16. *Semi-empirical Formula for the Seismic Characteristics of the Ground.*

By Kiyoshi KANAI,

Earthquake Research Institute.

(Read July 17, 1956; Feb. 26, Mar. 26, 1957.—Received Mar. 31, 1957.)

1. Introduction

That there is usually, for each kind of ground, both a predominant period and a particular spectral response, was first determined[1] in Japan from some of the earliest seismograph records. This principle was stated near the end of the nineteenth century, shortly after the invention of the seismograph. The seismic characteristics of the ground have become clear from field observations[2], theoretical work[3], and studies of damage statistics[4] by many authors. The present paper presents a semi-empirical formula for the seismic characteristics of the ground based on these three kinds of results.

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1) S. SEKIYA, *Transact. Seism. Soc., Japan*, **12** (1888), 83: F. OMORI, *Pub. Earthq. Inv. Comm.*, **10** and **11** (1902).

2) N. NASU, *Rep. Imp. Earthq. Inv. Comm.*, 100, A (1925), 313 (in Japanese): K. SUYEHIRO, *Bull. Earthq. Res. Inst.*, **1** (1926), 59 (in Japanese); **7** (1929), 467: A. IMAMURA, *ditto*, **7** (1929), 489 (in Japanese): M. ISHIMOTO, *ditto*, **9** (1931), 159; **10** (1932), 171; **12** (1934), 234; **13** (1935), 592; **14** (1936), 240; **15** (1937), 536: T. SAITA and M. SUZUKI, *ditto*, **12** (1934), 517: N. NASU and T. HAGIWARA, *ditto*, **14** (1936), 290: T. HAGIWARA and S. OMOTE, *ditto*, **16** (1938), 632: T. MINAKAMI, *ditto*, **22** (1944), 92: S. SAKUMA, *ditto*, **26** (1948), 67: S. MIYAMURA, *ditto*, **26** (1948), 101: K. KANAI and others, *ditto*, **31** (1953), 227, 305; **32** (1954), 361; **33** (1955), 109; **34** (1956), 61: T. MINAKAMI, *Rep. Special Comm. Fukui Earthq.*, Tokyo, 1950, 79-92. Others are represented in the cases to be made reference.

3) K. SEZAWA, *Bull. Earthq. Res. Inst.*, **8** (1930), 1: K. SEZAWA and G. NISHIMURA, *ditto*, **8** (1930), 321: K. SEZAWA and K. KANAI, *ditto*, **10** (1932), 1, 273; **13** (1935), 251: R. TAKAHASI and K. HIRANO, *ditto*, **19** (1941), 534 (in Japanese): R. TAKAHASI, *ditto*, **33** (1955), 259. Others are represented in the cases to be made reference.

4) T. MATUZAWA, *Rep. Imp. Earthq. Inv. Comm.*, 100, A (1925), 163: G. KITAZAWA, *Journ. Seism. Soci., Japan*, **3** (1950), 32 (in Japanese): H. KAWASUMI, *Journ. Architect. Inst., Japan*, 777 (1951), 10 (in Japanese): R. TANABASHI and H. ISHIZAKI, *Bull. Disast, Prev. Res. Inst.*, **5** (1953): T. SAITA, Iwanami Shoten, Tokyo, 1935, (in Japanese): S. OMOTE, *Bull. Earthq. Res. Inst.*, **24** (1946), 77; **27** (1949), 57, 63: S. MIYAMURA, *ditto*, **24** (1946), 99; S. OMOTE and S. MIYAMURA, *ditto*, **29** (1951), 183: Rep. Damage Fukui Earthq., Tokyo (1951) (in Japanese). Others are represented in the cases to be made reference.

2. Peculiarity of the ground

In most cases, the amplitude-period relation of earthquake motion is represented by a resonance type curve. The period corresponding to peak amplitude is comparatively short (0.1–0.4 sec) on hard ground and comparatively long (0.4–0.8 sec) on soft ground. From the analyses of many seismograms, we also know that the amplitude-period relation of an earthquake is quite similar to the relation between period and number of waves, with the peaks of the two curves occurring at practically the same period[5]. In this paper, we will often use the "frequency-period" relation instead of the amplitude-period relation because we have many data of the former type. Strictly speaking, the amplitude-period relation differs somewhat for different earthquakes. The "peak period" lengthens and the spectral response curve flattens as the epicentral distance or the earthquake magnitude increases. However, this variation of peak period is less than $\pm 20\%$[6].

In Figs. 1 through 7 are shown the numbers of small earthquakes and their peak periods for seven places with differing ground characteristics. Six of the places are in Tokyo: Hongo, Aoyama, Marunouti, Kanda, Komagome and Komaba. Noge-yama is in Yokohama. The

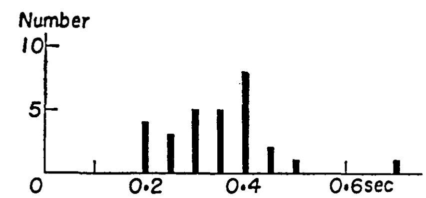

Fig. 1. The numbers of earthquakes and their peak periods at Hongo in Tokyo.

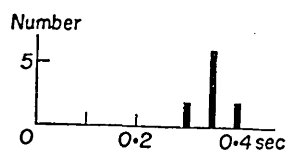

Fig. 2. The numbers of earthquakes and their peak periods at Aoyama in Tokyo.

