# Sensor Networks With Mobile Access: Optimal Random Access and Coding

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***Abstract*—We consider random access and coding schemes for sensor networks with mobile access (SENMA). Using an orthogonal code-division multiple access (CDMA) as the physical layer, an opportunistic ALOHA (O-ALOHA) protocol that utilizes channel state information is proposed. Under the packet capture model and using the asymptotic throughput as the performance metric, we show that O-ALOHA approaches the throughput equal to the spreading gain with an arbitrarily small power at each sensor. This result implies that O-ALOHA is close to the optimal centralized scheduling scheme for the orthogonal CDMA networks. When side information such as location is available, the transmission control is modified to incorporate either the distribution or the actual realization of the side information. Convergence of the throughput with respect to the size of the network is analyzed. For networks allowing sensor collaborations, we combine coding with random access by proposing two coded random access schemes: spreading code dependent and independent transmissions. In the low rate regime, the spreading code independent transmission has a larger random coding exponent (therefore, faster decay of error probability) than that of the spreading code dependent transmission. On the other hand, the spreading code dependent transmission gives higher achievable rate.**

***Index Terms*—Coding, mobile access point, random access, sensor network, throughput.**

## I. Introduction

### A. Multiple Access in SENMA

WE CONSIDER the design of multiple access and coding for large scale sensor networks with mobile access (SENMA) points [1]. As an architecture illustrated in Fig. 1, SENMA has two types of nodes: a large number of low-power sensors randomly deployed in a wide geographical area, and a few more powerful mobile access points (APs) that are capable of performing more sophisticated tasks from information retrieval and processing to network maintenance. Examples of mobile APs include manned/unmanned aerial vehicles (UAVs), ground vehicles equipped with sophisticated terminals and power generators, or specially designed light nodes that can hop around in the network.

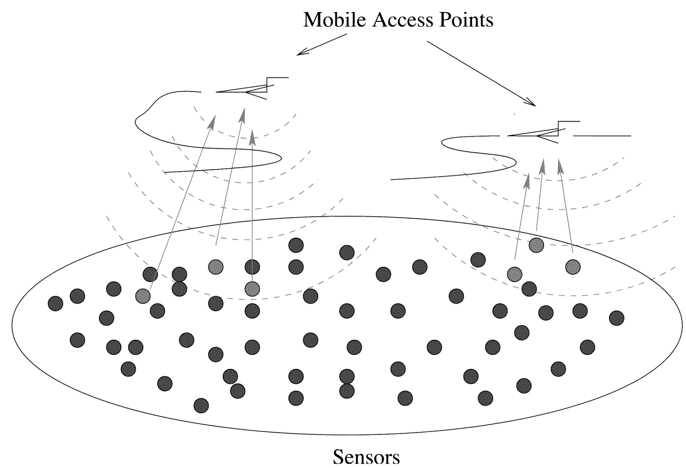


Fig. 1. Sensor network with mobile access points.

Architecturally, SENMA is the opposite of cellular network; while mobile nodes communicate with fixed base station in cellular networks, mobile APs communicate with stationary sensors in SENMA. The main reason that SENMA is a favorable choice for some applications is the power and energy efficiency for sensors. For battery operated microsensors, sensors should transmit or receive as infrequently as possible. It is, thus, crucial to shift networking functions away from sensors to a set of interconnected super nodes—the mobile access points in SENMA—that are less power and bandwidth constrained. The mobility of APs makes it possible that sensors communicate, for the most part, directly with access points. Consequently, as in cellular networks, most transmissions of sensors are for information delivery rather than network maintenance. For ad hoc sensor networks, in contrast, the transmission of protocol overhead by sensors can be a source of overwhelming power consumption. A key component of SENMA, then, is medium access control that governs the interaction between sensors and mobile APs.

In any multiaccess system, centralized scheduling provides the highest network throughput. In SENMA, centralized scheduling would mean that the mobile AP is able to address each sensor individually and is aware of the channel between itself and the sensor. The complexity of such schemes is unacceptable for large networks. A more practical choice, although in general suboptimal, is distributed medium access, where each sensor schedules its transmission locally. The design of distributed access protocol for SENMA, however, is nontrivial. The large number of sensor nodes, the lack of central control,

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the channel fading, and node duty cycle, all make the design especially challenging. The multiaccess protocol for SENMA should have certain basic properties like high throughput, efficient channel utilization and power efficiency. While the data rate from each sensor is low, the time allowed for the mobile AP to collect data can be constrained, especially in military applications. For large scale sensor networks, battery operated sensors have limited power and range. It is, therefore, necessary that the sensor transmits only under favorable fading conditions. The protocol should also be easy to implement. Each node should involve minimal processing and rely as little as possible on feedback.

### B. Main Results and Organization

In this paper, we consider a general form of distributed access, referred to as opportunistic ALOHA (O-ALOHA), that incorporates channel state information. Specifically, each sensor transmits a packet with a probability $s(\gamma)$ that is a function of its own channel state $\gamma$. The transmission control $s(\cdot)$ is chosen to optimize the network throughput. Details are presented in Section II. We then investigate the fundamental limits of SENMA using asymptotic throughput as the performance metric. Based on a physical layer using orthogonal code-division multiple access (CDMA), we show in Section III that, in the presence of fading, the optimal O-ALOHA performs as well as the optimal centralized scheduling. The use of channel state information and multiuser diversity are crucial to this result. By trading network size for power, we also show that the maximum asymptotic throughput can be achieved with arbitrarily small transmitting power of each sensor. An interesting feature of sensor networks is collaborative transmission. Because random access does not guarantee that all packets are received by the mobile access point, we propose in Section IV two coded O-ALOHA schemes for reliable information retrieval. Achievable rates and error exponents of these schemes are characterized. Finally, numerical results are presented in Section V.

