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Sensitivity Analysis of a Comprehensive Model for a Miniature-Scale Linear Compressor for Electronics Cooling

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Abstract

A comprehensive model of a linear compressor for electronics cooling was previously presented by Bradshaw et al. (2011). The current study expands upon this work by first developing methods for predicting the resonant frequency of a linear compressor and for controlling its piston stroke. Key parameters governing compressor performance – leakage gap, eccentricity, and piston geometry – are explored using a sensitivity analysis. It is demonstrated that for optimum performance, the leakage gap and frictional parameters should be minimized. In addition, the ratio of piston stroke to diameter should not exceed a value of one to minimize friction and leakage losses, but should be large enough to preclude the need for an oversized motor. An improved linear compressor design is proposed for an electronics cooling application, with a predicted cooling capacity of 200 W a cylindrical compressor package size of diameter 50.3 mm and length 102 mm.

Keywords: linear compressor, sensitivity study, miniature system, electronics cooling, loss analysis

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Nomenclature

BDC	Bottom Dead Center	[-]
c	Damping factor	[N sec m^{-1}]
$\dot{E}_d$	Exergy destroyed	[W]
f	Frequency	[Hz]
f	Dry friction coefficient	[-]
g	Leakage gap between piston and cylinder	[m]
h	Enthalpy	[kJ kg^{-1}]
k	Stiffness	[N m^{-1}]
M	Mass	[kg]
N	Normal force from piston to cylinder wall	[N]
P	Pressure	[kPa]
$\dot{Q}$	Heat transfer	[W]
$\dot{Q}_{cool}$	Cooling capacity	[W]
s	Entropy	[kJ kg^{-1} K^{-1}]
T_o	Ambient temperature	[K]
$\mathbf{T}_p$	Piston oscillation period	[s]
T_w	Compressor shell temperature	[K]
TDC	Top Dead Center	[-]
V	Volume	[m^3]
$\mathbf{V}$	Gas velocity	[m s^{-1}]
V_d	Displaced volume	[m^3]
$\dot{W}$	Work over a cycle	[W]

x_d	Distance between piston and valve plate at TDC	[m]
x_p	Instantaneous compressor piston position	[m]
x_s	Compressor stroke	[m]

Greek Letters

ò	Eccentricity of spring force	[m]
η	Efficiency	[-]
ω	Frequency	[rad sec^{-1}]
ζ	Damping ratio	[-]

Subscripts

cv1	Control volume 1
cv2	Control volume 2
eff	Effective
f	Friction
gas	Gas
leak	leakage
mov	Moving
n	Natural
res	Resonance
rms	Root mean square
tot	Total

1. Introduction

A comprehensive simulation model for a miniature-scale linear compressor was recently developed by Bradshaw et al. (2011). The model was also validated against experiments conducted on a prototype linear compressor constructed for the purpose. It was found that the overall performance metrics predicted by the compressor model are highly sensitive to the leakage gap g, eccentricity ò, dry friction coefficient **f**, and motor efficiency η_{motor}. Figure 1 depicts the major components and design parameters of a linear compressor. The geometry of the piston is directly related to both the friction and leakage of a compressor. Therefore, for a fixed displaced volume, some piston diameter and stroke combinations will provide higher efficiency than others. The impact of changes to these parameters proves useful when designing a linear compressor, and warrants further investigation.

A linear compressor has two major practical limitations, which restrict its implementation in practical systems. Both the resonant frequency and stroke are sensitive to changes in geometry and operating conditions (Cadman & Cohen, 1969; Park et al., 2004; Pollak et al., 1979; Unger & Novotny, 2002). This poses a challenge not only to compressor design but also to modeling efforts. The ability to predict and control these two parameters provides a useful tool for linear compressor design efforts.

A method for calculating the resonant frequency of a linear compressor is developed here. An approach to numerical control is also provided that ensures compressor operation at the desired stroke. A series of sensitivity studies are presented, which highlight the sensitivity to leakage gap and eccentricity as well as piston geometry. Finally, an improved compressor design is formulated for an electronics cooling application using results from the model.

2. Resonant Frequency of a Linear Compressor

The resonant frequency of the linear compressor depends on the mechanical springs selected in the design as well as the operating conditions. To calculate the resonant frequency of a linear compressor, the stiffness associated with both the mechanical springs and the operating conditions must be estimated. The stiffness of the mechanical springs is typically reported by the manufacturer. The stiffness associated with the operating conditions is the stiffness from gas compression. Using these stiffness values, an estimate of the resonant frequency of oscillation is obtained from the following expression:

$$\omega_{res} = \omega_n \sqrt{1 - 2\zeta^2} \quad (1)$$

where the damping ratio is defined as follows (Rao, 2004)

$$\zeta = \frac{c_{eff}}{2M_{mov}\omega_n} \quad (2)$$

and ω_n is given by,

$$\omega_n = \sqrt{\frac{k_{eff}}{M_{mov}}} \quad (3)$$

The effective stiffness is defined as:

$$k_{eff} = k_{mech} + k_{gas} = k_{mech} + \frac{(P_{CV,1} - P_{CV,2})A_p}{dx_p} \quad (4)$$

where $P_{CV,1} - P_{CV,2}$ is the differential pressure acting on the piston between the compression chamber and compressor shell. The effective damping is defined as:

$$c_{eff} = \frac{E_{cycle}}{\omega_{res} x_s^2 \pi} = \frac{W_f}{\omega_{res} x_s^2 \pi} + \frac{W_{gas}}{\omega_{res} x_s^2 \pi} \tag{5}$$

with work due to friction and work done on the gas defined as:

$$\begin{aligned} W_{gas} &= -\int P_{CV,1} dV \\ W_f &= 4\mathbf{f}\int_0^{x_s} N dx \end{aligned} \tag{6}$$

where the normal force is defined as:

$$N = \left(\frac{1}{L_p}\right)\left(k_{mech}\grave{o}(x_p - \grave{o}\theta)\right) \tag{7}$$

Equation (1) describes the resonant frequency of a one-dimensional system. The model presented previously in Bradshaw et al. (2011) utilizes a two degree of freedom system to describe the piston motion. These two degrees of freedom are the desired piston translation and the undesired rotation of the piston within the cylinder. However, it has been observed that the resonant frequency of the linear compressor is predicted accurately using a one degree of freedom approach (Pollak et al., 1979; Cadman & Cohen, 1969). In addition, Equations (1)-(7) are linearized based on the desired input stroke. This approach estimates the damping and stiffness of the system when the stroke is at a desired condition. By linearizing these equations, the dependence on instantaneous stroke is removed. Therefore, the resonant frequency can be calculated external to the comprehensive linear compressor model. Figure 2 compares the resonant frequency predicted from this approach and the experimentally obtained resonant frequencies. The experimental data presented were obtained using the methodology outlined in previous work (Bradshaw et al., 2011). The experimental prototype linear compressor in this work was tested at evaporation pressures between 537 and 561 kPa,at pressure ratios of 1.11 to 1.26, and with a constant superheat of 4.5 K. The predictions match the experimental measurements well, with a Mean Absolute Error (MAE) of 0.5%.

3. Stroke Control of a Linear Compressor

A consequence of the free-piston design of the linear compressor is that the stroke of the compressor changes with power input and operating conditions. A reciprocating compressor, on the other hand, has a stroke that is fixed by the kinematics of the device and remains constant regardless of operating conditions or power input. The desired piston stroke is one of the necessary specifications in the design of a linear compressor. Therefore, the utility of the linear compressor model developed in Bradshaw et al. (2011) is enhanced here by allowing for the desired stroke to be specified. Since the piston driving force is an input to the two-degree-of-freedom piston vibration model used to determine the piston behavior, a numerical algorithm is used to adjust the piston driving force until the desired piston stroke is achieved.. The stroke is controlled to within 0.1 μm for each geometric configuration examined.

The numerical algorithm utilized to control the linear compressor stroke is known as Brent's method (1971). This algorithm adjusts the force input to the piston iteratively until the desired stroke is obtained. Brent's method uses a combination of the Secant and Bisection methods for finding the roots. The algorithm depicted in Figure 3 presents the modified overall compressor model algorithm. The method requires two different guessed input values of force to begin. For each input value the same calculation algorithm is used. Using the resonant frequency model presented in Section 2, the compressor resonant frequency is calculated in the first step. Using a guess for force input, the model is solved in the manner previously described by Bradshaw et al. (2011). The calculated stroke is then compared with the input value and an updated value calculated using Brent's method. This calculated force is then entered into the model, which iterates until the desired stroke is achieved.

4. Losses and Efficiency Definitions

The overall isentropic efficiency gives a measure of the overall performance of the linear compressor:

$$\eta_{o,is} = \frac{\dot{m}(h_{2,s} - h_1)}{\dot{W}_{in}} \tag{8}$$

where the power input is calculated as a combination of motor heat loss and force applied to the piston, as follows:

$$\dot{W}_{in} = \left(F_{drive}\right)_{rms}\left(\dot{x}_p\right)_{rms} + \dot{Q}_{motor} \tag{9}$$

While the overall isentropic efficiency provides a measure of the total losses in a compressor, a separate estimation of major loss components provides useful insight into the compressor operation. Leakage and friction are typically two sources of greatest loss within a positive displacement compressor and are quantified here.