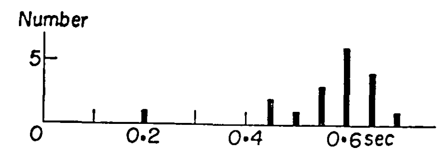

Fig. 3. The numbers of earthquakes and their peak periods at Marunouti in Tokyo.

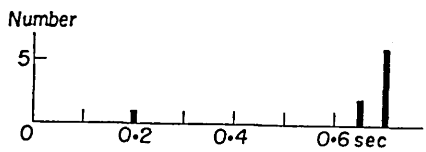

Fig. 4. The numbers of earthquakes and their peak periods at Kanda in Tokyo.

5) K. KANAI and M. SUZUKI, *Bull. Earthq. Res. Inst.*, **32** (1954), 189: Subsoil Res. Team, Earthq. Res. Inst., *ditto*, **33** (1955), 500–532. (in Japanese)

6) M. ISHIMOTO, *ditto*, **15** (1937), 536: T. MINAKAMI and S. SAKUMA, *ditto*, **26** (1948), 61.

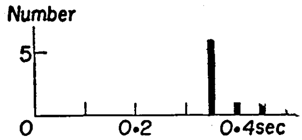

Fig. 5. The numbers of earthquakes and their peak periods at Komagome in Tokyo.

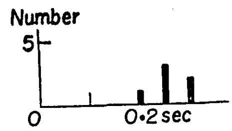

Fig. 6. The numbers of earthquakes and their peak periods at Komaba in Tokyo.

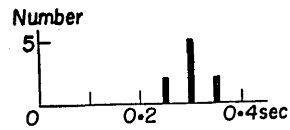

Fig. 7. The number of earthquakes and their peak periods at Nogeyama in Yokohama.

peak period in each figure may be considered the predominant period of the ground of that place. These predominant periods closely approximate the peak periods obtained from microtremor measurements at the respective places[7].

3. Empirical formula for the seismic characteristics of the ground

Since in most cases the amplitude-period relation of earthquake motion is a resonance type curve[8], we will write the following formula for the seismic characteristics of the ground:

$$U_s=\frac{c_1U_0}{\sqrt{\left\{1-\left(\frac{T}{T_0}\right)^2\right\}^2+\left\{\tau\frac{T}{T_0}\right\}^2}}, \tag{1}$$

where U_s=absolute amplitude of earthquake motion at free surface,
U_0=absolute amplitude of seismic waves reaching the bottom boundary of the surface layer,
T_0=predominant period of the ground,
T=period of seismic waves,
τ=apparent damping coefficient of the surface vibration,
c_1=a coefficient which depends upon the impedance ratio of the two media and is independent of T_0.

Damping may be caused by both dissipation of vibration energy to the bottom medium and solid viscosity in the surface layer. The former depends on both the ratio of the impedance of the surface layer to that of the bottom medium and the natural period of the surface layer. We will now estimate the values of c_1 and τ, using data obtained by many kinds of observation.

7) K. KANAI, T. TANAKA and K. OSADA, *Bull. Earthq. Res. Inst.*, **32** (1954), 199. Subsoil Research Team, Earthq. Res. Inst., *ditto*, **33** (1955), 492–495. (in Japanese)

8) *loc. cit.*, 2), 7).

Firstly we estimate the damping coefficient τ by the method[9] commonly used in determining damping coefficient from a resonance curve, that is:

$$\tau \doteqdot \frac{T_0(T_1-T_2)}{T_1T_2}\,, \tag{2}$$

in which T_0 is the resonance period and T_1, T_2 are the periods corresponding to an amplitude of $1/\sqrt{2}$ times the resonance amplitude.

The values of τ and T_0 at many places were thus obtained from the frequency period curves of both natural earthquakes and micro-

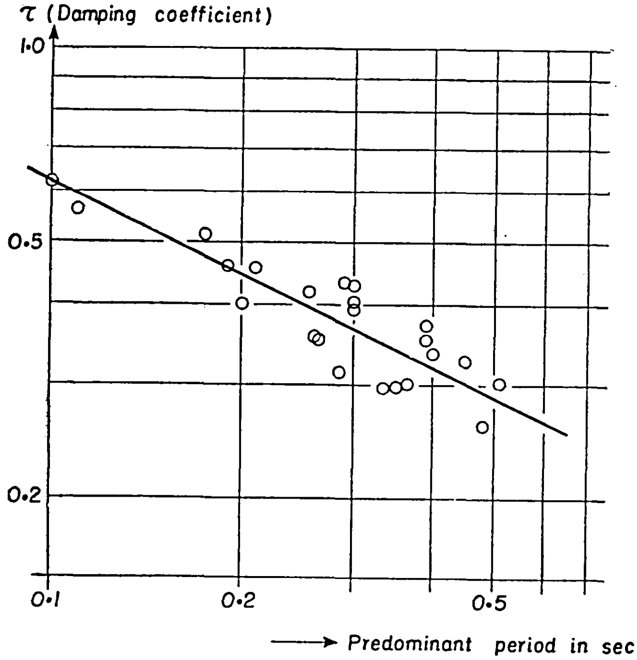

Fig. 8.

tremors[10]. These values are plotted in Fig. 8, from which we obtain an empirical formula,

$$\tau=\frac{0.2}{\sqrt{T_0}}\,. \tag{3}$$

The results of many comparative observations of earthquakes tell

9) K. KANAI and S. YOSHIZAWA, *Bull. Earthq. Res. Inst.*, **30** (1952), 121.
10) *loc. cit.*, 7): K. KANAI, T. TANAKA and K. OSADA, *ditto*, **35** (1957), 10.

us that, as the predominant period increases, the maximum amplitude of earthquake displacement increases and the maximum amplitude of earthquake acceleration decreases.

