C. H. Jenkins
L. Kjerengtroen
H. Oestensen
Mechanical Engineering Department
South Dakota School of Mines and Technology
Rapid City, SD 57701

Sensitivity of Parameter Changes in Structural Damage Detection

Structural damage detection by nondestructive methods is highly desirable. Changes in modal parameters such as frequency, damping, and mode shape are particularly inviting. Evidence is presented here that reveals that static deflection can, in many cases, be a more sensitive predictor of structural damage than frequency. The reasons for this are illuminated within, and hinge on very fundamental issues about the very nature of structural response. Furthermore, static deflection measurements are often easier to make, with higher levels of accuracy than dynamic measurements. Comparisons are made between theoretical models and experimental results for simple structures, with extensions given to more complex structures.

INTRODUCTION

There is currently great interest in nondestructive methods for use in structural damage detection. Change in dynamic response, which may appear as changes in modal parameters like frequency, damping, and mode shape, is an inviting option. Particularly for structures that receive dynamic excitation as a matter of course, e.g., rotating machinery, aircraft, bridges, etc., such a method would seem fortuitous. However, most results in this area published to date have been less than promising.

After review of many of these results, including our own, we suggest that for numerous structures, a "distributed" phenomenon like frequency response (distributed due to dependence on inertia or mass) is relatively insensitive to localized damage, such as that caused by a fatigue crack, notch, impact, and the like. Further complications occur when damage occurs at or near nodal points. Extracting useful data experimentally is not trivial. A much simpler method, applicable to many structures, involves changes in static deflection. We show that in many cases, static deflection shows significantly greater sensitivity to local damage than does frequency.

PREVIOUS WORK

Table 1 provides a representative sample, compiled from reported results, of changes in transverse fundamental frequency with damage in a beam structure. In all cases, damage was induced by a crack or saw cut perpendicular to a long edge (see Fig. 1), such that the beam was sectioned halfway through ($a/h = 0.5$). Clearly, this would represent significant damage in a nonredundant member, a situation that would lead to (possibly catastrophic) failure in real structures. However, the reported frequency changes were generally

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Table 1. Comparison of Reported Maximum Changes in Fundamental Frequency for Beams with Edge Damage (a/h = 0.5), Calculated and Experimental

Reference	Calculated Δf_1	Experiment Δf_1	Notes
Gudmundson (1982)	≤10%	≤10%	$l_1/L = 0.2$
Christides and Barr (1984)	≤10%	≤10%	$l_1/L = 0.5$
Spyrakos et al. (1990)	—	≤5%	$l_1/L = 0.4$
Collins et al. (1992)	≤3%	—	$l_1/L = 0.0$
Liang et al. (1992)	≤5%	—	$0.1 \le l_1/L \le 0.3$
Davini et al. (1993)	≤10%	≤10%	$l_1/L = 0.35$
Krawczuk and Ostachowicz (1993)	≤5%	—	$l_1/L = 0.3$
Kjerengtroen and Jenkins (1994)	≤11%	≤12%	$l_1/L = 0.4$

10% or less, which is not large considering that these results were either from computer models or controlled laboratory experiments. Real world effects, such as noise, stick-slip friction in joints, redundancy of members, etc., quickly overwhelm small changes in frequency (see, e.g., Toksoy and Aktan, 1993).

It is instructive to take an edge-damaged beam similar to those above and measure changes in static deflection versus increasing damage. Such changes are compared with those in fundamental frequency for the same beam in Fig. 2 (Kjerengtroen and Jenkins, 1994; Oestensen, 1994). [In this and the following, we refer to tip deflections caused by a tip load. The beam was an 457 mm (18 in.) long by 13 mm (0.5 in.) square steel beam, damaged by a saw cut approximately 14% in from one end.] For this simple structure, static deflection was considerably more sensitive to damage than fundamental frequency.

It should be noted that higher order frequencies have been reported to be more sensitive indicators of damage than fundamental frequency. While this may be true in certain cases, we did not find that to be true for the structures discussed herein. For example, we show experimental data in Fig. 3 for changes in the first five transverse natural frequencies of a cantilever beam with increasing damage. Furthermore, the decreased amplitudes and increased nodal points that accompany higher modes complicates their use in real situations.

PARAMETER PERTURBATION

Consider a structural system modeled simply as an assemblage of coupled spring-mass dashpot oscillators (Fig. 4). The equation of motion for vibrations of such a system is given in indicial notation as (e.g., see Meirovitch, 1986; Chondros and Dimarogonas, 1989)

$$\sum_{ij} [m_{ij}\ddot{x}_j(t) + c_{ij}\dot{x}_j(t) + k_{ij}x_j(t)] = 0, \qquad (1)$$

where m, c, and k are the lumped mass, damping, and stiffness of the system, respectively, $i, j = 1, 2, \ldots, n$, and n is the number of degrees of freedom x. This is written in matrix notation in the usual way as,

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{0\}. \qquad (2)$$

In the limit as $n \to \infty$, the response of the discrete model should converge to that of the continuous model.