The frictional losses in the compressor are determined as the scalar product of the frictional force and the piston velocity. These two quantities oscillate about a mean value of zero and always point in opposite directions. Therefore, to estimate the mean power dissipated via friction over a compression cycle requires the use of the root mean square (rms), and the scalar product reduces to:

$$\dot{W}_f = \left(\mathbf{f}N(x_p)\right)_{rms}\left(\dot{x}_p\right)_{rms} \tag{10}$$

In the prototype linear compressor, the leakage losses represent mass that has been pushed into the compressor shell instead of the discharge line; the same amount of mass also returns to the compression chamber due to mass being conserved. The net loss represents the work required to move this mass of gas both into and back out of the compressor shell, and represents a loss in available energy. The volumetric efficiency is a traditional measure of leakage used. However, in a linear compressor this metric becomes more complicated as the volumetric efficiency depends on the stroke and resonant frequency in addition to mass flow rate of the compressor The volumetric efficiency of the linear compressor is a ratio of the actual mass flow of the compressor to the theoretical maximum mass flow:

$$\eta_{vol} = \frac{\dot{m}}{\rho f_{res} x_s A_p} \tag{11}$$

This definition provides difficulty in isolating only leakage losses when varying piston stroke and resonant frequency. Therefore, to estimate the leakage loss, an exergy analysis is used. Using the compressor control volumes shown in Figure 4, the amount of exergy destroyed in moving the leakage mass into the compressor shell and subsequently back into the compression volume is:

$$\dot{E}_{d,leak} = \dot{W}_{leak} = T_o \dot{m}_{leak} (s_{cv,2} - s_{cv,1}) + \left(\frac{T_o}{T_w} \right) \dot{m}_{leak} (h_{cv,2} - h_{cv,1}) \tag{12}$$

The exergy destruction shown in Equation (12) gives one measure of leakage losses in a linear compressor. (13)

5. Sensitivity Studies

Sensitivity analyses conducted using the comprehensive linear compressor model are presented here for two studies. The first is a study that focuses specifically on leakage and friction independent of changes in other parameters. This is accomplished by examining the sensitivity to changes in the leakage gap g as well as spring eccentricity ò. The second study considers changes in the piston geometry and scaling of the compressor, specifically the piston diameter at various displacement volumes. The ranges of parameters investigated are first discussed, followed by the results of the studies.The earlier study by Bradshaw et al. (2011) highlighted four parameters that significantly impact the overall performance metrics of the compressor: motor efficiency η_{motor} , dry friction coefficient **f**, spring eccentricity ò, and leakage gap g. For simplicity, changes to η_{motor} are not considered here, as changes in this parameter would provide little insight into compressor design; the higher the linear motor efficiency, the better is the compressor overall performance. Thus, the efficiency is fixed at 90% for the rest of this work. The operating conditions are set at 20 ^{o}C, 40 ^{o}C, and 5 ^{o}C for evaporating, condensing, and superheat temperatures, respectively. This operating condition represents a typical electronics cooling application. In addition, the clearance gap between the piston at TDC and the valve plate, x_{dead} is fixed at 3 mm, illustrated in Figure 1.

5.1.Leakage and Friction

The dry friction coefficient **f** and spring eccentricity ò both relate to friction between the piston and cylinder. These two parameters are coupled by the normal force acting on the piston as shown in Equation (7). Thus, it is only necessary to vary one parameter to study the impact of friction. Therefore, only the spring eccentricity is examined, besides the leakage gap g. The ranges of the values used in this study attempt to represent an extreme set of conditions. For the eccentricity, this range spans from 0.1 cm to 0.9 cm, where the upper limit represents an extreme situation. The leakage gap ranged from 1 μm to 23 μm, which spans a realistic range of values for compressor leakage gaps (Kim & Groll, 2007; Jovane et al., 2006). In addition, the stroke is fixed at 2.54 cm (1 in) for this study.

The leakage mass flow rate as a function of leakage gap at different eccentricity values is shown in Figure 5. The leakage flow rate increases as the leakage gap increases and does not display a significant dependency on piston eccentricity. An increase in leakage mass flow rate results in a decrease in the volumetric efficiency of the device; this is shown in Figure 6, which presents the compressor volumetric efficiency for different eccentricities as a function of leakage gap. This decrease in efficiency corresponds to a decrease in mass flow rate from the compressor as a result of increased leakage. Both the leakage mass flow rate and the volumetric efficiency show a negligible dependence on eccentricity. It may be concluded that the flow parameters are not affected significantly by changes in frictional characteristics within the compressor but are highly sensitive to changes in leakage gap width.

The overall compressor performance is presented in Figure 7, in terms of the overall isentropic efficiency at different leakage gap and eccentricity values. As the eccentricity decreases, the overall isentropic efficiency increases proportionally. This is a result of increased frictional losses at higher eccentricity values. The compressor performance is also reduced at higher leakage gap widths, since the compressor mass flow rate decreases with larger leakage gaps.

Figure 8 shows the net frictional losses as a function of leakage gap width for each of the three eccentricity values considered. The frictional losses are seen to be largely independent of the leakage gap width, with a slight increase in loss as the leakage gap is reduced, and a strong dependence on eccentricity. This behavior is due to the dry friction model utilized which best represents the prototype compressor developed in Bradshaw et al. (2011). In contrast, a thin-film friction model is typically utilized for modeling compressors which are oil-lubricated. . Conversely, Figure 9 shows that leakage losses, increase as the leakage gap increases, with little dependence on eccentricity. Thus, at larger leakage gaps, the leakage losses tend to dominate over frictional losses and the volumetric and overall isentropic efficiencies decrease. However, at smaller leakage gaps (<10 μm), the frictional losses tend to dominate. This, coupled with the high sensitivity of the frictional losses to the eccentricity, generates a higher level of sensitivity of the overall isentropic efficiency at smaller leakage gaps.

These results show the high level of sensitivity of the linear compressor performance to both the leakage gap between the piston and cylinder and the eccentricity of the spring. Reducing the leakage gap and the eccentricity of the applied spring force would improve the compressor performance. The lower limit on both these parameters is dictated by manufacturing constraints. It is also noted that at the smallest leakage gap widths, a stronger coupling would exist between leakage and friction that is not accounted for by the dry friction model utilized in the current work.

5.2. Piston Geometry and Scaling

The geometry of a compressor piston can have a large impact on the overall performance of the device (Kim & Groll, 2007; Rigola et al., 2005). The impact of scaling of the linear compressor is also of interest to the goal of miniaturizing the compression technology. To explore the impact of scaling, three compressor displacement volumes of 2, 3, and 6 cm^3 are examined. For each displaced volume, the piston diameter is varied from 0.8 to 1.7 cm. Fixing the compressor displacement volume and varying the piston diameter requires modification of the compressor stroke. Therefore, the stroke-to-diameter ratio is investigated for three displacement volumes.

The volumetric efficiency of the linear compressor is low at small values of stroke-to-diameter ratio, but increases asymptotically as the value of this ratio is increased, as illustrated in Figure 10. This behavior is largely unaffected by the choice of displacement volumes. This behavior is the result of a higher dead volume relative to the displaced volume as the piston diameter increases. Therefore, as the stroke-to-diameter ratio decreases, the dead volume increases in relation to the piston surface area. This increase in dead volume results in a reduction in volumetric performance.

The effects of displaced volume and stroke-to-bore ratio on the frictional and leakage losses of the linear compressor are shown in Figure 11 and Figure 12. The frictional losses in the linear compressor increase at a super-linear (i.e. greater than linear) rate with an increase in stroke-to-diameter ratio. An increase in displacement volume leads to increased frictional losses. As the stroke-to-diameter ratio increases, the mean piston velocity increases as a result of the increase in stroke. In addition, the frictional force generated by the contact between the piston and cylinder increases as a result of an increase in the normal force, shown in Equation (7). Thus, as seen from Equation (10) where two of the three factors increase, the frictional losses increase at a super-linear rate as the compressor stroke-to-diameter ratio increases.

The leakage losses show a different behavior with the stroke-to-diameter ratio: they decrease slightly until a stroke-to-diameter ratio of between 3 and 4, and then increase rapidly as this ratio is increased further. As with the frictional losses, the leakage losses increase with an increase in the displacement volume. As the stroke-to-bore ratio is reduced from a value of 3, the leakage losses slightly increase, whereas the losses increase sharply as this ratio increases beyond a value of 4. This behavior is a result of two competing factors: the change in piston resonant frequency, which changes the time period over which leakage can occur, and the overall leakage area. Figure 13 shows the compressor piston resonant frequency as a function of the stroke-to-diameter ratio. As the stroke-to-bore ratio increases, the resonant frequency decreases. Since the resonant frequency of the piston is related to the piston oscillation period by:

$$f_{res} = \frac{1}{\mathbf{T}_p} \tag{14}$$

An increase in stroke-to-bore ratio increases the piston oscillation period. An increase in the piston period generates a larger time period over which leakage can occur. Leakage within the compressor is also proportional to the area within the leakage gap:

$$\dot{m}_{leak} = \rho \mathbf{V} A_{leak} \tag{15}$$

The leakage area is determined by the piston diameter, with larger piston diameters corresponding to larger leakage areas. When the stroke-to-diameter ratio is small, the piston period is also small, but the leakage area is large. When the stroke-to-diameter ratio is large, on the other hand, the piston period is large but the leakage area is small. This trade-off is largely dominated by piston period but does exhibit an optimum at a stroke-to-diameter ratio of between 3 and 4. While the leakage losses tend to increase sharply after a stroke-to-bore diameter of between 3 and 4, this is not apparent in the volumetric efficiency. The reason for this is that the magnitude of the leakage losses is relatively small even at the maximum. Therefore, there is little impact on the volumetric efficiency.

In contrast to the volumetric efficiency, the overall isentropic efficiency decreases with an increase in stroke-to-diameter ratio, as illustrated in Figure 14. In addition, the performance improves with a decrease in displacement volume.

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Based on the results presented above for the volumetric performance, leakage and friction losses, the overall isentropic performance would also be expected to exhibit a trade-off between volumetric efficiency and frictional and leakage losses. Figure 14 shows a degradation in overall isentropic efficiency with an increase in stroke-to-diameter ratio due to increased friction and leakage losses. The degradation in volumetric efficiency seen in Figure 10 at small stroke-to-diameter ratios does not seem to manifest as a decrease in the overall isentropic efficiency under these conditions. The volumetric efficiency decreases due to an increase in dead volume, but the overall compressor performance seems to be unaffected such an increase in dead volume. This peculiar behavior may be attributed to the unique free-piston design of the linear compressor. In a typical positive displacement compressor, the gas trapped in the dead volume is re-expanded during the suction stroke and the energy required to compress this volume is lost. In a free-piston design, the gas trapped in the dead volume is re-expanded during the suction stroke as well. However, in a linear compressor the spring-mass-damper system of the free piston has the unique ability to almost fully recapture the energy required to compress the gas trapped in the dead volume. This ability to recapture the energy back into the piston mechanism during re-expansion translates into a net reduction in power consumption at the cost of a lower mass flow rate. It is acknowledged that this behavior is also dependent on the frictional and leakage characteristics. The previous study showed a high level of sensitivity of the overall performance to both eccentricity and leakage gap. Thus, a change in either of these parameters may change the trends presented in this section.

The overall isentropic efficiency trends also show that as the displaced volume is decreased the overall performance tends to increase. This suggests that as the compressor is scaled down for lower capacity applications the performance would tend to increase. This trend is a result of the net frictional losses decreasing as the displaced volume decreases; which shows the ability of the linear compressor to scale effectively to miniature-scale for electronics cooling applications. However, it is acknowledged that this does not represent a comprehensive survey of the impacts of scaling as the miniaturization of the linear motor needs to be explored.