The average quantitative relationships which were obtained in a previous study[11] are as follows:

$$D_{\max.} \propto T_0^{1.7\pm0.2}\ , \tag{4}$$

$$A_{\max.} \propto T_0^{-0.6\pm0.1}\ , \tag{5}$$

in which $D_{\max.}$ and $A_{\max.}$ are the largest amplitude of displacement and acceleration of earthquake motion respectively.

Assuming the exponents of (4) and (5) to be 1.5 and -0.5, respectively. We obtain from (4) the following empirical formula:

$$U_{s(\max.)} = c_2 T_0^{1.5}\ , \tag{6}$$

where c_2 is a coefficient depending upon the nature of incident waves and the physical properties of surface layer and bed rock, but independent of T_0. Also, from (1) and (3), the amplitude in the case of resonance becomes:

$$U_{s(\text{res.})} = c_1 U_{0(\text{res.})} \frac{T_0^{0.5}}{0.2}\ . \tag{7}$$

Equating (6) and (7),

$$U_{0(\text{res.})} = \frac{0.2c_2}{c_1} T_0\ . \tag{8}$$

If equipartition of energy applies to the seismic waves arriving at the bottom boundary[12], the relation of amplitude to period can be expressed as:

$$U_0 = cT\ , \tag{8'}$$

where c is a constant.

Thus (8) is compatible with (8′) and represents the special case of resonance. Hence (8′) will be used as a basic factor below.

Substituting (3) and (8′) in (1), we obtain the empirical formula for the spectral response of ground,

11) K. KANAI, *Bull. Earthq. Res. Inst.*, **32** (1954), 211.

12) The velocity spectra of the large number of strong motion seismograms obtained in the U.S.A.* have the form of several fluctuations superposed on a horizontal line parallel to the period axis. It appears that the reason for this flatness is that the observing places are located on thick alluvium.

* J. L. ALFORD, G. W. HOUSNER and R. R. MARTEL, 1st Tech. Rep., Office Naval Res., N6onr 244-25, Proj. Desig. Nr-081-095.

$$U_s=\frac{c_1cT}{\sqrt{\left\{1-\left(\frac{T}{T_0}\right)^2\right\}^2+\left\{\frac{0.2}{\sqrt{T_0}}\left(\frac{T}{T_0}\right)\right\}^2}}\,. \tag{9}$$

Substituting (3) only, we obtain the amplification of ground,

$$\frac{U_s}{U_0}=\frac{c_1}{\sqrt{\left\{1-\left(\frac{T}{T_0}\right)^2\right\}^2+\left\{\frac{0.2}{\sqrt{T_0}}\left(\frac{T}{T_0}\right)\right\}^2}}\,. \tag{9'}$$

4. Theoretical interpretation of the empirical formula

Here we shall examine the simplest case, in which a plane distortional wave propagated vertically upward in an elastic semi-infinite medium is partly transmitted and partly reflected at the bottom boundary of the superficial visco-elastic layer and is reflected at the free surface. This is one of the multiple reflection problems of elastic waves in a superficial layer[13]. The wave amplitude at the free surface of a multi-layered ground may be expressed as:

$$U_s=\frac{2U_0}{\sqrt{\{F(T,\,T_{0m},\,\alpha_m,\,\xi_m)\}^2+\{G(T,\,T_{0m},\,\alpha_m,\,\xi_m)\}^2}}\,, \tag{10}$$

where, m represents the layer number and α and ξ represent, respectively, the impedance ratio of neighboring layers and the viscous coefficient of a layer[14].

In the case of a simple layer, assuming that

$$\frac{p\xi}{\mu_1}\ll 1\,, \tag{11}$$

the wave amplitude at the free surface may be written as:

$$U_s=\frac{2U_0}{\sqrt{\{F(T,\,T_0,\,\alpha)\}^2+\{G(T,\,T_0,\,\alpha,\,\xi)\}^2}}\,, \tag{12}$$

where

$$T_0=\frac{4H}{v_1},\quad \alpha=\frac{\rho_1v_1}{\rho_2v_2},\quad p=\frac{2\pi}{T}\,, \tag{13}$$

in which subscripts 1 and 2 represent surface layer and bottom medium, respectively, and μ, ρ, v and H represent rigidity, density, velocity and

13) K. KANAI, *Bull. Earthq. Res. Inst.*, **31** (1953), 219: K. KANAI, and S. YOSHIZAWA, *ditto*, **31** (1953), 275.

14) K. KANAI, *ditto*, **28** (1950), 31.

thickness of surface layer, respectively.

Values of ξ/v^3 for various materials in the ground are always of the same order[15], i.e. approximately 10^{-6} C.G.S. Using this value and considering the range of $v_1=50$ to 500 m/s, $T=0.1-1.0$ sec, the range of $p\xi/\mu$ becomes 0.05 to 1.5.

