Sensitivity Analysis of Characteristic Parameters of Railway Electric Traction System

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***Abstract*—The modal analysis, which decouples the contact wire and rail track conductors, is used to develop an integrated model of single rail track and contact wire. The characteristic impedance and propagation constant matrices of integrated model have been computed. The authors have studied effects of various parameters of interest on characteristic impedance and propagation constant using sensitivity analysis. The sensitivity functions of series impedance and shunt admittance related to contact wire and catenary have been developed and validated in this paper.**

***Index Terms*—railway traction lines, characteristic parameters, sensitivity analysis, parameter sensitivity**

I. Introduction

Railway electric traction system is a fast and green transportation system with enhanced load carrying capacity. The analysis and design of power and signaling system of railway electric traction system needs mathematical model of the system. The authors in this paper have used the multi-conductor transmission line theory to develop the mathematical model of the electric traction system and to compute the characteristic impedance and propagation constant of this system. This approach can then be extended to find the voltage and current distribution of the system. The characteristic impedance and propagation constant are the functions of line parameters i.e. self and mutual impedances and admittances of contact wire and rail track. The numerical values of these parameters are not accurately known under practical conditions as they are dependent on varying/uncertain parameters like permeability, supply frequency and shape of the conductor. In order to study the effect of these uncertain parameters on the characteristic parameters of the railway electric traction system, the authors have carried out the sensitivity analysis of characteristic impedance and propagation constant with respect to the above mentioned parameters of interest for the first time in the literature. In a rail track ferromagnetic material, both saturation and hysteresis are observed. This affects the permeability of rail track. The permeability also depends upon the composition of rail track material which may not remain constant throughout the length of the track. Further, to enhance the performance of the traction system, it is desirable to consider a conductor with a cross-section that will produce uniform magnetic field on its surface. Also, the supply side contains the harmonic frequency components due to the power electronic devices which are inherent to the modern electric traction drives. The variations in these parameters of interest motivated the authors to carry out the sensitivity analysis of characteristic parameters of the system wrt these parameters.

The complete mathematical model of railway electric traction lines, which include the contact wire and rail track, has received little attention in the literature. Most of the traction studies are limited to railway track modeling. R.J. Hill *et al.* [1] has computed frequency dependent self and mutual admittance for a single track power-rail, considering it as a distributed transmission line. They have computed the impedance by numerical calculations and compared with analytic impedance models based on the Carson eqns. for a stratified weakly conducting ground. A. Mariscotti has applied multi conductor transmission lines theory to railway traction system to find the distribution of the traction return current in AC and DC electric railway systems and carried out sensitivity analysis of rail current and rail to rail voltage wrt parameters of interest [2]. He has also carried out modeling of track circuit signaling system which may be helpful in automatic train control [3]. Measurement of currents at power-supply frequency and validation of a multi-conductor transmission-line model is done for (2x25kV) electric railway system with autotransformer [4]. An electromagnetic field model for identification of distributed self and mutual shunt admittances using multi-conductor transmission lines (MTL) is formulated and used to study the parametric behavior of single track admittance [5]. Andrea Mariscotti *et al.* [6] have analyzed the published experimental data and formulae for the determination of the electric parameters of railway traction lines. He observed that the experimental results conform with the

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calculated results in frequency range of 50Hz-50 kHz. He also focused on the CCITT simplified method for the evaluation of induced voltages in electric traction system [7].

II. Configuration of Railway Electric Traction Lines

In modern overhead electric traction system, the catenary is fed at 25kV, 1-phase ac through the feeding posts which are positioned at frequent intervals alongside the track. The catenary energizes the contact wire which is kept at constant height and in the right position with the help of droppers. The pantograph, a high-strength tubular steel structure which can be raised or lowered, is used to make contact with the overhead contact wire to draw energy to feed the drives of the locomotive. Fig. 1 shows the physical model of railway electric traction system.

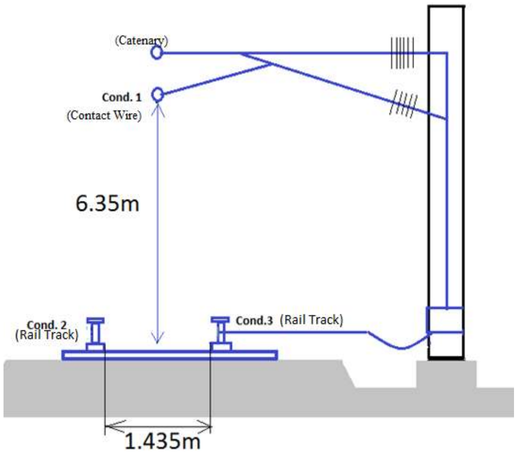

Figure 1. Physical model of railway electric traction system

III. Mathematical Model

A mathematical model based on the electromagnetic properties of rail track is necessary for the analysis and design of railway electric power and signaling systems. The studies related to propagation of voltages and currents signals over the railway electric traction lines need the series impedance and shunt admittance matrices per unit length. These matrices state the electrostatic and electromagnetic performance of the system and depend on the physical and electrical characteristics of the rail track conductor, the geometric arrangement of the conductors and rail track, height above the earth level, permeability and the earth resistivity [8]. The accuracy of the result depends, therefore, on accuracy with which the basic series impedance and shunt admittance matrices of railway electric traction system have been formed.