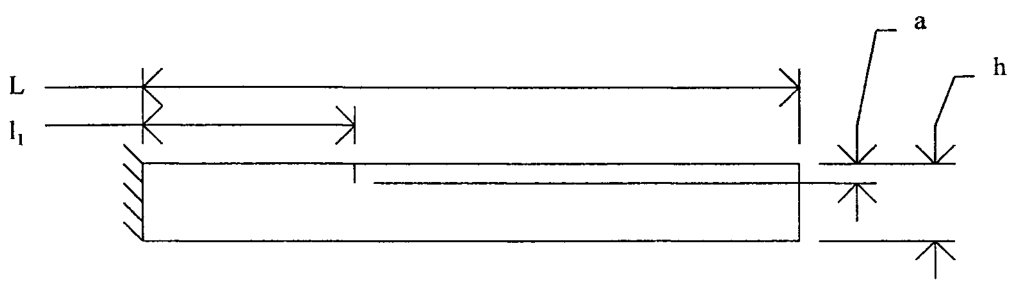

FIGURE 1 Definition sketch of damaged cantilever beam.

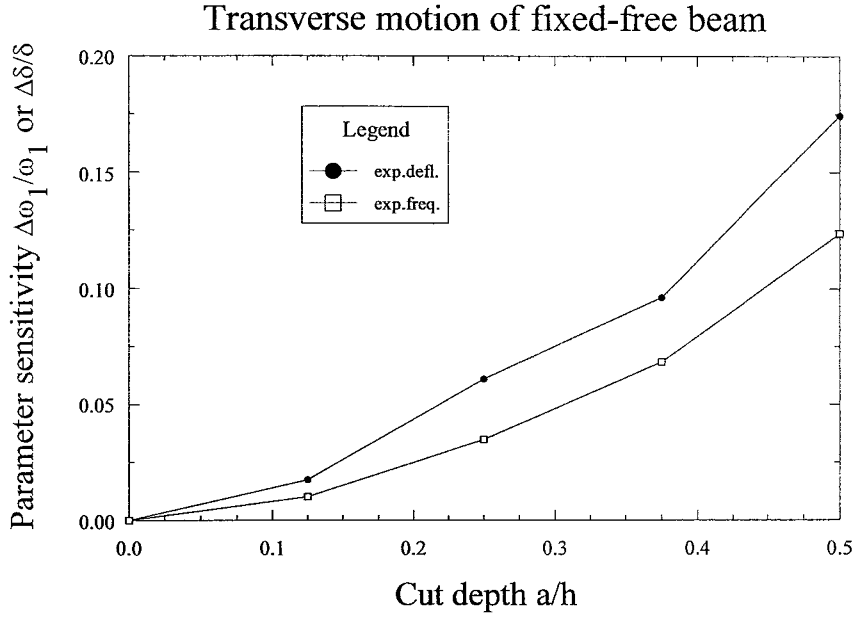

FIGURE 2 Experimental comparison of frequency and deflection sensitivity with increasing damage for a cantilever beam.

For the undamped case, solution by modal superposition results in the eigenvalue problem

$$[K - \lambda_i M]\{X\}_i = \{0\}, \tag{3}$$

with natural circular frequencies $\lambda_i = \omega_i^2$ given by

$$|K - \lambda_i M| = 0. \tag{4}$$

Natural frequencies can also be found via the Rayleigh quotient with suitable trial choices for the modal vectors $\{\mathbf{X}\}_i$,

$$\lambda_i = \frac{\{\mathbf{X}\}_i^T[K]\{\mathbf{X}\}_i}{\{\mathbf{X}\}_i^T[M]\{\mathbf{X}\}_i}, \tag{5}$$

where the superscript T indicates transposition. If

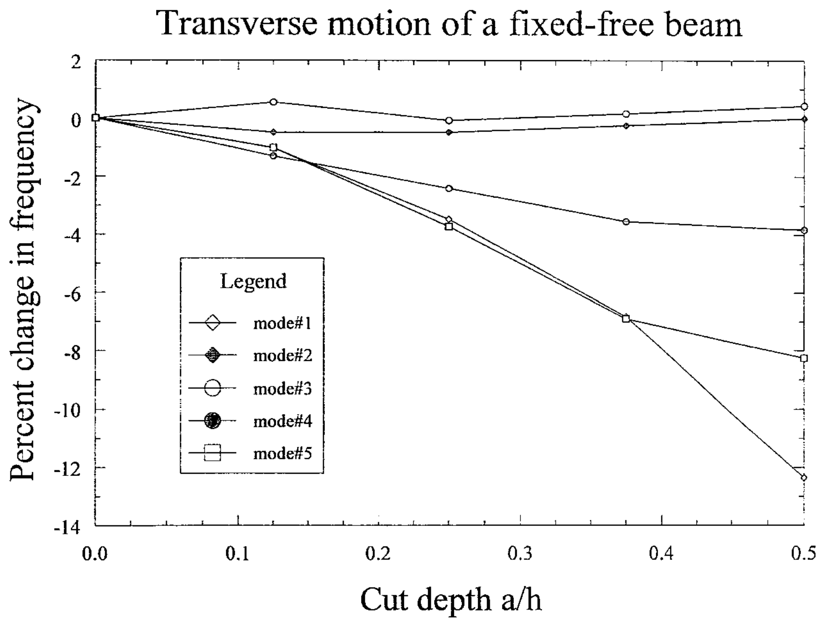

FIGURE 3 Comparison of mode number in frequency sensitivity with increasing damage.

