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SENSITIVITY OF LAGEOS ORBITS TO GLOBAL GRAVITY FIELD MODELS

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ABSTRACT. Precise orbit determination is an essential task when analyzing SLR data. The quality of the satellite orbits strongly depends on the models used for dynamic orbit determination. The global gravity field model used is one of the crucial elements, which has a significant influence on the satellite orbit and its accuracy. We study the impact of different gravity field models on the determination of the LAGEOS-1 and -2 orbits for data of the year 2008. Eleven gravity field models are compared, namely JGM3 and EGM96 based mainly on SLR, terrestrial and altimetry data, AIUB-CHAMP03S based uniquely on GPS-measurements made by CHAMP, AIUB-GRACE03S, ITG-GRACE2010 based on GRACE data, and the combined gravity field models based on different measurement techniques, such as EGM2008, EIGEN-GL04C, EIGEN51C, GOCO02S, GO-CONS-2-DIR-R2, AIUB-SST. The gravity field models are validated using the RMS of the observation residuals of 7-day LAGEOS solutions. The study reveals that GRACE-based models have the smallest RMS values (i.e., about 7.15 mm), despite the fact that no SLR data were used to determine them. The coefficient C_{20} is not always well estimated in GRACE-only models. There is a significant improvement of the gravity field models based on CHAMP, GRACE and GOCE w.r.t. models of the pre-CHAMP era. The LAGEOS orbits are particularly sensitive to the long wavelength part of the gravity fields. Differences of the estimated orbits due to different gravity field models are noticeable up to degree and order of about 30. The RMS of residuals improves from about 40 mm for degree 8, to about 7 mm for the solutions up to degrees 14 and higher. The quality of the predicted orbits is studied, as well.

Keywords: satellite geodesy, SLR, LAGEOS, orbit determination, Earth gravity field models

1. INTRODUCTION

Precise orbit determination is one of the essential tasks when analyzing Satellite Laser Ranging (SLR) observations. The proper choice of the processing models and of the deterministic orbit parameters is critical for the quality of the resulting satellite orbit. The global gravity field model is one of the crucial elements, which has a significant impact on the satellite orbits and their accuracy. The impact of different gravity field models on the determination of LAGEOS-1 and -2 orbits is subsequently analyzed.

Many new Earth gravity field models were developed in the first decade of the 21st century. After the launch of the CHAllenging Minisatellite Payload (CHAMP) mission

(Reigber et al., 1998) the accuracy of the gravity field models could be significantly improved w.r.t. the models from the pre-CHAMP era. Current gravity field missions, such as the Gravity Recovery And Climate Experiment (GRACE) (Tapley et al., 2004) and the Gravity field and steady-state Ocean Circulation Explorer (GOCE) (Drinkwater et al., 2006), allow it to determine the gravity field with improved accuracy and resolution. Low degrees of gravity field models can especially be derived using SLR observations to geodetic satellites: LAGEOS-1 and -2, ETALON-1, -2, Stella, Starlette, Ajisai (e.g., Deleflie et al., 2009, Cheng and Tapley, 1999, Mitrovica et al., 1993)

Earth gravity field models are usually validated by analyzing degree difference variances of gravity field recoveries (see, e.g., Jäggi et al., 2011a) or by comparing the coefficients of different gravity field models (ICGEM, 2011). The external validation ensures the independent evaluation of gravity field models. The orbits of spherical satellites are sensitive to the models used, because of the high precision achieved and because of the dynamic orbit representation. The LAGEOS observations are e.g., accurate at the mm-level. This is why geodetic satellites are well suited for comparing gravity field models. Lejba and Schillak (2011) performed the validation of five Earth gravity field models using the RMS of the observation residuals for LAGEOS, AJISAI, Stella and Starlette. Here, the gravity field models are validated using LAGEOS-1 and -2 observations. Predicted orbits are analyzed, as well, by studying the deterministic orbit parameters, and by investigating the impact of the maximum degree and order of the gravity field models on the determined orbits.

In Section 2 we describe the LAGEOS arcs and the data used for the study. In Section 3 the sensitivity of the LAGEOS orbits is studied as a function of the maximum degree (and order) of the gravity field models. In Section 4 different gravity fields are validated based on three methods, namely by analyzing the RMS of the observation residuals, by comparing the estimated and predicted orbits (with and without estimating Helmert transformation parameters) and, last but not least, by analyzing the deterministic orbit parameters. Section 5 compares orbits obtained with different gravity field models. Section 6 summarizes the findings and contains the conclusions.

The experiments based on SLR data were performed using the Bernese GNSS Software (BSW) (Dach et al., 2007).

2. MODELS AND DATA CHARACTERIZATION

The two LAGEOS satellites and ETALON-1 and -2 are the only geodetic satellites routinely used by the International Laser Ranging Service (ILRS) Analysis and Combination Centers (Pearlman et al., 2002) to derive the global parameters for the realization of International Terrestrial Reference Frame (ITRF), i.e., the station coordinates, origin, scale, Length-of-Day (LoD), and the polar motion coordinates x and y as official products. The observations to the LAGEOS satellites allow it to estimate stable positions of the SLR tracking sites. The LAGEOS satellites are of particular importance, because of their height of about 5800 km above the Earth's surface, which renders dynamic orbit determination simple. The orbits are in particular close to insensitive to atmospheric drag. Moreover, the small area-to-mass ratio (6.9 cm^2/kg) minimizes the impact of non-gravitational forces on the satellite orbits. Thanks to the height of the satellites, LAGEOS-1 and -2 may be tracked by virtually all SLR stations. Even though the ETALON satellites have a comparable area-to-mass ratio (9.3 cm^2/kg) they are not as widely used as the LAGEOS satellites. The average number of observations to ETALON satellites is about ten times smaller than to the LAGEOS satellites (Sośnica et al., 2011). However, the ETALON satellites are less sensitive to Earth gravity field coefficients, because of their higher attitude; therefore we use observations to LAGEOS.

The 7-day arcs were computed using all available observations to LAGEOS from all SLR stations in 2008. The BSW, Version 5.1 with SLR extensions (Thaller et al., 2009) was used. The models are described in Table 1, whereas Table 2 contains the list of the estimated parameters. For each 7-day LAGEOS solution one set of six initial osculating orbital parameters is estimated together with deterministic accelerations, namely a constant acceleration in along-track direction and once-per-revolution parameters (*sine* and *cosine* terms) in along-track and out-of-plane directions. Deterministic orbit parameters are intended to absorb non-gravitational accelerations acting on the satellites e.g., direct solar radiation pressure, the Earth's albedo re-radiation of the solar radiation, thermal effects (i.e. Yarkovksy and Yarkovsky-Schach effects (Rubincam, 1987)), as well as poorly modeled gravitational accelerations, e.g., due to ocean tides and gravity field variations. Other estimated parameters are: station coordinates, Earth Rotation Parameters (ERP), and range biases for selected stations.

Data of the year 2008 were used for the comparison of gravity field models. All in all 139,000 SLR normal points are available in this year. Their number per week varies between 1932 and 3804. Although most of the SLR stations are situated in the northern hemisphere, the number of observations from both hemispheres is comparable (see Fig. 1) due to the high productivity of Yarragadee (Australia, 7090), Mt. Stromlo (Australia, 7825), San Juan (Argentina, 7406), and Concepción (Chile, 7405). In the northern hemisphere, the largest amount of data was collected by Zimmerwald (Switzerland, 7810) and Herstmonceux (United Kingdom, 7840).

