Semi-Definite Programming Algorithms for Sensor Network Node Localization With Uncertainties in Anchor Positions and/or Propagation Speed

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***Abstract*—Finding the positions of nodes in an ad hoc wireless sensor network (WSN) with the use of the incomplete and noisy distance measurements between nodes as well as anchor position information is currently an important and challenging research topic. However, most WSN localization studies have considered that the anchor positions and the signal propagation speed are perfectly known which is not a valid assumption in the underwater and underground scenarios. In this paper, semi-definite programming (SDP) algorithms are devised for node localization in the presence of these uncertainties. The corresponding Cramér–Rao lower bound (CRLB) is also produced. Computer simulations are included to contrast the performance of the proposed algorithms with the conventional SDP method and CRLB.**

***Index Terms*—Node localization, range measurements, semi-definite programming, sensor networks.**

I. Introduction

A WIRELESS sensor network (WSN) consists of a number of sensors spread across a geographical area. These sensor nodes are small in size and inexpensive and have limited processing, storage, sensing and communication capabilities. WSNs are useful for a wide range of monitoring and control applications in the military, environmental, health and commercial aspects [1]–[4]. Due to the mostly arbitrary node deployment, the sensor locations are often unknown. As a result, determining the physical positions of the sensor nodes is an important problem in the WSNs.

The task of WSN localization is to determine the positions of sensor nodes in a network given incomplete and noisy pairwise time-of-arrival (TOA), time-difference-of-arrival, received signal strength and/or angle-of-arrival measurements [4], [5], which are acquired by the sensors during communications with their neighbors. A standard assumption is that the positions of some nodes, called anchors, are known exactly, so that it is possible to find the absolute positions of the remaining nodes in the WSN. In this work, we focus on node localization with the use of the pairwise distances obtained from multiplying the signal propagation speed with the TOA measurements, which has received significant attention in the literature [6]–[18]. In the presence of Gaussian disturbance, the maximum-likelihood estimator (MLE) for WSN location estimation is devised in [6] which corresponds to a multivariable nonlinear optimization problem and is hard to implement in practice. The MLE can be realized by stochastic optimization methods such as genetic algorithm and simulated annealing [7] but they involve intensive computations with no guarantee of attaining the global optimum point. Analogous to the MLE, Costa *et al.* [8], [9] have proposed to minimize the stress function, which is a metric multidimensional scaling (MDS) technique, via an iterative and distributed procedure with proper initial position estimates of unknown-location nodes. Alternatively, it is possible to relax the MLE formulation to a semi-definite programming (SDP) problem [10], [11] in order to provide a high-fidelity approximate solution that can be obtained in a globally optimum fashion with reduced computational efforts. Apart from SDP, second-order cone programming (SOCP) [12], [13] relaxation is another convex optimization [19] technique for node localization. Although SOCP has a simpler structure and the potential to be solved faster than SDP, its relaxation is weaker than that of SDP which implies an inferior estimation performance. On the other hand, the pairwise distance information is transformed into the relative coordinates of nodes in the classical MDS [14], [15] approach. Unlike metric MDS, classical MDS is much less computationally demanding because only eigenvalue decomposition and simple matrix operations are involved in the positioning procedure. A subspace-based WSN localization approach has been devised in [16] which generalizes our work in single source positioning [17], and this methodology can be considered as an alternative to the classical MDS technique. Inspired by [20], a linear least squares node positioning algorithm which allows distributed processing has been devised in [18].

However, most WSN localization studies [6]–[12], [14]–[18] concentrate on the case where the anchor positions and/or the propagation speed are perfectly known. In this paper, we devise novel SDP algorithms for node localization using noisy pairwise TOA or distance measurements in the presence of these uncertainties. A representative application scenario is node positioning for underwater WSNs [21]–[26]. In a typical underwater sensor network [26], there are three types of nodes, namely, surface buoys, anchors and ordinary or unknown-position nodes.

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Surface buoys drift on the water surface and they can get their absolute locations from global positioning system (GPS) or by other means. As radio frequency waves are heavily attenuated under water, the anchors localize themselves through communications with the buoys instead of equipping with GPS receivers, and this indicates that anchor positions are subject to errors. Note that even GPS-based positioning cannot give error-free location solutions as well. As in conventional WSNs, the ordinary nodes communicate with each other as well as the anchors to estimate their positions as they do not have wireless connections with the buoys. On the other hand, the standard choice for the underwater WSN communication is to utilize acoustic waves but the speed of sound is a function of temperature, pressure, salinity and depth in the oceans [27], [28], which implies that the signal propagation speed is also subject to uncertainties. While in underground WSNs [29] and in-solid scenarios [30] where seismic/vibrational sensor data are processed, the propagation speed is unknown and depends strongly on the propagation medium [31], [32]. In fact, localization of single or noncollaborative sources with anchor location errors have been addressed in [33]–[37] which show that positioning accuracy will be improved when the receiver location uncertainty is taken into account. Recently, a pioneering work for the scenario of WSNs has been presented in [13]. On the other hand, joint estimation of single source position and propagation speed has been studied in [38]–[40]. In [24], the propagation speed is treated as one of the to-be-calibrated parameters in the application of underwater ultrasound imaging.

The rest of the paper is organized as follows. Assuming that both the distance errors and anchor position errors are Gaussian distributed, the MLE for node localization with anchor location uncertainty is first developed in Section II. A new SDP relaxation algorithm for approximating the MLE is then derived. We also present its simplified form when the anchor position errors are independently and identically distributed and make a connection to the standard SDP algorithm [10] which assumes perfect anchor position information. In addition, further approximation on the developed algorithm based on the edge-based semi-definite programming (ESDP) [41] which allows a more computationally efficient realization is suggested. In Section III, we proceed our SDP development to the case of unknown propagation speed where estimation of both node positions and propagation speed is performed. Section IV integrates the development in Sections II and III to devise the SDP algorithm when there are uncertainties in both anchor positions and signal propagation speed. As Cramér–Rao lower bound (CRLB) for node localization with uncertainties is not available in the literature, we have provided its derivation in Section V. The proposed WSN positioning algorithms are evaluated by comparing with the standard SDP approach as well as CRLB in Section VI. Finally, conclusions are drawn in Section VII.

II. Node Localization With Anchor Position Errors

To start with, we would like to introduce the notations used in this paper. Bold upper case symbols denote matrices and bold lower case symbols denote vectors. We use $\{\cdot\}^o$ to represent the true value while its variable is $\{\cdot\}$ and its estimate is $\hat{\{\cdot\}}$. The $\mathbf{0}_{m\times n} \in \mathbb{R}^{m\times n}$ and $\mathbf{0}_n \in \mathbb{R}^{n\times n}$ are zero matrices and $\mathbf{I}_n \in \mathbb{R}^{n\times n}$ is the identity matrix. For two symmetric matrices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A} \succeq \mathbf{B}$ is equal to $\mathbf{A} - \mathbf{B} \succeq \mathbf{0}$ which indicates that $\mathbf{A} - \mathbf{B}$ is positive semi-definite. Trace operator of matrix $\mathbf{A}$ is denoted by $\mathrm{Tr}(\mathbf{A})$. The ^{T} and $^{-1}$ denote matrix transpose and inverse operators, respectively, and $\|\mathbf{x}\|_2$ represents the 2-norm of a vector $\mathbf{x}$. Consider a network of m sensors in a two-dimensional space. Let $\mathbf{x}_i^o = [x_i^o, y_i^o]^T$, $i = 1, 2, \ldots, m$, be the true position of the ith node. Without loss of generality, we assume that the first k of them, $\mathbf{x}_1^o, \mathbf{x}_2^o, \ldots, \mathbf{x}_k^o$, are the anchor positions while $\mathbf{x}_{k+1}^o, \mathbf{x}_{k+2}^o, \ldots, \mathbf{x}_m^o$, correspond to the unknown-position sensors. In this section, we consider that there is position uncertainty in the anchor information and our task is to find better estimates of the k anchor positions as well as the $n = (m-k)$ unknown-sensor locations, or to estimate $\mathbf{X}^o = [\mathbf{x}_1^o, \mathbf{x}_2^o, \ldots, \mathbf{x}_m^o] \in \mathbb{R}^{2\times m}$. After the development of the SDP algorithm for general Gaussian anchor position errors, we will consider the special cases of uncorrelated errors and perfect knowledge of $\mathbf{x}_1^o, \mathbf{x}_2^o, \ldots, \mathbf{x}_k^o$. By further relaxing the constraints in the proposed SDP algorithm, we have also provided its computationally efficient approximation using the ESDP.

A. SDP Algorithm Development

Denote t_{ij}^o and r_{ij}^o, $i, j = 1, 2, \ldots, m$, as the one-way propagation time taken for the radiated signal to travel from the ith node to jth node and their distance, respectively, and let c^o be the known signal propagation speed. In the absence of measurement error, a simple relation between them is then

$$t_{ij}^o = \frac{r_{ij}^o}{c^o}, \quad i, j = 1, 2, \ldots, m \tag{1}$$

where

$$r_{ij}^o = r_{ji}^o = \sqrt{\left(x_i^o - x_j^o\right)^2 + \left(y^o - y_j^o\right)^2} = \|\mathbf{x}_i^o - \mathbf{x}_j^o\|_2, \quad i, j = 1, 2, \ldots, m. \tag{2}$$

In the presence of distance errors and anchor position errors, our observations are

$$d_{ij} = r_{ij}^o + e_{ij}, \quad i \neq j, \quad i, j = 1, 2, \ldots, m \tag{3}$$

and

$$\mathbf{a}_i = \mathbf{x}_i^o + \mathbf{u}_i, \quad i = 1, 2, \ldots, k. \tag{4}$$

Each $\mathbf{a}_i \in \mathbb{R}^{2\times 1}$ represents an erroneous anchor position. The disturbances e_{ij} and $\mathbf{u}_i \in \mathbb{R}^{2\times 1}$ are assumed to be independent zero-mean Gaussian processes with variances and covariance matrices σ_{ij}^2 and $\mathbf{\Phi}_i \in \mathbb{R}^{2\times 2}$, respectively. Note that we only have an incomplete set of $\{d_{ij}\}$ due to limited communication ranges between nodes. Without loss of generality, we assume that all the distance measurements between anchors are not available.