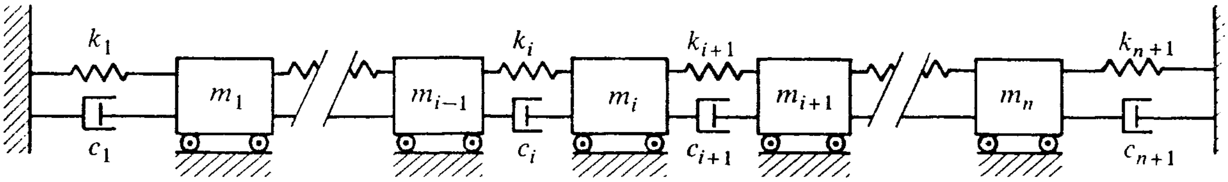

FIGURE 4 Definition sketch for lumped-parameter model (after Meirovitch, 1986).

the choices for modal vectors are exact, the natural frequencies are identical to those found from (4) above.

For a small change in the stiffness matrix, $[\Delta K]$, and mass matrix, $[\Delta M]$, the perturbed natural frequency can be found from $\lambda_i + \Delta\lambda_i$ by the Rayleigh quotient,

$$\lambda_i + \Delta\lambda_i = \frac{\{\mathbf{X}^*\}_i^T[K + \Delta K]\{\mathbf{X}^*\}_i}{\{\mathbf{X}^*\}_i^T[M + \Delta M]\{\mathbf{X}^*\}_i}, \qquad (6)$$

where $\{\mathbf{X}^*\}_i$ is the modal vector resulting from reduced stiffness (damaged springs) and mass. Now using (5), and regarding effects of changes in the mass matrix $[\Delta M]$ and modal vectors $\{\mathbf{X}^*\}$ to be an order of magnitude or more smaller than changes in $[K]$, the relationship between change in stiffness and change in frequency is given approximately by,

$$\Delta\lambda_i = \frac{\{\mathbf{X}\}_i^T[\Delta K]\{\mathbf{X}\}_i}{\{\mathbf{X}\}_i^T[M]\{\mathbf{X}\}_i}. \qquad (7)$$

The frequency sensitivity is found from

$$\frac{\Delta\omega_i}{\omega_i} = \frac{\omega_i^*}{\omega_i} - 1 = \sqrt{\frac{\lambda_i + \Delta\lambda_i}{\lambda_i}} - 1. \qquad (8)$$

The static deflection sensitivity, i.e., the percent change in static deflection δ due to local damage, is found from

$$\frac{\Delta\delta}{\delta} = 1 - \frac{\delta^*}{\delta} = 1 - \frac{F/k_{eq}^*}{F/k_{eq}} = 1 - \frac{k_{eq}}{k_{eq}^*}, \qquad (9)$$

where as before, * quantities represent damaged values. In (8) and (9) the reduction in frequency and stiffness is correctly accounted for by resultant negative values.

SENSITIVITY TO LOCAL DAMAGE: LONGITUDINAL MOTION

To investigate the relative sensitivity of deflection and vibration frequency to local damage, we first consider longitudinal motion of a fixed–free beam, represented initially by only 2 degrees of freedom (DOF; Fig. 5). For the undamaged beam, let $k_1 = k_2 = k$, $m_1 = m_2 = m$. Natural frequencies for the 2 DOF system are given by, say, (5) as $\lambda_1 = 0.382$ k/m and $\lambda_2 = 2.62$ k/m, where $\{\mathbf{X}\}_i^T = \{1 \pm 1.618\}$. (To match the eigenvalues for the limiting case of the continuous system, which are 1.57 and 3.14, respectively, the discrete eigenvalues must be multiplied by $(2n + 1)/2$; this is shown in the Appendix.) For purposes of calculating static deflection sensitivity, the equivalent spring constant is given by $k_{eq} = k/2$.

Let the beam now incur some damage such that, say, the first spring element, k_1, is reduced in stiffness to $k_1^* = k_1 - \varepsilon k_1 = (1 - \varepsilon)k$, where ε is a small number. We back out the value of ε by setting the "damage ratio" k_{eq}^*/k_{eq} equal to some arbitrary measure of damage, i.e., a value less than one, say 0.9: in that case, $\varepsilon = 0.25$.