### C. Related Work

The idea of using centralized channel state information in multiple access was considered by Knopp and Humblet [2], Tse and Hanly [3], and others, all in an information theoretic setting. The main conclusion is twofold. First, significant gains can be realized by scheduling transmissions based on users' channel states. Knopp and Humblet showed that the optimal centralized scheduler, in the sense of maximizing the sum rate, is a deterministic single user transmission [2]. The distributed use of channel state information was considered by Telatar and Shamai [4] and Viswanath *et al.* [5], again using the sum rate as the performance metric. They concluded that the the use of distributed scheduling incurs little loss. Qin and Berry [6] proposed the use of channel state information in ALOHA coupled with power control at the transmitter. Using a simple threshold policy under the collision channel model, they demonstrate the effect of multiuser diversity. In particular, they show that, the throughput of their scheme grows (with the number of users) at the same rate as that of the centralized scheduler. The throughput achieved, however, is only a fraction $(1/e)$ of that of the centralized scheduler [7].

We adopt the framework established in [8] and [9] for using channel state information in transmission control. Our focus is on a practical implementation using orthogonal CDMA as the physical layer with a fixed number of spreading codes. We also characterize the rate of convergence of the throughput of finite networks to the asymptotic results in [8] and [9].

The literature on medium access control for ad hoc sensor network is extensive [10], [11]. Self-organizing medium access for sensor networks or SMACS [12] is a distributed protocol, which enables a collection of nodes to discover their neighbors and schedule communications. They consider a flat topology and do not require the use of mobile nodes. In the Eavesdrop and Register (EAR) algorithm [12], mobile nodes are incorporated into the system, which continuously monitor the stationary network and initiate handshaking procedures when desired. These mobile nodes maintain a partial registry of stationary nodes, and are not responsible for data transfer in the network. Heinzelman *et al.* in [13] proposed LEACH, a medium access control (MAC) protocol for sensor networks, where nodes are grouped into clusters with one node in each cluster acting as the cluster-head. The cluster-head is stationary and is responsible for collecting data from the nodes and deliver them to the base station. In the message ferrying (MF) scheme proposed in [14], mobile nodes (termed as ferries) take responsibility of carrying messages between disconnected stationary nodes. The authors mainly consider proactive routing by exploiting nonrandomness in the mobility of the message ferries. Similarly, in [15], mobile entities called MULEs are present in the sensor environment. MULEs pick up data from the sensors when in close range and drop off the data at access points. A MAC protocol for wireless ad hoc networks with mobile relaying nodes has also been considered by Bansal and Liu in [16], however, their focus is on delay and routing. Other MAC protocols for wireless networks are based on contention resolution and involve overhead in the form of requests or transmission of user ID [11].

## II. Opportunistic ALOHA

### A. Protocol Discipline

In this section, we describe the working of the O-ALOHA protocol [8], [9]. We consider a network where $n$ sensors communicate to a mobile AP over a common channel. Time is slotted into equal intervals of one unit, corresponding to the size of one packet. Slot $t$ occupies the time interval $[t, t+1)$.

The slot structure is shown in Fig. 2. The network is assumed to operate in the time-division duplex (TDD) mode. At the beginning of each slot, the mobile AP transmits a beacon that is used by each sensor to gain synchronization and estimate its channel gain. By reciprocity, we assume that this is the channel gain from the sensor to the mobile AP. We denote $\gamma_i^{(t)}$ as the channel from the $i$th sensor to the AP in slot $t$. During the data transmission period, each sensor transmits its packet with a probability $s(\gamma_i^{(t)})$. Function $s(\cdot)$ is called the transmission control that maps the channel state to a probability. Since the protocol mandates that the probability of transmission be a function of the channel state, it is called O-ALOHA.

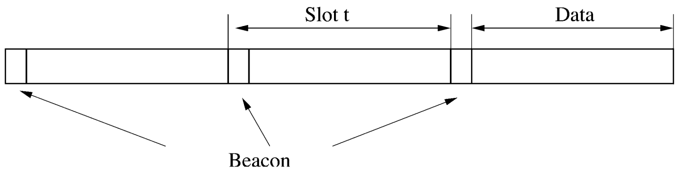


Fig. 2. Slot structure.

### B. Data Transmission and Reception

We assume that the physical layer of the sensor network is based on CDMA with spreading gain $N$. It is assumed that there is a pool of $N$ orthogonal codes, and each transmitting sensor randomly selects one of these $N$ codes to transmit its data. The receiver at the mobile AP uses $N$ matched filters to demodulate the received data. By ignoring the interference from users transmitting on other spreading codes, we can, without loss of generality, focus on the output of a single matched filter. We assume that a packet is received successfully if the signal-to-interference-noise ratio (SINR) is greater than a threshold $\beta$. Specifically, in slot $t$, if $k$ sensors choose to transmit using a particular orthogonal code, and their channel gains are given by $\boldsymbol{\gamma}_k = (\gamma_1, \ldots, \gamma_k)$, then the outcome can be represented by $\boldsymbol{\theta}_k = (\theta_k^1, \ldots, \theta_k^k)$, where $\theta_k^{(i)}$ equal to one implies the success of the $i$th sensor and zero otherwise. The criterion for successful reception of sensor $i$ is well-approximated [17] by

$$\theta_k^i = \begin{cases} 1, & \frac{P_T \gamma_i}{\sigma^2 + \sum_{j=1, j\neq i}^{k} P_T \gamma_j} > \beta \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where $P_T$ is the transmitting power and $\sigma^2$ the variance of the background noise. The parameter $\beta$ is the minimum SINR needed for successful reception and is determined by factors such as type of modulation and receiver sensitivity. Depending on the value of $\beta$, more than one packet can be captured successfully. However, no more than $1 + \lfloor (1/\beta) \rfloor$ can be captured [17]. If we assume $\beta > 1$, then at most one packet can be captured per orthogonal code. Therefore, the maximum throughput possible per slot will be equal to the spreading gain $N$. The channel is defined by the conditional probability mass function $p(\boldsymbol{\theta}_k \,|\, \boldsymbol{\gamma}_k)$.