Let $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m]$ be the variable matrix for $\mathbf{X}^o$ and $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_k]$. Under Gaussian disturbance assumption, the MLE for $\mathbf{X}^o$ is achieved by maximization of the following probability:

$$p(\{d_{ij}\}, \mathbf{A}|\mathbf{X}). \tag{5}$$

As $\{e_{ij}\}$ and $\{\mathbf{u}_i\}$ are independent, (5) can be expressed as

$$p(\{d_{ij}\}, \mathbf{A}|\mathbf{X}) = p(\{d_{ij}\}|\mathbf{X}) \prod_{i=1}^{k} p(\mathbf{a}_i|\mathbf{x}_i). \tag{6}$$

Maximizing (6) is equivalent to the nonlinear least squares (NLS) problem

$$\min_{\mathbf{X}\in\mathbb{R}^{2\times m}} \sum_{i=k+1}^{m}\sum_{j=1}^{k}\frac{\delta_{ij}}{\sigma_{ij}^2}\|d_{ij}-\|\mathbf{x}_i-\mathbf{x}_j\|_2\|^2 + \sum_{\substack{i,j=k+1\\ i>j}}^{m}\frac{\delta_{ij}}{\sigma_{ij}^2}\|d_{ij}-\|\mathbf{x}_i-\mathbf{x}_j\|_2\|^2 + \sum_{i=1}^{k}(\mathbf{a}_i-\mathbf{x}_i)^T\mathbf{\Phi}_i^{-1}(\mathbf{a}_i-\mathbf{x}_i) \quad (7)$$

where $\delta_{ij}=1$ if the distance measurement is available and 0 otherwise. The first and second terms of (7) correspond to the distances between the anchors and unknown-position sensors, and distances among the unknown-position sensors, respectively, while the last term addresses the anchor position uncertainty. To simplify the expression, we define g_{ij}:

$$g_{ij}=\begin{cases}\delta_{ij}/2\sigma_{ij}^2, & i>k \text{ and } j>k\\ \delta_{ij}/\sigma_{ij}^2, & \text{otherwise.}\end{cases} \quad (8)$$

The objective function of (7) can now be written as

$$\begin{aligned}&\sum_{i=k+1}^{m}\sum_{j=1}^{k}\frac{\delta_{ij}}{\sigma_{ij}^2}|d_{ij}-\|\mathbf{x}_i-\mathbf{x}_j\|_2|^2 + \frac{1}{2}\sum_{j=k+1}^{m}\sum_{i=k+1}^{m}\frac{\delta_{ij}}{\sigma_{ij}^2}|d_{ij}-\|\mathbf{x}_i-\mathbf{x}_j\|_2|^2 + \sum_{i=1}^{k}(\mathbf{a}_i-\mathbf{x}_i)^T\mathbf{\Phi}_i^{-1}(\mathbf{a}_i-\mathbf{x}_i)\\ &= \sum_{i=k+1}^{m}\sum_{j=1}^{m}g_{ij}|d_{ij}-\|\mathbf{x}_i-\mathbf{x}_j\|_2|^2 + \sum_{i=1}^{k}\mathbf{a}_i^T\mathbf{\Phi}_i^{-1}\mathbf{a}_i-2\mathbf{a}_i^T\mathbf{\Phi}_i^{-1}\mathbf{x}_i+\mathbf{x}_i^T\mathbf{\Phi}_i^{-1}\mathbf{x}_i. \quad (9)\end{aligned}$$

Expanding (9) and dropping the constant terms which have no effects on the minimization, yields

$$\sum_{i=k+1}^{m}\sum_{j=1}^{m}g_{ij}\left[\|\mathbf{x}_i-\mathbf{x}_j\|_2^2-2d_{ij}\|\mathbf{x}_i-\mathbf{x}_j\|_2\right] + \sum_{i=1}^{k}\mathbf{x}_i^T\mathbf{\Phi}_i^{-1}\mathbf{x}_i-2\mathbf{a}_i^T\mathbf{\Phi}_i^{-1}\mathbf{x}_i. \quad (10)$$

In order to form a tight constraint in the later relaxation procedure, we would like to introduce two dummy variables γ_{ij} and r_{ij} for the first term and second term of (10), respectively. Then a constraint which relates γ_{ij} and $\mathbf{X}$ is

$$\begin{aligned}\gamma_{ij}=r_{ij}^2&=\|\mathbf{x}_i-\mathbf{x}_j\|^2\\ &=\mathbf{x}_i^T\mathbf{x}_i+\mathbf{x}_j^T\mathbf{x}_j-\mathbf{x}_i^T\mathbf{x}_j-\mathbf{x}_j^T\mathbf{x}_i\\ &=y_{ii}+y_{jj}-y_{ij}-y_{ji},\\ &i=k+1,k+2,\ldots,m,\quad j=1,2,\ldots,m \quad (11)\end{aligned}$$

where $y_{ij}=\mathbf{x}_i^T\mathbf{x}_j$ is the (i,j) entry of the matrix $\mathbf{Y}$ which is defined as

$$\mathbf{Y}:=\begin{bmatrix}\mathbf{X}^T\mathbf{X} & \mathbf{X}^T\\ \mathbf{X} & \mathbf{I}_2\end{bmatrix}. \quad (12)$$

Furthermore, we denote $\mathbf{\Xi}_i$:

$$\mathbf{\Xi}_i=\mathbf{x}_i\mathbf{x}_i^T. \quad (13)$$

The second last term of (10) will become

$$\sum_{i=1}^{k}\mathbf{x}_i^T\mathbf{\Phi}_i^{-1}\mathbf{x}_i=\sum_{i=1}^{k}\mathrm{Tr}\left(\mathbf{\Phi}_i^{-1}\mathbf{\Xi}_i\right). \quad (14)$$

For the sake of establishing a relationship between $\mathbf{\Xi}_i$ and y_{ii}, we utilize (13) to introduce

$$\mathrm{Tr}(\mathbf{\Xi}_i)=\mathbf{x}_i^T\mathbf{x}_i=y_{ii},\quad i=1,2,\ldots,k \quad (15)$$

as a further constraint. With the use of all developed constraints, the MLE of (7) is equivalent to the following formulation:

$$\begin{aligned}\min_{\mathbf{X},\mathbf{Y},\{\mathbf{\Xi}_i\},\{\gamma_{ij}\},\{r_{ij}\}}\ &\sum_{i=k+1}^{m}\sum_{j=1}^{m}g_{ij}\left[\gamma_{ij}-2d_{ij}r_{ij}\right] + \sum_{i=1}^{k}\left[\mathrm{Tr}\left(\mathbf{\Phi}_i^{-1}\mathbf{\Xi}_i\right)-2\mathbf{a}_i^T\mathbf{\Phi}_i^{-1}\mathbf{x}_i\right]\\ \text{s.t.}\quad &\gamma_{ij}=y_{ii}+y_{jj}-y_{ij}-y_{ji},\\ &i=k+1,k+2,\ldots,m,\quad j=1,2,\ldots,m\\ &r_{ij}^2=\gamma_{ij},\quad i=k+1,k+2,\ldots,m,\\ &j=1,2,\ldots,m\\ &\mathrm{Tr}(\mathbf{\Xi}_i)=y_{ii},\quad i=1,2,\ldots,k\\ &\mathbf{\Xi}_i=\mathbf{x}_i\mathbf{x}_i^T,\quad i=1,2,\ldots,k\\ &\mathbf{Y}=\begin{bmatrix}\mathbf{X}^T\mathbf{X} & \mathbf{X}^T\\ \mathbf{X} & \mathbf{I}_2\end{bmatrix}. \quad (16)\end{aligned}$$

We now relax (16) to a convex optimization problem as follows. The equality $r_{ij}^2=\gamma_{ij}$ in (16) will be replaced by the inequality $r_{ij}^2\le\gamma_{ij}$ to meet the convex specification. In fact, r_{ij} and γ_{ij} will increase and decrease in the minimization, respectively, a tight constraint is automatically achieved, and thus the inequality constraint will be forced to an equality. In addition, performing semi-definite relaxation on (12) and (13), the MLE of (16) is approximated as a convex optimization problem:

$$\begin{aligned}\min_{\mathbf{X},\mathbf{Y},\{\mathbf{\Xi}_i\},\{\gamma_{ij}\},\{r_{ij}\}}\ &\sum_{i=k+1}^{m}\sum_{j=1}^{m}g_{ij}\left[\gamma_{ij}-2d_{ij}r_{ij}\right] + \sum_{i=1}^{k}\left[\mathrm{Tr}\left(\mathbf{\Phi}_i^{-1}\mathbf{\Xi}_i\right)-2\mathbf{a}_i^T\mathbf{\Phi}_i^{-1}\mathbf{x}_i\right] \quad (17)\\ \text{s.t.}\quad &\gamma_{ij}=y_{ii}+y_{jj}-y_{ij}-y_{ji},\\ &i=k+1,k+2,\ldots,m,\quad j=1,2,\ldots,m \quad (18)\\ &r_{ij}^2\le\gamma_{ij},\\ &i=k+1,k+2,\ldots,m,\quad j=1,2,\ldots,m \quad (19)\\ &\mathrm{Tr}(\mathbf{\Xi}_i)=y_{ii},\quad i=1,2,\ldots,k \quad (20)\\ &\begin{bmatrix}\mathbf{\Xi}_i & \mathbf{x}_i\\ \mathbf{x}_i^T & 1\end{bmatrix}\succeq\mathbf{0}_3,\quad i=1,2,\ldots,k \quad (21)\end{aligned}$$

$$\mathbf{Y} \succeq \mathbf{0}_{m+2} \quad (22)$$

$$\mathbf{x}_i = [y_{i\,m+1}, y_{i\,m+2}]^T, \quad i = 1, 2, \ldots, m \quad (23)$$

$$\begin{bmatrix} y_{m+1\,m+1} & y_{m+1\,m+2} \\ y_{m+2\,m+1} & y_{m+2\,m+2} \end{bmatrix} = \mathbf{I}_2 \quad (24)$$

where all the constraints are tight except (21) and (22) which impose rank relaxation on the matrices. In the optimization literature, there are readily available solvers for finding the globally optimum SDP solution for (17)–(24), such as SeDuMi [42] and SDPT3 [43], [44].