The only nonzero term in $[\Delta K]$ is $\Delta k_{11} = -\varepsilon k$. The frequency and deflection sensitivities given by (8) and (9), respectively, are $\Delta\omega_1/\omega_1 = -0.0681$ and $\Delta\delta/\delta = -0.111$. Note that damaging one out of two springs is roughly equivalent to damaging one-half the length of the beam (excluding mass effects); static deflection is roughly the same for a crack of small width as it is for one of larger width.

For the continuous model, $k^*/k = (EA^*/L)/(EA/L) = A^*/A = 1 - \varepsilon$, and sensitivities are given by,

$$\frac{\Delta\omega_1}{\omega_1} = \frac{\omega_1^*}{\omega_1} - 1 = \frac{\sqrt{(\lambda_1 A^* E)/(\mu L^2)}}{\sqrt{(\lambda_1 AE)/(\mu L^2)}} - 1 = \sqrt{1 - \epsilon} - 1 \qquad (10)$$

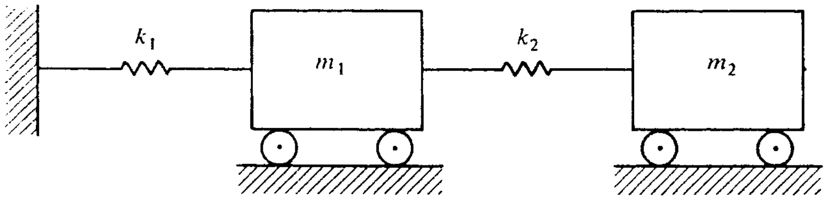

FIGURE 5 Two DOF lumped parameter model for longitudinal vibration of a fixed–free beam.

and

$$\frac{\Delta\delta}{\delta} = 1 - \frac{\delta^*}{\delta} = 1 - \frac{FL/A^*E}{FL/AE} = \frac{\epsilon}{\epsilon - 1}, \quad (11)$$

where μ = mass/length, E is the tensile modulus, and A and A^* are the undamaged and damaged cross-sectional areas, respectively. For the continuous model, $\Delta\omega_1/\omega_1$ is an upper limit because it assumes a uniform decrease in area A^*; in the longitudinal case, static deflection is insensitive to local or global reductions in area. With a damage ratio of 0.9, (10) and (11) give −0.0513 and −0.111, respectively.

To have the discrete model approach the continuous model, we increased the number of DOF (n). In Table 2 we provide results for frequency and deflection sensitivities as the number of DOF is increased for two damage ratios: k^*_{eq}/k_{eq} = 0.9 and 0.5 (equivalent to a crack halfway through the thickness), and for damage to spring $'k'_1$. Table 3 gives similar results for damage to spring $'k'_2$. Note that as n becomes large, the fundamental frequency of the discrete model approaches that of the continuous model, and the frequency sensitivity tends to zero, while deflection sensitivity remains constant. Also note that a single spring damaged among 10,000 springs roughly represents a crack of width 1/10,000 or 0.01 mm in 0.1 m. Frequency sensitivity results are plotted in Fig. 6 for the discrete fixed–free case, and similar results are plotted in Fig. 7 for the fixed–fixed case. Both figures also show for reference the static deflection sensitivity and continuous model frequency sensitivity limit for a damage ratio equal to 0.9.

For the case of longitudinal motion, deflection was more sensitive to local damage than frequency. This is easily verified if we rewrite (10) and (11) as

Table 2. Frequency and Deflection Sensitivities for Longitudinal Motion of Fixed–Free Beam for Two Values of Damage Ratio, k^*_{eq}/k_{eq} = 0.9 and 0.5; Spring k_1 Damaged

DOF	(cf. 1.571) $[(2n+1)/2]\lambda_1$	$k^*_{eq}/k_{eq} = 0.9$ $\Delta\omega_1/\omega_1$	$\Delta\delta/\delta$	$k^*_{eq}/k_{eq} = 0.5$ $\Delta\omega_1/\omega_1$	$\Delta\delta/\delta$
2	1.545	−0.0681	0.111	−0.281	1.0
3	1.545	−0.0704	0.111	−0.230	1.0
4	1.563	−0.0687	0.111	−0.191	1.0
5	1.569	−0.658	0.111	−0.161	1.0
10	1.571	−0.0512	0.111	−0.0902	1.0
20	1.571	−0.0342	0.111	−0.0475	1.0
50	1.571	−0.0169	0.111	−0.0196	1.0
100	1.571	−0.00917	0.111	−0.0099	1.0
200	1.571	−0.00478	0.111	−0.00498	1.0
500	1.571	−0.00196	0.111	−0.002	1.0
1000	1.571	−9.91E-4	0.111	−9.99E-4	1.0
5000	1.571	−2.0E-4	0.111	−2.0E-4	1.0
10,000	1.571	−9.99E-5	0.111	−9.99E-5	1.0

Table 3. Frequency and Deflection Sensitivities for Longitudinal Motion of Fixed–Free Beam for Two Values of Damage Ratio, k^*_{eq}/k_{eq} = 0.9 and 0.5; Spring k_2 Damaged

