## BRIEF COMMUNICATIONS

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# Sensitivity of the scale partition for variational multiscale large-eddy simulation of channel flow

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The variational multiscale method has been shown to perform well for large-eddy simulation (LES) of turbulent flows. The method relies upon a partition of the resolved velocity field into large- and small-scale components. The subgrid model then acts only on the small scales of motion, unlike conventional LES models which act on all scales of motion. For homogeneous isotropic turbulence and turbulent channel flows, the multiscale model can outperform conventional LES formulations. An issue in the multiscale method for LES is choice of scale partition and sensitivity of the computed results to it. This is the topic of this investigation. The multiscale formulation for channel flows is briefly reviewed. Then, through the definition of an error measure relative to direct numerical simulation (DNS) results, the sensitivity of the method to the partition between large- and small-scale motions is examined. The error in channel flow simulations, relative to DNS results, is computed for various partitions between large- and small-scale spaces, and conclusions drawn from the results.  [DOI: 10.1063/1.1644573]

The numerical method uses a Fourier basis in the streamwise and spanwise directions, and modified Legendre polynomials in the wall-normal direction. Full details of the numerical procedure can be found in Hughes *et al.*[1] The velocity field is decomposed into large- ($\overline{\boldsymbol{u}}$) and small-scale ($\boldsymbol{u}'$) components

$$\boldsymbol{u}=\overline{\boldsymbol{u}}+\boldsymbol{u}', \tag{1}$$

according to the wave numbers in the streamwise and spanwise directions and the polynomial order in the wall-normal direction. The partition of the scales is denoted by a scalar $\overline{N}$. A mode, given by $(k_x, n_y, k_z)$, is part of the "large-scale" motions if

$$-\frac{\overline{N}}{2}<k_x<\frac{\overline{N}}{2}, \quad 0\leqslant n_y<\overline{N}, \quad -\frac{\overline{N}}{2}<k_z<\frac{\overline{N}}{2}, \tag{2}$$

where $k_x$ and $k_z$ are the wave numbers in the streamwise and spanwise directions, respectively, and $n_y$ is the polynomial order in the wall-normal direction. Modes which do not form part of the large-scale motions constitute the small-scale basis. All the results presented in the following use 32 modes in each spatial direction.

For all multiscale cases, the subgrid stress is calculated from

$$\boldsymbol{\tau}=2\nu_T\nabla^s\boldsymbol{u}', \tag{3}$$

where $\nu_T$ is the eddy viscosity and $\nabla^s\boldsymbol{u}'$ denotes the symmetrical part of the small scales velocity gradient. The eddy viscosity may depend on the large, small, or on all scales.

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For the static ''large–small'' multiscale version of the Smagorinsky model[1–3] the eddy viscosity is calculated from large scales

$$\nu_T=(C_s\Delta)^2|\nabla^s\overline{\boldsymbol{u}}|, \tag{4}$$

where $C_s$ is the Smagorinsky constant and $\Delta$ is a discretization-dependent length scale. For the static ''small–small'' version of the Smagorinsky model,[1–3] the eddy viscosity is calculated from small scales

$$\nu_T=(C_s\Delta)^2|\nabla^s\boldsymbol{u}'|. \tag{5}$$

The third multiscale model used in this study utilizes the dynamic procedure for calculating the Smagorinsky parameter, $C_s\Delta$.[4,5] Once the term $C_s\Delta$ has been computed, based on the flow field, the eddy viscosity is calculated from all scales (i.e., large plus small scales)

$$\nu_T=(C_s\Delta)^2|\nabla^s\boldsymbol{u}|. \tag{6}$$

This version will be denoted dynamic ''all-small.'' Details of the adopted implementation for computing the Smagorinsky parameter can be found in Hughes *et al.*[1] Once the eddy viscosity has been calculated, the subgrid stress is calculated according to Eq. (3). It is apparent that there are a number of possible ways to combine the dynamic procedure with the multiscale method. The rationale behind the choice made here is as follows: In our initial studies of static multiscale methods, we used the same $C_s$ as for the static Smagorinsky model (0.1 in all cases). We made no attempt to optimize $C_s$ for the multiscale cases. Analogously, here in our initial study of a dynamic multiscale procedure, we selected the $C_s\Delta$ obtained by the conventional dynamic procedure. Again, no attempt was made to optimize $C_s\Delta$ for the dynamic multiscale case. Clearly, to do so would offer potential further improvements and would constitute a worthwhile avenue of research. Studies have been initiated in which the Germano identity[4] is directly applied to the multiscale models. We hope to report upon this in the near future.