B. Simplified Algorithm for Uniformly Diagonal $\mathbf{\Phi}_i$

In particular, when the disturbances in the x and y coordinates are independently and identically distributed, that is, each $\mathbf{\Phi}_i$ is a diagonal matrix of the form $\mathbf{\Phi}_i = \kappa_i^2 \mathbf{I}_2$, the proposed SDP algorithm can be simplified to

$$\begin{aligned}
\min_{\mathbf{X},\mathbf{Y},\{\gamma_{ij}\},\{r_{ij}\}} \; & \sum_{i=k+1}^{m} \sum_{j=1}^{m} g_{ij} \left[\gamma_{ij} - 2d_{ij} r_{ij}\right] + \sum_{i=1}^{k} \kappa_i^{-2} \left(y_{ii} - 2\mathbf{a}_i^T \mathbf{x}_i\right) \\
\text{s.t.} \; & \gamma_{ij} = y_{ii} + y_{jj} - y_{ij} - y_{ji}, \\
& i = k+1, k+2, \ldots, m, \quad j = 1, 2, \ldots, m \\
& r_{ij}^2 \le \gamma_{ij}, \quad i = k+1, k+2, \ldots, m, \\
& \quad j = 1, 2, \ldots, m \\
& \mathbf{Y} \succeq \mathbf{0}_{m+2} \\
& \mathbf{x}_i = [y_{i\,m+1}, y_{i\,m+2}]^T, \quad i = 1, 2, \ldots, m \\
& \begin{bmatrix} y_{m+1\,m+1} & y_{m+1\,m+2} \\ y_{m+2\,m+1} & y_{m+2\,m+2} \end{bmatrix} = \mathbf{I}_2
\end{aligned} \quad (25)$$

where $\mathbf{\Xi}_i$ is replaced by y_{ii} directly. As a result, the constraints of (20) and (21) will also be dropped.

C. Connection to Existing SDP Relaxation

We now show that the SDP relaxation algorithm of (17)–(24) can be easily modified to the scenario of perfect anchor position information. When $\mathbf{\Phi}_i \to \mathbf{0}_2$, the matrix $\mathbf{\Xi}_i$ can be removed, and hence the SDP algorithm in the absence of anchor position uncertainty will become

$$\begin{aligned}
\min_{\mathbf{X},\mathbf{Y},\{\gamma_{ij}\},\{r_{ij}\}} \; & \sum_{i=k+1}^{m} \sum_{j=1}^{m} g_{ij} \left[\gamma_{ij} - 2d_{ij} r_{ij}\right] \\
\text{s.t.} \; & \gamma_{ij} = y_{ii} + y_{jj} - y_{ij} - y_{ji}, \\
& i = k+1, k+2, \ldots, m, \quad j = 1, 2, \ldots, m \\
& r_{ij}^2 \le \gamma_{ij}, \quad i = k+1, k+2, \ldots, m, \\
& j = 1, 2, \ldots, m \\
& \mathbf{Y} \succeq \mathbf{0}_{m+2} \\
& y_{ii} = \mathbf{a}_i^T \mathbf{a}_i, \quad i = 1, 2, \ldots, k \\
& \mathbf{x}_i = \mathbf{a}_i, \quad i = 1, 2, \ldots, k \\
& \mathbf{x}_i = [y_{i\,m+1}, y_{i\,m+2}]^T, \quad i = 1, 2, \ldots, m \\
& \begin{bmatrix} y_{m+1\,m+1} & y_{m+1\,m+2} \\ y_{m+2\,m+1} & y_{m+2\,m+2} \end{bmatrix} = \mathbf{I}_2.
\end{aligned} \quad (26)$$

By direct substitution of the last two equalities into the other constraints, (26) can be simplified to

$$\begin{aligned}
\min_{\mathbf{X},\mathbf{Y},\{\gamma_{ij}\},\{r_{ij}\}} \; & \sum_{i=k+1}^{m} \sum_{j=1}^{m} g_{ij} \left[\gamma_{ij} - 2d_{ij} r_{ij}\right] \\
\text{s.t.} \; & \gamma_{ij} = y_{ii} + y_{jj} - y_{ij} - y_{ji}, \\
& i, j = k+1, k+2, \ldots, m \\
& \gamma_{ij} = y_{ii} + \mathbf{a}_j^T \mathbf{a}_j - 2\mathbf{x}_i^T \mathbf{a}_j, \\
& i = k+1, k+2, \ldots, m, \quad j = 1, 2, \ldots, k \\
& r_{ij}^2 \le \gamma_{ij}, \quad i = k+1, k+2, \ldots, m, \\
& j = 1, 2, \ldots, m \\
& \mathbf{Y}_{uu} \succeq \mathbf{0}_{n+2} \\
& \mathbf{x}_i = [y_{i\,n+1}, y_{i\,n+2}]^T, \\
& i = 1, 2, \ldots, n \\
& \begin{bmatrix} y_{n+1\,n+1} & y_{n+1\,n+2} \\ y_{n+2\,n+1} & y_{n+2\,n+2} \end{bmatrix} = \mathbf{I}_2
\end{aligned} \quad (27)$$

where $\mathbf{X}_u \in \mathbb{R}^{2\times n}$ is extracted from $\mathbf{X} = [\mathbf{X}_a \quad \mathbf{X}_u]$ with $\mathbf{X}_a \in \mathbb{R}^{2\times k}$, and $\mathbf{Y}_{uu} \in \mathbb{R}^{(n+2)\times(n+2)}$ is a submatrix of $\mathbf{Y}$ in (12):

$$\mathbf{Y} = \begin{bmatrix} \mathbf{X}_a^T \mathbf{X}_a & \mathbf{X}_a^T \mathbf{X}_u & \mathbf{X}_a^T \\ \mathbf{X}_u^T \mathbf{X}_a & \mathbf{X}_u^T \mathbf{X}_u & \mathbf{X}_u^T \\ \mathbf{X}_a & \mathbf{X}_u & \mathbf{I}_2 \end{bmatrix} \quad (28)$$

with

$$\mathbf{Y}_{uu} = \begin{bmatrix} \mathbf{X}_u^T \mathbf{X}_u & \mathbf{X}_u^T \\ \mathbf{X}_u & \mathbf{I}_2 \end{bmatrix}.$$

It is worthy to note that the SDP algorithm of (27) is an alternative realization of the approximate MLE solution in [10].

D. Edge-Based SDP

As the arithmetic operation complexity of the SDP is at least $O(m^3)$ [41], it is desirable to have a more computationally efficient solution particularly when the network size is large. One recent SDP development which can achieve efficient and accurate estimation while retaining its key theoretical property is to relax the single semi-definite matrix cone into a set of small-size cones, and this is known as ESDP relaxation [41]. The ESDP version of our proposed algorithm is simply achieved by replacing the single $(m+2)$-dimensional matrix cone $\mathbf{Y}$ in (24) with at most $m(m-1)/2$ 4-dimensional matrix cones:

$$\begin{bmatrix} y_{ii} & y_{ij} & \mathbf{x}_i^T \\ y_{ji} & y_{jj} & \mathbf{x}_j^T \\ \mathbf{x}_i & \mathbf{x}_j & \mathbf{I}_2 \end{bmatrix} \succeq \mathbf{0}_4, \quad i, j = 1, 2, \ldots m, \quad i > j, \quad \delta_{ij} = 1. \quad (29)$$

That is, (17)–(21) and (29) correspond to the ESDP relaxation algorithm for node localization in the presence of anchor position uncertainty.