DOF	(cf. 1.571) $[(2n+1)/2]\lambda_1$	$k^*_{eq}/k_{eq} = 0.9$ $\Delta\omega_1/\omega_1$	$\Delta\delta/\delta$	$k^*_{eq}/k_{eq} = 0.5$ $\Delta\omega_1/\omega_1$	$\Delta\delta/\delta$
2	1.545	−0.0255	0.111	−0.0968	1.0
3	1.545	−0.0447	0.111	−0.141	1.0
4	1.563	−0.0527	0.111	−0.144	1.0
5	1.569	−0.0553	0.111	−0.134	1.0
10	1.571	−0.0488	0.111	−0.086	1.0
20	1.571	−0.0338	0.111	−0.047	1.0
50	1.571	−0.0169	0.111	−0.0196	1.0
100	1.571	−0.00917	0.111	−0.0099	1.0
200	1.571	−0.00478	0.111	−0.00497	1.0
500	1.571	−0.00196	0.111	−0.002	1.0
1000	1.571	−9.91E-4	0.111	−9.99E-4	1.0
5000	1.571	−2.0E-4	0.111	−2.0E-4	1.0
10,000	1.571	−9.99E-5	0.111	−9.99E-5	1.0

$$\frac{\Delta\omega_1}{\omega_1} = \sqrt{\frac{k^*}{k}} - 1, \quad (12)$$

$$\frac{\Delta\delta}{\delta} = 1 - \frac{k}{k^*}. \quad (13)$$

In Fig. 8 we plot the "relative sensitivity ratio" $(\Delta\omega/\omega)/(\Delta\delta/\delta)$ versus $[1 - (k^*/k)]$ for the continuous model. At best, frequency is seen to be only about half as sensitive as deflection, becoming even less so as damage increases.

It is clear from the above that frequency is a "distributed" phenomenon in the sense that the dynamical system properties are distributed with

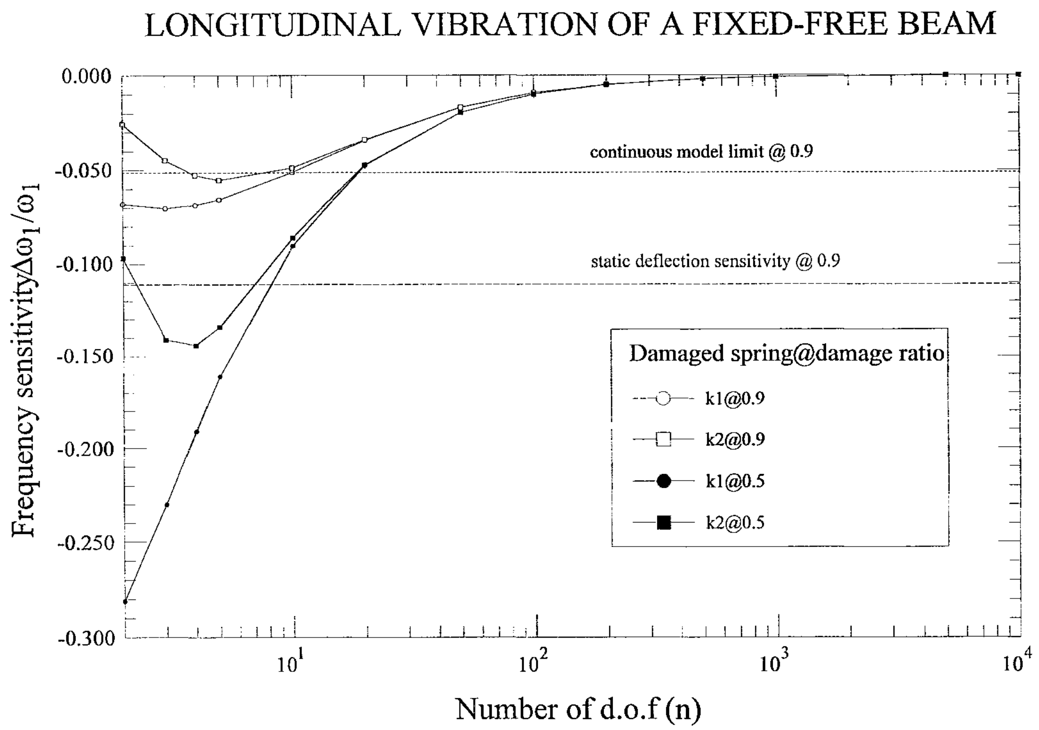

FIGURE 6 Frequency sensitivity for the longitudinal vibration of a lumped parameter fixed–free beam for various damage ratios and damaged springs, and extended in the limit toward the continuous model.