The relation between U_s/U_0 and $2\pi\xi/\mu_1 T_0$ in a previous paper[16] is shown in Fig. 9. The straight lines satisfactorily approximate the

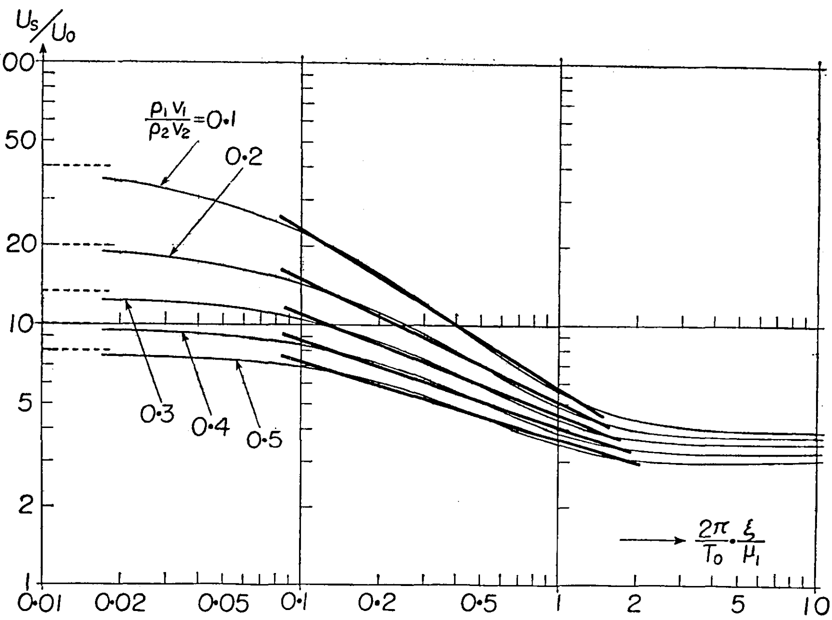

Fig. 9.

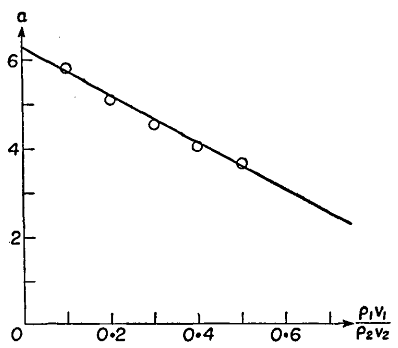

Fig. 10.

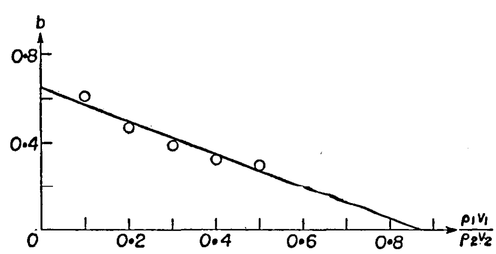

Fig. 11.

15) K. KANAI and K. OSADA, *Bull. Earthq. Res. Inst.*, **29** (1951), 514, Table II.
16) K. KANAI, *ditto*, **30** (1952), 35, Fig. 4.

curves in the range 0.05 to 1.5 of abscissa. The relation may be expressed as

$$\frac{U_s}{U_{0\,(\mathrm{Res.})}}=a\left(\frac{p_0\xi}{\mu_1}\right)^b . \tag{14}$$

By means of Figs. 10 and 11 the values of a and b are determined from Fig. 9, whence,

$$a=6.3-5.3\alpha\ , \tag{15}$$

$$b=-0.65+0.75\alpha\ . \tag{16}$$

Substituting (15) and (16) in (14), we get the following empirical formula:

$$\frac{U_s}{U_{0\,(\mathrm{Res.})}}=(6.3-5.3\alpha)\left(\frac{\mu_1 T_0}{2\pi\xi}\right)^{0.65-0.75\alpha} . \tag{17}$$

Since the form of seismic waves is complex, it is unlikely that large amplitudes will occur at short periods corresponding to higher harmonics of the surface layer[17]. Neglecting these harmonics, and considering the neighborhood of the fundamental resonance period of (12)[18], we can obtain the semi-theoretical formula for spectral response,

$$U_s=\frac{2kU_0}{\sqrt{\left\{1-\left(\frac{T}{T_0}\right)^2\right\}^2+\left\{\kappa\frac{T}{T_0}\right\}^2}}\ , \tag{18}$$

in which k is a function of the impedance ratio of two media and independent of T_0 and κ is the theoretical damping coefficient.

To evaluate k, we may utilize the special case where the thickness of the surface layer is much larger than the wave length: the condition $T/T_0\rightarrow 0$ may be substituted in (18). The amplitude at the surface will equal the product of three factors: transmission factor at the bottom boundary, $2/(1+\alpha)$; reflection factor at the free surface, 2; and amplitude of incident waves[19]. Then, (18) may be rewritten as

$$\frac{U_s}{U_0}=\frac{4}{1+\alpha}\left[\left\{1-\left(\frac{T}{T_0}\right)^2\right\}^2+\left\{\kappa\frac{T}{T_0}\right\}^2\right]^{-1/2} . \tag{19}$$

From (17) and (19), we obtain

17) K. KANAI and S. YOSHIZAWA, *Bull. Earthq. Res. Inst.*, **34** (1956), 167.