III. Node Localization With Unknown Propagation Speed

In this section, we consider that the speed of signal propagation, $c^o \in [c_l, c_u]$, is unknown, although its lower and upper bounds, namely, c_l and c_u, may be available, while the anchor position information is perfect, that is, $\mathbf{A} = \mathbf{X}_a^o$ is free of noise. Instead of distances between nodes, the TOA measurements are employed here and they are modeled as

$$t_{ij} = \frac{\left\|\mathbf{x}_i^o - \mathbf{x}_j^o\right\|_2}{c^o} + \eta_{ij}, \quad i \neq j, \quad i,j=1,2,\ldots,m \tag{30}$$

where $\{\eta_{ij}\}$ are the disturbances in $\{t_{ij}\}$ and they are assumed independent zero-mean Gaussian processes with variances $\{\lambda_{ij}^2\}$. The MLE for $\mathbf{X}_u^o$ and c^o is achieved by maximizing

$$p(\{t_{ij}\}|\mathbf{X}_u, c). \tag{31}$$

Following the development in Section II, particularly (10), the optimum solution can be obtained from the following NLS cost function by letting $\mathbf{x}_i^o = \mathbf{a}_i = \mathbf{x}_i$, $i = 1, 2, \ldots, k$:

$$\min_{c\in[c_l,\, c_u],\mathbf{X}_u} \sum_{i=k+1}^{m}\sum_{j=1}^{m} h_{ij}\left|t_{ij} - \frac{\|\mathbf{x}_i - \mathbf{x}_j\|_2}{c}\right|^2 \tag{32}$$

where

$$h_{ij} = \begin{cases} \delta_{ij}/2\lambda_{ij}^2, & i > k \text{ and } j > k \\ \delta_{ij}/\lambda_{ij}^2, & \text{otherwise.}\end{cases} \tag{33}$$

Denoting $\mathbf{s}_i = \mathbf{x}_i/c$, $\mathbf{S} = \mathbf{X}/c$ and $\mathbf{S}_u = \mathbf{X}_u/c$, we expand (32) to yield

$$\min_{c\in[c_l,\, c_u],\mathbf{S}_u} \sum_{i=k+1}^{m}\sum_{j=1}^{m} h_{ij} \left[t_{ij}^2 + \mathbf{s}_i^T\mathbf{s}_i + \mathbf{s}_j^T\mathbf{s}_j - \mathbf{s}_i^T\mathbf{s}_j - \mathbf{s}_j^T\mathbf{s}_i - 2t_{ij}\|\mathbf{s}_i - \mathbf{s}_j\|_2\right]. \tag{34}$$

A dummy matrix $\mathbf{Z} \in \mathbb{R}^{(m+2)\times(m+2)}$ where its (i,j) entry is $z_{ij} = \mathbf{s}_i^T\mathbf{s}_j$, $i,j = 1,2,\ldots,m$, is then introduced, which has the form of

$$\mathbf{Z} := \begin{bmatrix} \mathbf{S}^T\mathbf{S} & \mathbf{S}^T \\ \mathbf{S} & \mathbf{I}_2 \end{bmatrix}. \tag{35}$$

Let $\bar{v} = 1/c^2$ and $\bar{c} = 1/c$. Similar to (11), we define $l_{ij} = \|\mathbf{x}_i - \mathbf{x}_j\|_2/c$ and $\beta_{ij} = l_{ij}^2$. In doing so, the optimization problem of (32) is equivalent to

$$\begin{aligned}
&\min_{\mathbf{S},\mathbf{Z},\bar{c},\bar{v},\{l_{ij}\},\{\beta_{ij}\}} \sum_{i=k+1}^{m}\sum_{j=1}^{m} h_{ij}[\beta_{ij} - 2t_{ij}l_{ij}]\\
\text{s.t.}\quad & \beta_{ij} = z_{ii} + z_{jj} - z_{ij} - z_{ji}, \quad i = k+1, k+2, \ldots, m, \quad j = 1,2,\ldots,m\\
& l_{ij}^2 = \beta_{ij}, \quad i = k+1, k+2, \ldots, m, \quad j = 1,2,\ldots,m\\
& z_{ij} = \bar{v} \times \mathbf{a}_i^T\mathbf{a}_j, \quad i,j = 1,2,\ldots,k, \quad i > j\\
& \mathbf{s}_i = \bar{c} \times \mathbf{a}_i, \quad i = 1,2,\ldots,k\\
& 1/c_u \le \bar{c} \le 1/c_l\\
& (1/c_u)^2 \le \bar{v} \le (1/c_l)^2\\
& \bar{c}^2 \le \bar{v}\\
& \mathbf{Z} = \begin{bmatrix} \mathbf{S}^T\mathbf{S} & \mathbf{S}^T \\ \mathbf{S} & \mathbf{I}_2 \end{bmatrix}.
\end{aligned} \tag{36}$$

Note that $\bar{v}$ is employed to strengthen the relationship between z_{ii} and $\mathbf{a}_i$. The last three constraints are basically obtained from the physical limitations, and they are optional and will be removed if the bounds for c^o are not available. Without loss of information, these three constraints can be combined as

$$\begin{aligned} &(1/c_u) \le \bar{c}\\ &\bar{c}^2 \le \bar{v} \le (1/c_l)^2. \end{aligned} \tag{37}$$

With the use of (37), we perform relaxation on (36) and note that $l_{ij}^2 \le \beta_{ij}$ has the same effect of $l_{ij}^2 = \beta_{ij}$ to obtain the SDP relaxation algorithm for node localization with unknown propagation speed:

$$\begin{aligned}
&\min_{\mathbf{S},\mathbf{Z},c,\bar{c},v,\bar{v},\{l_{ij}\},\{\beta_{ij}\}} \sum_{i=k+1}^{m}\sum_{j=1}^{m} h_{ij}[\beta_{ij} - 2t_{ij}l_{ij}]\\
\text{s.t.}\quad & \beta_{ij} = z_{ii} + z_{jj} - z_{ij} - z_{ji},\\
&\qquad i = k+1, k+2, \ldots, m, \quad j = 1,2,\ldots,m\\
& l_{ij}^2 \le \beta_{ij}, \quad i = k+1, k+2, \ldots, m, \quad j = 1,2,\ldots,m\\
& \mathbf{Z} \succeq \mathbf{0}_{m+2}\\
& \mathbf{s}_i = [z_{i\,m+1}, z_{i\,m+2}]^T, \quad i = 1,2,\ldots,m\\
& \begin{bmatrix} z_{m+1\,m+1} & z_{m+1\,m+2} \\ z_{m+2\,m+1} & z_{m+2\,m+2} \end{bmatrix} = \mathbf{I}_2\\
& z_{ij} = \bar{v} \times \mathbf{a}_i^T\mathbf{a}_j, \quad i,j = 1,2,\ldots,k, \quad i > j\\
& \mathbf{s}_i = \bar{c} \times \mathbf{a}_i, \quad i = 1,2,\ldots,k\\
& (1/c_u) \le \bar{c}\\
& \bar{c}^2 \le \bar{v} \le (1/c_l)^2.
\end{aligned} \tag{38}$$

In principle, $\bar{c}^2 = \bar{v}$ when noise is absent. But in practice, their values will be different because of the inequalities in the SDP formulation, that is, $z_{ij},\ i,j = 1,2,\ldots,m$, tends to be larger than $s_i s_j$. From the empirical point of view, $\bar{c}$ is chosen as the scaling factor to retrieve $\mathbf{X}$ from $\mathbf{S}$ because $\mathbf{s}_i$ is proportional to $\mathbf{a}_i$ in (38). On the other hand, $1/\sqrt{\bar{v}}$ is a better choice than $1/\bar{c}$ for speed estimation. It is because $z_{ij},\ i,j = 1,2,\ldots,m$, is directly related with β_{ij} in the equality constraints of (38). The β_{ij} is essentially the estimate of t_{ij}^{o2}, while l_{ij} is the estimate of t_{ij}^o. As the inequality between β_{ij} and l_{ij}^2 is forced to be an equality, they are adjusted in a tight manner to estimate t_{ij}^o because only the square and cross multiplication terms of $\mathbf{s}_i$, which is represented by $z_{ij},\ i,j = 1,2,\ldots,m$, in (36) are involved in the optimization process. Nevertheless, instead of employing $\bar{v}$, after estimating $\mathbf{S}$, we substitute the node position estimates, $\hat{\mathbf{S}}$, in (32) and minimize the resultant expression to produce a more accurate estimate of c^o:

$$\hat{c} = \frac{\sum_{i=k+1}^{m}\sum_{j=1}^{m}\delta_{ij}\|\hat{\mathbf{s}}_i - \hat{\mathbf{s}}_j\|_2^2}{\sum_{i=k+1}^{m}\sum_{j=1}^{m}\delta_{ij}t_{ij}\|\hat{\mathbf{s}}_i - \hat{\mathbf{s}}_j\|_2}. \tag{39}$$

For its ESDP version, the $(m+2)$-dimensional matrix cone relaxation $\mathbf{Z}$ in (35) will be further relaxed to

$$\begin{bmatrix} z_{ii} & z_{ij} & \mathbf{s}_i^T \\ z_{ji} & z_{jj} & \mathbf{s}_j^T \\ \mathbf{s}_i & \mathbf{s}_j & \mathbf{I}_2 \end{bmatrix} \succeq \mathbf{0}_4, \quad i,j=1,2,\ldots m, \quad i>j, \quad \delta_{ij}=1. \tag{40}$$

IV. Node Localization With Combined Uncertainties

In this section, we will extend our study to the scenario when there are uncertainties in the anchor positions and signal propagation speed by utilizing the developments in Sections II and III. The optimum estimates of $\mathbf{X}^o$ and c^o are now obtained from:

$$\min_{c\in[c_l,\, c_u],\mathbf{X}} \sum_{i=k+1}^{m}\sum_{j=1}^{m} h_{ij}\left|t_{ij} - \frac{\|\mathbf{x}_i - \mathbf{x}_j\|_2}{c}\right|^2 + \sum_{i=1}^{k}(\mathbf{a}_i - \mathbf{x}_i)^T\boldsymbol{\Phi}_i^{-1}(\mathbf{a}_i - \mathbf{x}_i). \quad (41)$$