The imbalanced SLR network causes gaps in the groundtrack coverage of the satellites (see Fig. 2). The longest gaps are found over the Pacific and the Atlantic Oceans, over India, Africa, and, of course, over the polar regions, caused by orbital inclinations. Well distributed SLR observations are necessary for deriving and validating Earth gravity field models in the global sense. For lower geodetic satellites, the global coverage is much poorer with even larger ground-gaps. Therefore LAGEOS satellites are better suited for global validation of gravity field models, even though the average number of observations to e.g. Ajisai is higher than to LAGEOS.

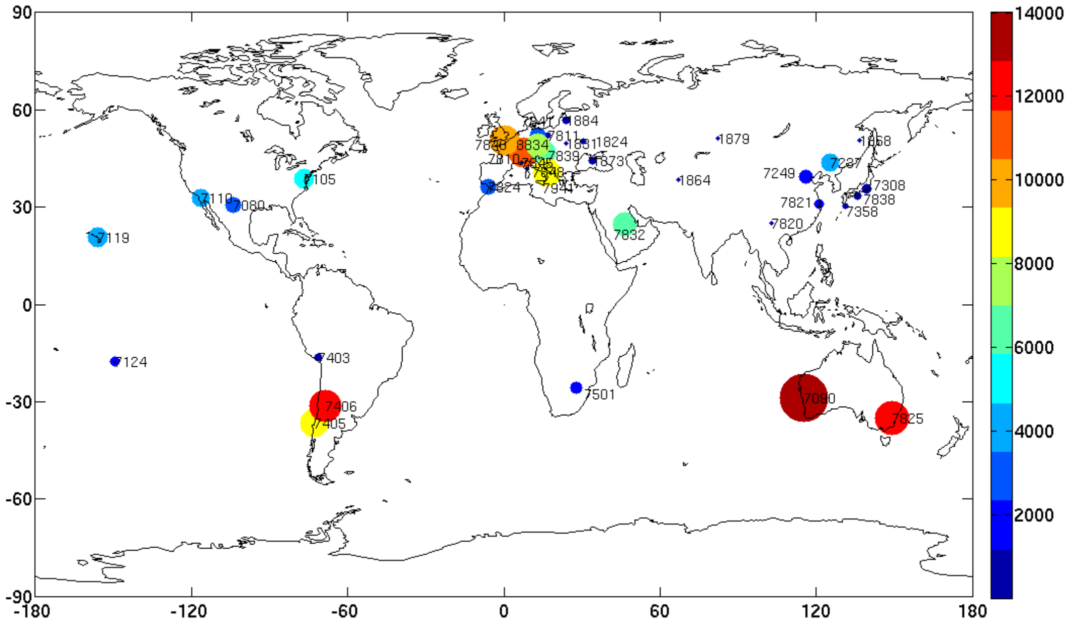


Fig. 1. Number of normal points to LAGEOS-1 and -2 in 2008

Table 1. Modelling standards of 7-day LAGEOS solutions

Type of model	Description
Length of arc	7 days
Data editing	2.5 sigma editing, maximum overall sigma: 25 mm
Satellite center of mass	Station and satellite specific
Troposphere delay	Mendes-Pavlis delay model + FCULa mapping function (Mendes and Pavlis, 2004)
Cut-off angle	3 degrees
Relativity	Light time corrections, IERS 2003 conventions (McCarthy and Petit, 2004)
Third-body	Earth's Moon, Sun, Venus, Mars, Jupiter, ephemeris: JPL DE405 (Folkner et al., 1994)
Tidal forces	Solid Earth tide model: IERS2000 (Kolaczek et al., 2000)
	Ocean tides: CSR4.0 up to degree and order 8
Subdaily pole model	IERS2000 (Kolaczek et al., 2000)
Nutation model	IAU2000 (Herring et al., 2002)
Center of mass corrections	Ocean tidal loading: FES2004 (Lyard et al., 2006)
	Atmospheric tidal loading: Ray and Ponte (2003)
Solar radiation pressure	Direct radiation: applied
Earth's albedo rad.:	Not applied
Numerical integration	Interval: 2 minutes; polynomial degree: 12; collocation method (Beutler, 2005)
Reference frame	SLRF2005 (a priori), (ILRS, 2011)
GM	$398.6004415 \cdot 10^{12}$ m^3 s^{-2}

Table 2. Estimated parameters

Type of parameter	Description
Station coordinates	All SLR stations
Orbit	Initial osculating orbital elements (1 set per 7-day arc), no a priori constrains
	Dynamic orbit parameters: Constant acceleration in along-track (S_0), once-per-rev terms for along-track (S_C, S_S), and out-of-plane (W_C, W_S,) (one set per 7-day arc), no a priori constrains
Earth rotation parameters	*x*- and *y*- coordinates of polar motion, LoD (1 set per day)
Range biases	For selected sites, different for LAGEOS-1 and -2 (1 bias per station, per 7-day arc)

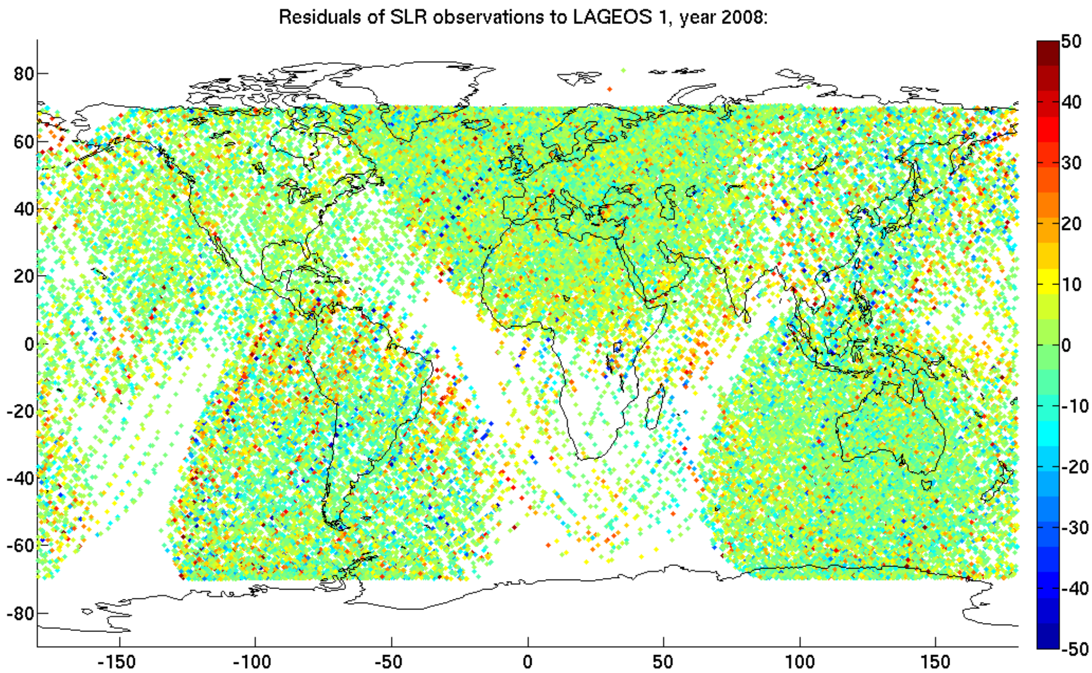


Fig. 2. Observation residuals to LAGEOS-1 (in mm)

Table 3. The quantity of LAGEOS weekly solutions sorted in ascending order of mean RMS of residuals for EGM2008 (mean values for 2008).