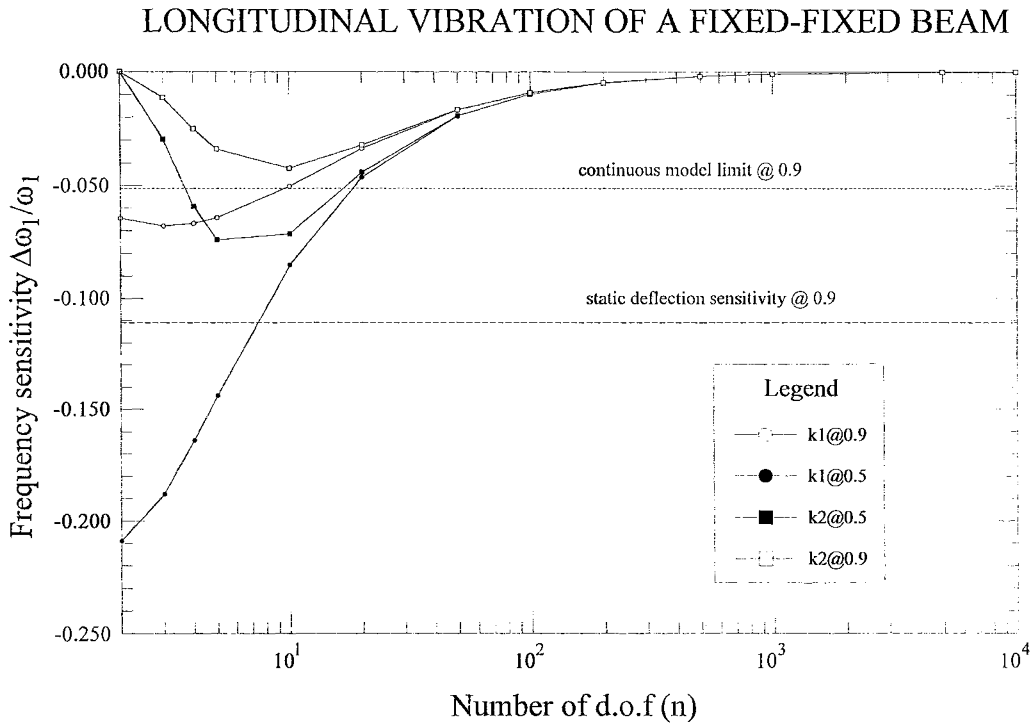

FIGURE 7 Frequency sensitivity for the longitudinal vibration of a lumped parameter fixed–fixed beam for various damage ratios and damaged springs, and extended in the limit toward the continuous model.

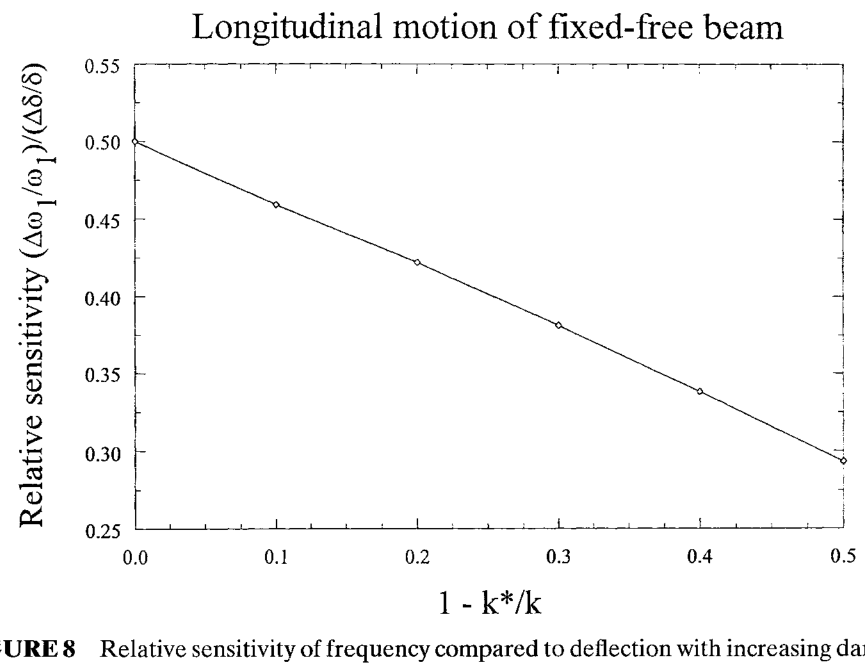

FIGURE 8 Relative sensitivity of frequency compared to deflection with increasing damage for the longitudinal motion of a fixed–free beam.

the mass; hence frequency is relative insensitive to local damage. On the other hand, deflection is a "local" property, independent of mass distribution. Local reductions in stiffness affect deflections globally, in a "weak-link" sense; hence, deflection has a relatively high sensitivity to local damage. [See also Springer et al. (1987) for a discussion of damage in longitudinally vibrating beams.]