Recall that the main inspiration on dealing with uncertain anchor positions in (15), which is a redundant constraint to relate y_{ii} with $\boldsymbol{\Xi}_i$, while $\mathbf{S}$ in (35) is introduced to tackle the unknown propagation speed. At first sight, it seems that we have two choices. One is based on the anchor position uncertainty framework with extension to unknown propagation speed and $\mathbf{X}$ is used to provide node position estimates. The second is based on unknown propagation speed formulation with taking anchor position errors into account and we estimate $\mathbf{S}$ and multiply it with $1/\bar{c}$. However, the latter approach is infeasible because we only have the erroneous anchor positions. That is, constraints between s_i and $\mathbf{a}_i$ as well as z_{ij} and $a_i a_j$ as in (38) cannot be applied. Furthermore, it is hard to implement the second term of (41) with $\mathbf{S}$ only as $\mathbf{S}$ should be multiplied with an unknown scale $\bar{c}$, which is a technical challenge for SDP technique, in order to compare with a_i. As a result, we base on the anchor position uncertainty formulation to define

$$\tilde{\mathbf{Z}} := \begin{bmatrix} \bar{v}\times\mathbf{X}^T\mathbf{X} & \mathbf{X}^T \\ \mathbf{X} & v\times\mathbf{I}_2\end{bmatrix} = \begin{bmatrix} \mathbf{S}^T\mathbf{S} & \mathbf{X}^T \\ \mathbf{X} & v\times\mathbf{I}_2\end{bmatrix} \quad (42)$$

where the (i,j) entries of $\tilde{\mathbf{Z}}$, namely, $\tilde{z}_{i,j}$, and $\mathbf{Z}$ are exactly the same for $i,j = 1,2,\ldots,m$. Then, (32) is now modified as

$$\min_{\mathbf{X},\tilde{\mathbf{Z}},\{\boldsymbol{\Xi}_i\},v,\{l_{ij}\},\{\beta_{ij}\}} \sum_{i=k+1}^{m}\sum_{j=1}^{m} h_{ij}[\beta_{ij} - 2t_{ij}l_{ij}] + \sum_{i=1}^{k}\left[\mathrm{Tr}\left(\boldsymbol{\Phi}_i^{-1}\boldsymbol{\Xi}_i\right) - 2\mathbf{a}_i^T\Phi_i^{-1}\mathbf{x}_i\right]$$

$$\begin{aligned}
\text{s.t.}\quad & \beta_{ij} = \tilde{z}_{ii} + \tilde{z}_{jj} - \tilde{z}_{ij} - \tilde{z}_{ji}, \\
& \qquad i = k+1, k+2,\ldots,m, \quad j = 1,2,\ldots,m \\
& l_{ij}^2 = \beta_{ij}, \quad i = k+1,k+2,\ldots,m, \quad j = 1,2,\ldots,m \\
& \boldsymbol{\Xi}_i = \mathbf{x}_i\mathbf{x}_i^T, \quad i = 1,2,\ldots,k \\
& c_l^2 \le v \le c_u^2 \\
& \tilde{\mathbf{Z}} = \begin{bmatrix} \bar{v}\times\mathbf{X}^T\mathbf{X} & \mathbf{X}^T \\ \mathbf{X} & v\times\mathbf{I}_2\end{bmatrix}
\end{aligned} \quad (43)$$

where the constraint of (20) is removed as there is no direct information of y_{ii} and the last constraint is essentially the physical boundary of propagation speed. Note that $\mathbf{S}$, which is subject to errors, does not appear. Performing SDP relaxation on (43), the algorithm for tackling the combined uncertainties is then

$$\min_{\mathbf{X},\tilde{\mathbf{Z}},\{\boldsymbol{\Xi}_i\},v,\{l_{ij}\},\{\beta_{ij}\}} \sum_{i=k+1}^{m}\sum_{j=1}^{m} h_{ij}[\beta_{ij} - 2t_{ij}l_{ij}] + \sum_{i=1}^{k}\left[\mathrm{Tr}\left(\boldsymbol{\Phi}_i^{-1}\boldsymbol{\Xi}_i\right) - 2\mathbf{a}_i^T\boldsymbol{\Phi}_i^{-1}\mathbf{x}_i\right]$$

$$\begin{aligned}
\text{s.t.}\quad & \beta_{ij} = \tilde{z}_{ii} + \tilde{z}_{jj} - \tilde{z}_{ij} - \tilde{z}_{ji}, \\
& \qquad i = k+1,k+2,\ldots,m, \quad j = 1,2,\ldots,m \\
& l_{ij}^2 \le \beta_{ij}, \quad i = k+1,k+2,\ldots,m, \quad j = 1,2,\ldots,m \\
& c_l^2 \le v \le c_u^2 \\
& \begin{bmatrix} \boldsymbol{\Xi}_i & \mathbf{x}_i \\ \mathbf{x}_i^T & 1\end{bmatrix} \succeq \mathbf{0}_3, \quad i = 1,2,\ldots,k \\
& \tilde{\mathbf{Z}} \succeq \mathbf{0}_{m+2} \\
& \mathbf{x}_i = [\tilde{z}_{i\,m+1}, \tilde{z}_{i\,m+2}]^T, \quad i = 1,2,\ldots,m \\
& \begin{bmatrix} \tilde{z}_{m+1\,m+1} & \tilde{z}_{m+1\,m+2} \\ \tilde{z}_{m+2\,m+1} & \tilde{z}_{m+2\,m+2}\end{bmatrix} = \mathbf{I}_2.
\end{aligned} \quad (44)$$

Hence, the estimation of propagation speed is provided by $\sqrt{v}$, but the ambiguity causes it severely biased, so the refined estimate of c^o is calculated using (39), where $\hat{\mathbf{s}}_i$ replaced with $\hat{\mathbf{x}}_i$, with $\hat{\mathbf{x}}$ being the estimated position of both anchors and sensors. Similarly, for its ESDP version, the $(m+2)$-dimensional matrix cone relaxation $\tilde{\mathbf{Z}}$ in (44) will be further relaxed to

$$\begin{bmatrix} \tilde{z}_{ii} & \tilde{z}_{ij} & \mathbf{x}_i^T \\ \tilde{z}_{ji} & \tilde{z}_{jj} & \mathbf{x}_j^T \\ \mathbf{x}_i & \mathbf{x}_j & v\times\mathbf{I}_2\end{bmatrix} \succeq \mathbf{0}_4, \quad i,j = 1,2,\ldots m, \quad i > j, \quad \delta_{ij} = 1. \quad (45)$$

V. Cramér–Rao Lower Bound

In this section, the CRLBs for WSN node localization in the presence of anchor position uncertainty and/or unknown propagation speed are derived. We first consider the scenario of combined uncertainties. Let $\mathbf{z} = [t_{1k+1}t_{1k+2}\cdots t_{m-1m}\mathbf{a}_1^T\mathbf{a}_2^T\cdots\mathbf{a}_k^T]^T$ be a vector which contains all available observations of $\{t_{ij}\}$ and $\mathbf{A}$. With the use of (5) and (31), we see that $\mathbf{z}$ is Gaussian distributed with mean $\boldsymbol{\mu}$ and covariance matrix $\mathbf{C}$:

$$\mathbf{z} \sim \mathcal{N}(\boldsymbol{\mu}, \mathbf{C}) \quad (46)$$

where

$$\boldsymbol{\mu} = \begin{bmatrix} \frac{r_{1k+1}^o}{c^o} & \frac{r_{1k+2}^o}{c^o} & \cdots & \frac{r_{m-1m}^o}{c^o} & \mathbf{x}_1^{oT} & \mathbf{x}_2^{oT} & \cdots & \mathbf{x}_k^{oT}\end{bmatrix}^T$$

$$\mathbf{C} = \mathrm{blkdiag}\ (\mathbf{C}_{11}, \mathbf{C}_{22})$$

$$\mathbf{C}_{11} = \mathrm{diag}\ \left(\sigma_{1k+1}^2, \sigma_{1k+2}^2, \ldots, \sigma_{m-1m}^2\right)$$

and

$$\mathbf{C}_{22} = \mathrm{blkdiag}\ (\boldsymbol{\Phi}_1, \boldsymbol{\Phi}_2, \ldots, \boldsymbol{\Phi}_k)$$

with diag $\cdot$ and blkdiag $\cdot$ denoting the diagonal and block diagonal matrices, respectively. The Fisher information matrix (FIM) for $[x_1^o\ y_1^o\ x_2^o\ y_2^o\ \cdots\ x_m^o\ y_m^o\ c^o]^T$, denoted by $\mathbf{I}_{\text{combined}}$, is then

$$\mathbf{I}_{\text{combined}} = \mathbf{H}\mathbf{C}^{-1}\mathbf{H}^T \quad (47)$$

where

$$\mathbf{H} = \begin{bmatrix} \frac{\partial \boldsymbol{\mu}}{\partial x_1^o} & \frac{\partial \boldsymbol{\mu}}{\partial y_1^o} & \frac{\partial \boldsymbol{\mu}}{\partial x_2^o} & \frac{\partial \boldsymbol{\mu}}{\partial y_2^o} & \cdots & \frac{\partial \boldsymbol{\mu}}{\partial x_m^o} & \frac{\partial \boldsymbol{\mu}}{\partial y_m^o} & \frac{\partial \boldsymbol{\mu}}{\partial c^o} \end{bmatrix}^T = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{h}_{21} & \mathbf{0}_{1\times 2k} \end{bmatrix}$$

with [see the equations shown at the bottom of the page]. Taking the inverse of $\mathbf{I}_{\text{combined}}$, the CRLB for the parameters is then obtained from its diagonal elements.