6. An Improved Linear Compressor Design for Electronics Cooling

Based on the results obtained in this work, an improved linear compressor design is formulated for an electronics cooling application. A compressor for such an application should be compact, as package size in electronic equipment is often a constraint. The performance of the previous prototype design by Bradshaw et al. (2011) can be readily improved, at a lower total compressor package size. The modified design presented here provides an improved overall efficiency and reduced package size.

From the leakage and friction sensitivity studies it was concluded that both the leakage (leakage gap *g*) and frictional parameters (spring eccentricity ò, and dry friction coefficient **f**) should be minimized. However, practical limitations prohibit the reduction of these parameters to zero. Therefore, a realistic set of values was selected based on input from compressor manufacturers and previous studies on compressor modeling (Dagilis & Vaitkus, 2009; Kim et al., 2009). A leakage gap width of 4 μm was selected as a lower limit on a clearance fit for a mass produced product. A dry friction coefficient of 0.2 represents a reasonable and low value for an engineering polymer (*e.g.* PEEK, PTFE, or Rulon) and steel. A value for the spring eccentricity of 0.5 cm also appears to be realistically attainable; lower values may be manufacturable and would correspondingly alter the design. Table 1 summarizes these values as well as additional design parameters for the linear compressor.

Similar to the design presented in Bradshaw et al. (2011), the improved linear compressor design utilizes an off-the-shelf motor from HW2 Technologies. First, a cooling capacity of 200 W was selected to represent a desktop computer cooling application. The design conditions utilized are the same as presented in the sensitivity studies, 20 ^{o}C, 40 ^{o}C, and 5 ^{o}C for evaporating, condensing, and superheat temperatures, respectively. The comprehensive compressor model was then run using the design parameters listed in Table 1 with various stroke-to-diameter ratios necessary to achieve the required 200 W of cooling. The required stroke-to-diameter ratio and the corresponding continuous driving force dictate the choice of an appropriate H2W Technologies motor (P/N NCM02-17-035-2F). The modeling results from the present work show an increase in compressor performance with a decrease in the stroke-to-diameter ratio. However, the force required to operate the compressor piston increases with a decrease in this ratio. An increase in force requires a larger linear motor, which translates into a larger overall package size. Thus, there is a trade-off between package size and performance. For the current design, a stroke-to-diameter ratio of 0.4 leads to an acceptable overall package size.

The proposed design of the improved miniature linear compressor is shown in Figure 15. The overall performance of the compressor as well as the corresponding refrigeration system performance are listed in Table 2. While the predicted cooling capacity of the linear compressor prototype is lower than with the previous design from Bradshaw et al. (2011), the capacity-to-volume ratio has increased significantly. Therefore, this design lends itself to greater miniaturization of linear compressors.

7. Conclusions

The sensitivity of the performance of a linear compressor to changes in its geometric parameters is analyzed, leading to insights in the design methodology of linear compressors. These insights allow for a further refinement of the design prototype presented in Bradshaw et al. (2011). Methods for predicting the resonant frequency of a linear compressor as well as for numerical control of the stroke of the device are presented. In addition, a loss analysis is presented which quantifies the work lost due to friction and leakage.

The sensitivity studies conducted showed that the linear compressor is highly sensitive to changes in the leakage gap between the piston and cylinder as well as the spring eccentricity; both parameters should be minimized for optimal performance. Therefore, it is important to quantify and control these parameters in any compressor design that is mass-produced to maximize performance.

The present work also illustrates the ability of the linear compressor to be readily scaled to smaller capacities. Other types of positive-displacement compressors suffer from performance limitations upon miniaturization due to manufacturing tolerances. The small number of moving parts in the proposed linear compressor design, along with its insensitivity to dead volume, make it an ideal technology for electronics cooling applications. The ability to handle larger amounts of dead volume without performance degradation could also allow this technology to be used to control the capacity of the refrigeration system. Capacity control is a critical need for high-performance refrigeration systems in electronics cooling and should be further investigated.

The results from the sensitivity analysis are used to inform the scalable design procedure for a linear compressor, and an improved linear compressor design for electronics cooling is presented with an overall package size of 50.3 mm diameter by 102 mm length and a predicted refrigeration capacity of 200 W.

8. Acknowledgement

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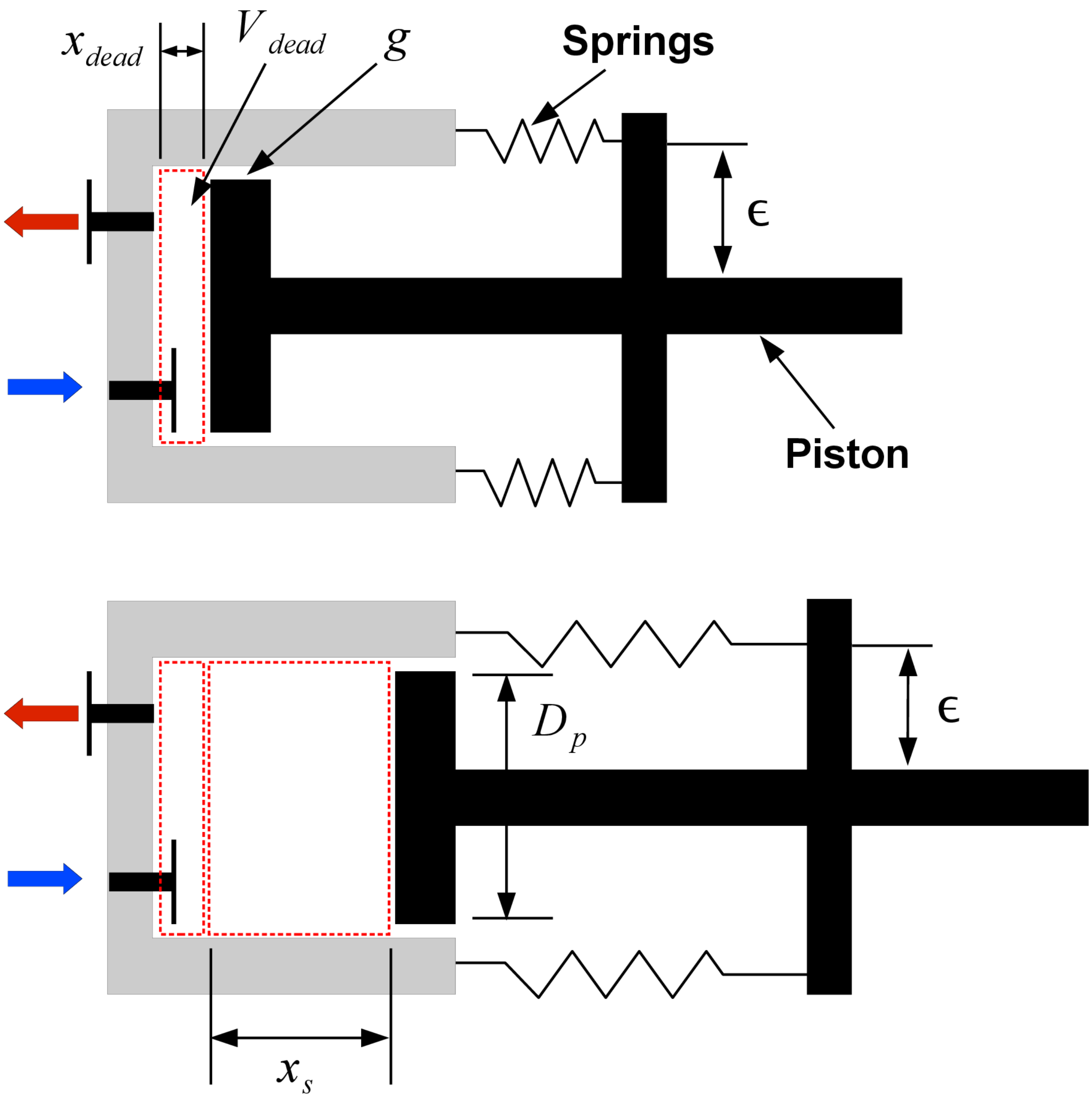
Table of Tables

Table of Figures

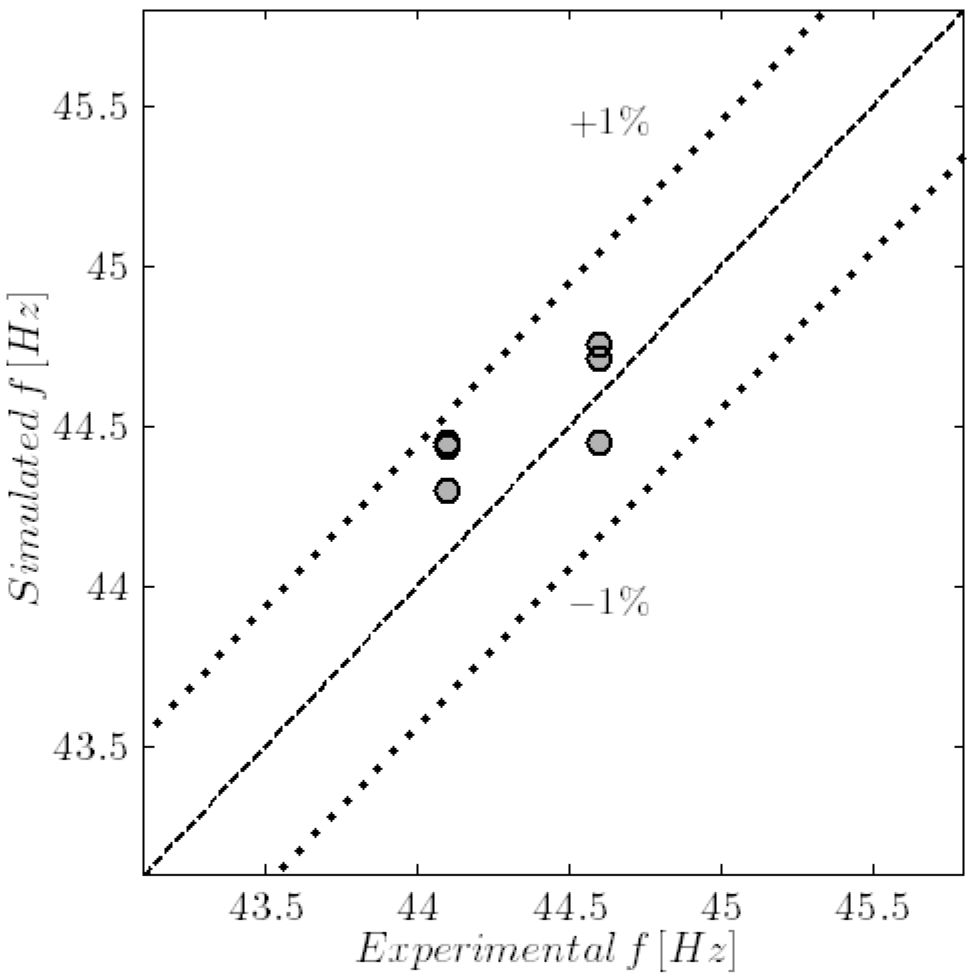
Figure 15: Improved linear compressor design with predicted cooling capacity of 200 W.