Note that the transmitting power and channel gain jointly determine the outcome. Thus, for arbitrary transmitting power $P_T$, a user with sufficiently high channel gain can still be successfully decoded. The optimal transmission control ensures that such events happen with high probability.

## III. Performance Limits of O-ALOHA

### A. Throughput and Asymptotic Throughput

We now give an expression for the throughput of the system described. The throughput is the average number of packets successfully decoded per slot for a given spreading code. This is a function of the channel state distribution, reception model, and transmission control.

We shall assume that the underlying channel state information $\gamma_i^{(t)}$ has a cumulative distribution function (CDF) $F(\gamma)$ and is independent across sensors and from slot to slot. The throughput depends not only on $F(\gamma)$ but also on the transmission control $s(\gamma)$. The unconditional probability of transmission is given by

$$p_s = \int s dF. \tag{2}$$

Assuming $p_s \neq 0$, the (*a posteriori*) channel state distribution seen at the receiver is

$$T(\gamma) = \frac{1}{p_s} \int_0^{\gamma} s dF \tag{3}$$

which determines the probability of successful reception [8].

We generalize the multipacket reception model first considered in [18] to incorporate channel state. Let $C_k(T(\cdot))$ be the expected number of received packets conditioned on $k$ users transmitting, i.e.,

$$C_k(T(\cdot)) \triangleq \sum_{i=1}^{k} E\left\{ \theta_k^{(i)} \,\middle|\, k \text{ users transmit} \right\}. \tag{4}$$

For a system with $n$ sensors, the throughput[1] per slot (per spreading code) is given by

$$\lambda_n(s(\cdot)) = \sum_{k=1}^{n} \binom{n}{k} (1 - qp_s)^{n-k} (qp_s)^k C_k(T(\cdot)) \tag{5}$$

where $q$ is the probability of choosing a particular spreading code conditioned on a sensor transmitting. The power of using transmission control is that it allows us to manipulate the *a posteriori* distribution $T(\cdot)$ through $s(\cdot)$. The ideal approach is to obtain a transmission control which would lead to an *a posteriori* distribution that achieves the best throughput. The problem, however, is complicated because the transmission control also affects the probability of transmission $p_s$.

Now, we define the notion of the asymptotic throughput of a network. For a finite number of users $n$ in a network, the throughput given the channel state distribution, transmission control and reception model is given by (5). The asymptotic throughput can be defined as

$$\lambda_\infty(s(\cdot)) \triangleq \lim_{n\to\infty} \lambda_n(s(\cdot)). \tag{6}$$

This quantity is of value for "large" networks, and we can obtain transmission controls that have good asymptotic behavior based on this metric. The maximum asymptotic throughput (MAT) of the system is the best possible asymptotic throughput achievable among all possible sequences of transmission controls. This is given by

$$\lambda_\infty^*(s(\cdot)) = \sup_{s_n(\gamma)} \lim_{n\to\infty} \sum_{k=1}^{n} \binom{n}{k} \times (1 - qp_s)^{n-k} (qp_s)^k C_k(T(\cdot)). \tag{7}$$

### B. Optimal Transmission Control

Our strategy to obtain optimal transmission control is to shape the *a posteriori* channel state distribution to a "target" distribution whose asymptotic throughput is large and can be characterized analytically. Let the underlying channel state be distributed according to $F(\cdot)$. Let $T(\cdot)$ be a distribution function that is

[1]This is also the maximum stable throughput [8].

absolutely continuous with respect to $F(\cdot)$, i.e., $T(\cdot) \ll F(\cdot)$. From the Radon–Nikodym theorem, there exists a nonnegative function $(dT/dF)$ such that, for any measurable set $A$

$$\mu_T(A) = \int_A \frac{dT}{dF} dF. \tag{8}$$

We choose the sequence of transmission controls as

$$s_n(\gamma) = q \min\left(\frac{x}{n}\frac{dT}{dF}, 1\right) \tag{9}$$

where we incorporate $q$—the probability of choosing a spreading code—into the transmission control $s_n(\gamma)$. The following theorem characterizes the asymptotic throughput of such a system.

*Theorem 1:* If the sequence of transmission controls is chosen as

$$s_n(\gamma) = q \min\left(\frac{x}{n}\frac{dT}{dF}, 1\right) \tag{10}$$

where $x$ is the average number of transmissions per slot, then the asymptotic throughput is given by

$$\lambda_\infty(x, s_n(\cdot)) = e^{-xq} \sum_{k=1}^{\infty} \frac{(xq)^k}{k!} C_k(T(\cdot)). \tag{11}$$

*Proof:* Follows from the proof of Proposition 4 in [8].