When there is only anchor position uncertainty, our observation vector is the same as (46) but now the speed c^o is known. The corresponding FIM for $[x_1^o\ y_1^o\ x_2^o\ y_2^o\ \cdots\ x_m^o\ y_m^o]^T$, denoted by $\mathbf{I}_{\text{anchor}}$, is modified from (44) as

$$\mathbf{I}_{\text{anchor}} = \mathbf{H}_{11}\mathbf{C}_{11}^{-1}\mathbf{H}_{11}^T + \mathbf{H}_{12}\mathbf{C}_{22}^{-1}\mathbf{H}_{12}^T. \tag{48}$$

On the other hand, the observation vector for the unknown speed only scenario will become

$$\grave{\mathbf{z}} \sim \mathcal{N}(\grave{\boldsymbol{\mu}}, \mathbf{C}_{11}) \tag{49}$$

where

$$\grave{\mathbf{z}} = \begin{bmatrix} t_{1k+1}^o & t_{1k+2}^o & \cdots & t_{m-1m}^o \end{bmatrix}^T$$

and

$$\grave{\boldsymbol{\mu}} = \begin{bmatrix} \frac{r_{1k+1}^o}{c^o} & \frac{r_{1k+2}^o}{c^o} & \cdots & \frac{r_{m-1m}^o}{c^o} \end{bmatrix}^T.$$

The corresponding FIM for $[x_{k+1}^o\ y_{k+1}^o\ x_{k+2}^o\ y_{k+2}^o\ \cdots\ x_m^o\ y_m^o\ c^o]^T$, denoted by $\mathbf{I}_{\text{speed}}$, will be

$$\mathbf{I}_{\text{speed}} = \mathcal{H}\mathbf{C}_{11}^{-1}\mathcal{H}^T \tag{50}$$

where

$$\mathcal{H} = \begin{bmatrix} \mathcal{H}_{11} \\ \mathbf{h}_{21} \end{bmatrix}$$

and $\mathcal{H}_{11}$ is the same as $\mathbf{H}_{11}$ but with the first $2k$ rows removed.

VI. Simulation Results

Computer simulation has been conducted to evaluate the performance of the proposed SDP node positioning approach in the uncertain scenarios. Comparison with the standard SDP algorithm based on MLE [10] which assumes perfect anchor position information and/or the corresponding CRLBs is also made. We utilize the Matlab toolbox YALMIP [45] to realize all SDP algorithms where the solver SDPT3 [43], [44] is employed. Unless stated otherwise, we consider a WSN of 18 sensors with 8 of them are anchors and its configuration is depicted in Fig. 1. The anchor positions are (20, 20) m, (20, −20) m, (−20, 20) m, (−20, −20) m, (20, 0) m, (0, 20) m, (−20, 0)m and (0, −20) m, while the unknown-position sensors are located at (−8.6237, −1.2310) m, (−10.4088, 17.5334) m, (12.3117, −13.0601) m, (10.6205, −7.1419) m, (−2.6837, −10.9620) m, (3.1923, 10.4146) m, (1.1929,5.6211) m, (−11.6372, −10.8073) m, (11.3331, 7.2338) m and (−12.5562, 10.7131) m. In this WSN geometry, nodes are partially connected and the maximum communication range between nodes is set to be 25 m which corresponds to an average node degree [46] of 8.67. It is noteworthy that we simply follow [47] and [48] to place the anchors on the perimeter of the network because their experimental studies show that this will yield more accurate estimation performance. However, as pointed out by [49], the estimation performance also depends on the connectivity and uniformity of the WSN. That is, more accurate position estimates will be obtained when the network is isotropic and/or the average node degree is large whereas the estimated results will be poorer if it is anisotropic and/or the average node degree is small. Further-

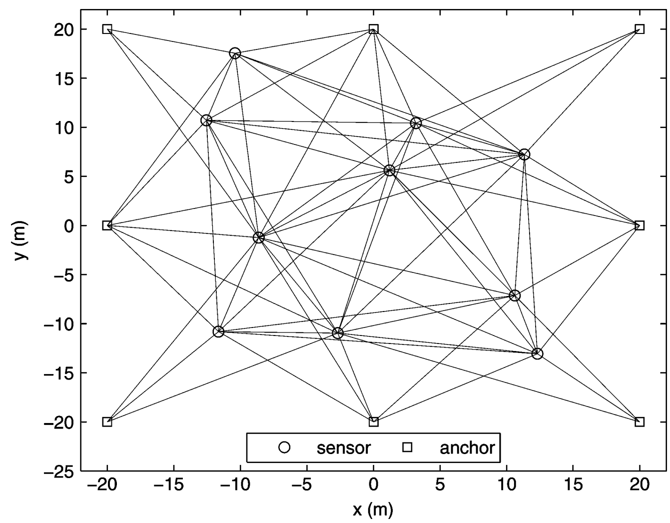

Fig. 1. Geometry of sensor network.

$$\mathbf{H}_{11} = \frac{1}{c^o}\begin{bmatrix} \frac{\mathbf{x}_1^o-\mathbf{x}_{k+1}^o}{r_{1k+1}^o} & \frac{\mathbf{x}_1^o-\mathbf{x}_{k+2}^o}{r_{1k+2}^o} & \cdots & \frac{\mathbf{x}_1^o-\mathbf{x}_m^o}{r_{1m}^o} & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \frac{\mathbf{x}_2^o-\mathbf{x}_{k+1}^o}{r_{2k+1}^o} & \frac{\mathbf{x}_2^o-\mathbf{x}_{k+2}^o}{r_{2k+2}^o} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\mathbf{x}_m^o-\mathbf{x}_1^o}{r_{m1}^o} & 0 & 0 & \cdots & \frac{\mathbf{x}_m^o-\mathbf{x}_{m-1}^o}{r_{mm-1}^o} \end{bmatrix}$$

$$\mathbf{H}_{12} = \begin{bmatrix} \mathbf{I}_{2k} & \mathbf{0}_{2k\times 2m-k} \end{bmatrix}^T$$

and

$$\mathbf{h}_{21} = \frac{-1}{c^{o2}}\begin{bmatrix} r_{1k+1}^o & r_{1k+2}^o & \cdots & r_{m-1m}^o \end{bmatrix}$$

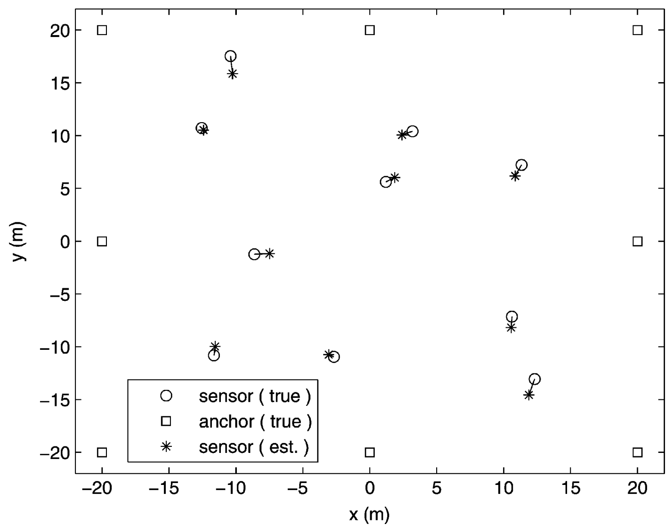

Fig. 2. Single trial performance of the standard SDP algorithm [10] in the presence of anchor position uncertainty.

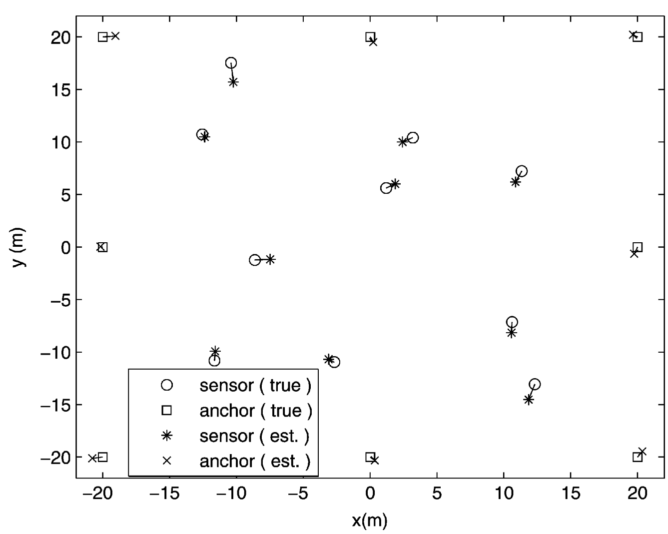

Fig. 3. Single trial performance of the proposed SDP algorithm in the presence of anchor position uncertainty.

more, the study of [49] indicates that placing the anchors on the boundary generally gives better node localization performance than the randomly deployment scenarios. We have performed empirical study on different network geometries and the findings generally agree with [49]. For the mean-square error (MSE) performance evaluation, only the estimates for the unknown-position nodes are involved in the computation as the standard algorithm cannot fine tune the anchor positions, and all the results are based on averages of 500 independent runs. The range errors $\{e_{ij}\}$ in (3) and TOA errors $\{\eta_{ij}\}$ in (30) are zero-mean white Gaussian variables with standard deviations $\{\sigma_d r_{ij}^o\}$ and $\{\lambda_t t_{ij}^o\}$, respectively, which means that a larger range or longer arrival time will correspond to a larger variance, and we scale the values of σ_d and λ_t to obtain different noisy conditions. Unless stated otherwise, all anchor position covariance matrices are assigned as $\mathbf{\Phi}_i = \kappa_i^2 \mathbf{I}_2$ with $\kappa_i^2 = -10$ dBm^2 for all i.