SENSITIVITY TO LOCAL DAMAGE: TRANSVERSE MOTION

A crack in a beam undergoing transverse motion can be simulated mathematically by a rotational spring. For such a model, the frequency and deflection sensitivities can be given as (Chondros and Dimarogonas, 1980; Dimarogonas and Paipetis, 1983; Kjerengtroen and Jenkins, 1994),

$$\frac{\Delta\omega_1}{\omega_1} = 10.7\frac{h}{L}G\left(\frac{l_1}{L}\right)J\left(\frac{a}{h}\right) \qquad (14)$$

and

$$\frac{\Delta\delta}{\delta} = 16\frac{h}{L}\left(1-\frac{l_1}{L}\right)^2 J\left(\frac{a}{h}\right), \qquad (15)$$

where $G(l_1/L)$ is a function that accounts for crack location, $J(a/h)$ is a dimensionless local compliance function, and we take the sensitivities in an absolute value sense for convenience. [It should be noted that (14) is restricted to specific boundary conditions.] Figure 9 shows Eqs. (14) and (15) replotted on the experimental data of Fig. 2 discussed earlier. Again we note the relatively higher sensitivity of deflection versus fundamental frequency.

Further verification is possible through use of finite element analysis. A cantilever beam 457 mm (18 in.) long with a 12.7 mm (0.5 in.) square cross section, and having a rotational spring at the fixed end to simulate damage, was modeled using Images® software. The rotational stiffness was determined from linear elastic fracture mechanics (Liang et al., 1992). A tip load was applied to determine deflection sensitivities, while eigenvalue analysis was employed for frequency sensitivity. Both sensitivities are plotted in Fig. 9.

Examination of Fig. 9 reveals that theoretical frequency predictions slightly underpredicted experimental values for all cases, while there was a mix of slight under- and overprediction for deflections. In every case, however, deflection was always a better predictor of damage than frequency.

MORE COMPLEX STRUCTURES

There has been recent interest in applying dynamic response techniques to more complex structures such as portal frames (Chondros and Dimarogonas, 1989; Akgun and Ju, 1990). The increased sensitivity of deflection over frequency is exhibited in these types of structures as well. As a simple example, we took three 457 × 12.7 × 12.7 mm beams and assembled them as a portal frame, with a rotational spring simulating damage at the fixed end of the left column (Fig. 10). Deflections were measured under the action of a point load as shown. Frequencies were found for the first symmetrical in-plane (breathing) mode and the first asymmetrical in-plane mode. Results are plotted in Fig. 11. Once again, deflection changes outperformed frequency changes in the prediction of structural damage. (Additional results for this and more complicated structures are in preparation and will be presented in a forthcoming article by the authors.)

CONCLUSIONS

Robust nondestructive evaluation of structural health is a worthy but challenging opponent. It may well turn out that no one technique will dominate, but that different techniques, or combinations of techniques, will be required for different situations. At least for the simple structures investigated here, it was shown that static deflection is a more sensitive evaluator of structural damage than frequency change. This was verified with fundamental considerations, numerical analysis, and an experiment. Static deflection testing may also be considerably more tractable in many situations than dynamic response testing. The extension to more complex structures is currently under investigation by the authors.

What we have attempted to point out in this article are two fundamental ideas: first, static deflection is a local phenomenon, while dynamic response (frequency in particular) is a global or distributed phenomenon; we used the limit of a lumped mass model to demonstrate this effect. This leads us to our second point: static deflection is fundamentally more sensitive to damage than natural frequency, at least for the simple structures

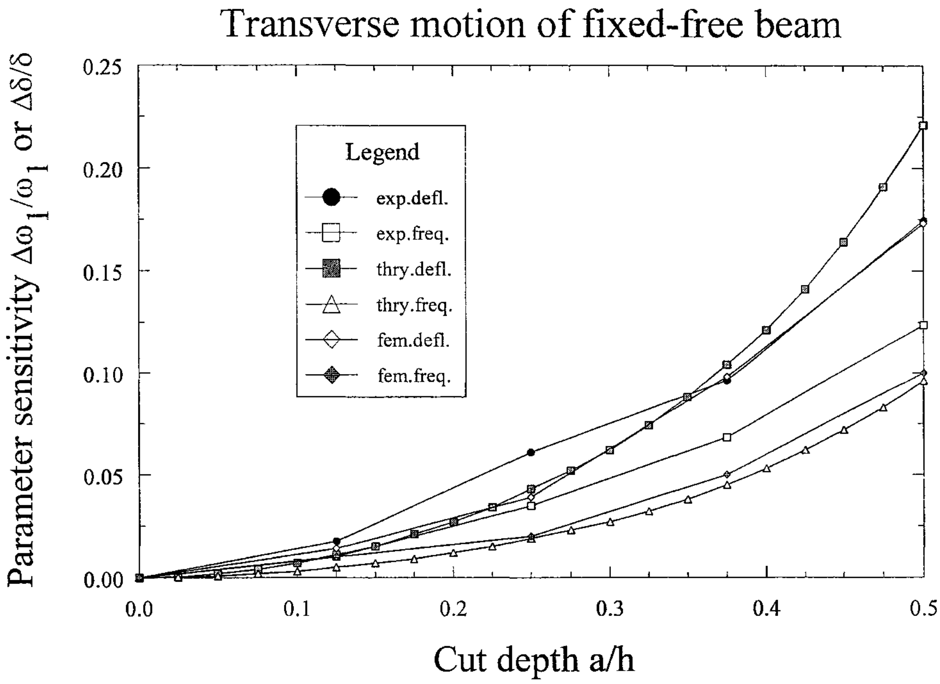

FIGURE 9 Comparison of frequency and deflection sensitivities, both theoretical and experimental, for the transverse motion of a cantilever beam.

we investigated. This was demonstrated by numerical analysis and by experiment.