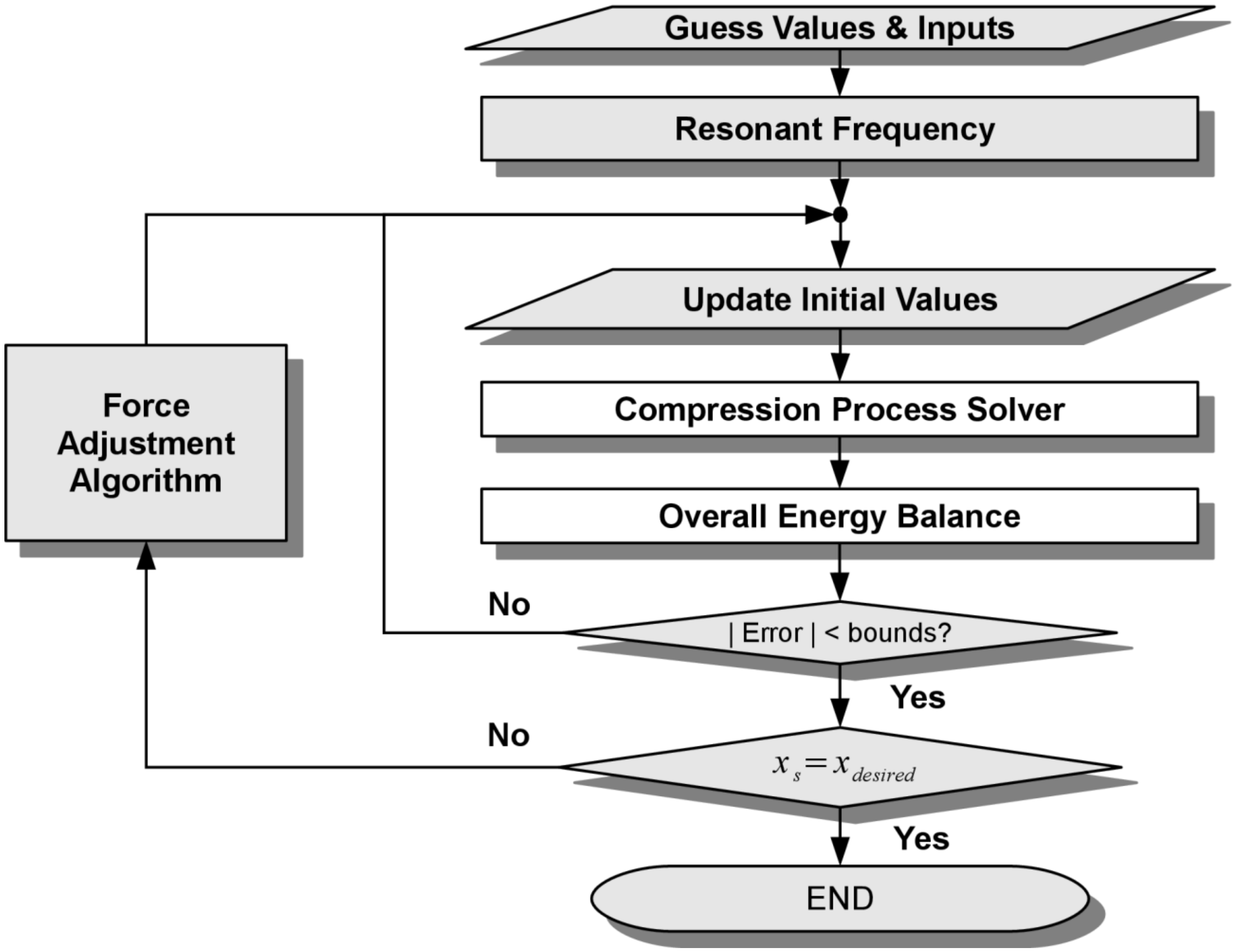
	k_{mech}	**f**	D_p	g	f_{res}	ò
Design	N/mm	-	cm	μm	Hz	cm
Current Work	30.6	0.2	1.35	4	60	0.5
Bradshaw et al. (2011)	23.0	0.35	1.217	13	44.5	1.145

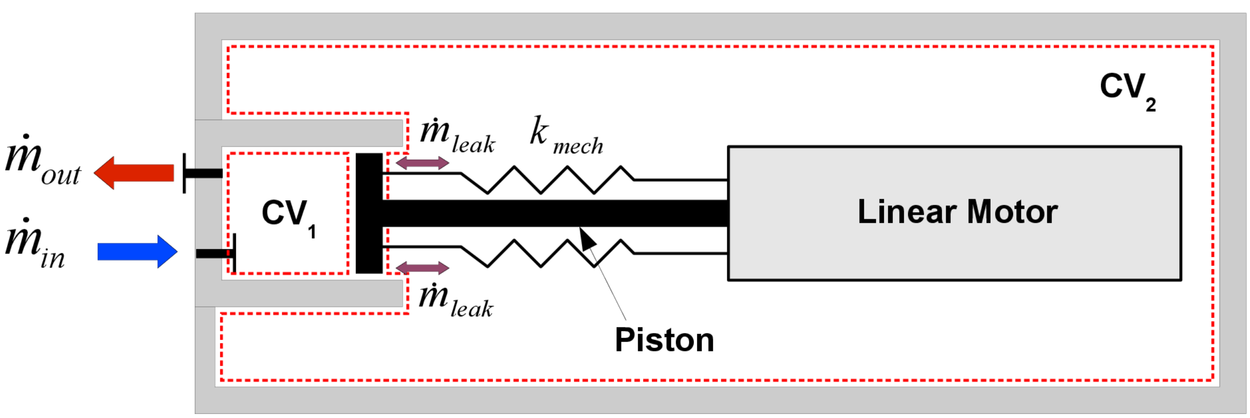
	V_d	x/D	$\dot{Q}_{cool}$	η_{vol}	$\eta_{o,is}$
Design	cm^3	-		-	-
Current Work	2.00	0.4	200	0.96	0.86
Bradshaw et al. (2011)	3.09	2	520	0.4	0.08