A. Series Impedance Matrix

The series impedance matrix, Z can be given by (1)

$$Z = R_c + R_e + j\,(X_c+X_e+X_g) \quad (1)$$

where suffix 'c' denotes the quantities relating to conductor, 'e' denotes the quantities relating to earth and 'g' denotes the quantities relating to geometric arrangement of the conductors The resistance matrix R_c is a diagonal matrix whose off diagonal elements are zero. The resistance of the catenary conductor, R_{c11},is given by (2),

$$R_{c11} = S \times R_o \quad (2)$$

where R_o is dc resistance of the conductor, S is skin effect ratio and is given by (3) :

$$S = \frac{1+\sqrt{\left(1+\frac{x^4}{48}\right)}}{2}, \; x = \pi d\sqrt{\frac{2f\mu\times10^{-5}}{\rho}} \quad (3)$$

where d is the diameter of catenary in mm, f is frequency of supply in Hz, μ is permeability of conductor material and ρ is the resistivity of material in micro-ohm cm.

The resistance of rail track conductors, R_{c22} and R_{c33} is given by (4)[9]:

$$R_{c22}=R_{c33}=\frac{1}{\pi a^2\sigma_r}\left[\frac{1}{4}+\sqrt{2}\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right]\left[\frac{a}{2\delta_r}+\frac{3\delta_r}{32a}\right] \quad (4)$$

where, σ_r is the conductivity of rail track, θ is the hysteresis angle , δ_r is skin depth and 'a' is the equivalent radius of rail track and may be calculated considering the rail track as an equivalent I structure[9].

$$X_{c11} = \omega\frac{\mu_0}{8\pi} \quad (5)$$

The reactance of track conductors X_{c22} and X_{c33} is given by (6)[9].

$$X_{c22}=X_{c33}=\frac{1}{\pi a^2\sigma_r}\left[\sqrt{2}\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right]\left[\frac{a}{2\delta_r}-\frac{3\delta_r}{32a}\right] \quad (6)$$

The resistance and reactance due to earth path, R_e and X_e, are computed using Carson's formula [10] given by (7.1) and (7.2) respectively.

$$R_e = [10^{(-4)} \quad (8\,\pi\,P\,f)] \quad (7.1)$$
$$X_e = [10^{(-4)} \quad (8\,\pi\,Q\,f)] \quad (7.2)$$

P and Q are calculated using Carson's series [10] in terms of two parameters c_{ij} and the angle subtended between the conductor and the image, θ_{ij}.

Inductive reactance due to geometry of conductors above the ground, X_g is given by (8.1),

$$X_g = 2\pi f\,(\mu_0\mu_r)\,F \quad (8.1)$$

where, F is Maxwell's coefficient matrix, whose elements are given by (8.2),

$$\ln(2H_i/r_i),\; i=1,2,3 \;;\; \ln(x_{ij}/u_{ij}),\; i=1,2,3,\; j=1,2,3,\; i\neq j \quad (8.2)$$

Each term in the matrix represents the distance between two conductors or the conductor and the image .

B. Shunt-Admittance Matrix

The shunt-admittance matrix, Y is a function of the physical geometry of the conductor relative to the earth plane. Further, because the rail track conductor is on the ground, it has appreciable leakage currents while the catenary conductor is located at a sufficient height from the ground and from the rail track conductors hence the leakage currents from catenary to ground and from catenary to rail conductors are assumed to be zero [8].

The admittance matrix is given by (9):

$$Y=G+j\omega C \tag{9}$$

where, G is the conductance matrix. The elements of conductance matrix corresponding to the catenary are zero and the elements corresponding to rail track are given by (10) [5].

$$g_{22}=g_{33}=\frac{\pi\sigma_r}{\ln 2b/a};\ g_{23}=g_{32}=\frac{\pi\sigma_r}{\ln 2} \tag{10}$$

where, σ_r is conductivity of rail track, b is distance between two rail track.

The capacitance matrix $[C]$ is given by (11) ,

$$[C]=2\pi\varepsilon_0\,[F]^{-1} \tag{11}$$

IV. Modal Theory

In order to compute the characteristic parameters of railway electric traction lines, modal theory has been applied to decouple the phase quantities to modal quantities using the transformation given by(12)[11]-[12].

$$Pq=\lambda q \tag{12}$$

where, P is the product matrix and is given by (13)

$$P=[Z][Y] \tag{13}$$

λ is the eigen value of P and q is the corresponding eigen vector.

Thus, as given by (14), phase voltage and currents can be transformed into modal quantities which make the product matrix [Z][Y] diagonal.

$$[Q]^{-1}[Z]\,[Y][Q]=[\lambda] \tag{14}$$

where $[Q]$ is the transformation matrix of phase voltages, constituted by the eigenvectors associated with each of the eigen values, λ_i , of $[P]=[Z]\ [Y]$. The eigen value matrix, λ can be given by (15),

$$[\lambda_i]=\mathrm{diag}(\lambda_1,\ ....,\lambda_n) \tag{15}$$

The propagation constant $\gamma^{(k)}$ of the kth mode is equal to the square root of the kth eigen value of [Z] [Y] and is given by (16),

$$\gamma^{(k)}=\lambda_k^{1/2}=\alpha^{(k)}+j\beta^{(k)} \tag{16}$$

where, $\alpha^{(k)}$ is the attenuation constant and $\beta^{(k)}$ is the phase coefficient. The phase velocity $v^{(k)}$ is obtained from (17)

$$v^{(k)}=\omega/\beta^{(k)} \tag{17}$$

where ω is the angular velocity of phase voltages.