This is not to say that static deflection tests will always be more robust that dynamic response tests. We can think of many cases where static deflection tests would be impractical or impossible. What we hope to have shown is that there are fundamental reasons not to exclude static deflections tests from the damage detection toolbox.

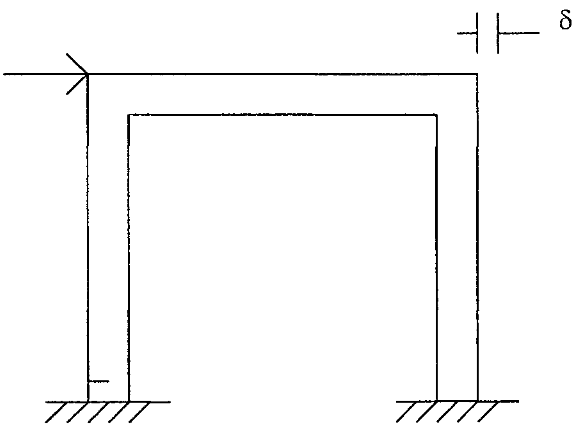

FIGURE 10 Definition sketch of a damaged portal frame.

APPENDIX

To compare eigenvalues for the discrete and distributed models, we can simply take the limit of the discrete model as the number of DOF (n) goes to ∞, then compare this to the distributed model and adjust for any missing terms. For example, in the fixed–free longitudinal case, the discrete eigenvalue is given by (Blevins, 1984)

$$2\sin\left[\frac{(2i-1)}{(2n+1)}\frac{\pi}{2}\right], \tag{A.1}$$

where i is the mode number.

In the limit as $n \rightarrow \infty$, the denominator of the sine argument becomes large, the argument itself becomes small, and we replace the sine function by its argument in the usual small angle approximation. We then set twice the sine function argument equal to the distributed eigenvalue (Blevins, 1984)

$$(2i-1)\frac{\pi}{2}, \tag{A.2}$$

giving

$$C\frac{2(2i-1)}{(2n+1)}\frac{\pi}{2} = (2i-1)\frac{\pi}{2}, \tag{A.3}$$

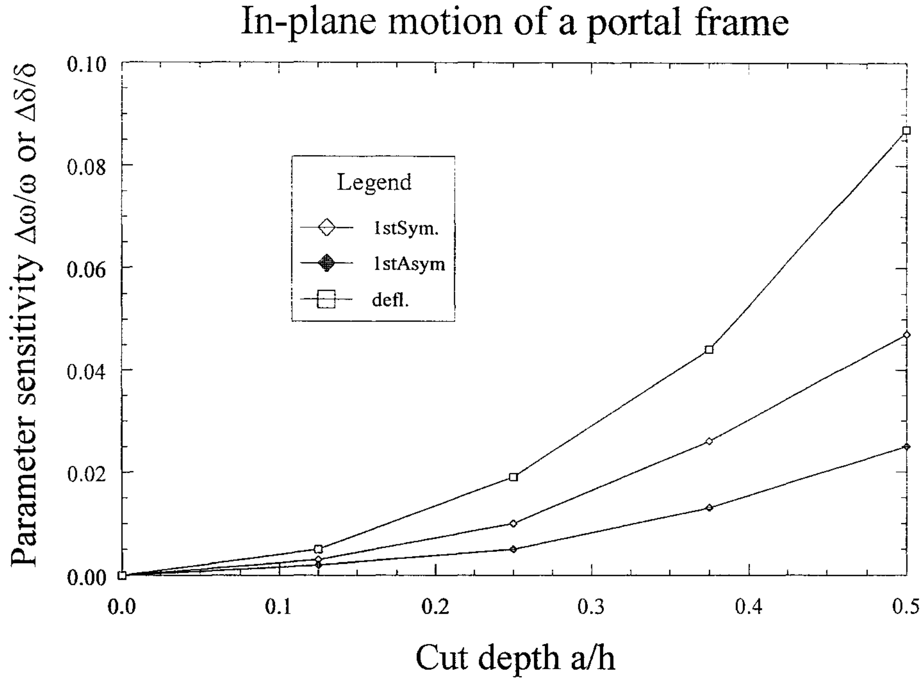

FIGURE 11 Comparison of frequency and deflection sensitivity with increasing damage for a portal frame.

where the constant C has been included to enforce the equality. By inspection then, the constant $C = (2n + 1)/2$.

A similar treatment of the fixed–fixed longitudinal case would result in $C = n + 1$.

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