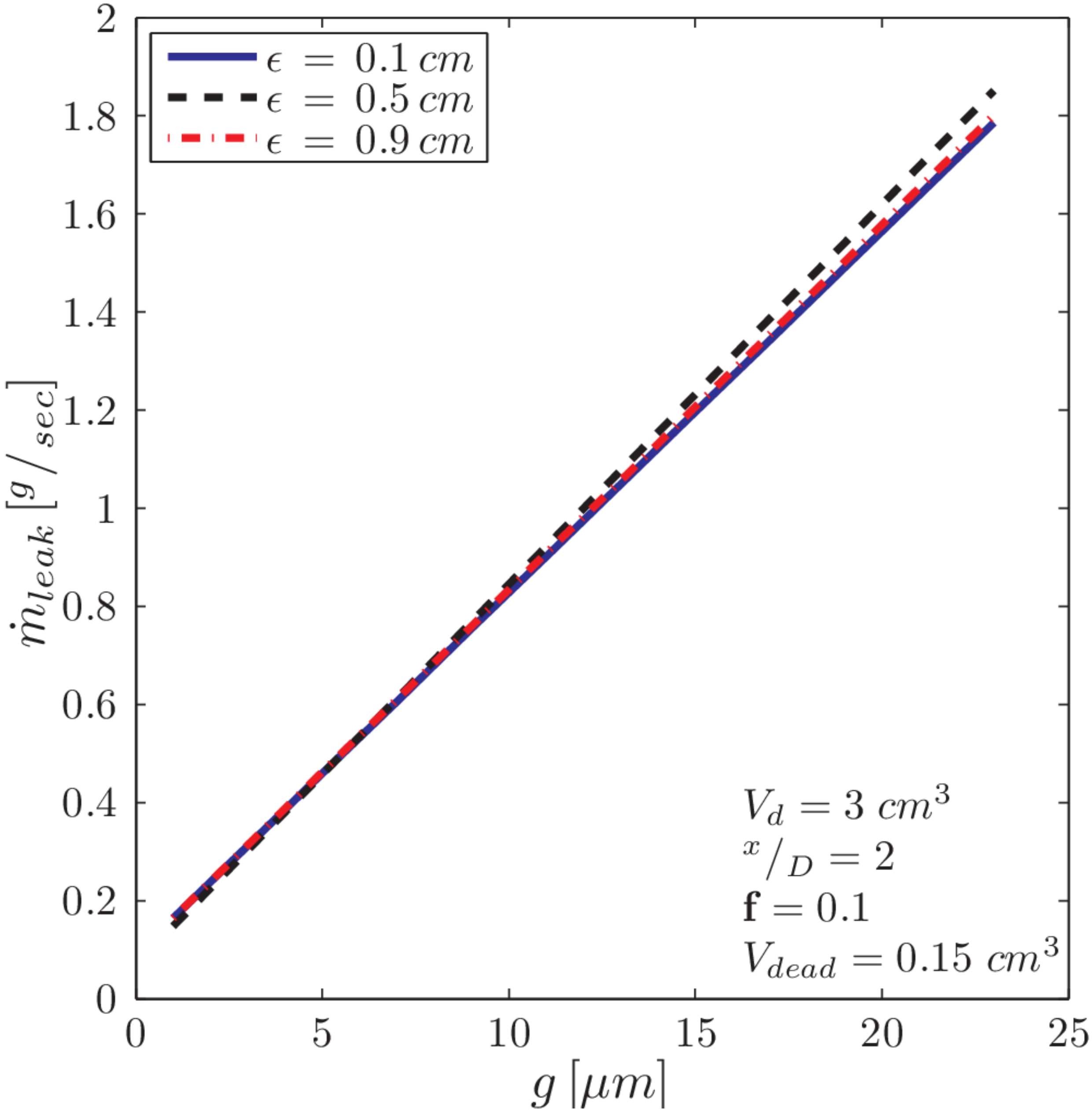


x_{dead}
V_{dead}
g
Springs
ϵ
Piston
ϵ
D_p
x_s

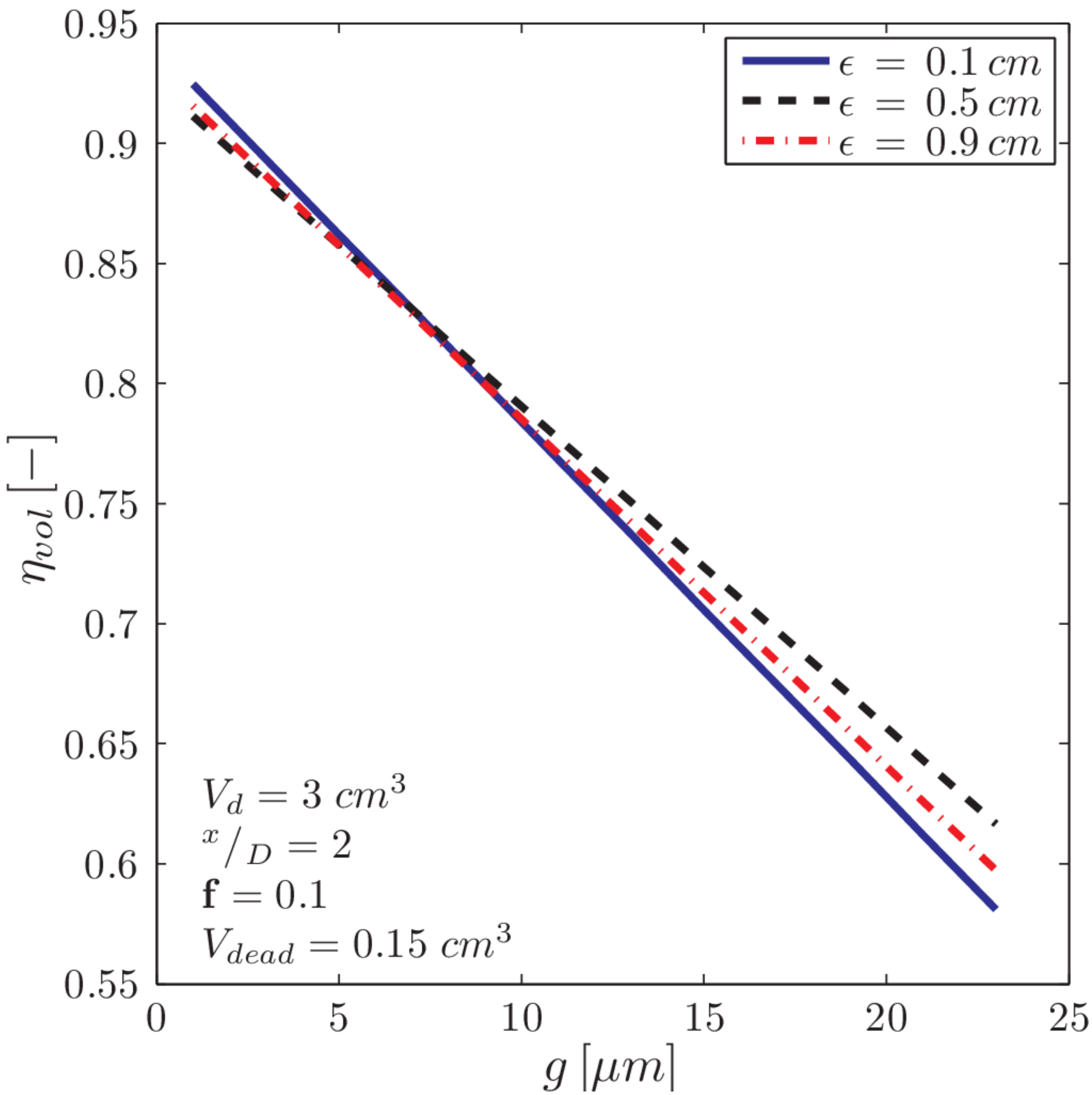

45.5
45
44.5
44
43.5
+1%
−1%
Simulated f [Hz]
43.5
44
44.5
45
45.5
Experimental f [Hz]


Guess Values & Inputs
Resonant Frequency
Update Initial Values
Compression Process Solver
Overall Energy Balance
Force Adjustment Algorithm
No
| Error | < bounds?
Yes
No
$x_s = x_{desired}$
Yes
END

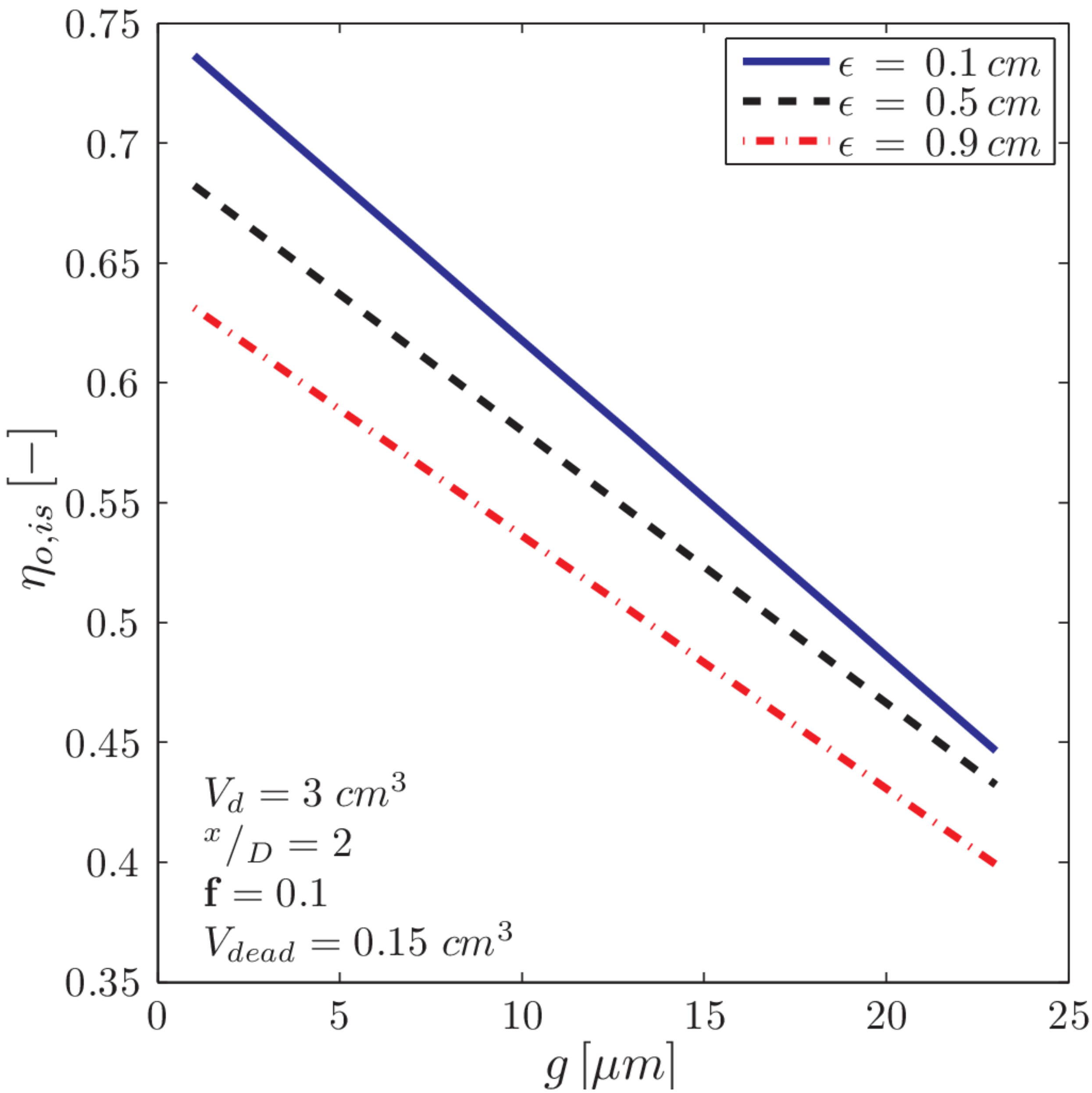

$\dot{m}_{out}$
$\dot{m}_{in}$
CV$_1$
CV$_2$
$\dot{m}_{leak}$
k_{mech}
$\dot{m}_{leak}$
Linear Motor
Piston



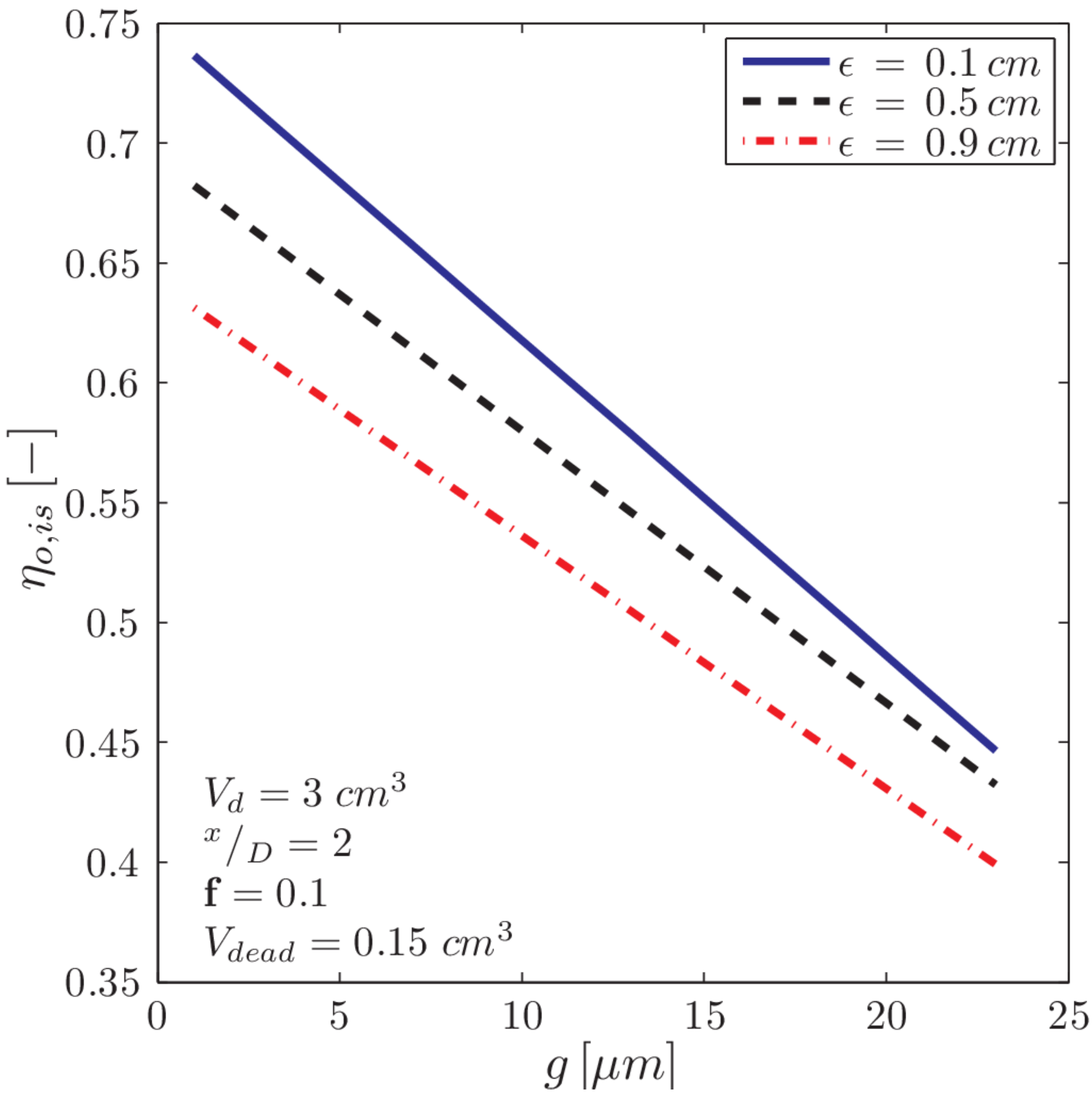
$\epsilon = 0.1\,cm$
$\epsilon = 0.5\,cm$
$\epsilon = 0.9\,cm$
$V_d = 3\ cm^3$
$^x/_D = 2$
$\mathbf{f} = 0.1$
$V_{dead} = 0.15\ cm^3$
$\dot{m}_{leak}\ [^g/_{sec}]$
$g\ [\mu m]$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
5
10
15
20
25