An intuitive interpretation of the above theorem is that, for a large network, the specific choice of transmission control makes the number of transmissions distributed as a Poisson process with mean $xq$ and channel state distribution $T(\cdot)$. To obtain the optimal transmission control, a direct approach is to maximize $\lambda_\infty(x, s_n(\cdot))$ with respect to $T(\cdot)$ and $x$. Such an approach, unfortunately, is not tractable. We take an indirect approach by considering the class of distributions with a roll-off $\delta$, which has been analyzed in [17] and [19] and proving the optimality using such a class of target distributions. The following theorem shows the asymptotic optimality of O-ALOHA.

*Theorem 2:* Let $T(\gamma)$ be a target distribution with roll-off $\delta$ defined by

$$T(\gamma) = 1 - \left(\frac{\gamma_0}{\gamma}\right)^\delta 1_{\gamma > \gamma_0}$$

where $\gamma_0 > 0$ is any constant and $1_{(\cdot)}$ is the indicator function. Assume that $T(\cdot) \ll F(\cdot)$. For any $P_T > 0$, the transmission control given by

$$s_{n,i}(\gamma) = q_i \min\left(\frac{dT}{dF}\frac{x}{n}, 1\right) \tag{12}$$

where $q_i > 0$ is the probability of choosing the $i$th code, has asymptotic throughput $\lambda_\infty(x, s_{n,i}(\cdot))$ satisfying

$$\sum_{i=1}^{N} \lim_{x\to\infty, \delta\to 0} \lambda_\infty(x, s_{n,i}(\gamma)) = N. \tag{13}$$

*Proof:* Refer to the Appendix.

When the threshold $\beta > 1$, the above theorem implies the asymptotic optimality of O-ALOHA given in (12). Specifically, with a sufficiently large mean transmissions $x$, the throughput of O-ALOHA approaches $N$, which is the maximum throughput possible using any scheduler. The fact that the maximum throughput can be achieved with arbitrary $P_T$ demonstrates the idea of trading the size of the network for power: for any power $P_T$ at the transmitter, one can choose a sensor network large enough and the mean transmission high enough such that there are always $N$ packets captured.

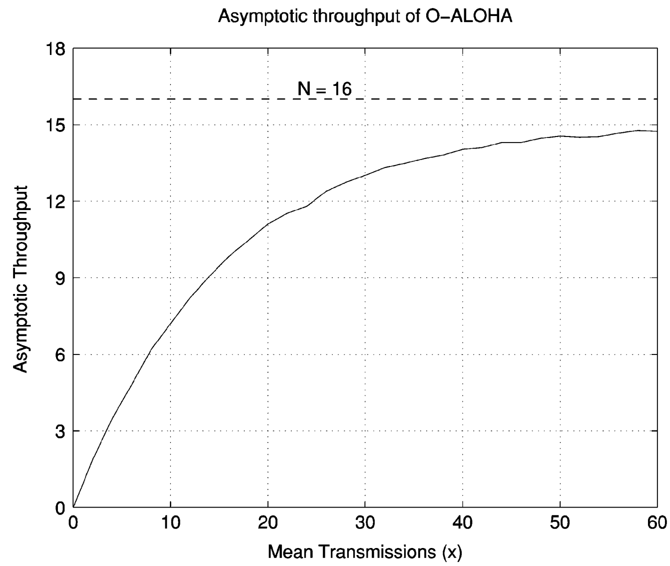


Fig. 3. Asymptotic throughput of O-ALOHA with $N = 16, \delta = 0.01, \beta =$ 4 dB.

The assumption that $T(\cdot) \ll F(\cdot)$ is reasonable for practical fading models. Typically, prior channel state distributions with infinite support (Rayleigh, Rician, etc.) satisfy this requirement. For distributions with finite support, a truncated roll-off distribution can be used in (12).

One should be cautioned that, although O-ALOHA is throughput optimal, it is not necessarily energy efficient because the large number of transmissions required to achieve the maximum throughput. In practice, a sufficiently high throughput can be realized using a moderate value of mean transmissions as shown in Fig. 3. The centralized scheduler is in general more energy efficient. See [20] for such protocols for SENMA.

### C. Convergence of Throughput

In the previous sections, we characterized the limiting throughput. However, from a practical standpoint, we are interested in the case where both $n$ and $x$ are finite. In this section, we characterize the rate of convergence of the throughput of the finite user system.

*Theorem 3:* Given the *a priori* channel state distribution $F(\gamma)$, any target distribution $T(\cdot) \ll F(\cdot)$, and the design parameter $x$, if $\sup\{(dT/dF)\} < \infty$, then

$$|\lambda_n(x) - \lambda_\infty(x)| = O\left(\frac{x^2}{2n}\frac{d^2}{dx^2}(\lambda_\infty(x))\right). \tag{14}$$

*Proof:* Refer to the Appendix.

The rate of convergence with respect to the size of the networks is $(1/n)$. One may notice that the condition $\sup\{(dT/dF)\} < \infty$ may exclude some practical distributions. For example, a Rayleigh-fading model that is shaped to be a target distribution with roll-off violates this condition. However, by truncating the target distribution, the expression for rate of convergence is still valid. An example of such truncation is discussed in Section V.

### D. Incorporating Side Information

In some applications, additional side information is available at the sensor. For example, the distance between the sensor and the mobile AP can be computed via the use of geolocation devices. Even without explicit measurement, one may obtain the distance distribution from the way sensors are deployed and the mobility pattern of the mobile AP. We consider here two types of transmission controls: one with the realization of the side information, the other with only the distribution. We shall use the location as an example of side information.