The characteristic impedance Z_0, is given by (18)

$$Z_o=\sqrt{Z/Y}=Q\,\lambda^{-1/2}\,Q^{-1}\ Z \tag{18}$$

V. Parameter Senstivity

Sensitivity analysis is an effective method to predict the effect of a parameter on the response of the system. Mathematically, it is given by (19) [13]:

$$\hat{S}_p^F=\frac{\partial F/F}{\partial p/p}=\left(\frac{p}{F}\right)\left(\frac{\partial F}{\partial p}\right)=\frac{\partial(\log F)}{\partial(\log P)} \tag{19}$$

The (19) is applicable for small parameter variations; however, for large parameter variations the same equation can be used in steps, assigning small variations to the parameters until the values corresponding to the system are reached. In any physical system, generally more than one parameter changes simultaneously, for eg., in railway traction system the permeability of the rail track, the earth resistivity, the supply frequency and height of the catenary conductor etc. change simultaneously. Therefore, in order to carry out the complete sensitivity, the Jacobian matrix can be developed as given by (20).

$$J_{0P_i}^{F}=\begin{Bmatrix}\partial F_1/\partial P_1\partial F_1/\partial P_2\ldots\partial F_1/\partial P_N\\ \vdots \qquad\qquad \vdots\\ \partial F_A/\partial P_1\partial F_A/\partial P_{2\ldots\ldots}\partial F_A/\partial P_N\end{Bmatrix} \tag{20}$$

A. Sensitivity Functions

The normalised sensitivity of characteristic impedance, Z_o, and propagation constant, γ with respect to the generic parameter x_i is given by (21) – (24),

$$\hat{S}_{x_i}^{Z_o}=\frac{x_i}{Z_o}\,S_{x_i}^{Z_o} \tag{21}$$

$$\hat{S}_{x_i}^{\gamma}=\frac{x_i}{\gamma}\,S_{x_i}^{\gamma} \tag{22}$$

where,

$$S_{x_i}^{Z_o}=\frac{\partial[Z_0]}{\partial x_i}=\frac{1}{2}\left(Z^{-1/2}\ \frac{\partial[Z]}{\partial x_i}Y^{1/2}-Z^{1/2}Y^{-3/2}\ \frac{\partial[Y]}{\partial x_i}\right) \tag{23}$$

$$S_{x_i}^{\gamma}=\frac{\partial[\gamma]}{\partial x_i}=\frac{1}{2}\left(Z^{-1/2}\ \frac{\partial[Z]}{\partial x_i}Y^{1/2}+Z^{1/2}Y^{-1/2}\ \frac{\partial[Y]}{\partial x_i}\right) \tag{24}$$

In order to determine the normalised sensitivity of Z_o and γ, the sensitivity functions of [Z] and [Y] are calculated as given by (25) and (26) respectively,[11]

$$[S_{x_i}^{Z}]=\frac{\partial[Z]}{\partial x_i}=\frac{\partial \mathrm{Re}}{\partial x_i}+\frac{\partial \mathrm{Rc}}{\partial x_i}+\left(\frac{\partial \mathrm{Xe}}{\partial x_i}+\frac{\partial \mathrm{Xc}}{\partial x_i}+\frac{\partial \mathrm{Xg}}{\partial x_i}\right) \tag{25}$$

$$[S_{x_i}^{Y}]=\frac{\partial[Y]}{\partial x_i}=\frac{\partial \mathrm{G}}{\partial x_i}+\left(\frac{\partial \mathrm{C}}{\partial x_i}\right);\ \ i=1,2,\ .....,N_p \tag{26}$$

where N_p represents the number of parameters of interest. From (13), the sensitivity functions of the matrix [P] are obtained as given by (27):

$$[S_{x_i}^{P}]=[S_{x_i}^{Z}]\ [Y]+[Z]\ [S_{x_i}^{Y}] \tag{27}$$

The sensitivity matrix of [P] is used to find the sensitivities of all the eigen values λ_k of [P], as given by (28) [14],

$$[S_{x_i}^{\lambda}]=\frac{\{[S_{xi}^{P}][Q_k]\}*[S_k]}{[Q_k]*[S_k]} \tag{28}$$

where $[Q_k]$, $[S_k]$ are the eigenvectors of [P] and $[P_t]$ respectively, associated with the eigen value λ_k , and the asterisk indicates the scalar product of two vectors.

From (16), the sensitivity functions of the modal propagation constants $\gamma^{(k)}$ are given by (29)-(32),

$$S_{x_i}^{\gamma(k)}=S_{x_i}^{\alpha(k)}+jS_{x_i}^{\beta(k)}=(2\gamma^{(k)})^{-1}\,S_{x_i}^{\lambda(k)} \tag{29}$$

$$S_{x_i}^{\alpha(k)}=R_e\{(2\gamma^{(k)})^{-1}S_{x_i}^{\lambda(k)} \tag{30}$$

$$S_{x_i}^{\beta(k)} = I_m\{(2\gamma^{(k)})^{-1} S_{x_i}^{\lambda(k)}\} \quad (31)$$

$$S_{x_i}^{v(k)} = \frac{v^{(k)}}{\beta^{(k)}} S_{x_i}^{\beta(k)} \quad (32)$$

Finally, the normalised sensitivity functions are obtained as given by (33) - (35):

$$\hat{S}_{x_i}^{\alpha(k)} = \frac{x_i}{\alpha^{(k)}} S_{x_i}^{\alpha(k)} \quad (33)$$

$$\hat{S}_{x_i}^{\beta(k)} = \frac{x_i}{\beta^{(k)}} S_{x_i}^{\beta(k)} \quad (34)$$

$$\hat{S}_{x_i}^{v(k)} = \frac{x_i}{v^{(k)}} S_{x_i}^{v(k)} = \hat{S}_{x_i}^{\beta(k)} \quad (35)$$

VI. Numerical Results

The characteristic parameters of the system are computed using the specifications for catenary and rail track as given in Appendix A. Using (12)- (18), the characteristic impedance, Z_o, propagation constant, γ, and modes of propagation are computed and presented in Table I, II and III.