Gravity field model up to degree and order	RMS of observation residuals [mm]	Comparison of estimated and predicted orbits			
		Scale [ppb]	RMS radial prediction [mm]	RMS along-track prediction [mm]	RMS out-of-plane prediction [mm]
140	7.13	-0.03	29.7	398.0	199.2
70	7.13	-0.03	29.7	398.0	199.2
30	7.13	-0.03	29.7	398.0	199.2
20	7.14	-0.04	29.7	399.2	199.6
16	7.16	-0.03	29.9	411.0	205.0
14	7.73	-0.05	31.0	448.8	222.4
12	18.19	0.19	41.6	1522.1	769.8
8	39.35	1.95	142.2	2485.7	1634.1

3. MAXIMUM DEGREE AND ORDER

The impact of the maximum degree and order of the gravity field model on orbit determination of LAGEOS-1 and -2 is studied. EGM2008 was selected, because this model is recommended by the IERS 2010 conventions (Petit and Luzum, 2011).

Table 3 shows that the LAGEOS orbits are very sensitive to the gravity field parameters up to degree and order 14. Differences in the solutions related to the degrees 8, 12, and 14, are extremely large. Small differences between degree 14, 16, 18 and 20 are also visible (but they are only at a level of 0.5 mm). The difference in the RMS values for solutions up to degree 20 and 30 is only of the order of 0.01 mm. Increasing the degree and order of gravity field above 30 has no impact on the resulting satellite orbit. In order not to loose any gravity information and not to degrade the quality of estimated orbits, we recommend the value of 30 for LAGEOS solutions. This recommendation is also given by ILRS Annalysis Working Group, although in the IERS2010 conventions the recommended value is 20 (Petit and Luzum, 2011).

4. VALIDATION OF EARTH GRAVITY FIELD MODELS

Eleven gravity field models were compared, namely JGM3 (Tapley et al., 1996) and EGM96 (Lemoine et al., 1998) based on SLR, terrestrial, altimeter data, GPS, DORIS, optical and other observations of artificial satellites, AIUB-CHAMP03S (Prange, 2011) based uniquely on GPS-measurements made by CHAMP, AIUB-GRACE03S (Jäggi et al., 2011a), and ITG-GRACE2010 (Mayer-Gürr et al., 2011) based on GRACE data, and combined gravity field models based on different measurement techniques, such as EGM2008 (Pavlis et al., 2008), EIGEN-GL04C (Förste et al., 2008), EIGEN51C (Bruinsma et al., 2010), GOCO02S (Goiginger et al., 2011), GO-CONS-2-DIR-R2 (Bruinsma et al., 2010), AIUB-SST-only (Jäggi et al., 2010). Most gravity field models were downloaded from the International Centre for Global Earth Models (ICGEM, 2011), see Tab. 4. The models were used up to degree and order 70 in this section. The spherical harmonics C_{21} and S_{21} were not taken from particular models, but they were handled with a consistent manner w.r.t. IERS 2003 Conventions. All compared models were derived using the same value of gravity constant, but different value of Earth radius (R_E). In the analysis the value of R_E was taken as for GRS-80 and all ITRF realizations, and for gravity field models having different values of R_E the total potential has been rescaled correspondingly.

Table 4. List of compared gravity field models

Gravity field model	Year	Max. degree	Coeff. drift	SLR	CHAMP	GRACE	GOCE	Ground data
JGM3	1994	70	C_{20}	X				X
EGM96	1996	360	C_{20}	X				X
EIGEN-GL04C	2006	360	d/o 4	X		X		X
EGM2008	2008	2190	-			X		X
EIGEN51C	2010	359	d/o 4	X	X	X		X
ITG-GRACE2010	2010	180	-			X		
AIUB-CHAMP03S	2010	100	-		X			
AIUB-GRACE03S	2011	160	d/o 30			X		
GO-CONS-2-DIR-R2	2011	240	-			(X)	X	
GOCO02S	2011	250	-	X	X	X	X	
AIUB - SST - only	-	120	-		X		X	

4.1. RMS OF THE OBSERVATION RESIDUALS

The RMS of the observation residuals from the weekly solutions is used as an indicator of the quality of the underlying gravity field models – when using otherwise the same set of models and data. The RMS characterizes the precision of fitting the observations if the set of adjusted parameters are indistinguishable. However, all compared models were derived using different standards and conventions. Therefore it is not possible to guarantee that the residuals are not reflecting the omission errors from the missing or unmodeled elements stemming from, e.g., older class ocean tide models. In the precise orbit determination it is crucial to take into account the total Earth potential as the sum of solid Earth potential, potential induced by the oceans and the atmosphere. In this study we neglect the variations of the potential induced by atmospheric gravity and we use the same ocean tide model together with different gravity field models. However, from other studies we know that the differences of ocean tide model cause variations in RMS of observations residuals up to 1 mm (for hydrodynamic tidal models with assimilation from observed tidal data from altimetry missions).

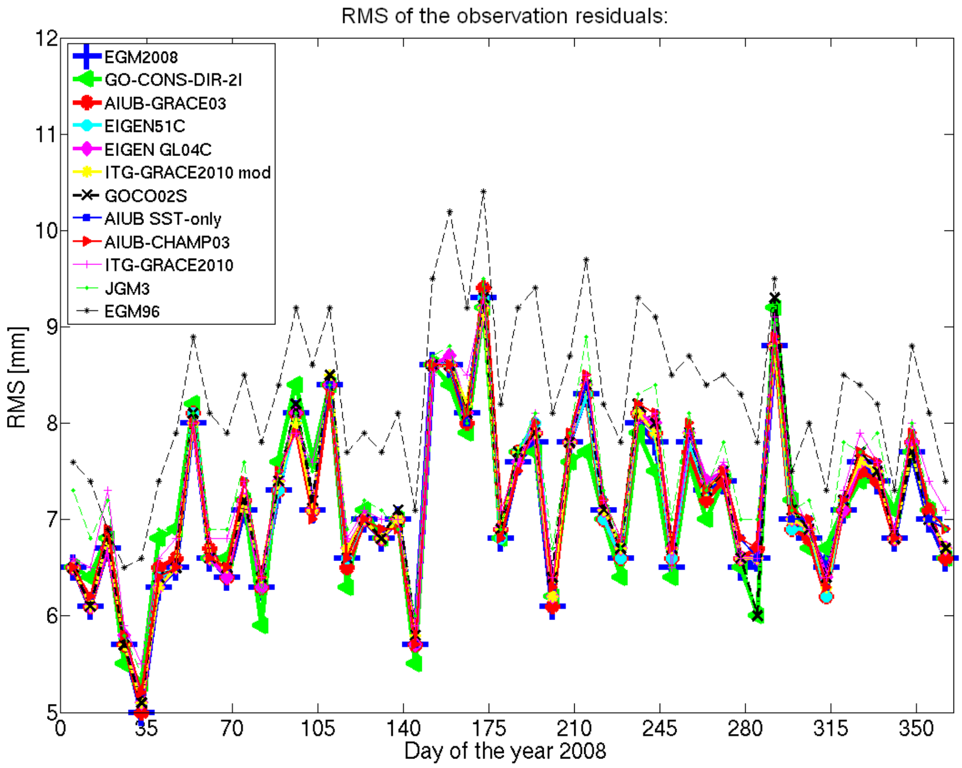


Fig. 3. RMS of the observation residuals for different gravity field models.