Let $r$ represent the location parameter distributed with probability density function (pdf) $g(\cdot)$. By location independent transmission (LIT) control we mean that the transmission control $s(\cdot)$ is independent of $r$; it is only a function of $g(\cdot)$. Let $f(\gamma\,|\,r)$ be the pdf of the channel state conditioned on location $r$ and $t(\cdot)$ the pdf of the target distribution. The LIT control is given by

$$s_n(\gamma) = q\min\left(\frac{t(\gamma)}{\int f(\gamma\,|\,r)g(r)dr}\frac{x}{n}, 1\right). \tag{15}$$

Note that the above LIT control can be obtained prior to the sensor deployment and is, therefore, easy to implement.

By location aware transmission (LAT) control we mean that the sensor has access to the location parameter $r$. To obtain the target distribution with roll-off characteristics, it is sufficient to consider the transmission control of the following form:

$$s_n(\gamma, r) = q\min\left(\frac{t(\gamma)}{f(\gamma\,|\,r)}\frac{x}{n}, 1\right). \tag{16}$$

This can be verified by following the steps in the proof of Theorem 1 and integrating over $r$. Since the transmission control of each sensor is different and dependent on their respective locations, LAT is more complicated to implement and would demand more processing on the part of the sensors. However, as we shall see in Section V, the LAT control has some desirable properties.

## IV. Coded Random Access

Under certain circumstances, sensors may be able collaborate in their transmission. The assumption here is that all nodes agree upon the transmission of a particular message, which makes it possible that the transmissions are coded to cope with packet loss. In this section, we characterize $C$, the number of bits per slot, that can be reliably transmitted back to the mobile AP. We propose two schemes combining coding and random access for this purpose and characterize their achievable rates and error exponents.

### A. Spreading Code Independent Transmission

We assume that the entire network has agreed to transmit a particular codeword from the same binary codebook of size $\lceil 2^{MR} \rceil \times M$, where $M$ is the codeword length and $R$ is the rate of the codebook. The bits in the codeword are transmitted sequentially in time. In the $i$th slot, the mobile AP requests the transmission of the $i$th bit of the codeword $\mathcal{C}$ in its beacon. All the transmitting sensors transmit the requested bit of the codeword in their packets. We assume that those packets received successfully (depending on the SINR threshold) are decoded without error. If no packet is received in a particular slot, the corresponding bit is erased. We then have an erasure channel, where the probability of erasure is the probability that no packet was received successfully in a slot. Assuming that the probability of choosing the $i$th spreading code $q_i$ is equal for all $N$ codes, the channel between the sensor network and the mobile AP for the spreading code independent transmission can be viewed as an erasure channel with erasure probability

$$P_{\text{ind}}(x) = \left(1 - \frac{\lambda_\infty(x)}{N}\right)^N \tag{17}$$

where $\lambda_\infty(x)$ is the asymptotic throughput of the sensor network. To see (17), note in Theorem 1 that the asymptotic throughput per spreading code is given by

$$\lambda_\infty(x, i) = e^{-xq_i}\sum_{k=1}^{\infty}\frac{(xq_i)^k}{k!}C_k(T(\cdot)) \tag{18}$$

where $C_k(T(\cdot))$ is the expected number of captures conditioned on $k$ sensors transmitting. Since we have assumed that the SINR threshold $\beta > 1$, at most one packet can be captured per spreading code. Therefore, $C_k(T(\cdot))$ is the probability of capture conditioned on $k$ users transmitting [17]. Thus, $\lambda_\infty(x, i)$ is the unconditional probability of capture from the $i$th spreading code. Since $q_i = q_j,\ \forall i, j$

$$\lambda_\infty(x, i) = \frac{\lambda_\infty(x)}{N}, \quad \forall i. \tag{19}$$

The probability of erasure is the probability that there are no captures from any of the $N$ matched filters and is, hence, given by (17). Since channel state is i.i.d across slots, the transmission of bits is also independent identically distributed (i.i.d.). Therefore, the achievable rate of the binary erasure channel [21] for spreading code independent transmission given $x$ is

$$C_{\text{ind}}(x) = 1 - \left(1 - \frac{\lambda_\infty(x)}{N}\right)^N \ \text{bits/slot}. \tag{20}$$

For a codebook size $\lceil 2^{MR} \rceil \times M$, the probability of detection error can be upper bounded by error exponent expressions [22]

$$P_e^{\text{ind}}(M, x) \le \exp(-M\log_e(2)\mathcal{E}_{\text{ind}}(R, x)) \tag{21}$$

$$\mathcal{E}_{\text{ind}}(R, x) = \tag{22}$$

$$\begin{cases} -(1-R)\log_2\left(\frac{P_{\text{ind}}(x)}{1-R}\right) - R\log_2\left(\frac{1-P_{\text{ind}}(x)}{R}\right), & R < 1 - P_{\text{ind}}(x). \\ 0, & R \ge 1 - P_{\text{ind}}(x). \end{cases} \tag{23}$$

The error exponents $\mathcal{E}_{\text{ind}}(R, x)$ and the achievable rate given in (20) can be optimized with respect to $x$.

### B. Spreading Code Dependent Transmission

In the previous coding scheme, it is evident that by transmitting only one bit per slot, the orthogonality of the spreading codes is not being utilized. In this section, we propose a modified scheme which utilizes the fact that transmissions using different orthogonal codes are independent.