18) *loc. cit*, 13), 34, Figs. 2-5.

19) C. G. KNOTT, Phil. Mag., **48** (1899), 64: H. KAWASUMI and T. SUZUKI, *Journ. Seism. Soc. Japan*, **4** (1932), 277. (in Japanese)

$$\kappa=\frac{4}{(1+\alpha)(6.3-5.3\alpha)}\left(\frac{\mu_1 T_0}{2\pi\xi}\right)^{-0.65+0.75\alpha} . \tag{20}$$

To reconcile the semi-theoretical formula of (19) and the empirical formula of (9′), two conditions are necessary:

(I)
$$c_1=\frac{4}{1+\alpha}, \tag{21}$$

(II)
$$0.2\,T_0^{-0.5}=\frac{4}{(1+\alpha)(6.3-5.3\alpha)}\left(\frac{\mu_1 T_0}{2\pi\xi}\right)^{-0.65+0.75\alpha} . \tag{22}$$

Since (22) is an identity, it may be determined by solution of the equation

$$T_0^{-0.5}=T_0^{-0.65+0.75\alpha}, \tag{22′}$$

whence

$$\alpha=1/5 . \tag{23}$$

Equation (23) tells us that the observation points at which the empirical data incorporated in (9′) were obtained have an average vibration impedance of 1/5 that of bed rock. This is a plausible value in consideration of the average condition of the observation points. Substituting (23) in (22), we obtain

$$\frac{\xi}{\mu_1}=0.02 . \tag{24}$$

This is also a plausible value.

Now, if we adopt in (20) the generally applicable values of $\xi/v_1^3=10^{-6}$ and $\rho_1=2$, we obtain a semi-theoretical formula as follows:

$$\kappa \doteqdot \frac{4}{6+\alpha}\left(\frac{T_0}{\pi v_1}\times 10^6\right)^{-0.65+0.75\alpha} . \tag{20′}$$

Finally then, if we assume $\alpha=1/5$, $\xi/\mu_1=0.02$, $U_0=cT$, we get from (19) the semi-empirical formula for spectral response of ground vibration,

$$U_s=\frac{cT}{0.3\sqrt{\left\{1-\left(\frac{T}{T_0}\right)^2\right\}^2+\left\{\frac{0.2}{\sqrt{T_0}}\left(\frac{T}{T_0}\right)\right\}^2}}, \tag{25}$$

where c is a constant.

The amplification of ground vibration is expressed by

$$\frac{U_s}{U_0}=\frac{1}{0.3}\left[\left\{1-\left(\frac{T}{T_0}\right)^2\right\}^2+\left\{\frac{0.2}{\sqrt{T_0}}\left(\frac{T}{T_0}\right)\right\}^2\right]^{-1/2} . \tag{25′}$$

Therefore, the resonance amplitude of ground vibration becomes

$$U_{s(\mathrm{Res.})}=\frac{50\,T_0^{1.5}}{3}\times c\,. \tag{26}$$

Comparing (25), (25′) with (9), (9′) it is seen that $c_1=10/3$.

5. Application limits of the semi-empirical formula

The semi-empirical formula, equation (25), expresses the amplitude at free surface as influenced by multiple reflection in the stratified visco-elastic layer. There are some limits of ground conditions for the application of the formula.

(I) The case of a thick stratified layer.

In the case of a thick stratified layer, the phenomenon of superposition of transmitted and reflected seismic waves will be unlikely. Therefore the amplitude at ground surface is the product of three factors as illustrated in (18) and (19). This is a special case of (25), (25′).

The Arvin-Tehachapi earthquake of July 1952 gave a good example of this case[20]. The campus of the California Institute of Technology is located on several hundred feet of alluvium and the Seismological Laboratory of the Institute rests on an outcrop of bed-rock; the ratio of maximum displacements was nearly two to one.

(II) The case of multiple stratified layers.

In the case of multiple stratified layers, since the waves reflected and refracted at various boundaries interfere with one another, the spectral response of surface amplitude is very irregular[21]. Except in the extremely special case where there are nodes both at the bottom boundary and at the first boundary below the surface, the maximum amplitudes are not as large as for a single surface layer[22]. Therefore, the empirical formula (25) is not applicable in this case.

(III) The case of exposed rock.

Usually, the predominant period observed on rock covers the range

20) G. W. HOUSNER, Geotechnique, Dec. 1954, 153.

21) *loc. cit.*, 16): K. KANAI, *Monthly Meeting, Earthq. Res. Inst.*, (July 1956).

22) The largest amplitude is nearly $2\rho_n v_n/\rho_1 v_1$ and its period is $4H_1/v_1$, in which $\rho_n v_n$ and $\rho_1 v_1$ are the vibrational impedance of the bed rock and the first layer, respectively, and H_1 is the thickness of the first layer. This phenomenon has sometimes appeared in actual earthquakes as well as in micro-tremors. This problem will be considered in a forthcoming paper.

M. ISHIMOTO, *Bull. Earthq. Res. Inst.*, **13** (1935), 592 (in Japanese): K. KANAI, *Monthly Meeting, Earthq. Res. Inst.*, (Sept. 1956).

between 1/30 and 1/5 sec. This range is explained in terms of multiple reflection in a thin weathered layer of the rock.