Figure 3 and Table 5 show the RMS of the observation residuals for different gravity field models. Although the solutions look similar at first sight and most of the differences are not statistically significant, the gravity field models may be associated with three groups: The first group with a slightly higher RMS contains models from the pre-CHAMP era, namely EGM96 and JGM3 with RMS values of 8.29 mm and 7.42 mm, respectively. A special attention should be paid to ITG-GRACE2010S. Although this model is from the post-CHAMP era, it induces a rather big RMS of 7.32 mm. ITG-GRACE2010S is the only model with non-zero values for the harmonic coefficients C_{11}, S_{11}, and C_{10}. These coefficients correspond to the translation of the geocenter w.r.t. the origin of the terrestrial reference frame (x, y, and z components, respectively, Beutler (2005)). ITG-GRACE2010S generates a constant shift of about 14 mm of the orbit w.r.t. geocenter. According to IERS Conventions 2003, gravity field coefficients of degree 1 should not be used in the gravity field determination. Therefore, this model was used twice in our solution, once in the way published and once with the coefficients C_{11}, S_{11}, and C_{10} set to zero (indicated as ITG-GRACE10 mod). This modification reduces the RMS from 7.32 mm to 7.18 mm (see Tab. 5).

In the second group there are gravity field models based on kinematic orbits of Low Earth Orbiters (LEOs), estimated using the Global Positioning System (GPS). This group consists of AIUB-CHAMP03S (based uniquely on CHAMP data) and of AIUB SST-only model (based on CHAMP and GOCE (GPS-only)). The RMSs are 7.22 mm and 7.21 mm for these models, respectively.

The last group contains the models based either on GRACE K-band observations or on combined techniques with highly weighted GRACE contributions (GOCO02S, ITG-GRACE10 mod, EIGEN-GL04C, EIGEN51C, AIUB-GRACE03S, GO-CONS-2-DIR-R2, and EGM2008). These models show more or less a comparable quality with the smallest RMS errors for AIUB-GRACE03S, GO-CONS-2-DIR-R2 and EGM2008 (7.15 mm, 7.14 mm and 7.13 mm, respectively).

Table 5. Quality of LAGEOS 7-day solutions in descending order of the mean RMS of the observation residuals (mean values for 2008)

Gravity field model	RMS of observation residuals [mm]	Comparison of estimated and predicted orbits including Helmert transformation			
		Scale [ppb]	RMS radial prediction [mm]	RMS along-track prediction [mm]	RMS out-of-plane prediction [mm]
EGM96	8.29	-0.06	29.8	400.4	199.8
JGM3	7.42	-0.05	29.7	398.6	199.1
ITG-GRACE2010S	7.32	-0.03	29.9	396.8	198.6
AIUB-CHAMP03S	7.22	-0.04	29.7	398.0	199.2
AIUB SST-only	7.21	-0.04	29.7	397.9	199.1
GOCO02S	7.20	-0.04	29.6	397.5	198.8
ITG-GRACE10 mod	7.18	-0.04	29.7	398.0	199.2
EIGEN-GL04C	7.17	-0.04	29.7	397.6	198.9
EIGEN51C	7.16	-0.04	29.7	397.9	199.2
AIUB-GRACE03S	7.15	-0.04	29.7	397.8	198.9
GO-CONS-2-DIR-R2	7.14	-0.04	29.5	398.2	199.0
EGM2008	7.13	-0.03	29.7	398.0	199.2

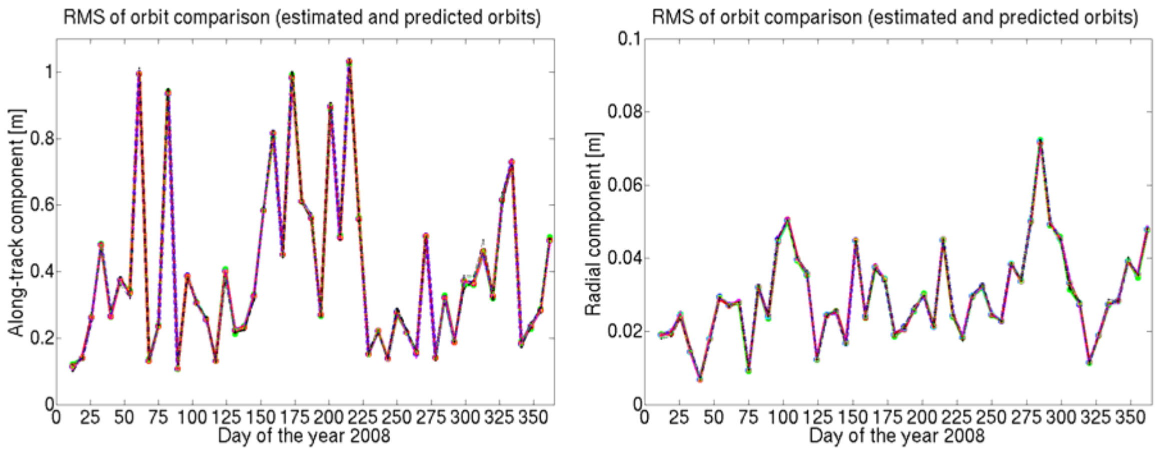
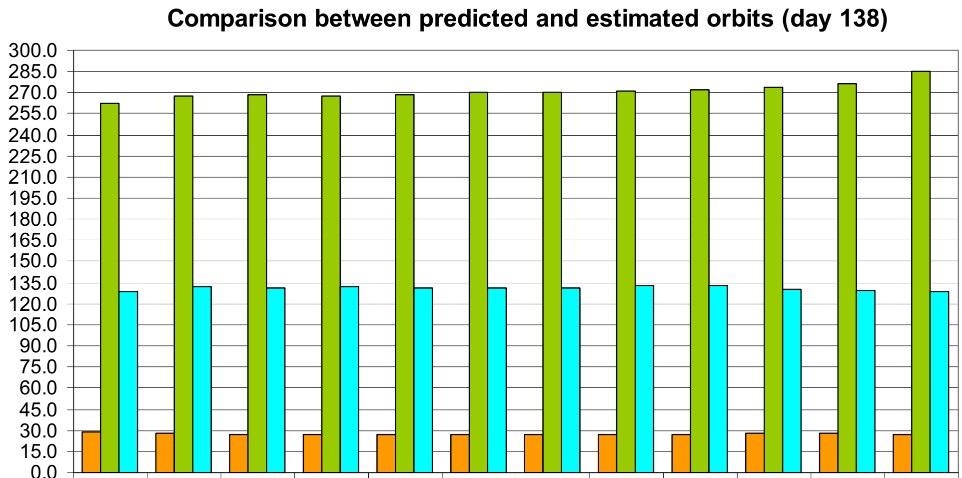


Fig. 4. RMS of the Helmert transformation between estimated and predicted orbits for along-track and radial components. Note the different scales.