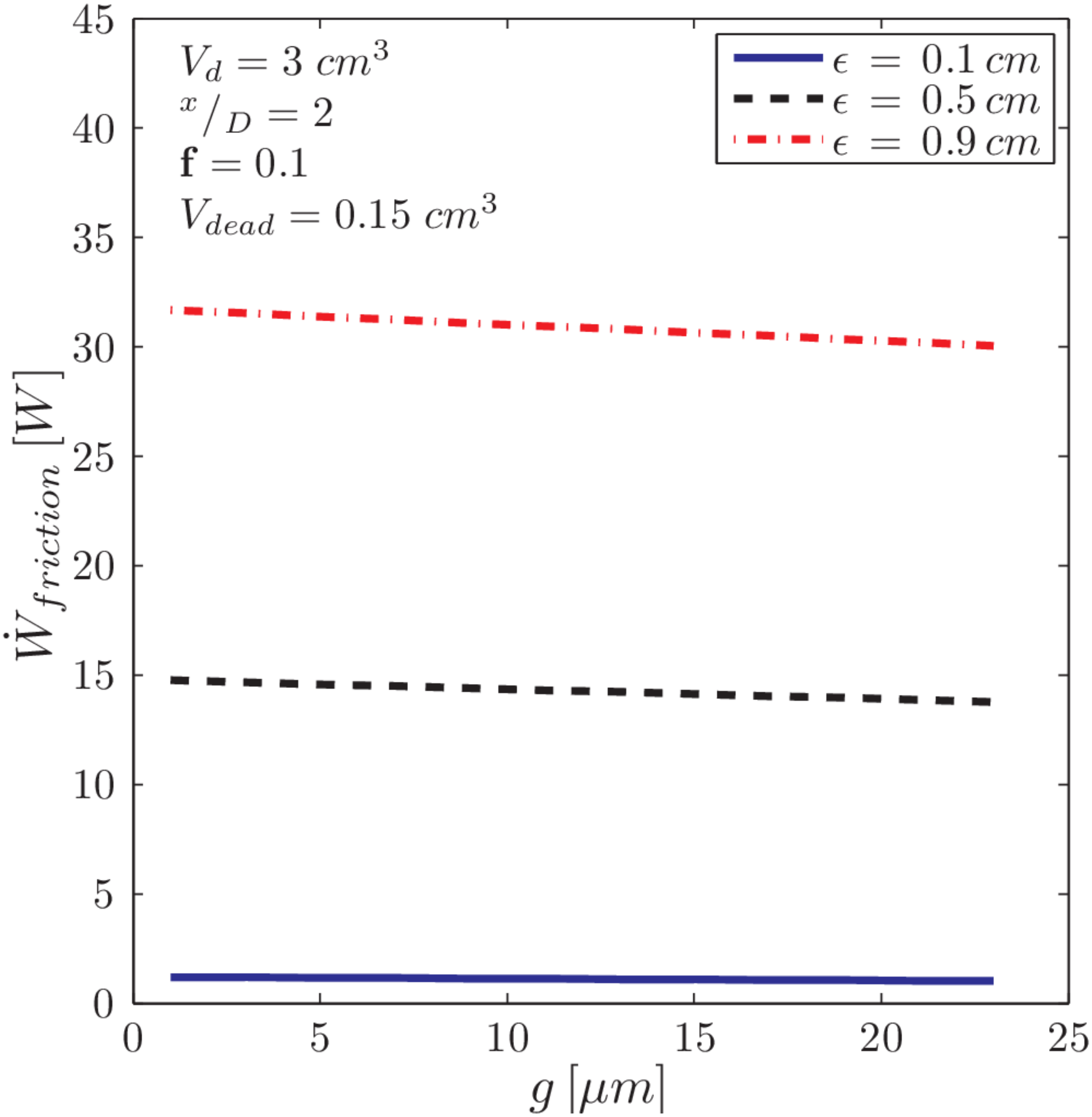
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0
5
10
15
20
25
$\eta_{vol}\ [-]$
$g\ [\mu m]$
$\epsilon = 0.1\,cm$
$\epsilon = 0.5\,cm$
$\epsilon = 0.9\,cm$
$V_d = 3\ cm^3$
$^x/_D = 2$
$\mathbf{f} = 0.1$
$V_{dead} = 0.15\ cm^3$



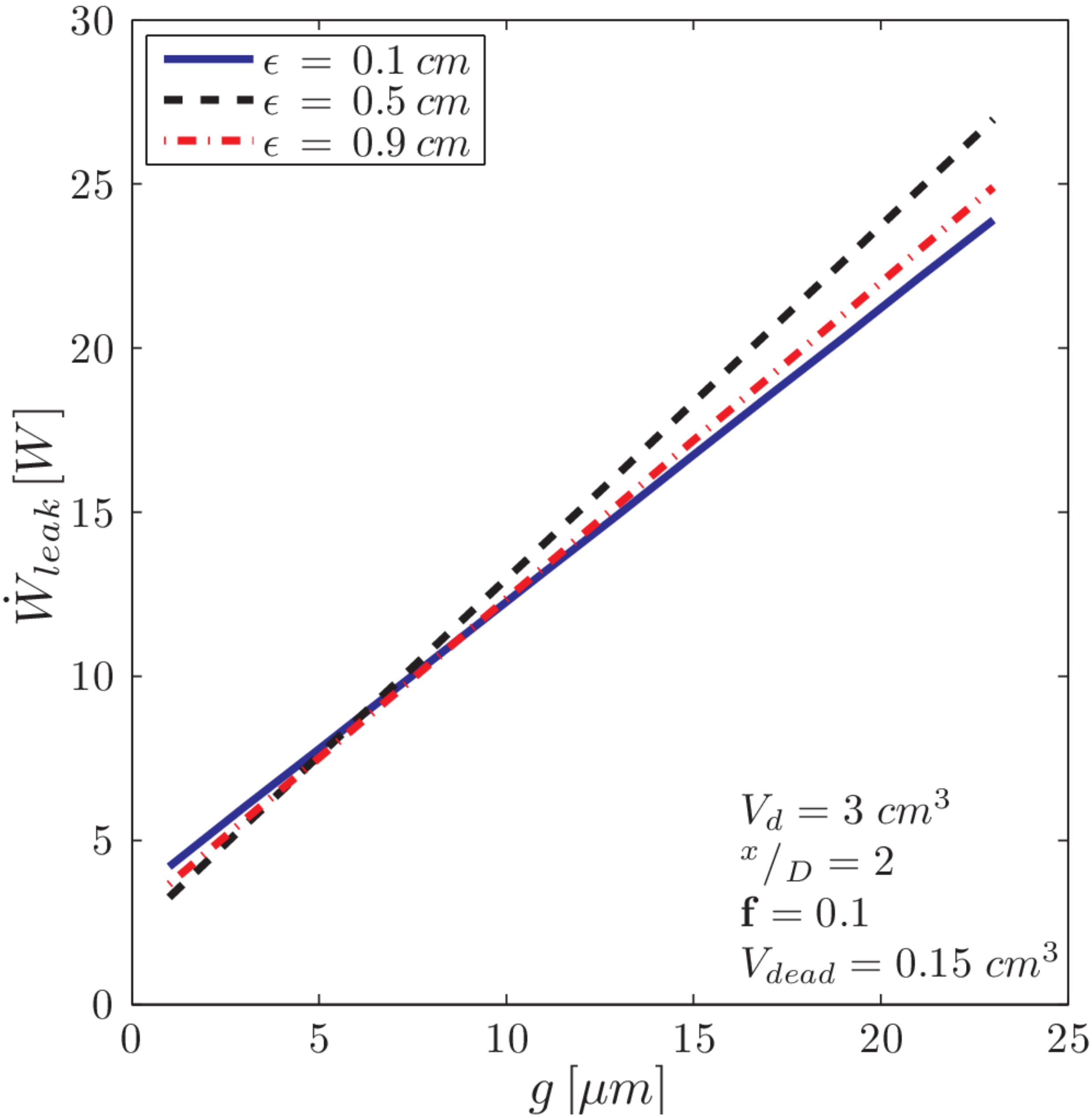
$\epsilon = 0.1\ cm$
$\epsilon = 0.5\ cm$
$\epsilon = 0.9\ cm$
$V_d = 3\ cm^3$
$^x/_D = 2$
$\mathbf{f} = 0.1$
$V_{dead} = 0.15\ cm^3$
$\eta_{o,is}\ [-]$
$g\ [\mu m]$
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0
5
10
15
20
25