The semi-empirical formula (25′) may be applicable to the usual older rock outcrop.

(IV) The case of fresh exposed rock and bed rock.

On the other hand, the vibration characteristics of fresh exposed rock and bed rock are quite different[23]. The peak of the spectral response curve sometimes appears at 0.3 sec to 0.6 sec, and sometimes the curve is flat. The details of this problem will be shown in a forthcoming paper.

6. Statistical study of earthquake damage

(I) The case of wooden houses.

Recently, in Japan, in order to clarify the question of how earthquake damage varies with ground conditions, investigations have been carried out from various standpoints based on experiences in great destructive earthquakes.

Figs. 12 and 13 show the relations between damage to wooden houses and number of stories at twelve places in Tokyo in the great Kwanto earthquake of 1923.[24] Fig. 12 applies to the up-town region

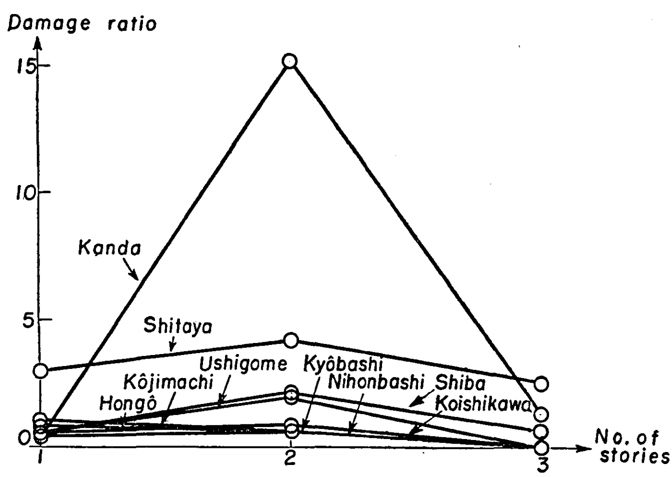

Fig. 12. Relation between damage to wooden houses and number of stories in up-town Tokyo in the 1932 earthquake.

23) K. KANAI, T. TANAKA and K. OSADA, *Monthly Meeting Earthq. Res. Inst.*, (Sept. 1956; Jan. and Feb., 1957).

24) G. KITAZAWA, *Rep. Imp. Earthq. Inv. Comm.* No. 100, Ca, 1.

where the ground consists mainly of Diluvium or Tertiary formations and the predominant period is about 0.2 to 0.4 sec, while Fig. 13 applies to the down-town region where the ground consists mainly of thick alluvium and the predominant period is about 0.5 to 0.8 sec.

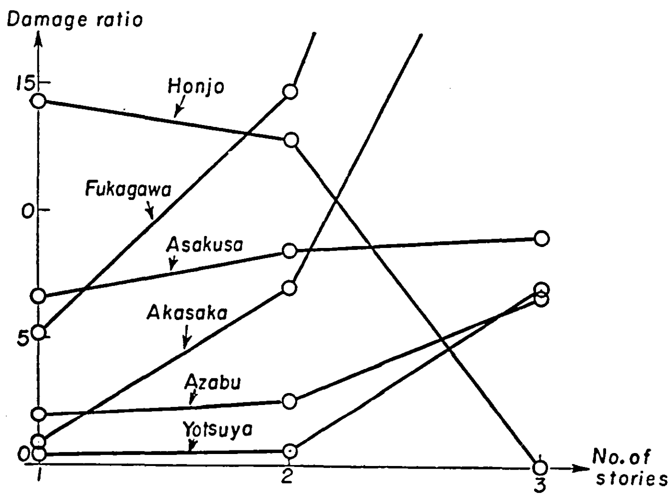

Fig. 13. Relation between damage to wooden houses and number of stories in down-town Tokyo in the 1923 earthquake.

Fig. 12 tells us that two-storied Japanese style wooden houses on firm ground had a larger damage rate than one- or three-storied houses. On the contrary, Fig. 13 shows that on soft ground the damage rate increased with the increase in number of stories, excepting in the Honjo district.

These facts may be explained in terms of the relation of the natural period of the house to the predominant period of the ground.

The larger damage rates on soft ground than on hard ground are due to the larger duration and amplitude of earthquake motion as well as to the unequal settling of foundations[25].

(II) The case of brick buildings.

Fig. 14 shows the relation between damage rate of brick buildings and number of stories in the 1923 earthquake[26] at three places in Tokyo.

The notations rigid, soft and very soft in Fig. 14 indicate that the

25) K. KANAI, *Bull. Earthq. Res. Inst.*, **29** (1951), 215. (in Japanese)

26) Y. SATO, *Rep. Imp. Earthq. Inv. Comm.* No. 100, Ca, 550.

ground consists respectively, of Diluvium or Tertiary formation, 10 to 20 m thickness of alluvium, and 30 to 50 m thickness of alluvium.

It will be seen from Fig. 14 that, the softer the ground, the less the earthquake damage of brick buildings, and that the damage rate is only slightly affected by the number of stories. These facts may be explained qualitatively as follows: (i) The natural period of one- to three-storied brick buildings is comparable to the predominant period of hard ground. (ii) The softer the ground, the larger the dissipation of earthquake vibration energy of buildings to the ground[27].