4.2. COMPARING PREDICTED AND ESTIMATED ORBITS

In this section the estimated orbits are compared with the prediction of the orbits from the previous week. The first comparison was performed through a 7-parameter Helmert transformation (see Fig. 4) and the second one without such a transformation. The scale and the RMS of orbit differences for the first type of comparison are provided in Table 5. The

RMS associated with the radial residuals is the smallest one, due to radial sensitivity of SLR. In the first approach the differences do not exceed 0.4 mm, 3.6 mm, and 1.2 mm in the radial, along-track and out-of-plane components, respectively. These values correspond to 1.3%, 0.9% and 0.6% of the absolute RMS values, respectively. It explicitly indicates that the uncertainties in gravity field models are not the limiting factor for the orbit predictions. More important are probably the quality of surface load modelling, tidal forces, albedo, relativistic effects (Lucchesi, 2005), and thermal re-radiation effects, such as Yarkovsky and Yarkovsky-Schach effect (Rubincam, 1987). The along-track component of the orbit comparison between predicted and estimated orbits is evidently worse during the eclipsing periods. In 2008 there is one eclipsing period for LAGEOS-1 lasting from 110 day till 203 day of the year and three eclipsing periods for LAGEOS-2, namely in days: 44-95, 170-219 and 263-366. During eclipsing periods, the orbit prediction in along-track is especially weak mainly due to Yarkovsky-Schach effect and thermal induced positive net accelerations in along-track direction (Appleby, 1998).



	EGM96	AIUB-GRACE 03S	ITG-GRACE 10 mod	EIGEN 51C	EIGEN-GL04C	GOCO 02S	EGM 2008	AIUB SST	AIUB-CHAMP 03S	JGM3	GO-CONS-2-DIR-R2	ITG-GRACE 2010S
RMS radial prediction [mm]	28.6	27.7	27.2	27.3	27.3	27.3	27.3	27.2	27.2	28.1	27.7	27.0
RMS along-track prediction [mm]	262.1	267.3	268.2	268.0	268.4	270.0	270.3	270.9	271.7	273.4	276.8	285.0
RMS out-of-plane prediction [mm]	128.9	132.1	131.4	131.8	131.6	131.5	131.2	132.7	132.8	130.4	129.8	129.0

Fig. 5. Predicted and estimated orbits without Helmert transformation (in ascending order of 3D residual).

The comparison between the predicted and the estimated orbit without Helmert transformation (Fig. 5) shows the smallest 3D RMS for EGM96 (i.e., 293 mm compared to 314 mm for the worst model), even though this model has the largest RMS of the observation residuals in the weekly solutions. The result might be explained because the coefficients of the gravity field models up to degree and order about 30 are significant for the RMS of the observation residuals, whereas the orbit prediction is determined mainly by the very low harmonic coefficients (up to degree/order of about 10) and other modeling issues, e.g., tidal forces, loading effects, non-gravitational forces acting on satellites, and the different realization of the reference frame in every week (different set of SLR tracking sites). The quality of the very low gravity field coefficients is usually too optimistic for the GRACE based models (in particular for degrees 2 and 3 in the GRACE K-band only models (Jäggi et al., 2010)). Better results are achieved when using SLR observations to spherical satellites. In order to prove this assumption, the solutions for the EGM2008 and EGM96 models were used only up to degree and order 8 and 5, respectively. Figure 6 shows that the solution up to degree and order 8 is comparable for EGM96 and EGM2008 (39.34 mm and 39.35 mm, respectively); whereas the solution up to degree 5 is slightly better for the EGM96 based to some extend on the SLR observation technique (38.33 mm and 38.61 mm for EGM96 and EGM2008, respectively).

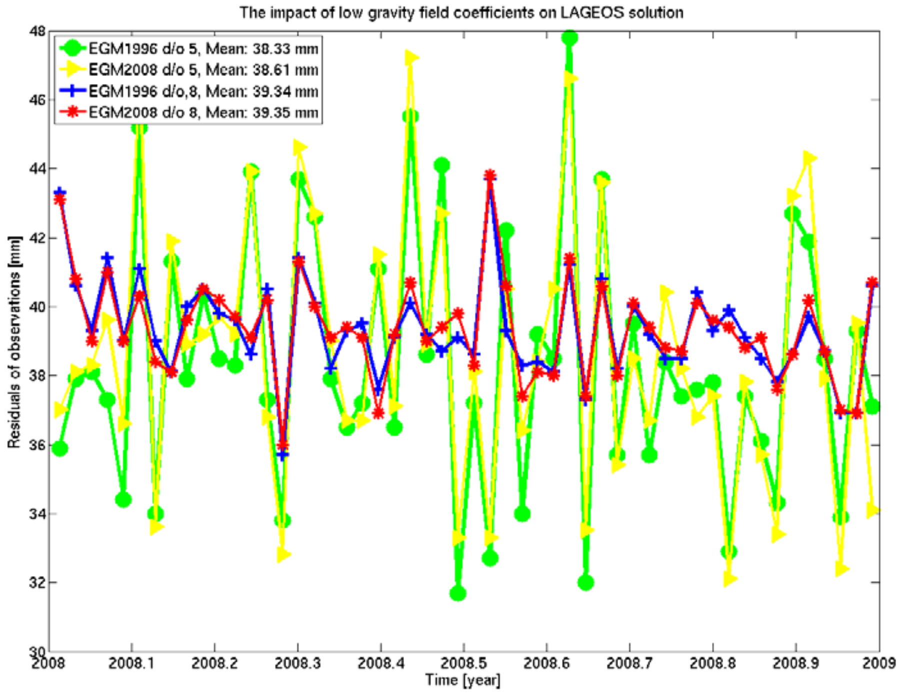


Fig. 6. The RMS of observation residuals for EGM2008 and EGM96 used up to degree and order 5 and 8, respectively.

In the case of GOCO02S, a combined model established in 2011 based on SLR, CHAMP, GRACE and GOCE data, the SLR observations contribute only to the coefficient C_{20} (Maier et al., 2011). A better quality could probably be achieved by using the SLR observations to geodetic satellites up to higher degrees (see Section 3).

The second and third model in the comparison between estimated and predicted orbits (see Fig. 5) are AIUB-GRACE03S and ITG-GRACE10 mod (with coefficients of d/o one reduced to zero). The largest RMS of the comparison is for ITG-GRACE2010S.

4.3. COMPARING EMPIRICAL ORBIT PARAMETERS

The empirical orbit parameters estimated using different gravity field models do not show significant differences in the along-track component (S_0) and in the out-of-plane *cosine* (W_C) term (Fig. 7). The only noticeable differences are seen in the *sine* term (W_S) of the out-of-plane component (Fig. 8). For AIUB-GRACE03S this parameter differs significantly from the other models (and is significantly different from zero). Moreover, for LAGEOS-1 the *sine* term in the out-of-plane direction has a positive value, whereas a negative value results for LAGEOS-2 (about $+2 \cdot 10^{-9}$ for LAGEOS-1 and $-3 \cdot 10^{-9}$ for LAGEOS-2, see Fig. 8).

