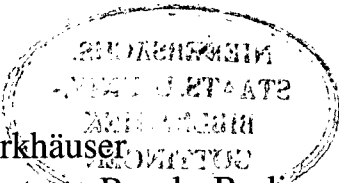


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Semiconcave Functions, Hamilton–Jacobi Equations, and Optimal Control



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