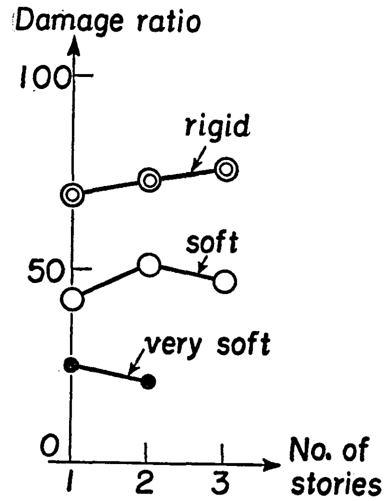

Fig. 14. Relation between damage to brick buildings and number of stories in Tokyo in the 1923 earthquake.

(III) The case of reinforced concrete buildings.

Fig. 15 shows the relation between damage rate of reinforced concrete buildings and number of stories in the 1923 earthquake[28] at three places in Tokyo. The meanings of the notations rigid, soft and very soft are the same as in Fig. 14.

From Fig. 15, it will be seen that, on rigid ground, the curve of damage rate

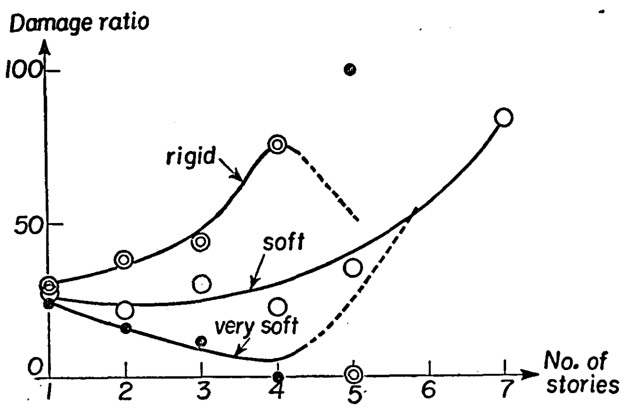

Fig. 15. Relation between damage to reinforced concrete buildings and number of stories in Tokyo in 1923 earthquake.

27) K. KANAI, *Bull. Earthq. Res. Inst.*, **27** (1949), 97: K. KANAI and S. YOSHIZAWA, *ditto*, **29** (1951), 209.

28) N. NAGATA, *Rep. Imp. Earthq. Inv. Comm.*, No. 100, Cb, 211: loc. cit., 26).

versus number of stories seems to have a peak at four-story buildings. On the other hand, the average natural period of 1923 Japanese reinforced concrete buildings may be estimated from the formula[29] $T=(0.07-0.09)\times N$, in which T is natural period in seconds and N is the number of stories; the period of four-story buildings comes out to be 0.3 to 0.4 sec. This is also the approximate range of the predominant period of hard ground in Tokyo. Paragraph Fig. 15 also shows that, in very soft ground, the larger the number of stories the larger the damage rate. If we consider that the predominant period in very soft ground is larger than 0.7 sec, buildings would have to be higher than eight-stories to have a chance of resonance. The relation between the damage rate in very soft ground and the number of stories is somewhat complex because the spectral response of such ground is complex.

Thus the variations in damage rates of wooden, brick and reinforced concrete buildings in the Kwanto earthquake of 1923 can be qualitatively explained in terms of vibrational characteristics of the ground.

7. Discussion

Let $B(T)$, $G(T)$ and $S(T)$ be respectively, the spectral response of incident waves transmitted to bed rock, the vibration characteristics of the ground, and the vibration characteristics of the structure.

The spectral response of actual earthquake motion at ground surface is expressed by

$$B(T)\times G(T)\,. \tag{27}$$

Then, (25) and (25′) correspond to $G(T)$, and, if (8′) is valid, $B(T)=cT$. In general, $B(T)$ will vary depending on the magnitude, epicentral distance and other parameters. But, unless $B(T)$ takes an extremely unusual form, the influence of $G(T)$ on the vibration at ground surface will be considerable. (One of the special cases is when $B(T)$ is an infinite simple harmonic wave train. Then the vibration of both ground and structure has the same period as $B(T)$).

The spectral response of a structure on the ground surface in an earthquake may be written as

$$B(T)\times G(T)\times S(T)\,, \tag{28}$$

is which $S(T)$ depends both on the kind of construction and the vibrational conditions between foundation and ground. Therefore, the earth-

29) C. TANIGUCHI, *Journ. Architect. Inst. Japan*, Dec. 1949. (in Japanese)

quake motion of serious consideration for earthquake-proof construction is the vibration having a period approximate to the natural period of the structure. It is needless to say that vibration with large displacement like 1 m in which the period is 10 sec, and vibration with large acceleration like 1 g in which the period is 50 c/s, only slightly affect the vibration of the structure.

The theoretical interpretation of the empirical formula in Chapter 6 was made in terms of multiple reflection of body waves. But theoretically the resonance-type vibrational characteristics of the ground can be explained also in terms of surface waves[30].

If the velocities of *S*-waves and surface waves can be observed satisfactorily, it will not be difficult to find by which type of wave to characterize the vibration of ground.

8. Conclusion

By using the observational results of earthquakes as well as micro-tremors and adding a theoretical interpretation, we obtained a semi-empirical formula for the vibrational characteristics of the ground.

