1, paramodulation is into a subterm of an instance of x=x. All other uses of instances of x=x are for resolution. We believe this type of paramodulation can be avoided by making the transformation as shown in Figure 21. Lemma 4 allows us to assume that the equality is the next literal to resolve in clause C g(a)=g(b). Now the newly introduced paramodulation is into a level 2 subterm of a true negative equality which may descend from another equality axiom. This introduces a host of new cases; further, this case is recursive — we must handle the case in which an arbitrary subterm of a descendant of an equality axiom is paramodulated. All of these cases have not yet been analyzed.

Of course, functional reflexive axioms cannot be eliminated entirely under the present semantic restrictions. Consider the clauses 1. -P(x,x), 2. P(f(a),f(b)). and 3. a=b. We may chose an interpretation in which the first clause is false and the other two are true. Then the only semantic refutations possible require paramodulation from a functional reflexive axiom. It has been shown in practice [17] that inclusion of the functional reflx xive axioms severely degrades the performance of the program. Some strategies for paramodulation are complete without them [1,5], and we conjecture that there is some modified version of semantic paramodulation for Horn sets that is complete without these axioms.

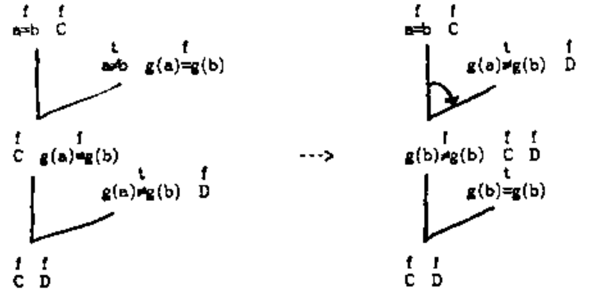

Figure 21 -- F.tf

3. Related Results

T. Brown and Li da Fa have reported independently on similar research. Brown [2] proves the completeness of a version of semantic resolution and paramoduiation for Horn sets. His conditions on resolution are the same as ours, but his conditions on paramoduiation are weaker. There is a restriction only on the into parent — it must either be a false positive unit or have the equality true and the remaining literals false. (Brown defines paramoduiation differently so that all occurrences of a term in the into clause are replaced.) His method o[proof is induction using the excess literal parameter introduced by Bledsoe, Li da Fa [6] has reported a completeness proof with conditions similar to ours but he assumes a very limited class of Herbrand interpretations — namely those in which two terms are equal if and only if they are the same sequence of symbols. His proof is immediate. We are not aware of any reports of experimental results with either of these methods.

4. Experimentation

We plan to conduct extensive experimentation with semantic paramoduiation using NUTS (Northwestern University Theorem-proving System) [10], an LMA-based theorem prover [7,8], Of course, we will use various techniques for efficient evaluation of clauses [eg., 4] and methods for insuring that later substitutions in the deduction don't eliminate all false instances of an earlier false parent [11,12].

While it will be interesting enough to compare Horn semantic paramoduiation with other paramoduiation strategies, the main emphasis in our experiments will be to determine, if possible, what kinds of interpretations lead to good performance. Experience has shown [4,12] that for resolution, choosing the wrong interpretation leads to little or no improvement over simple set-of-support resolution. This is also true for paramoduiation. For example, if we have all unit equalities (very often the case) and the interpretation assigns only the negative clause to be false, then the only allowable semantic paramodulations are those from any of the (true) positive equality units into the one false negative unit. Moreover, the resulting negative unit also will be false. Such an interpretation allows exactly the same paramodulations as if we had chosen the one negative clause to be the set of support. On the other hand, if we choose an interpretation in which the negative clause is true, but some special hypotheses are false, then the only initial paramodulations allowed are paramodulations from or into the false equality units which result in false paramodulants or between pairs of these false special hypotheses. If we have chosen a good interpretation, there will be fewer of these than one would get by allowing those special hypotheses to be in a set of support

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