The differences in the out-of-plane *sine* term correspond to the different values of the second zonal harmonic C_{20} (see Fig. 9, left). In the case of AIUB-GRACE03S, the harmonic coefficients and their drifts up to degree and order 30 are derived mostly from GRACE K-band observations without any regularizations. This may lead to rather poor estimates of the C_{20} value and its drift and therefore also big differences w.r.t. other models (see Fig. 9, right).

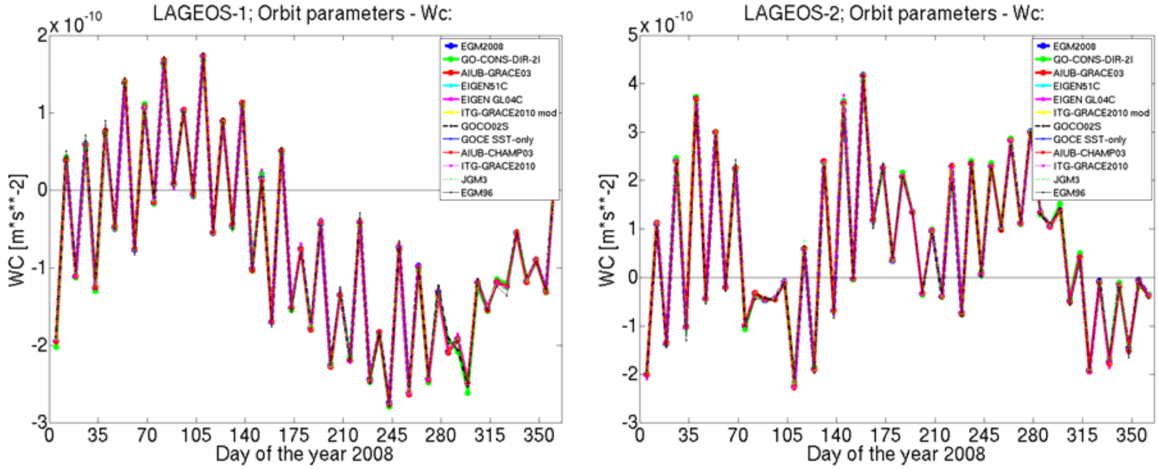


Fig. 7. The empirical orbit out-of-plane *cosine* parameter for LAGEOS-1 (left) and LAGEOS-2 (right). Note the different scales.

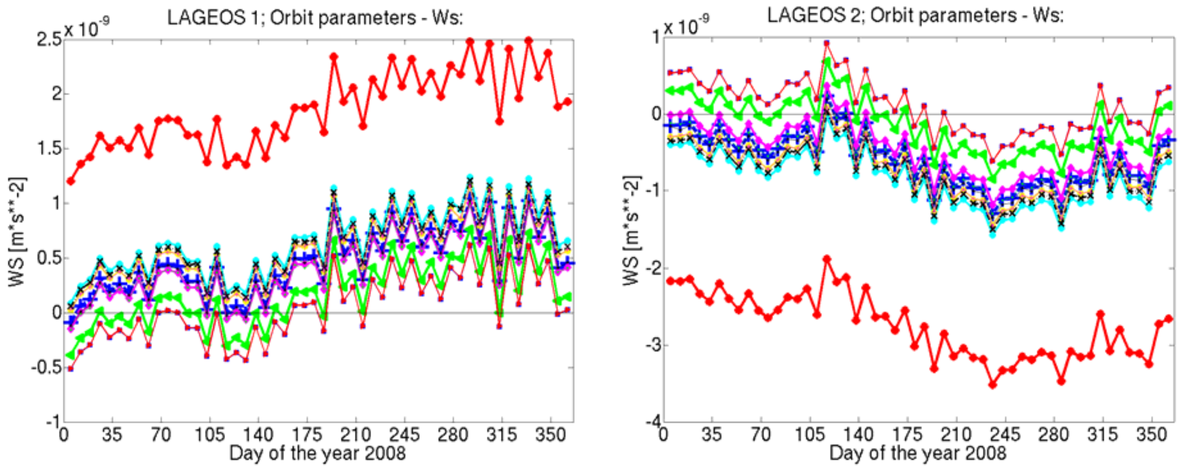


Fig. 8. The empirical orbit out-of-plane *sine* parameter for LAGEOS-1 (left) and LAGEOS-2 (right). Note the different scales.

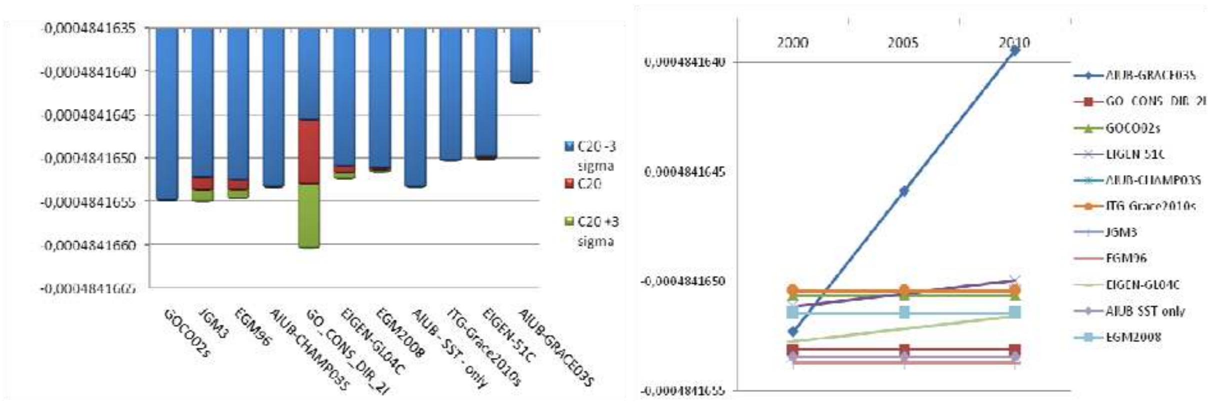


Fig. 9. C_{20} (normalized) for different gravity field models referred to epoch 2008.5 (left) and the value C_{20} for the time span 2000-2010 (right). In case of ITG-GRACE2010S and JGM3, C_{20} was transformed from zero-tide to tide-free model.

The Gaussian version of first-order perturbation theory due to second zonal spherical harmonic is expressed according to Beutler (2005) by Eq (1):

$$\begin{Bmatrix} R \\ S \\ W \end{Bmatrix} = \frac{3}{2}\frac{GM\,R_E^{\,2}C_{20}}{r^4}\begin{Bmatrix} 1-\frac{3}{2}\sin^2 i+\frac{3}{2}\sin^2 i\,\sin^2 u \\ \sin^2 i\,\sin 2u \\ \sin 2i\,\sin u \end{Bmatrix} \quad \textbf{(1)}$$

Jäggi et al. (2011b) explain the relation between C_{20} and out-of-plane *sine* component for LAGEOS' orbits. Moreover, they attribute the signs for LAGEOS-1 and -2 and the ratio of the numerical values to the inclination angles (110^o and 53^o for LAGEOS-1 and -2, respectively). The equation shows the relationship between the perturbing acceleration caused by C_{20} in the radial (*R*), along-track (*S*), and out-of-plane (*W*) directions as a function of the argument of latitude *u* and inclination *i* (with *G* – gravity constant, *M* – mass of Earth, R_E – Earth radius, *r* – length of satellite state vector). It explicitly shows the correlation between C_{20} and the *sine* term of the once-per-rev out-of-plane acceleration. The above equation also shows that there is no direct correlation between C_{20} and the argument of latitude *u* for the other terms of the estimated empirical *once-per-rev* accelerations (Colombo 1989; Tapley et al., 1993) – as confirmed by our experiment. However, there would be a correlation between along-track *twice-per-rev sine* term, if only such a term was estimated.