The general equation obtained semi-theoretically, which expresses the spectral response of ground vibration, is

$$U_s=\frac{4U_0}{1+\alpha}\left[\left\{1-\left(\frac{T}{T_0}\right)^2\right\}^2+\left\{\kappa\frac{T}{T_0}\right\}^2\right]^{-1/2}, \tag{19}$$

in which

$$\kappa \doteqdot \frac{4}{6+\alpha}\left(\frac{T_0}{\pi v_1}\times 10^6\right)^{-0.65+0.75\alpha}. \tag{20'}$$

Most of the data obtained by the observations of earthquakes and micro-tremors satisfy the following formula:

$$U_s=\frac{cT}{0.3}\left[\left\{1-\left(\frac{T}{T_0}\right)^2\right\}^2+\left\{\frac{0.2}{\sqrt{T_0}}\left(\frac{T}{T_0}\right)\right\}^2\right]^{-1/2}, \tag{25}$$

(25) is a special case of (19).

If we give a numerical value to c according to engineering judgement, we can obtain a practial formula for engineering purposes.

From (25), the largest amplification of ground is as follows:

$$U_{s(\text{Res.})}=\frac{50T_0^{1.5}}{3}\times c\,. \tag{26}$$

30) R. STONELEY, *M.N.R.A.S. Geophs. Suppl.*, **4** (1937), 39: K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **15** (1937), 577, 845: K. TAZIME, *Journ. Physics of the Earth*, **5**, No. 1 (1957), 43.

It must be borne in mind that (25) explains qualitatively the damage to buildings caused by the Kwanto earthquake of 1923.

In conclusion, the author wishes to express his thanks to Prof. R. Takahasi and Prof. H. Kawasumi for their kind discussion in the course of this investigation. Also his thanks are due to Miss S. Yoshizawa who assisted him in preparing this paper.

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16. 地盤の震動特性に関する半実験式

地震研究所 金 井 清

地震動及び常時微動の観測結果を使い，理論的な検討を加えて，地盤の震動特性に関する半実験式を作つた．即ち，地盤の増巾度を表わす式は $(U_s/U_0)_{\max}$ であり，その一般式は次式となる

$$\left(\frac{U_s}{U_0}\right)_{\max}=(6.3-5.3\alpha)\left(\frac{\mu_1 T_0}{2\pi\xi}\right)^{0.65-0.75\alpha}. \tag{17}$$

又，地震時の地盤の震動スペクトルを表わす一般式は

$$U_s=\frac{4cT}{1+\alpha}\left[\left\{1-\left(\frac{T}{T_0}\right)^2\right\}^2+\left\{\kappa\frac{T}{T_0}\right\}^2\right]^{-1/2} \tag{19}$$

で表わされる．但し

$$\kappa=\frac{4}{(1+\alpha)(6.3-5.3\alpha)}\left(\frac{\mu_1 T_0}{2\pi\xi}\right)^{-0.65+0.75\alpha}. \tag{20}$$

今日までの観測結果から κ についての平均の値を求めると $\kappa\doteqdot\frac{0.2}{\sqrt{T_0}}$ となり，地震時の地盤の震動スペクトルは次式となる．

$$U_s=\frac{cT}{0.3\sqrt{\left\{1-\left(\frac{T}{T_0}\right)^2\right\}^2+\left\{\frac{0.2}{\sqrt{T_0}}\left(\frac{T}{T_0}\right)\right\}^2}}, \tag{25}$$

c を仮定すれば工学的の実用式になる訳である．

地盤の最大増巾度は次式となる．

$$U_{s(\text{Res.})}=\frac{50T_0^{1.5}}{3}\times c. \tag{26}$$

以上の半実験式は，地震観測の結果によく合うばかりでなく，関東地震の震害と地盤との関係にもこれによつて或程度の説明がつく．

基盤に到達した入射波のスペクトル・レスポンスを $B(T)$，地盤固有のスペクトル・レスポンスを $G(T)$，構造物自体のそれを $S(T)$ とすると，地震時における地表面の震動のスペクトルは実際的には次のように表わされる．

$$B(T)\times G(T), \tag{27}$$

而して，(25′) 式は $G(T)$ に当り，(25) 式は $B(T)=cT$ と仮定したときの $B(T)\times G(T)$ に当る．

地震の大きさ，震央巨離，その他の条件によつて，$B(T)$ がちがう事は想像される．しかし，$B(T)$ が余程特別なものでない限り，$G(T)$ の影響は相当に大きい筈である．(極端な場合として，入射波が無限長調和波ならば，地盤も構造物も入射波と同じ単一周期の振動をするのは無論である．)

次に，地表面上の構造物の地震時の震動のスペクトルは実際的には次のように表わせる．

$$B(T)\times G(T)\times S(T)\,, \tag{28}$$

但し，$S(T)$ は上部構造の種類並びに基礎と地面との間の震動的条件に関係する．従つて，耐震構造上に重大な意味をもつ地震動は，構造物の周期に近い震動である．[1 m の大振巾の地震動でも周期が 10 秒のものとか，(加速度は 40 gal になる)，1 g の大加速度振巾の地震動でも，その周期が 50 c/s（変位振巾は 0.1 mm）のものとかは，実在構造物に対して破壊的作用をしないであろう．]