TABLE I. CHARACTERISTIC IMPEDANCE MATRIX

Characteristic Impedance, Z_0 in ohms
$\begin{bmatrix} 545.99 - 57.47j & 0.65 + 0.50j & 0.65 + 0.50j \\ 0.65 + 0.50j & 7.13 + 6.92j & 0.49 - 0.55j \\ 0.65 + 0.50j & 0.49 - 0.55j & 7.13 + 6.92j \end{bmatrix}$

TABLE II. PROPAGATION CONSTANT

γ_{11} (/m)	$(10)^{-3}[0.0001 + 0.0013j]$
$\gamma_{22} = \gamma_{33}$ (/m)	$(10)^{-3}[0.2497 + 0.2413j]$

TABLE III. MODES OF PROPAGATION

Ground Mode	Aerial Mode I	Aerial Mode II
$0.124 - 0.013j$	$-0.000 - 0.000j$	$1.000 + 0.000j$
$0.706 + 0.000j$	$0.707 + 0.000j$	$-0.0005 - 0.003j$
$0.706 + 0.000j$	$-0.707 + 0.000j$	$-0.0005 - 0.003j$

From the Table III, it is observed that there are three modes of propagation for a catenary and rail track lines as shown in Fig. 2. The three modes are independent to each other. The mode related to maximum attenuation and lowest velocity of attenuation is called ground mode whereas the mode having minimum attenuation and maximum velocity is called aerial mode 1 and the other is called aerial mode 2. For ground mode, current is flowing in all of the three conductors. In aerial mode 1, only two outer conductors take part in signal propagation. Here current is entering the rail track conductor 2 and leaving through rail track conductor 3. As the resistivity of catenary conductor is much smaller as compared to rail track, current coming out from catenary is much higher compared to current flowing through track conductors for aerial mode 2.

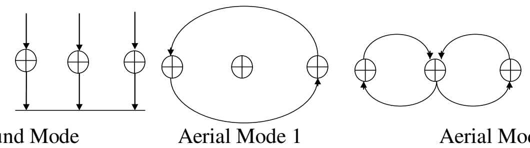

Figure 2. Modes of propagation

Fig. 3-Fig. 6 shows the variation of characteristic impedance of contact wire and rail track conductor respectively with respect to frequency for two frequency ranges. It is noted that characteristic impedance of contact wire decreases wrt the frequency up to around 170 kHz and then starts increasing at higher frequencies whereas the characteristic impedance of rail track conductor increases exponentially with the frequency up to nearly 400 kHz and then it becomes approximately constant at around 620 ohms. It is also observed that propagation constant of both contact wire and rail track conductor increases linearly with the frequency as shown in Fig. 7 and Fig. 8. Fig. 9 and Fig. 10 show that characteristic impedance and propagation constant of rail track conductor varies linearly with the permeability of rail track conductor.

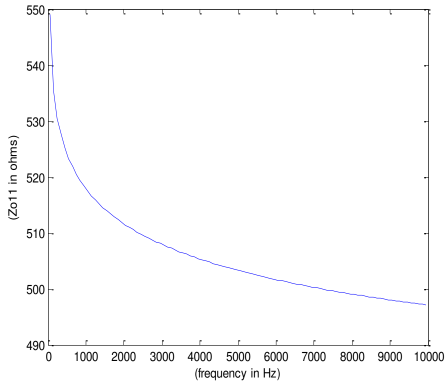

Figure 3. Variation of Z_{o11} with frequency(0-10k Hz.)

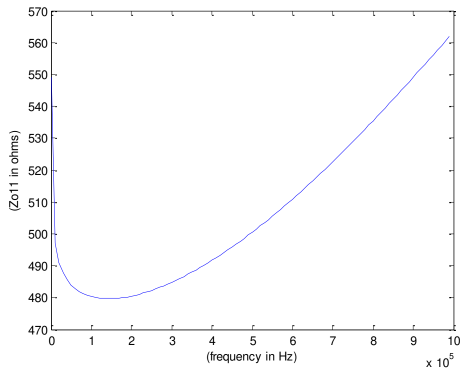

Figure 4. Variation of Z_{o11} with frequency(0-1M Hz.)

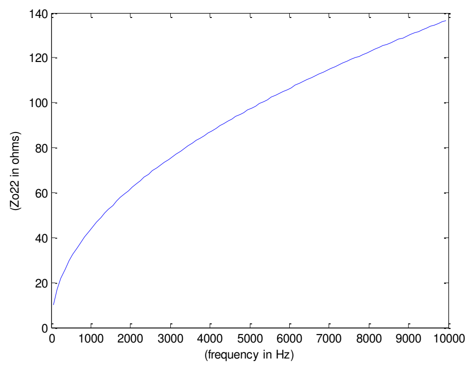

Figure 5. Variation of Z_{o22} with frequency(0-10k Hz.)

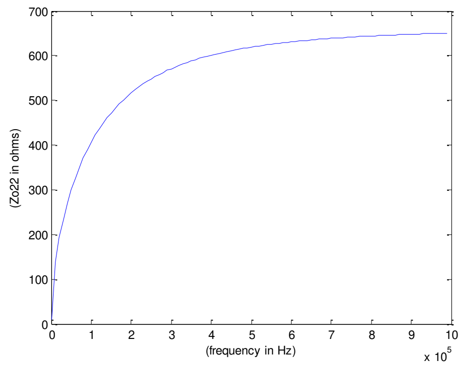

Figure 6. Variation of Z_{o22} with frequency(0-1M Hz.)