We also conclude that the out-of-plane *sine* parameter W_S is responsible for the LAGEOS solutions' insensitivity to the quality of the C_{20} coefficient of the Earth's gravity field, when W_S parameter is estimated. For confirming this fact, we concucted an additional experiment and computed solutions without estimating W_C, W_S parameters. As expected, Figure 10 and Table 6 show a significant degradation of the solution without estimating the W_S, W_C parameters w.r.t. the solution estimating these parameters (Fig. 3 and Tab. 5). The RMS of the observation residuals for AIUB-GRACE03S grew to 30.74 mm (compared to the 7.15 mm including the estimation of these parameters). The RMS of the comparison between estimated and predicted orbits in the out-of-plane direction is about three times worse for AIUB-GRACE03S than for the other models. The smallest RMS of observation residuals is achieved for the AIUB-CHAMP03S and AIUB SST-only models – 10.51 mm and 10.52 mm, respectively, but even these values are larger than the previous ones (7.22 mm and 7.21 mm). It is not possible to distinguish on the basis of RMS of observation residuals (Table 6), which gravity field models come from pre-CHAMP era and which models are based on CHAMP, GRACE and GOCE data. It shows that LAGEOS orbits are extremely sensitive to all inexactnesses and variations of C_{20}. Even gravity field models based on the SLR observations do not show better quality than models based on other observation types. Comparing Fig. 8 and Fig. 10 one can see that the models with value of W_S closest to zero also have the smallest value of RMS in the solutions, where W_S and W_C were not estimated. The result implies that GPS-based gravity field models provide the most reliable values of C_{20} for 2008, even without estimating the drift of the gravity field parameters. The discrepancies for the single models are suggestions for some serious missmodeling during the development of the gravity field model. Fig. 8 (right) shows a clear seasonal signal for W_S. Moreover, there is a good agreement between the out-of-plane *sine* term (Fig. 8, right) and series of monthly C_{20} values in 2008 (e.g. CSR, 2011) due to the strong mathematical correlations from Eq (1). This behaviour suggests that not considered time-dependent variations in C_{20} are responsible for orbit missmodeling, especially in out-of-plane direction.

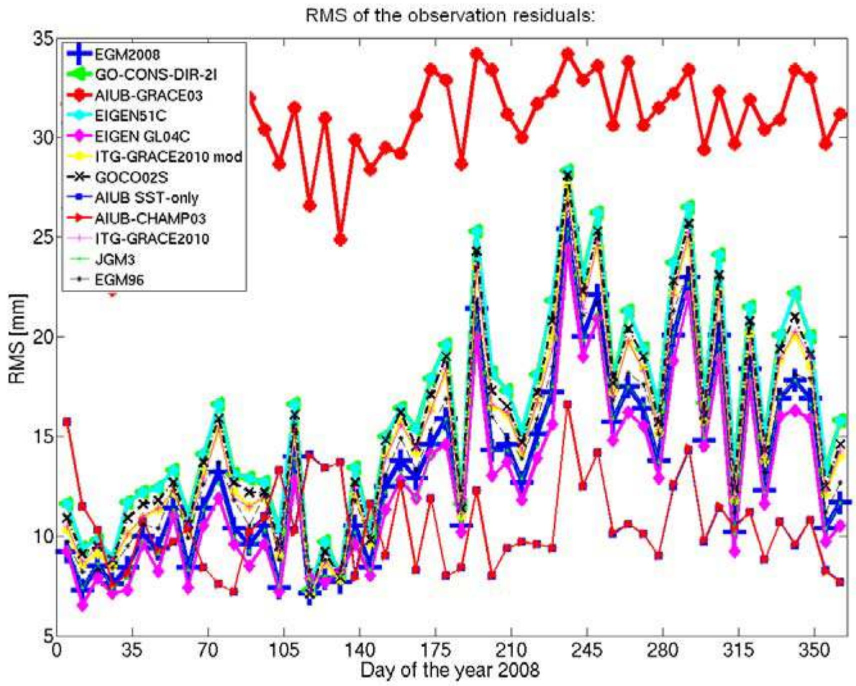


Fig. 10. RMS of the residuals for eleven gravity field models without estimating W_S and W_C .

Table 6. LAGEOS weekly solutions without estimating W_C and W_S (mean values for 2008)

Gravity field model	RMS of observation residuals [mm]	Comparison of estimated and predicted orbits			
		Scale [ppb]	RMS radial prediction [mm]	RMS along-track prediction [mm]	RMS out-of-plane prediction [mm]
EGM96	14.33	-0.10	32.7	413.1	319.6
JGM3	13.28	-0.08	32.3	408.9	304.7
ITG-GRACE2010S	15.05	-0.07	33.8	415.0	353.7
AIUB-CHAMP03S	10.51	-0.07	33.8	415.0	353.7
AIUB SST-only	10.52	-0.03	30.1	394.5	231.6
GOCO02S	15.55	-0.09	34.0	419.0	369.7
ITG-GRACE10 mod	15.01	-0.09	33.6	416.5	354.4
EIGEN-GL04C	12.56	-0.06	32.0	405.7	388.6
EIGEN51C	16.19	-0.09	34.4	422.3	388.6
AIUB-GRACE03S	30.74	-0.21	52.8	572.8	1011.2
GO-CONS-2-DIR-R2	16.20	-0.09	34.4	422.3	388.6
EGM2008	13.40	-0.07	32.6	409.1	312.5

From the results in this section we conclude that not estimating either C_{20} or the out-of-plane *once-per-revolution* orbit parameter may lead to substantially degraded solutions, because of a insufficient a priori value for C_{20}. The time variability of C_{20} (e.g., CSR, 2011) cannot be described accurately enough by a static gravity field or by a constant value and a drift of C_{20}. It is necessary to estimate either C_{20} or W_S and W_C empirical parameter or to use time-variable gravity field models (e.g. monthly solutions). However, from other experiments we know that such monthly gravity field models derived from GRACE, imply a bigger RMS of the observation residuals for LAGEOS than static gravity fields. This is mostly due to the poorer estimates of sectorial gravity field coefficients in such time-variable GRACE-based gravity field models.

Besides the correlations shown above, the drift of right ascension of ascending node (Ω) is also correlated with the *sine* term of out-of-plane once-per-rev parameter (Beutler, 2005). Seasonal variations of C_{20} induce corresponding variations of the drift of right ascension of ascending node. Moreover, the drift of the ascending node also influences the LoD, if the LoD is derived from satellite obsrvations. The LAGEOS satellites have different inclination angles, so the change rate of Ω has a different impact on LoD. For the solution using only one satellite, there is a strong correlation between LoD and the drift in Ω. For multi-satellite solution the impact of C_{20} on the drift in Ω and on LoD is smaller (Bloβfeld, 2011).