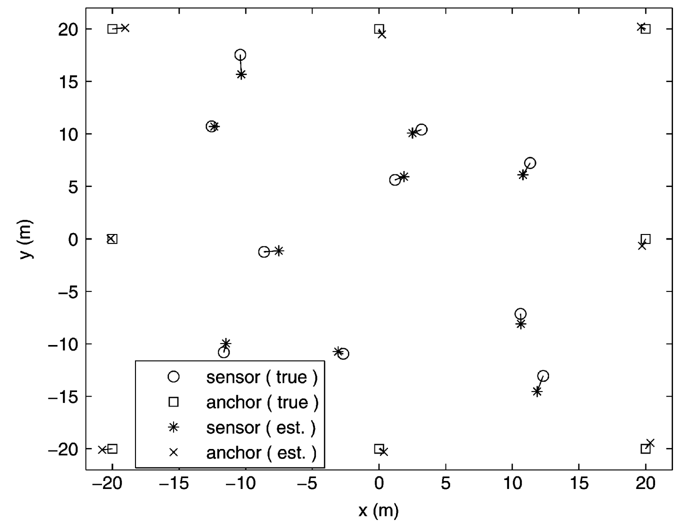

Fig. 4. Single trial performance of the proposed ESDP algorithm in the presence of anchor position uncertainty.

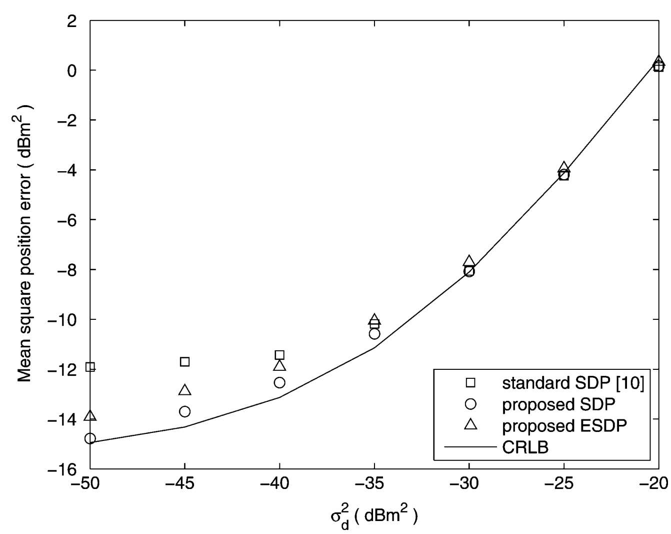

Fig. 5. Mean-square position error versus σ_d^2 in the presence of anchor position uncertainty.

In the first experiment, we investigate the performance of the SDP algorithms in the presence of anchor position uncertainty. Figs. 2 to 4 show the estimation results for a single trial at $\sigma_d^2 = -20$ dBm^2 using the standard as well as proposed SDP and ESDP algorithms, respectively. We cannot see obvious difference between their performance except that our approach is able to estimate the anchor positions as well. Fig. 5 shows the MSEs of the position estimates versus σ_d^2 where we can see the superiority of the proposed SDP and ESDP methods over the standard one particularly for smaller noise conditions, although the two SDP algorithms give nearly the same performance when $\sigma_d^2 \geq -30$ dBm^2. It is also observed that the performance of our SDP method is close to the CRLB while the ESDP version only degrades the tighter SDP scheme by less than 0.5 dBm^2. The MSE results versus κ_i^2 at $\sigma_d^2 = -50$ dBm^2 are plotted in Fig. 6. Apart from higher estimation performance of the proposed SDP and ESDP methods, we see that the improvement over the standard one increases with the anchor position error.

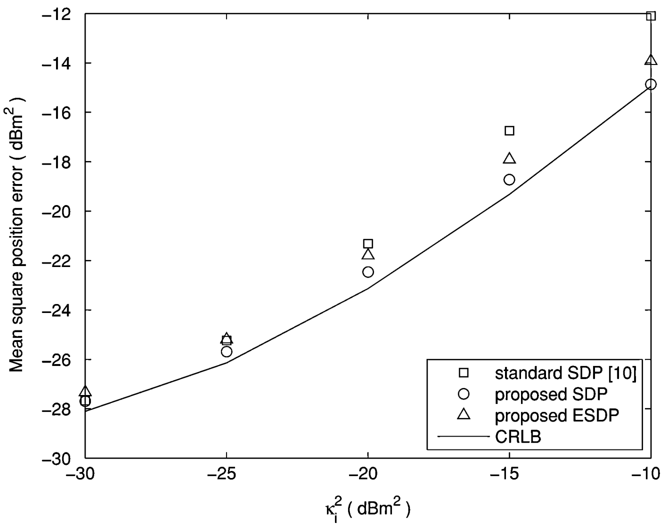

Fig. 6. Mean-square position error versus κ_i^2 at $\sigma_d^2 = -50$ dB.

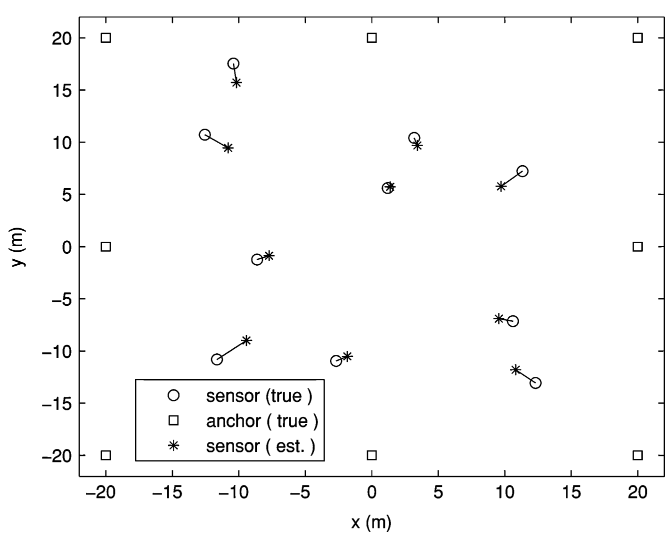

Fig. 7. Single trial performance of the standard SDP algorithm [10] for unknown propagation speed.

In the second experiment, the performance of the standard and proposed SDP algorithms for unknown signal propagation speed situation is studied. The true propagation speed is set to be 360 ms^{-1} while its upper and lower bounds are $c_l = 120$ ms^{-1} and $c_u = 400$ ms^{-1}. As the former cannot perform speed estimation, we use a random number uniformly distributed between c_l and c_u as its speed estimate. Single trial estimation results at $\sigma_d^2 = \lambda_t^2/c^{o2} = -20$ dBm^2 is shown in Figs. 7 and 8 which illustrates the superiority of the proposed method. Figs. 9 and 10 plot the MSEs of the position and speed estimates versus λ_t^2/c^{o2}, respectively, at $\sigma_d^2 = -20$ dBm^2. Note that the speed estimate of the standard algorithm is not included. We see that accurate position and speed estimation is achieved by the proposed SDP scheme as its performance is very close to the corresponding CRLBs. We have also illustrated in Fig. 10 that the speed estimate derived from $\bar{v}$ is of poorer accuracy. It

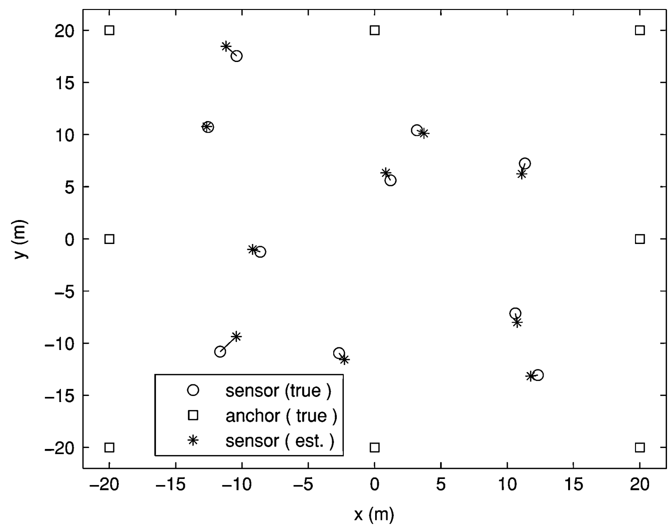

Fig. 8. Single trial performance of the proposed SDP algorithm for unknown propagation speed.

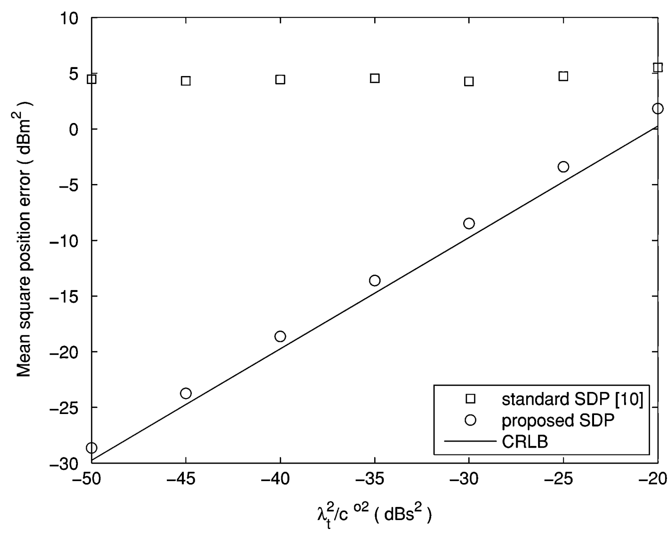

Fig. 9. Mean-square position error versus λ_t^2/c^{o2} for unknown propagation speed.

is worthy to point out that the ESDP variant cannot give satisfactory performance in this scenario, which may be due to the severer ambiguity effect for rank relaxed matrices with smaller sizes and the scaling error. As a larger matrix limits the freedom of its elements in stronger sense which leads to a better estimation performance in the SDP algorithm while the numerous smaller matrices in the ESDP scheme provide a higher degree of freedom and can produce unsatisfactory result in the presence of the scaler variable $\bar{v}$. As a result, the estimation performance of the latter is not included.