Turbulent channel flows at $\mathrm{Re}_\tau=180$, $\mathrm{Re}_\tau=395$, and $\mathrm{Re}_\tau=590$ have been previously computed using the static large–small and small–small multiscale formulations.[1,3] Here, results are presented at $\mathrm{Re}_\tau=395$ for the dynamic all-small formulation ($\bar{N}/N'=0.5$), the conventional dynamic Smagorinsky model, and DNS data.[6] Full details of the numerical formulation and the channel configuration can be found in Hughes *et al.*[2] Mean velocity profiles are compared in Fig. 1. The multiscale results are so close to the DNS results that it is difficult to distinguish the two responses. The velocity fluctuations in each spatial direction are shown in Fig. 2. Both LES models perform well in predicting the streamwise fluctuations. However, the multiscale model is significantly better than the conventional dynamic model in the other two spatial directions.

To examine the sensitivity of the multiscale formulations to the scale partition, the deviation of the LES results from the DNS results is quantified for the mean velocity and the velocity fluctuations. All quantities are nondimensional and are averaged in the streamwise and spanwise directions, and in time. All results presented here are for $\mathrm{Re}_\tau=395$. An initial study was performed for the $\mathrm{Re}_\tau=180$ case and the results and conclusions were similar to those for $\mathrm{Re}_\tau=395$. Consequently, the $\mathrm{Re}_\tau=180$ results are not shown. For the static large–small, and small–small Smagorinsky models, $C_s=0.1$. Note that no wall damping function is used for any calculations.

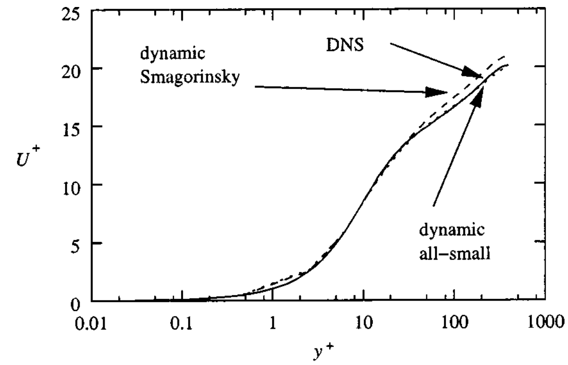


FIG. 1. Mean streamwise velocity profile at $\mathrm{Re}_\tau=395$. For the multiscale result $\bar{N}=16$.

The first quantity examined is the mean flow. The error $e$ in the mean flow is defined as

$$e=\left(\frac{1}{\delta}\int_{-\delta/2}^{\delta/2}(U^+_{\mathrm{DNS}}-U^+_{\mathrm{LES}})^2dy\right)^{1/2}, \tag{7}$$

where $U^+$ is the mean velocity, $\delta$ is the channel height, and the $y$ direction is normal to the wall. The error is examined for the previously outlined multiscale cases: The large–small model; the small–small model; and the dynamic all-small model. For each LES model, the error is shown as a function of the scale partition $\bar{N}$. The error in the mean velocity profile is shown in Fig. 3. For the static multiscale models, we have included results for scale partitions $\bar{N}=16,18,...,32$. For the dynamic multiscale model, we have included results for $\bar{N}=8,10,...,24$. These ranges include the optimal locations and are sufficient to determine the sensitivity of the results as we move away from the optimal locations. In addition to the error at each computed scale partition, Fig. 3 includes a quadratic least-squares fit for each case. For the static large–small and small–small cases, the error drops rapidly when the partition ratio is close to 0.65. Increasing the size of the large-scale space (increasing $\bar{N}/N'$), the error increases steadily as the no model case is approached ($\bar{N}/N'=1$). Decreasing the size of the large-scale space, the error increases rapidly away from $\bar{N}/N'=0.65$. While the static large–small and small–small formulations can yield very accurate results, there is sensitivity to the partition between large and small scales. This is evident in the form of the least-squares fitted quadratic polynomial for the two static cases. The parabolas are steep away from the optimal partition and the minima are considerably below the parabolas. Clearly, the data are more ''V'' shaped that parabolic. The smallest error for the dynamic multiscale case is close to $\bar{N}/N'=0.5$. In contrast to the static large–small and small–small formulations, the dynamic multiscale model is relatively insensitive to the scale partition. This is manifest in the ''flatness'' of the fitted quadratic polynomial and the lower minimum. While