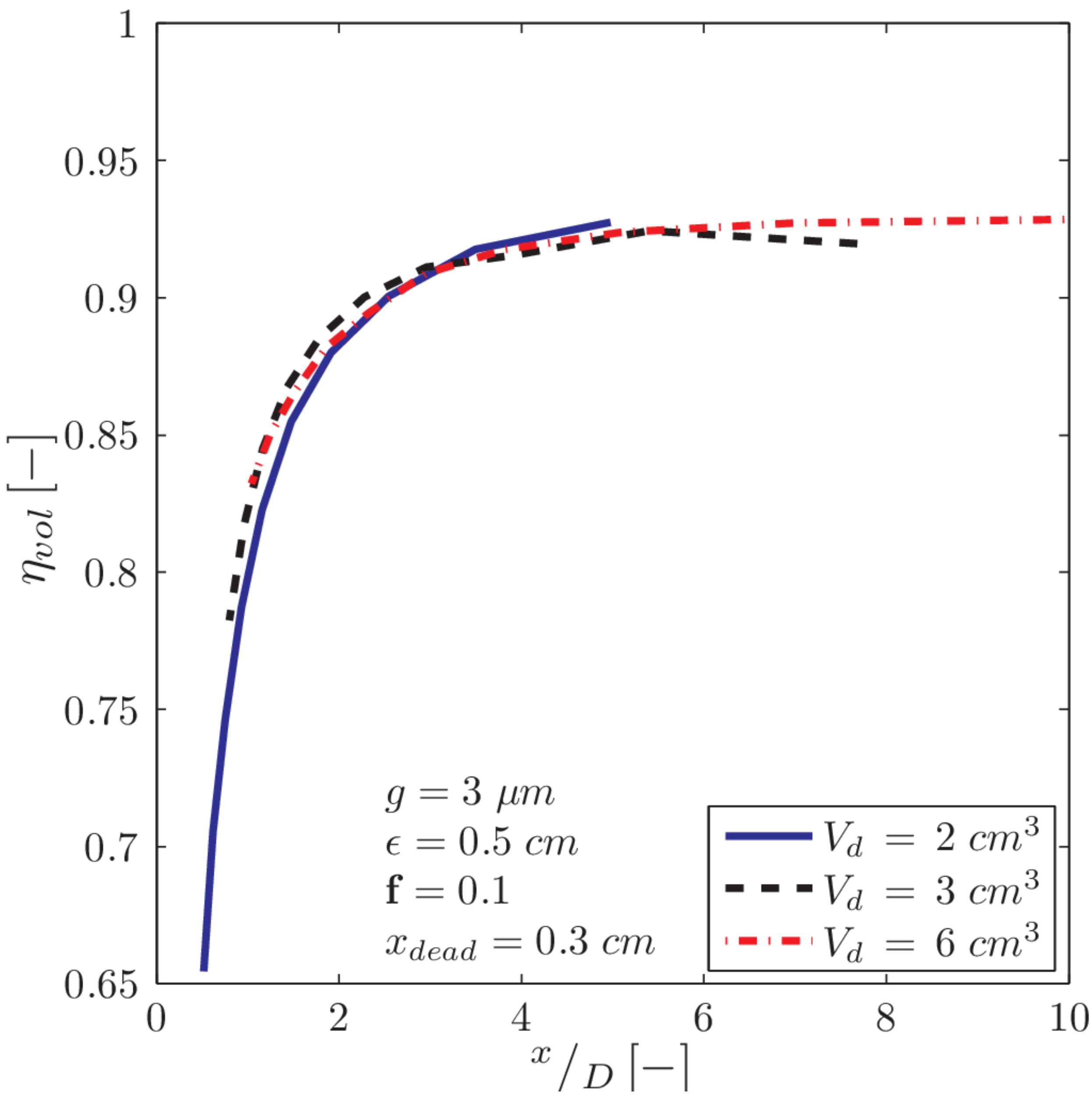


$\epsilon = 0.1\,cm$
$\epsilon = 0.5\,cm$
$\epsilon = 0.9\,cm$
$V_d = 3\ cm^3$
$^x/_D = 2$
$\mathbf{f} = 0.1$
$V_{dead} = 0.15\ cm^3$
$\eta_{o,is}\,[-]$
$g\,[\mu m]$
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0
5
10
15
20
25

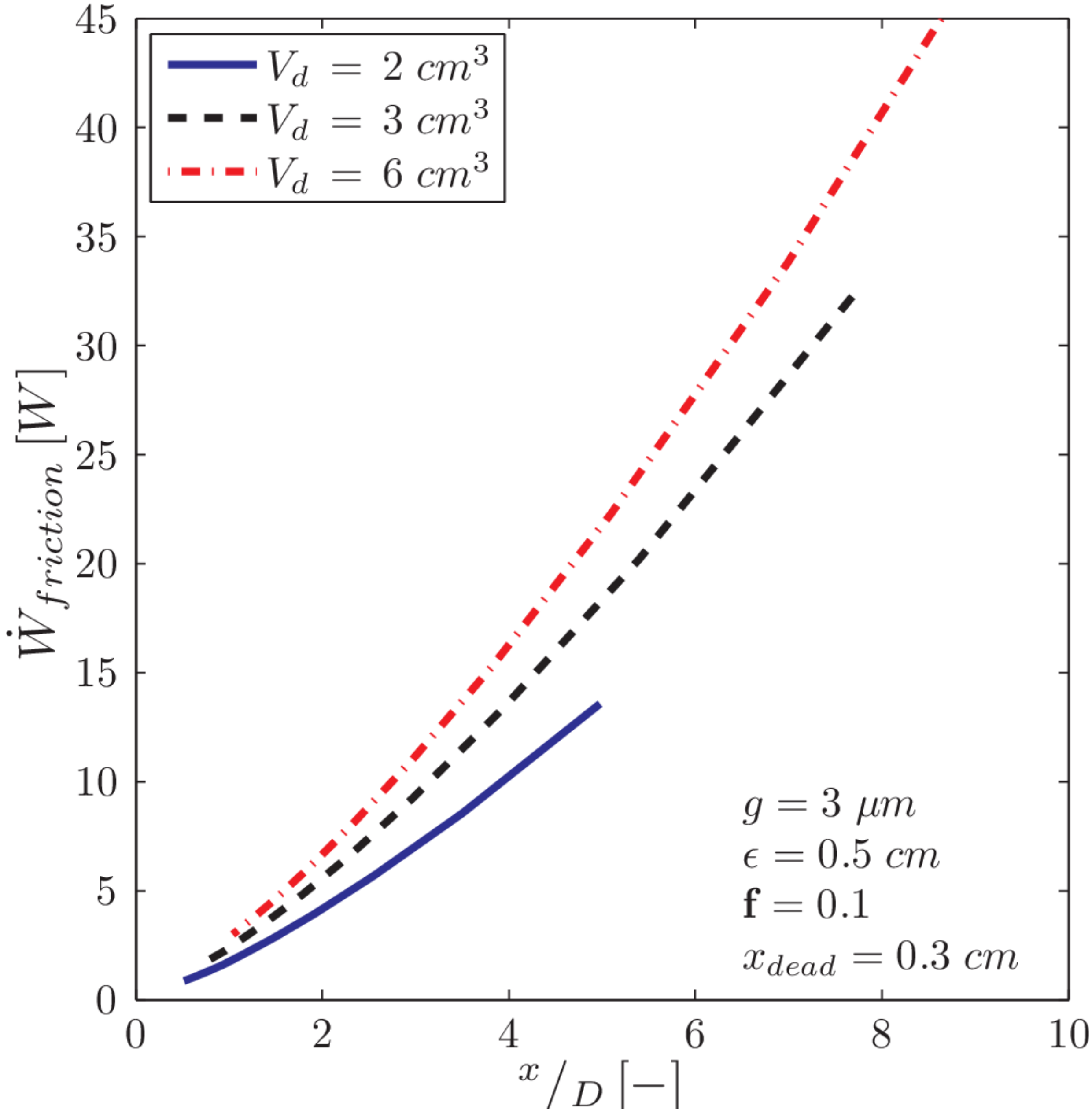


$V_d = 3\ cm^3$
$^x/_D = 2$
$\mathbf{f} = 0.1$
$V_{dead} = 0.15\ cm^3$
$\epsilon\ =\ 0.1\ cm$
$\epsilon\ =\ 0.5\ cm$
$\epsilon\ =\ 0.9\ cm$
$\dot{W}_{friction}\ [W]$
$g\ [\mu m]$
0
5
10
15
20
25
30
35
40
45