The structure for the codebook in this case is similar. Each codeword is divided into blocks of $N$ bits, where $N$ is the spreading gain. Therefore, each codeword of length $M = M'N$ can be visualized as a two-dimensional array $M' \times N$, where $M'$ is the number of blocks and $N$ is the number of bits per block. The codebook size is, therefore, $\lceil 2^{M'NR} \rceil \times M'N$. The spreading codes available for transmission are ordered from 1 to $N$. If the $k$th message in the codebook is to be sent to the mobile AP, then the encoding is as follows. In the $i$th slot, the mobile AP requests transmission of the $i$th block. Every sensor that decides to transmit using spreading code $j$, transmits the $(i, j)$ bit of the $k$th codeword. In every slot, therefore, one block of the codeword is transmitted. The number of slots required to transmit a codeword is, therefore, $M'$. Here, a bit is erased if there is no capture at the output of a particular matched filter. The channel between the sensors and the mobile AP can, therefore, be viewed as an erasure channel with erasure probability

$$P_{\text{dep}}(x) = 1 - \frac{\lambda_\infty(x)}{N}. \tag{24}$$

Recall that $(\lambda_\infty(x)/N)$ is the probability of successful reception from a particular matched filter. It is again assumed that the packets received successfully are decoded without error. Therefore, the probability of erasure of a bit in a codeword is the probability that no packet was received successfully using a particular spreading code. Since the transmission of each bit in a slot is using a different orthogonal code, there is no interbit-interference and, therefore, transmissions within a slot are i.i.d. provided each spreading code is chosen with equal probability. Since channel state is i.i.d. across slots, transmissions across slots are also i.i.d. Therefore, the achievable rate in bits/channel use of the Spreading code dependent transmission is $(\lambda_\infty(x)/N)$. Since there are $N$ channel uses per slot, the achievable rate given $x$, in bits per slot is

$$C_{\text{dep}}(x) = \lambda_\infty(x) \text{ bits/slot} \tag{25}$$

and the error exponent is given by

$$\mathcal{E}_{\text{dep}}(R, x) = \tag{26}$$
$$\begin{cases} -(1-R)\log_2\left(\frac{P_{\text{dep}}(x)}{1-R}\right) - R\log_2\left(\frac{1-P_{\text{dep}}(x)}{R}\right), & R < 1 - P_{\text{dep}}(x). \\ 0, & R \geq 1 - P_{\text{dep}}(x). \end{cases} \tag{27}$$

### C. Comparison of the Two Schemes

We shall now compare the two schemes proposed above in terms of the achievable rates and the error exponents. For a given average transmissions per slot $x$ and spreading gain $N$, the achievable rates are given by

$$C_{\text{ind}}(x) = 1 - \left(1 - \frac{\lambda_\infty(x)}{N}\right)^N \text{ bits/slot} \tag{28}$$
$$C_{\text{dep}}(x) = \lambda_\infty(x) \text{ bits/slot}. \tag{29}$$

From (20), it is clear that $C_{\text{ind}}(x)$ is always less than 1. However, depending on the spreading gain and parameters of the transmission control such as mean transmissions and roll-off factor, $C_{\text{dep}}(x)$ can take values much greater than 1. The following proposition characterizes the limiting value for the ratio of the two achievable rates.

*Proposition 1:* Let $\eta(x) \triangleq (C_{\text{dep}}(x))/(C_{\text{ind}}(x))$. For any $x$

$$\lim_{N\to\infty} \eta(x) = \frac{\lambda_\infty(x)}{1 - e^{-\lambda_\infty(x)}} \geq 1. \tag{30}$$

*Proof:* Refer to the Appendix.

From the above theorem, it is clear that the capacity of the spreading code dependent transmission is uniformly better than that of the spreading code independent transmission.

We shall now compare the error exponents of the two coding schemes for a fixed codebook size. We know that for a given codeword length, the number of slots required for transmission for the spreading code independent scheme is $N$ times greater than that required by the spreading code dependent transmission. Therefore, for comparison of error probability, it is necessary to choose the mean transmissions per slot appropriately, so that the total number of transmissions required is identical. Assuming $x$ mean transmissions per slot for the spreading code dependent scheme, we consider the ratio of error exponents normalized to the same energy used for each message

$$\alpha(R, N, x) \triangleq \frac{\mathcal{E}_{\text{ind}}\left(R, \frac{x}{N}\right)}{\mathcal{E}_{\text{dep}}(R, x)}. \tag{31}$$

We know that the spreading code dependent scheme has a higher capacity compared with the spreading code independent scheme. However, in low rate regimes, it can be shown that the spreading code independent scheme has a better error exponent.

*Proposition 2:* For given mean transmissions $x$, target distribution $T(\cdot)$, and $\beta > 1$

$$\lim_{R\to 0, N\to\infty} \alpha = \frac{xC_1(T(\cdot))}{\sum_{k=1}^{\infty} \frac{e^{-x}x^k}{k!} C_k(T(\cdot))} \geq x. \tag{32}$$

*Proof:* Refer to the Appendix.

Therefore, in the limiting case, $\alpha \geq 1$, if $x \geq 1$. Since $x$ is the mean transmissions per orthogonal code, it would be at least equal to 1. This implies that for low-rates, the spreading code independent scheme can have a greater error exponent than the spreading code dependent scheme.