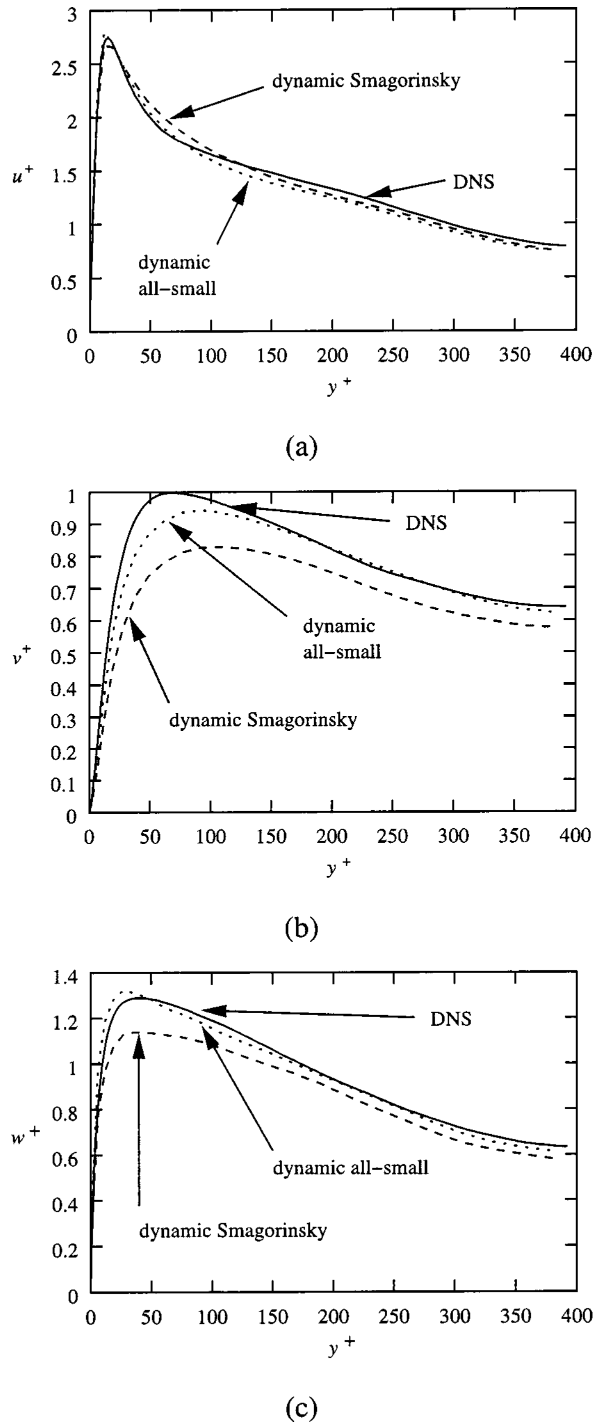


FIG. 2. RMS velocity fluctuations in the (a) streamwise, (b) wall-normal and (c) spanwise directions at $\mathrm{Re}_\tau=395$. For the multiscale results $\bar{N}=16$.

the minimum error for the dynamic multiscale model is slightly larger than that for the other static cases, there exists a broad range of partitions for which the dynamic multiscale model produces satisfactory results. The relative insensitivity of the dynamic multiscale model can be attributed to its enhanced ability to adapt and respond to the flow conditions. Reference plateaus for the static Smagorinsky model, a coarse DNS (i.e., no model) and the dynamic Smagorinsky model are included for comparison purposes. As may be inferred from Fig. 1, the conventional dynamic model is quite accurate for this case.