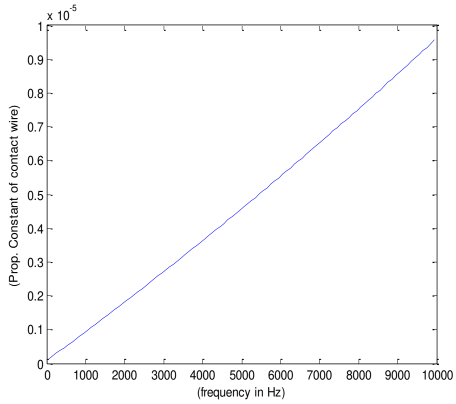

Figure 7. Variation of γ_{11} with frequency(0-10 kHz.)

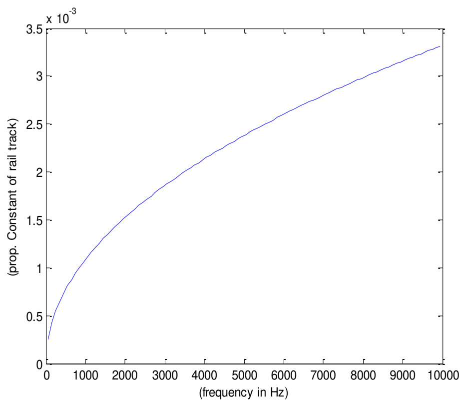

Figure 8. Variation of γ_{22} with frequency(0-10kHz.)

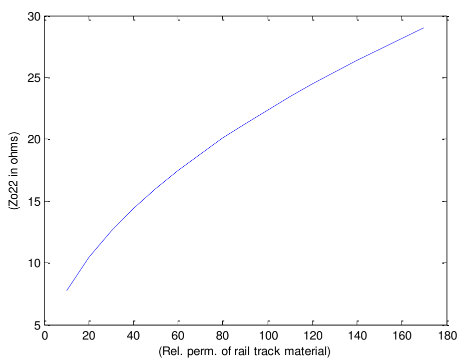

Figure 9. Variation of Z_{022} with μ_{rr}

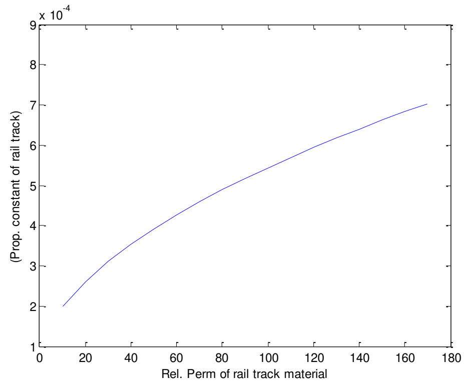

Figure 10. Variation of γ_{22} with μ_{rr}

Further, the effect of catenary parameters and rail track parameters on characteristic impedance and propagation constant are studied using sensitivity model. The sensitivity analysis of the series impedance and the shunt admittance, on which the characteristic impedance, propagation constant matrix, velocity of propagation and attenuation depend, has been carried out for the system under consideration. The sensitivity functions for series impedance and shunt admittance have been developed, with respect to radius of rail track, supply frequency and permeability of rail track material, first time in the literature and are presented in Appendix B. The results for normalized sensitivity are presented in Table IV. In order to validate the results of Table IV, normalized sensitivity has also been computed using difference equations. The results are presented in Table V and it is observed that the sensitivity computed by two methods conform to each other.

TABLE IV. COMPUTED VALUES OF NORMALIZED SENSITIVITY FROM SENSITIVITY FUNCTIONS DEVELOPED

Parameter → Normalised sensitivity	Equivalent Radius of Rail Track	Supply Frequency	Permeability Of Rail Track
$\hat{S}_p^{Rc22} = \hat{S}_p^{Rc33}$	-1.0266	0.5	0.5
$\hat{S}_p^{Xc22} = \hat{S}_p^{Xc33}$	-0.974	0.5	0.5
$\hat{S}_p^{Xg22} = \hat{S}_p^{Xg33}$	-0.389	1	1
$\hat{S}_p^{Xg11}$	0	1	0
$\hat{S}_p^{Xg12} = \hat{S}_p^{Xg13} = \hat{S}_p^{Xg21} = \hat{S}_p^{Xg31}$	0	1	0
$\hat{S}_p^{Xg23} = \hat{S}_p^{Xg32}$	0	1	1
$\hat{S}_p^{Xyc11}$	0	1	0
$\hat{S}_p^{Xyc22} = \hat{S}_p^{Xyc23}$	0.409	1	0
$\hat{S}_p^{Xyc13} = \hat{S}_p^{Xyc12} = \hat{S}_p^{Xyc31} = \hat{S}_p^{Xyc21}$	0	1	0
$\hat{S}_p^{Xyc23} = \hat{S}_p^{Xyc32}$	0	1	0
$\hat{S}_p^{G22} = \hat{S}_p^{G33}$	0	0	0
$\hat{S}_p^{G23} = \hat{S}_p^{G32}$	0.310	0	0