## V. Numerical Results

### A. Channel Model

We now consider a practical Rayleigh-fading channel with path loss. We assume that all the sensors are located in a disc of radius 1. As shown in Fig. 4, the mobile AP is assumed to be at a distance $h$ above the center of the disc. Let $r_i$, the radial distance

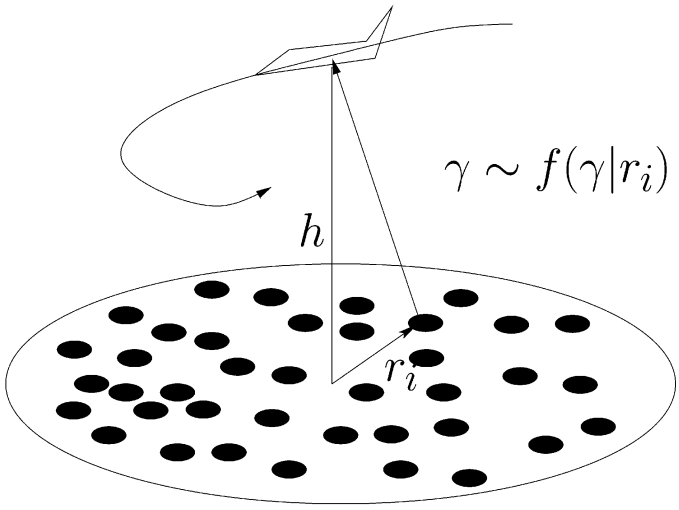


Fig. 4. Sensor deployment.

of sensor $i$, be modeled as a random variable that is distributed with p.d.f $g(r) = \mathrm{sqrt}(r)$, $0 \leq r \leq 1$. The propagation channel gain between sensor $i$ and the base station is modeled as

$$\gamma_i^{(t)} = \frac{R_{it}^2}{r_i^2 + h^2} \tag{33}$$

where $R_{it}$ is Rayleigh distributed. The attenuation due to distance is based on a free-space path loss model [23]. Let $f(\gamma)$ be the pdf of $\gamma_i^{(t)}$. We also use $f(\gamma \mid r)$ to denote the probability density function of the channel state given the radial distance $r$.

### B. Asymptotic Throughput and Convergence

Fig. 3 shows the asymptotic throughput of the O-ALOHA protocol with transmission control as given in (12). Thus, for a small value of roll-off factor $\delta$, it is possible to obtain a sufficiently high throughput (close to spreading gain) for moderate values of mean transmissions $x$.

We now look at the convergence of the throughput of finite user systems to the asymptotic throughput. In order to satisfy the condition in Theorem 3, we consider a transmission control as described in (9) with the target pdf given by a truncated roll-off distribution

$$t(\gamma) = \frac{\delta}{\gamma_0^{-\delta} - \gamma_1^{-\delta}} \frac{1}{\gamma^{1+\delta}} 1_{[\gamma_0 \leq \gamma \leq \gamma_1]} \tag{34}$$

where $\gamma_0, \gamma_1$ are positive constants. The throughputs of this scheme are compared with a simple TDMA scheduling scheme, where in each slot a particular sensor is chosen *a priori* to transmit using each orthogonal spreading code. Therefore, in each slot, we have $N$ sensors transmitting independently. The sensors scheduled to transmit are chosen *a priori*. Therefore, the channel state information is not utilized. In this case, the probability of success for a sensor is given by

$$p = P_r(\mathrm{SINR} > \beta) = \frac{P_T}{\beta\sigma^2}\left(e^{\frac{-h^2\beta\sigma^2}{P_T}} - e^{\frac{-(h^2+1)\beta\sigma^2}{P_T}}\right) \tag{35}$$

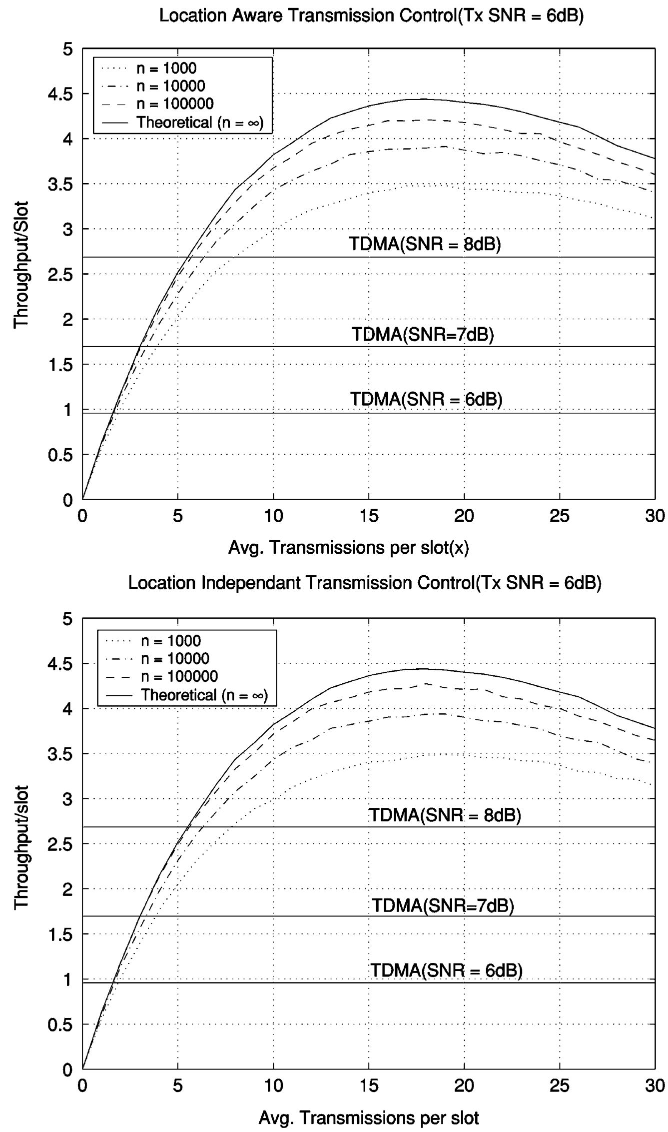


Fig. 5. Throughput of LAT and LIT controls with following parameters: $\beta = 4$ dB, $(P_T)/(\sigma^2) = 6$ dB, $\gamma_0 = 1.5$, $\gamma_1 = 14$, and $h = 2$.