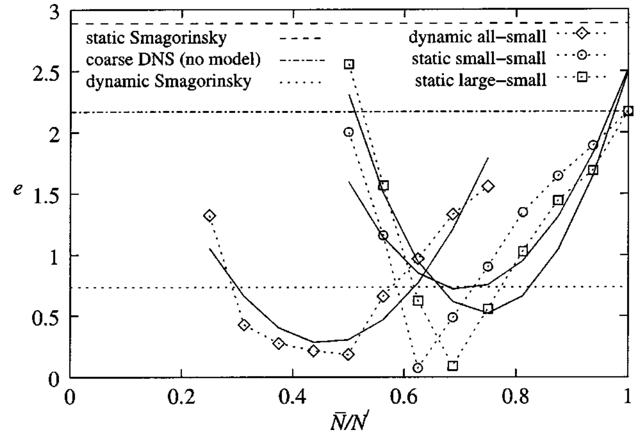


FIG. 3. Error in the mean velocity profile as a function of multiscale partition.

The error in the velocity fluctuations for the three multiscale models is shown in Fig. 4. Again, a quadratic polynomial has been fitted to the results. The error is calculated as

$$e=\left(\frac{1}{\delta}\int_{-\delta/2}^{\delta/2}((u^+_{\mathrm{DNS}}-u^+_{\mathrm{LES}})^2+(v^+_{\mathrm{DNS}}-v^+_{\mathrm{LES}})^2+(w^+_{\mathrm{DNS}}-w^+_{\mathrm{LES}})^2)dy\right)^{1/2}, \tag{8}$$

where $u^+$, $v^+$, and $w^+$ are the fluctuations in the streamwise, wall-normal and spanwise directions, respectively. The error results for the velocity fluctuations follow the same trend as the errors for the mean velocity profile. Reference plateaus are provided for a coarse DNS and the dynamic Smagorinsky model. The static Smagorinsky model plots off-scale in this case. The static large–small and small–small cases attain their minimum error at approximately $\bar{N}/N'=0.6$. As for the mean flow, the minimum error for the dynamic multiscale model is at a lower partition ratio, $\bar{N}/N'=0.5$. Again, the error increases slowly for the dynamic multiscale model as the partition moves away from the optimal point. The flat nature of the dynamic multiscale error across different partition ratios indicates its relative insensitivity to the scale partition. A broad band of partition ratios yield satisfactory results, both in terms of the mean flow and

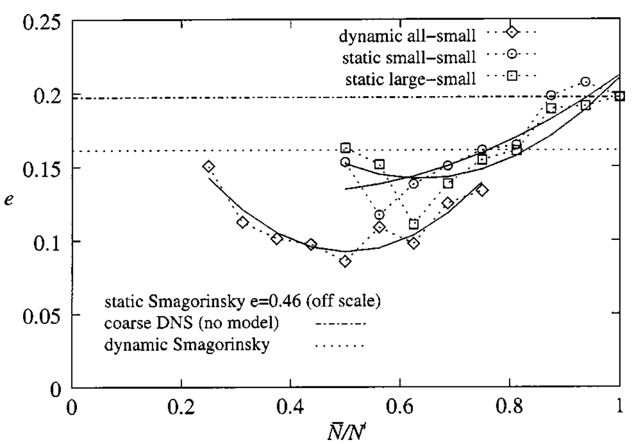


FIG. 4. Error in the velocity fluctuations as a function of multiscale partition.

velocity fluctuations. Hence, in future channel flow simulations with the dynamic multiscale method, a partition $\bar{N}/N'$ close to 0.5 could be expected to yield good results and is recommended; for the static multiscale methods, partitions $\bar{N}/N'$ in the range 0.6–0.7 may also be recommended. Smaller partitions are not recommended as they can behave erratically. This seems due to the very small fraction of large-scale modes which is approximately equal to $(\bar{N}/N')^3$. The static cases are again more sensitive to this than the dynamic case. As partitions approach the coarse DNS limit, $\bar{N}/N' \rightarrow 1$, all results behave fairly smoothly and almost monotonically. This may be seen for the static multiscale cases in Figs. 3 and 4. The dynamic multiscale model behaves similarly (not shown).

The sensitivity of the variational multiscale method for LES to the partition between large and small scales has been investigated and quantified. It has been shown that the multiscale method, in combination with a dynamic procedure for calculating the Smagorinsky parameter, is relatively insensitive to the chosen partition. For the static multiscale models, the computed results are highly accurate at the optimal partition ratio, but are more sensitive to the partition than the dynamic multiscale model.

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