TABLE V. COMPUTED VALUES OF NORMALIZED SENSITIVITY FROM DIFFERENCE EQUATIONS

Parameter → Normalised sensitivity	Equivalent Radius of Rail Track	Supply Frequency	Permeability of Rail Track
$\hat{S}_p^{Rc22} = \hat{S}_p^{Rc33}$	-1.057	0.516	0.528
$\hat{S}_p^{Xc22} = \hat{S}_p^{Xc33}$	-0.980	0.581	0.5
$\hat{S}_p^{Xg22} = \hat{S}_p^{Xg33}$	-0.392	0.991	1.01
$\hat{S}_p^{Xg11}$	0	1.006	0
$\hat{S}_p^{Xg12} = \hat{S}_p^{Xg13} = \hat{S}_p^{Xg21} = \hat{S}_p^{Xg31}$	0	1	0
$\hat{S}_p^{Xg23} = \hat{S}_p^{Xg32}$	0	0.900	1.005
$\hat{S}_p^{Xyc11}$	0	1	0
$\hat{S}_p^{Xyc22} = \hat{S}_p^{Xyc23}$	0.3894	0.910	0
$\hat{S}_p^{Xyc13} = \hat{S}_p^{Xyc12} = \hat{S}_p^{Xyc31} = \hat{S}_p^{Xyc21}$	0	1	0
$\hat{S}_p^{Xyc23} = \hat{S}_p^{Xyc32}$	0	1	0
$\hat{S}_p^{G22} = \hat{S}_p^{G33}$	0	0	0
$\hat{S}_p^{G23} = \hat{S}_p^{G32}$	0.331	0	0

From the Table IV, it is observed that the variation in the radius of rail track affects the resistance of the rail track the most while the mutual conductance between the rail track is least affected wrt radius of rail track. The other quantities viz. catenary resistance and reactance as well as the self and mutual capacitance of both rail track and catenary are insensitive to radius of rail track. Further, the sensitivity functions and hence, sensitivity of series impedance and shunt admittance matrix elements wrt frequency and permeability are constant.

The normalized sensitivity of characteristic impedance and propagation constant wrt same parameters of interest is calculated and presented in the Table VI.

TABLE VI. NORMALIZED SENSITIVITY OF CHARACTERISTIC IMPEDANCE, Z_o AND PROPAGATION CONSTANT, γ

Parameter → Normalised sensitivity	Equivalent Radius Of Rail Track	Supply Frequency	Permeability Of Rail Track
$\hat{S}_p^{Z011}$	-0.0008 + 0.0006i	-0.0715 + 0.0925i	-0.0001 - 0.0013i
$\hat{S}_p^{Z022} = \hat{S}_p^{Z033}$	-0.3088 + 0.0122i	1.4504 - 0.7840i	0.1221 - 0.2348i
$\hat{S}_p^{Y11}$	-0.0343 - 0.1262i	0.9769 - 0.1527i	0.0565 + 0.0148i
$\hat{S}_p^{Y22} = \hat{S}_p^{Y33}$	-0.3316 - 0.0096i	0.3953 + 0.0379i	0.1017 - 0.1820i

From the Table VI, it is noted that wrt supply frequency, the sensitivity of characteristic impedance of rail track conductors is higher than the sensitivity of characteristic impedance of contact wire whereas propagation constant of contact wire is more sensitive than propagation constant of rail track conductors. The permeability of rail track affects mainly the characteristic impedance and propagation constant of rail track conductor. The equivalent radius of rail track also has negligible effect on the characteristic parameters of contact wire conductors but appreciably effects the parameters related to rail track conductors.

VII. CONCLUSIONS

The mathematical model of integrated contact wire cum rail track has been developed using modal analysis. Characteristic impedance and the propagation constant have been computed and three modes of propagation have been identified. The normalized sensitivity functions related to characteristic impedance and propagation constant have been developed wrt parameters of interest first time in the literature. The results are validated using difference equations. It is observed that characteristic parameters are most sensitive wrt frequency whereas permeability of rail track has minimum effect on these parameters.

The variation of characteristic impedance and propagation constant has been drawn for different frequency range. Also the effect of permeability on these parameters has been studied. The studies shall be useful for the system and design engineer for the protection, signalling and automation schemes of railway electric traction lines.

APPENDIX A SPECIFICATIONS AND DATA USED.

supply frequency = 50Hz;
earth resistivity =100Ωm;
height of catenary above ground = 6.35m;
height of rail track above ground = 0.75m;
area of catenary conductor =157mm^2;
radius of rail conductor = 115mm;
permeability of rail conductor material = 18;
hysteresis angle = 0.4675radians;
contact wire resistivity = 2.78 10-8 Ωm;
rail track resistivity = 0.225× 10-6 Ωm;

APPENDIX B SENSITIVITY FUNCTIONS DEVELOPED

The sensitivity functions of series impedance and shunt admittance matrix are developed with respect to radius of rail track, supply frequency and permeability of rail material as under:

The sensitivity functions with respect to radius of rail track are given by B.1 – B.10:

$$\hat{S}_a^{Rc22} = \hat{S}_a^{Rc33} = \frac{a}{R_{c22}} * \frac{\partial R_{c22}}{\partial a} = -\left[1 + \frac{[6\delta_r/32a]}{[a/2\delta_r + 3\delta_r/32a]}\right] \quad \text{(B.1)}$$

$$\hat{S}_a^{Rc11} = \hat{S}_a^{Re11} = \hat{S}_a^{Re22} = \hat{S}_a^{Re33} = \hat{S}_a^{Re13} = \hat{S}_a^{Re12} = \hat{S}_a^{Re21} = \hat{S}_a^{Re31} = \hat{S}_a^{Re23} = \hat{S}_a^{Re32} = 0 \quad \text{(B.2)}$$

$$\hat{S}_a^{Xc22} = \hat{S}_a^{Xc33} = \frac{a}{X_{c22}} * \frac{\partial X_{c22}}{\partial a} = -\left[1 - \frac{[6\delta_r/32a]}{[a/2\delta_r + 3\delta_r/32a]}\right] \quad \text{(B.3)}$$

$$\hat{S}_a^{Xc12} = \hat{S}_a^{Xc13} = \hat{S}_a^{Xc11} = \hat{S}_a^{Xc21} = \widehat{S}_a^{Xc23} = \hat{S}_a^{Xc31} = \hat{S}_a^{Xc32} = 0 \quad \text{(B.4)}$$

$$\hat{S}_a^{Xe11} = \hat{S}_a^{Xe22} = \hat{S}_a^{Xe33} = \hat{S}_a^{Xe13} = \hat{S}_a^{Xe12} = \hat{S}_a^{Xe21} = \hat{S}_a^{Xe31} = \hat{S}_a^{Xe23} = \hat{S}_a^{Xe32} = 0 \quad \text{(B.5)}$$

$$\hat{S}_a^{Xg22} = \hat{S}_a^{Xg33} = \frac{a}{X_{g22}} * \frac{\partial X_{g22}}{\partial a} = -\frac{1}{\log_n 2h/a} \quad \text{(B.6)}$$

$$\hat{S}_a^{Xg12} = \hat{S}_a^{Xg13} = \hat{S}_a^{Xg11} = \hat{S}_a^{Xg21} = \widehat{S}_a^{Xg23} = \hat{S}_a^{Xg31} = \hat{S}_a^{Xg32} = \hat{S}_a^{G22} = \hat{S}_a^{G33} = 0 \quad \text{(B.7)}$$

$$\hat{S}_a^{G23} = \hat{S}_a^{G32} = \frac{a}{G_{23}} * \frac{\partial G_{23}}{\partial a} = \frac{1}{\log_n 2b/a} \quad \text{(B.8)}$$

$$\hat{S}_a^{C22} = \hat{S}_a^{C33} = \frac{a}{C_{22}} * \frac{\partial C_{22}}{\partial a} = \frac{1}{\log_n 2h/a} \quad \text{(B.9)}$$

$$\hat{S}_a^{C11} = \hat{S}_a^{C12} = \hat{S}_a^{C13} = \hat{S}_a^{C21} = \hat{S}_a^{C23} = \hat{S}_a^{C31} = \hat{S}_a^{C32} = 0 \quad \text{(B.10)}$$

Sensitivity functions wrt frequency are given by B.11 –B.17:

$$\hat{S}_{\omega}^{Rc22} = \hat{S}_{\omega}^{Rc33} = \frac{\omega}{R_{c22}} * \frac{\partial R_{c22}}{\partial \omega} = \frac{1}{2} - \frac{[3\delta_r/32a]}{[a/2\delta_r + 3\delta_r/32a]} \quad \text{(B.11)}$$

$$\hat{S}_{\omega}^{Rc11} = \frac{x^4}{S(4S-2)} \quad \text{(B.12)}$$

$$\hat{S}_{\omega}^{Re} = \left[1 + \frac{\omega}{P}\frac{\partial P}{\partial \omega}\right] \quad \text{(B.13)}$$

$$\frac{\partial P}{\partial \omega} = \frac{-\pi}{8}\frac{\partial S_4}{\partial \omega} + \frac{1}{2}\left(\log\frac{2}{\gamma\, c_{ij}}\right)\frac{\partial S_2}{\partial \omega} - \frac{1}{2}\frac{s_2}{c_{ij}}\frac{\partial C_{ij}}{\partial \omega} + \frac{1}{2}\theta ij\frac{\partial S_2'}{\partial \omega} - \frac{1}{2}\frac{\partial \sigma_1}{\partial \omega} + \frac{1}{2\sqrt{2}}\frac{\partial \sigma_2}{\partial \omega} \quad \text{(B.13.1)}$$

$$\frac{\partial S_4}{\partial \omega} = \frac{4n}{C_{ij}} S_4 \frac{\partial C_{ij}}{\partial \omega};\ \frac{\partial S_2}{\partial \omega} = \frac{2(2n+1)S_2}{c_{ij}}\frac{\partial C_{ij}}{\partial \omega};\ \frac{\partial S_2'}{\partial \omega} = \frac{2(2n+1)S_2'}{c_{ij}}\frac{\partial C_{ij}}{\partial \omega}; \quad \text{(B.13.2)}$$

$$\frac{\partial \sigma_1}{\partial \omega} = \frac{(4n-1)\sigma_1}{c_{ij}}\frac{\partial C_{ij}}{\partial \omega};\ \frac{\partial \sigma_2}{\partial \omega} = \frac{2(2n+1)\sigma_2}{c_{ij}}\frac{\partial C_{ij}}{\partial \omega};\ \frac{\partial C_{ij}}{\partial \omega} = \frac{C_{ij}}{2\omega} \quad \text{(B.13.2)}$$

$$\hat{S}_{\omega}^{Xe} = \left[1 + \frac{\omega}{Q}\frac{\partial Q}{\partial \omega}\right] \quad \text{(B.14)}$$