In the third experiment, we investigate the performance of the SDP algorithms in the presence of both uncertainties. Figs. 11 and 12 show the single trial results while Figs. 13 and 14 plot the MSEs of the position and speed estimates versus σ_t^2/c^{o2}. Although Fig. 12 indicates that all nodes are localizable, the latter figures illustrate the suboptimality of the proposed approach in this very challenging scenario. Nevertheless, the superiority of

TABLE I
COMPUTATIONAL TIME AND MEAN-SQUARE ERROR COMPARISON FOR PROPOSED SDP AND ESDP SCHEMES

$k+n$	Range (m)	Connectivity	SDP time (s)	ESDP time (s)	SDP MSE (m^2)	ESDP MSE (m^2)
8+2	48	8.5216	0.2802	0.3367	2.4935	2.4929
8+4	40	8.6007	0.3345	0.4500	1.6548	1.6622
8+8	30	8.8230	0.4905	0.6282	1.2497	1.2864
8+16	20	8.6188	1.2346	0.9322	1.0707	1.2013
8+32	12	6.6020	5.0113	1.5199	2.3573	3.5761

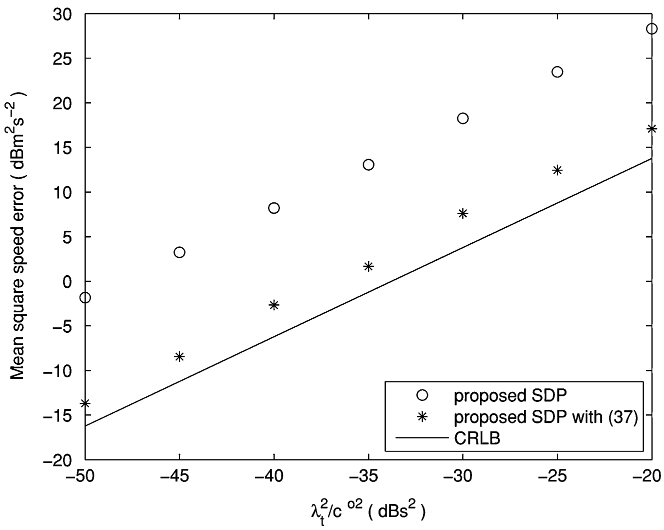

Fig. 10. Mean-square speed error versus λ_t^2/c^{o2} for unknown propagation speed.

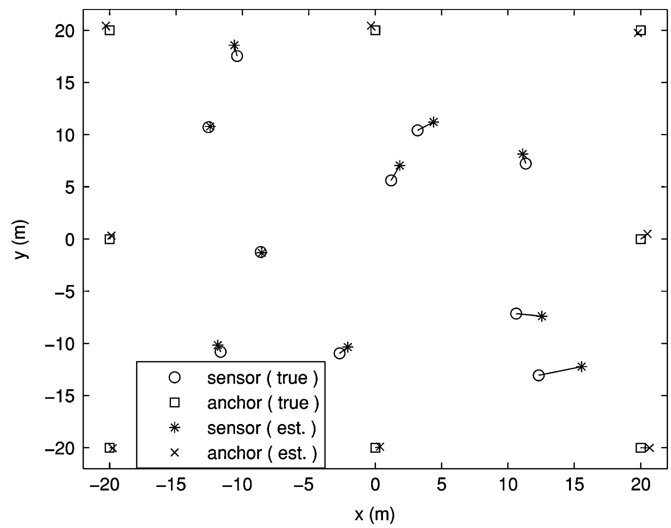

Fig. 12. Single trial performance of the proposed SDP algorithm in the presence of combined uncertainties.

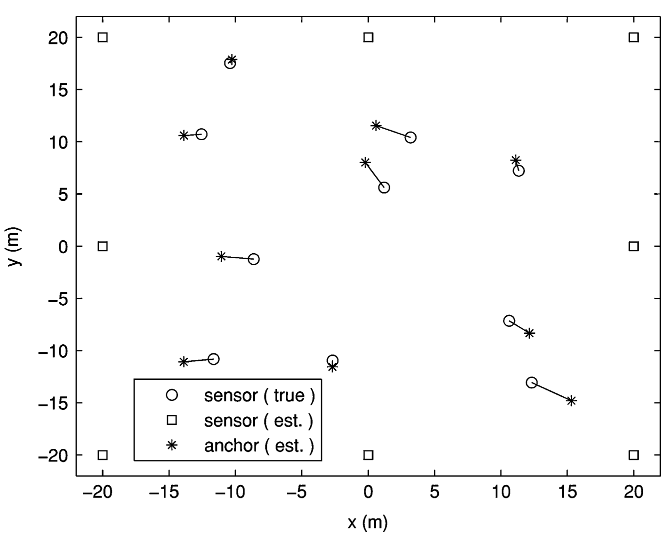

Fig. 11. Single trial performance of the standard SDP algorithm [10] in the presence of combined uncertainties.

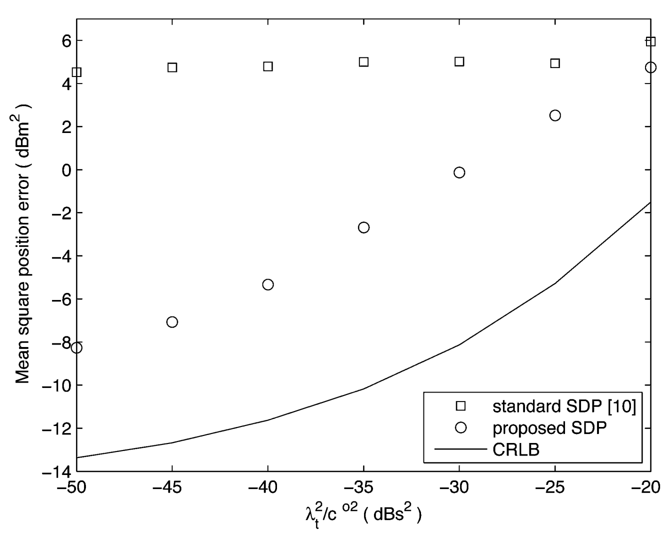

Fig. 13. Mean-square position error versus λ_t^2/c^{o2} in the presence of combined uncertainties.

our algorithm over the standard one is again demonstrated. Similar to the second experiment, the results of the ESDP variant are not included because of its poorer estimation performance.

Finally, the computation times and MSEs of the proposed SDP and ESDP algorithms for the anchor position uncertainty case are studied for different number of nodes, and the results are tabulated in Table I. The number of anchors is fixed at $k=8$ with the same positions as in the above tests. The unknown-position nodes are placed inside the 40 m $\times$ 40 m area where the communication range is governed by $0.3(40\times 40)/m$ and $\sigma_d^2 = -20$ dBm^2 is assigned. It is observed that for a larger WSN, the EDSP scheme is much more computationally efficient than the SDP method at the expense of a higher MSEs.

VII. CONCLUSION

Assuming Gaussian distributed disturbances, the nonconvex maximum likelihood estimation problems for sensor network node localization in the presence of anchor position and/or signal propagation speed uncertainties have been approximated

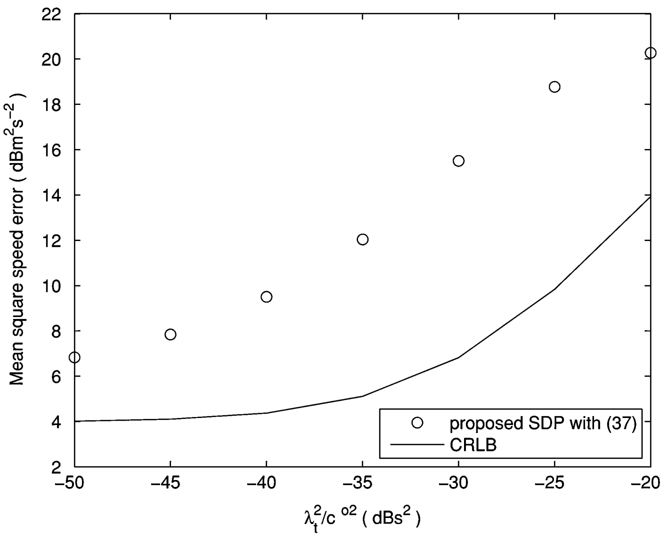

Fig. 14. Mean-square speed error versus λ_t^2/c^{o2} in the presence of combined uncertainties.

to convex optimization problems using the semi-definite programming (SDP) relaxation technique. It is shown that when only the anchor positions are of errors, the proposed SDP and its edge-based variant algorithms can give very accurate node localization performance. On the other hand, the performance of the SDP scheme is nearly optimal and suboptimal, respectively, when only the speed is unknown and in the presence of both uncertainties.

Our future works include optimal anchor placement in sensor networks and a good starting point is to analytically study the Cramér–Rao lower bound [49]–[51]. We will investigate the SDP methodology for node positioning with time-difference-of-arrival, angle-of-arrival [4], [5] and/or signal energy [52] measurements. Furthermore, it is interested to devise distributed SDP algorithms for node position estimation and tracking. Node localization in the presence of non-line-of-sight propagation is also a challenging research topic.

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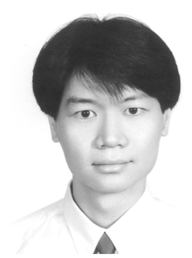

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