We conclude that gravity field models with poor estimates of C_{20} do not necessarily lead to poor LAGEOS orbits. C_{20} is correlated with the W_S parameter and the drift of ascending node, which influences the LoD estimation, thus W_S estimates may compensate for poor estimations in C_{20} and its drift. The *sine* term of out-of-plane empirical acceleration parameter absorbs the uncertainty of the C_{20} value. Using a gravity field model with a poorly established C_{20} does not necessarily lead to poor orbits or poor LoD parameters, but the parameters W_C and W_S have to be set up in orbit determination. These tests also reveal how extremely sensitivity of LAGEOS' orbits to C_{20}. Therefore, small variations in C_{20} may induce a serious degradation of precise satellite orbit determination.

5. ORBIT COMPARISON

Table 7. Mean RMS of orbit differences for 2008 due to different gravity fields. All values in mm, values < 5 mm in green, > 8 mm - in red

Gravity field model	EIGEN-GL04C	EGM 2008	EIGEN 51C	ITG-GRACE 2010S	GOCE SST	AIUB-CHAMP 03S	AIUB-GRACE 03S	ITG-GRACE 10 mod
JGM3	8.5	11.2	7.6	15.6	6.7	6.6	8.8	7.6
EIGEN-GL04C		3.8	2.1	14.3	6.2	5.9	1.8	1.2
EGM2008			5.6	15.3	9.8	9.5	3.7	4.3
EIGEN51C				14.1	4.4	4.2	2.4	1.9
ITG-GRACE2010S					14.7	14.6	14.5	14.1
AIUB SST						0.6	6.6	5.8
AIUB-CHAMP03S							6.4	5.5
AIUB-GRACE03S								2.2

Table 7 compares the LAGEOS orbits estimated using different gravity field models. The RMS for the orbits based on the ITG-GRACE2010S model is largest w.r.t. all other orbits (due to 14 mm constant orbital shift), but may be significantly reduced by setting C_{10}, C_{11} and S_{11} to zero (ITG-GRACE-10 mod). The models with the second largest differences to the other models are JGM3 and EGM96 with values above 6 mm. EGM2008 differs only slightly from GRACE-based models and significantly from CHAMP-based models (see Fig. 11). Orbits using AIUB-GRACE03S, ITG-GRACE10 mod, EIGEN-GL04C and EIGEN51C models are comparable in quality (all GRACE-based). The smallest values are achieved for the AIUB-CHAMP03S and the AIUB-SST-only models (the latter being the extension of the CHAMP-based model with GPS measurements from GOCE and thus almost identical for the low degrees). Gravity field models based on the same type of observations are of comparable reliability.

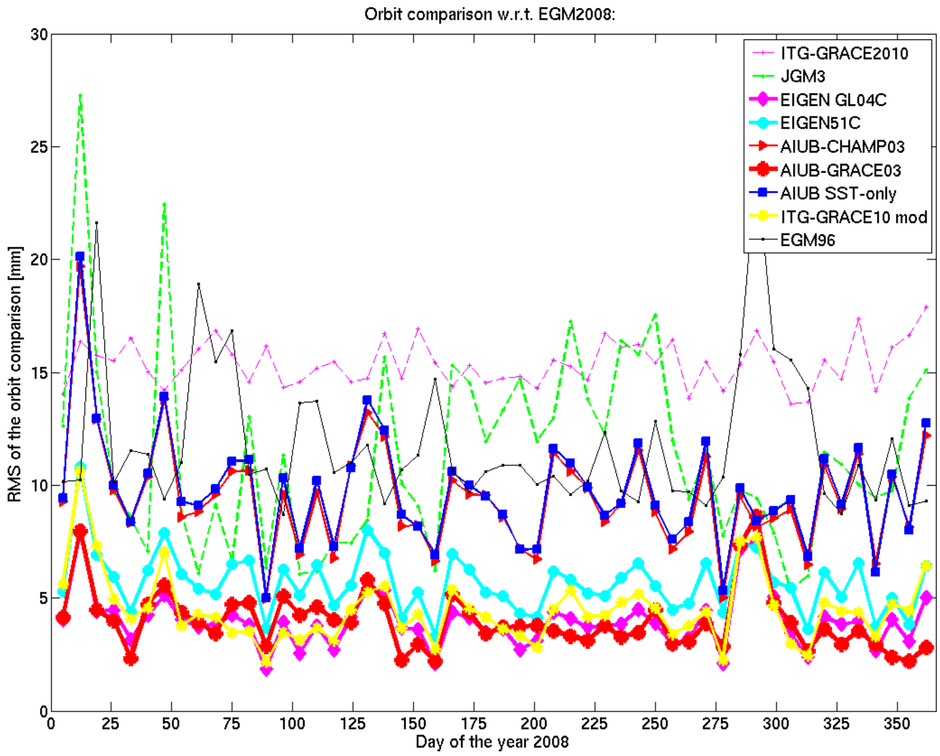


Fig. 11. Orbits based on different gravity fields compared to orbits based on EGM2008 (solutions with estimating W_S/W_C once-per-rev parameters)

6. CONCLUSIONS

We have shown that LAGEOS arcs of one week are sensitive up to degree 30 of the gravity field model. The smallest RMS to the SLR data of LAGEOS-1 and -2 is achieved with EGM2008, GO-CONS-2-DIR-R2 and AIUB-GRACE03S when estimating *once-per-rev* out-of-plane accelerations. Orbits based on JGM3 and EGM96 deviate w.r.t. orbits based on other models. A similar effect is observed for ITG-GRACE2010S when the coefficients of degree one are not set to zero. Thus, ITG-GRACE2010S should always be used with $S_{11} = C_{10} = C_{11} = 0$ (as generally recommended by IERS Conventions). The largest RMS of fit to the SLR data from LAGEOS-1 and -2 is obtained for AIUB-GRACE03S, when the *once-per-rev* out-of-plane accelerations W_S and W_C are not estimated, due to missmodeling of C_{20}. In case of not estimating *once-per-rev* out-of-plane accelerations, the smallest RMS of observation residuals from LAGEOS-1 and -2 is obtained for AIUB-CHAMP03S and AIUB-SST-only. The results tell that these models include a C_{20} value closest to the real mean value in 2008. Some of the lowest degree gravity field coefficients seem to be estimated better in SLR-based models. The SLR observation technique therefore should be considered when establishing combined gravity field models. The estimation of C_{20} and its drift based on GRACE K-band only may lead to unreliable results. The spherical harmonic coefficient C_{20}

and the *sine* term of the out-of-plane empirical acceleration parameter are strongly correlated and cannot be estimated together. The *sine* term of out-of-plane empirical acceleration parameter absorbs the uncertainty of the C_{20} value. Using a gravity field model with a poorly established C_{20} (due to the simultaneous estimation of W_C and W_S) does not necessarily lead to poor orbits, but the parameters W_C and W_S have to be set up in orbit determination, as well, in this case. It is very likely that considering only constant or linear behaviour of C_{20} is nowadays not sufficient anymore. If temporal variations should be considered e.g. by using monthly gravity field models, the time span of this study has to be clearly extended.

The results from this paper show that differences between LAGEOS orbits derived using modern gravity field models are rather small. The other error sources e.g. ocean tides, station loading displacement corrections and technical stations errors play much more important role for the quality of resulting LAGEOS orbits. However, the validation of very low gravity field coefficients can be successfully carried out using LAGEOS orbits due to the high precision of orbits, even though moderate sensitivity of LAGEOS to Earth gravity field.

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