$$\frac{\partial Q}{\partial \omega} = -\left(\log\frac{2}{\gamma\, c_{ij}}\right)\frac{\partial S_4}{\partial \omega} - \frac{1}{2}\frac{s_4}{c_{ij}}\frac{\partial C_{ij}}{\partial \omega} - \frac{1}{2}\theta ij\frac{\partial S_4'}{\partial \omega} + \frac{1}{\sqrt{2}}\frac{\partial \sigma_1}{\partial \omega} - \frac{\pi}{8}\frac{\partial S_2}{\partial \omega} + \frac{1}{\sqrt{2}}\frac{\partial \sigma_3}{\partial \omega} - \frac{1}{2}\frac{\partial \sigma_4}{\partial \omega} \quad \text{(B.14.1)}$$

$$\frac{\partial S_4'}{\partial \omega} = \frac{4n}{C_{ij}} S_4' \frac{\partial C_{ij}}{\partial \omega};\ \frac{\partial \sigma_3}{\partial \omega} = \frac{(4n-1)\sigma_3}{c_{ij}}\frac{\partial C_{ij}}{\partial \omega};\ \frac{\partial \sigma_4}{\partial \omega} = \frac{(4n)\sigma_4}{c_{ij}}\frac{\partial C_{ij}}{\partial \omega} \quad \text{(B.14.2)}$$

$$\hat{S}_{\omega}^{Xc22} = \hat{S}_{\omega}^{Xc33} = \frac{\omega}{X_{c22}} * \frac{\partial X_{c22}}{\partial \omega} = \frac{1}{2} + \frac{[3\delta_r/32a]}{[a/2\delta_r - 3\delta_r/32a]} \quad \text{(B.15)}$$

$$\hat{S}_{\omega}^{Xg11} = \hat{S}_{\omega}^{Xg12} = \hat{S}_{\omega}^{Xg13} = \hat{S}_{\omega}^{Xg21} = \hat{S}_{\omega}^{Xg22} = \hat{S}_{\omega}^{Xg23} = \hat{S}_{\omega}^{Xg31} = \hat{S}_{\omega}^{Xg32} = \hat{S}_{\omega}^{Xg33} = 1 \quad \text{(B.16)}$$

$$\hat{S}_{\omega}^{C11} = \hat{S}_{\omega}^{C12} = \hat{S}_{\omega}^{C13} = \hat{S}_{\omega}^{C21} = \hat{S}_{\omega}^{C22} = \hat{S}_{\omega}^{C23} = \hat{S}_{\omega}^{C31} = \hat{S}_{\omega}^{C32} = \hat{S}_{\omega}^{C33} = 1 \quad \text{(B.17)}$$

Sensitivity functions with respect to permeability of rail track are given by B.18 – B.25:

$$\hat{S}_{\mu r}^{Rc22} = \hat{S}_{\mu r}^{Rc33} = \frac{\mu_r}{R_{c22}} * \frac{\partial R_{c22}}{\partial \mu_r} = \frac{1}{2} - \frac{[3\delta_r/32a]}{[a/2\delta_r + 3\delta_r/32a]} \quad \text{(B.18)}$$

$$\hat{S}_{\mu r}^{Rc11} = \hat{S}_{\mu r}^{Xg11} = \hat{S}_{\mu r}^{Xg12} = \hat{S}_{\mu r}^{Xg13} = \hat{S}_{\mu r}^{Xg21} = \hat{S}_{\mu r}^{Xg31} = 0 \quad \text{(B.19)}$$

$$\hat{S}_{\mu r}^{Re11} = \hat{S}_{\mu r}^{Re22} = \hat{S}_{\mu r}^{Re33} = \hat{S}_{\mu r}^{Re13} = \hat{S}_{\mu r}^{Re12} = \hat{S}_{\mu r}^{Re21} = \hat{S}_{\mu r}^{Re31} = \hat{S}_{\mu r}^{Re23} = \hat{S}_{\mu r}^{Re32} = 0 \quad \text{(B.20)}$$

$$\hat{S}_{\mu r}^{Xe11} = \hat{S}_{\mu r}^{Xe22} = \hat{S}_{\mu r}^{Xe33} = \hat{S}_{\mu r}^{Xe13} = \hat{S}_{\mu r}^{Xe12} = \hat{S}_{\mu r}^{Xe21} = \hat{S}_{\mu r}^{Xe31} = \hat{S}_{\mu r}^{Xe23} = \hat{S}_{\mu r}^{Xe32} = 0 \quad \text{(B.21)}$$

$$\hat{S}_{\mu r}^{Xc22} = \hat{S}_{\mu r}^{Xc33} = \frac{\mu r}{X_{c22}} * \frac{\partial X_{c22}}{\partial \mu r} = \frac{1}{2} + \frac{[3\delta_r/32a]}{[a/2\delta_r - 3\delta_r/32a]} \quad \text{(B.22)}$$

$$\hat{S}_{\mu r}^{Xg22} = \hat{S}_{\mu r}^{Xg23} = \hat{S}_{\mu r}^{Xg32} = \hat{S}_{\mu r}^{Xg33} = 1 \quad \text{(B.23)}$$

$$\hat{S}_{\mu r}^{C11} = \hat{S}_{\mu r}^{C12} = \hat{S}_{\mu r}^{C13} = \hat{S}_{\mu r}^{C21} = \hat{S}_{\mu r}^{C22} = \hat{S}_{\mu r}^{C23} = \hat{S}_{\mu r}^{C31} = \hat{S}_{\mu r}^{C32} = \hat{S}_{\mu r}^{C33} = 0 \quad \text{(B.24)}$$

$$\hat{S}_{\mu r}^{G22} = \hat{S}_{\mu r}^{G23} = \hat{S}_{\mu r}^{G32} = \hat{S}_{\mu r}^{G33} = 0 \quad \text{(B.25)}$$

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