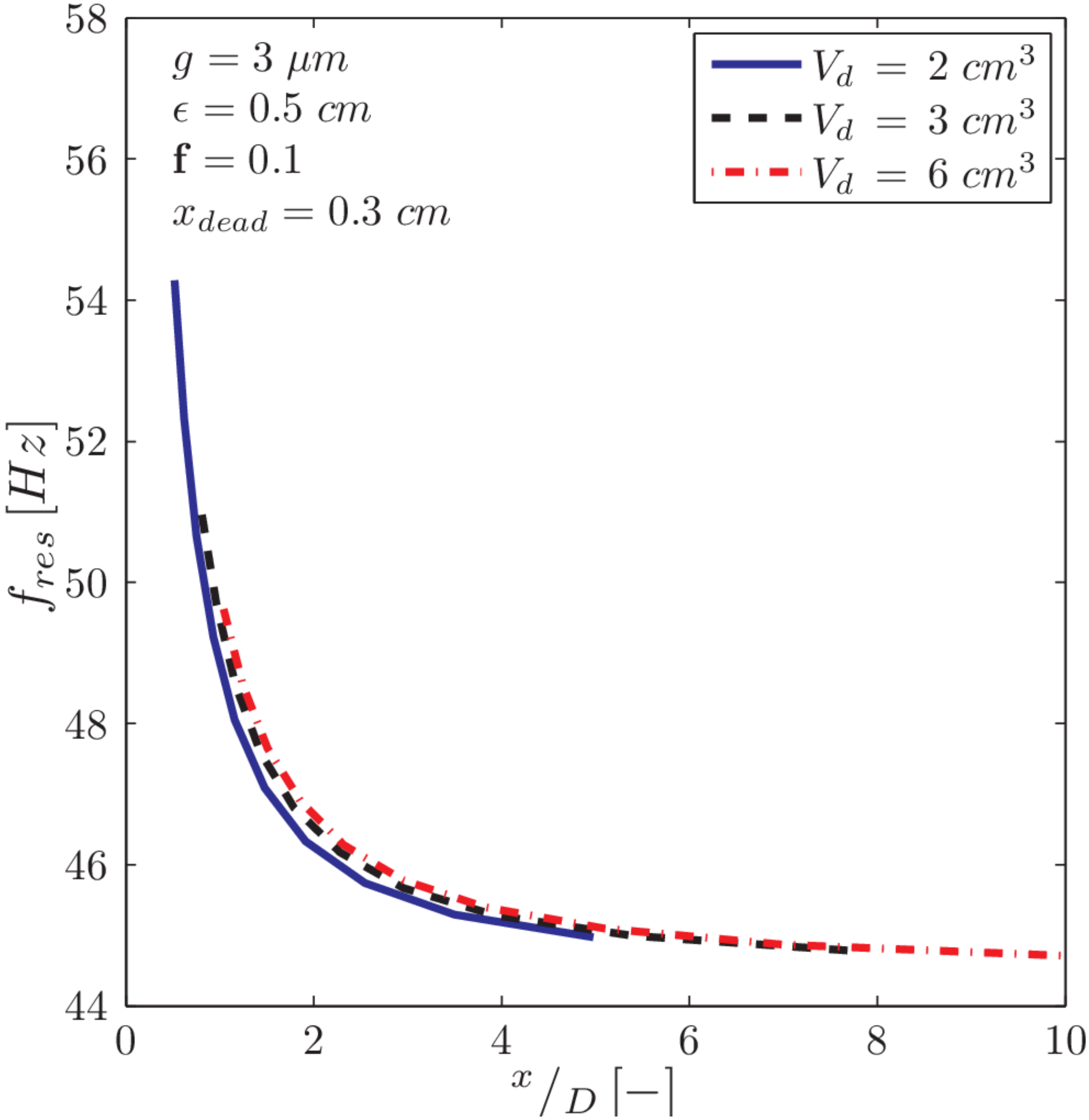

$\epsilon = 0.1\ cm$
$\epsilon = 0.5\ cm$
$\epsilon = 0.9\ cm$
$V_d = 3\ cm^3$
$x/D = 2$
$\mathbf{f} = 0.1$
$V_{dead} = 0.15\ cm^3$
$\dot{W}_{leak}\,[W]$
$g\ [\mu m]$
0
5
10
15
20
25
30

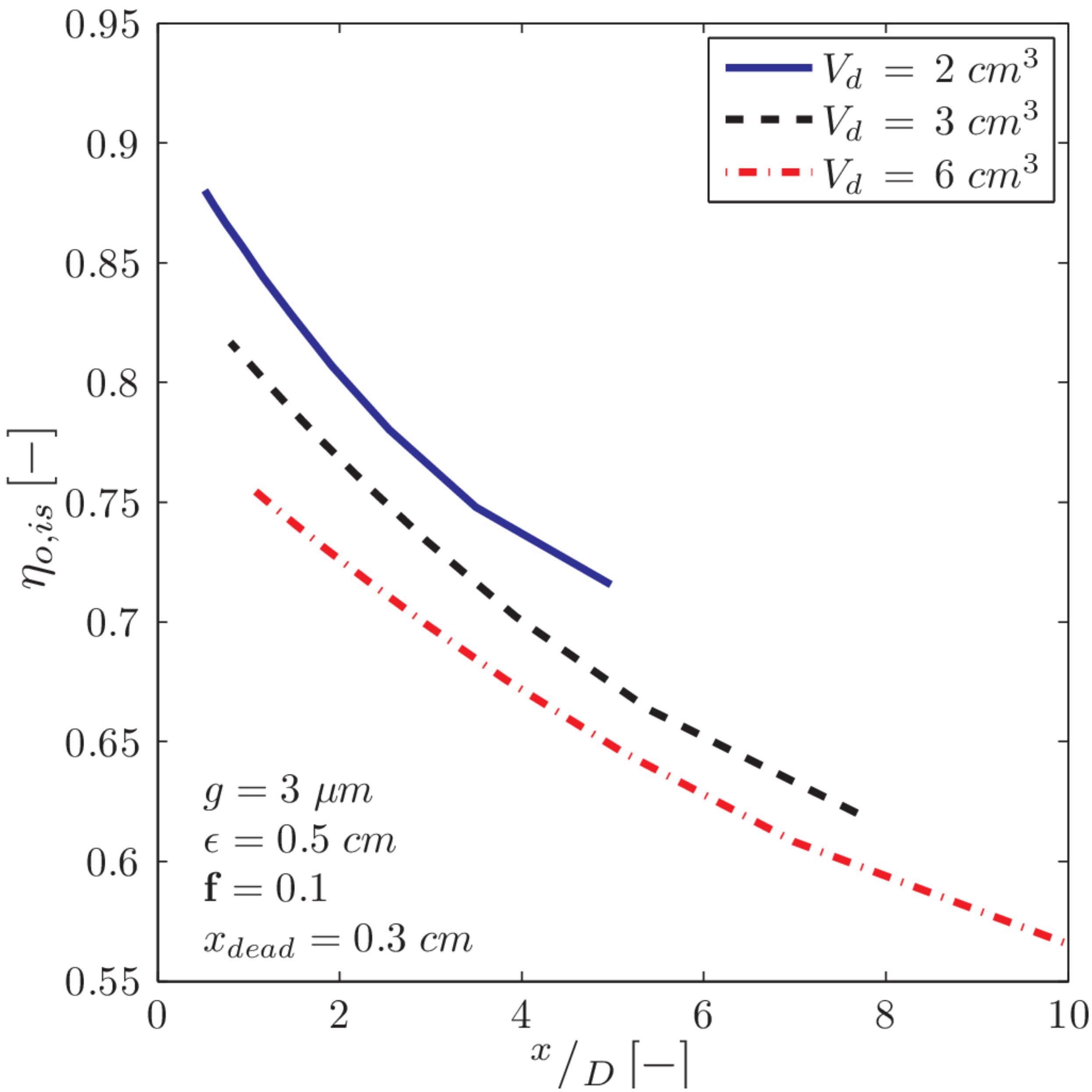


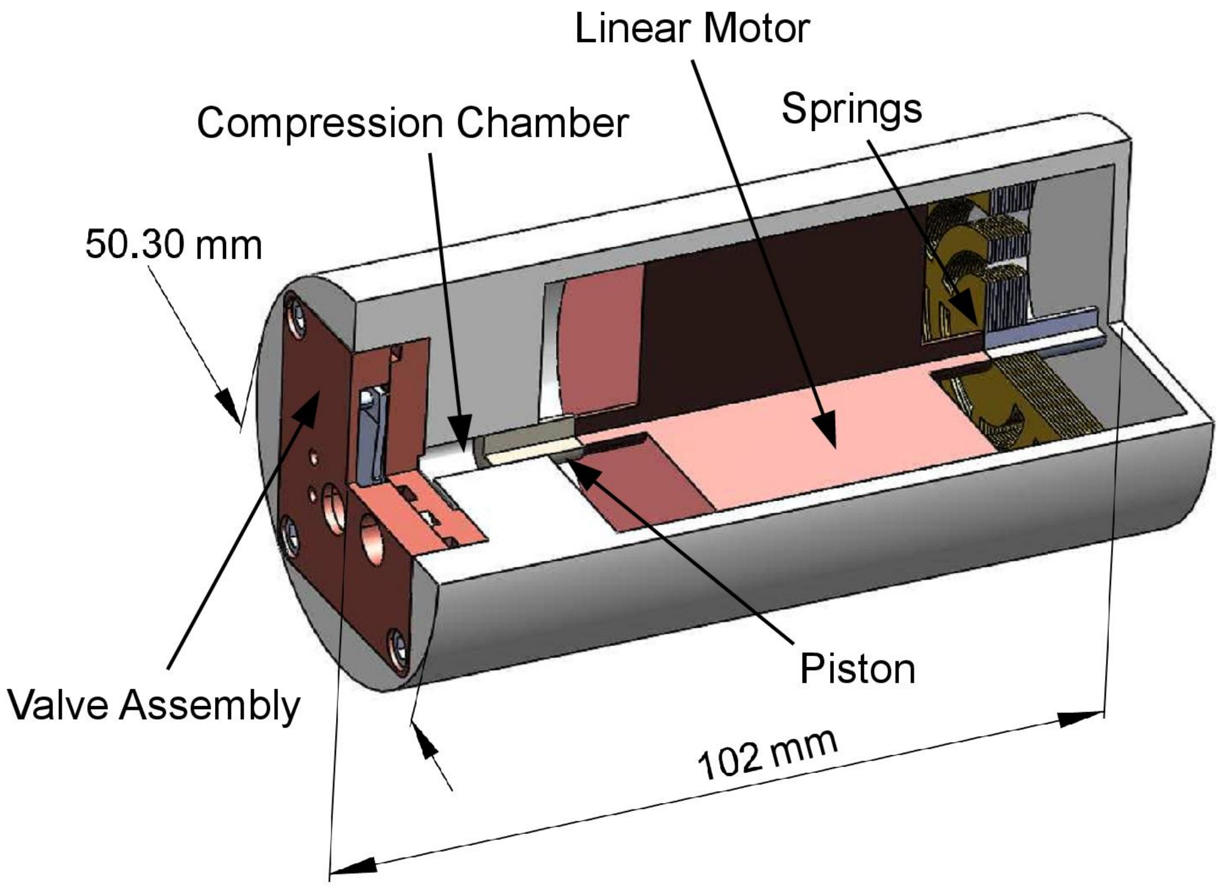
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0
2
4
6
8
10
η_{vol} $[-]$
x/D $[-]$
$g = 3\ \mu m$
$\epsilon = 0.5\ cm$
$\mathbf{f} = 0.1$
$x_{dead} = 0.3\ cm$
$V_d\ =\ 2\ cm^3$
$V_d\ =\ 3\ cm^3$
$V_d\ =\ 6\ cm^3$



$V_d = 2\ cm^3$
$V_d = 3\ cm^3$
$V_d = 6\ cm^3$
$g = 3\ \mu m$
$\epsilon = 0.5\ cm$
$\mathbf{f} = 0.1$
$x_{dead} = 0.3\ cm$
$\dot{W}_{friction}\ [W]$
$x/D\ [-]$
0
5
10
15
20
25
30
35
40
45
0
2
4
6
8
10


$g = 3\ \mu m$
$\epsilon = 0.5\ cm$
$\mathbf{f} = 0.1$
$x_{dead} = 0.3\ cm$
$V_d = 2\ cm^3$
$V_d = 3\ cm^3$
$V_d = 6\ cm^3$
$f_{res}\ [Hz]$
$x/D\ [-]$
58
56
54
52
50
48
46
44
0
2
4
6
8
10


0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
$\eta_{o,is}$ [−]
0
2
4
6
8
10
x/D [−]
$V_d = 2\ cm^3$
$V_d = 3\ cm^3$
$V_d = 6\ cm^3$
$g = 3\ \mu m$
$\epsilon = 0.5\ cm$
$\mathbf{f} = 0.1$
$x_{dead} = 0.3\ cm$


Linear Motor
Compression Chamber
Springs
50.30 mm
Piston
Valve Assembly
102 mm