where $\beta$ is the threshold of the reception model, $\sigma^2$ is the variance of the noise, and $h$ is the height of the mobile AP (see Fig. 4). This probability is obtained directly from the channel state CDF $F(\gamma)$. The throughput per slot is, therefore, given by

$$\lambda_{\mathrm{TDMA}} = Np = \frac{NP_T}{\beta\sigma^2}\left(e^{\frac{-h^2\beta\sigma^2}{P_T}} - e^{\frac{-(h^2+1)\beta\sigma^2}{P_T}}\right). \tag{36}$$

The throughput for the LIT and LAT controls are plotted in Fig. 5 versus the design parameter $x$, the average transmissions per slot. The throughput for the TDMA scheme described above is also plotted for different transmission powers. It is evident from the plots that, for the TDMA scheme, the throughput goes to zero as $P_T \to 0$, irrespective of the size of the network. However, from Theorem 3, we know that for O-ALOHA, given any nonzero $P_T$, it is possible to reach as close to the theoretical throughput as possible by increasing the size of the network. The convergence of finite-user networks to the asymptotic throughput can also be observed in the figures.

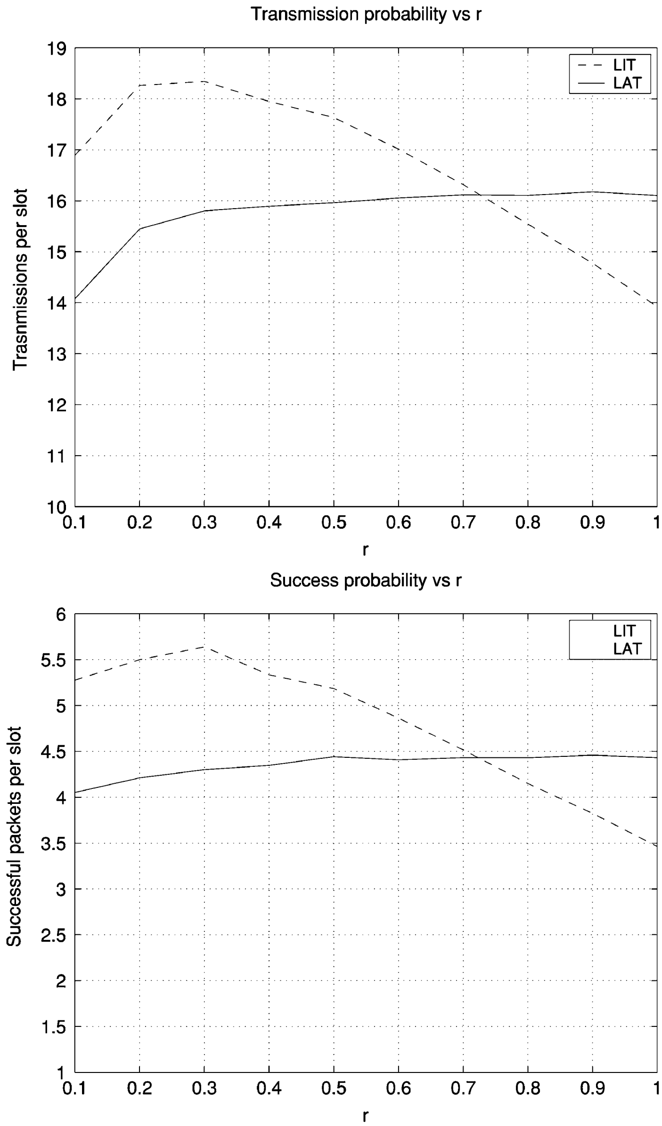


Fig. 6. Transmission and success patterns.

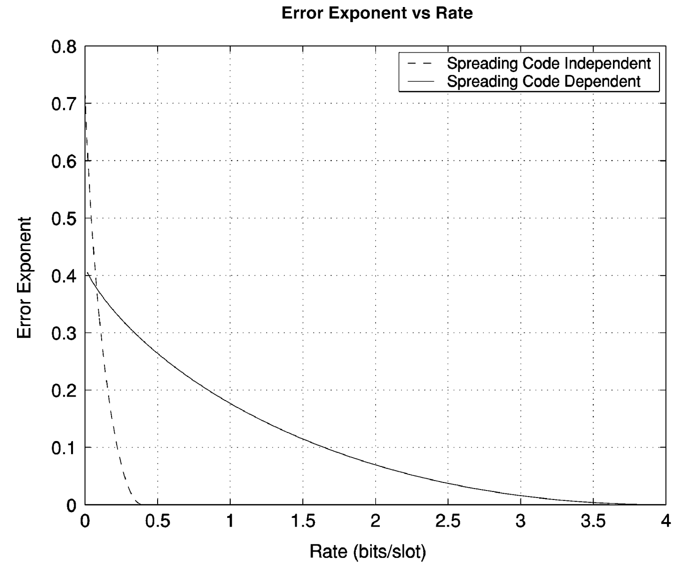


Fig. 7. Error exponents.

### C. LIT Versus LAT

We observe little difference in the throughputs of finite user systems using LIT and LAT controls. The difference between LIT and LAT is primarily in the distribution of transmitting and successful sensors as a function of radial distance $(r_i)$ to the mobile AP (see Fig. 4). If a sensor network employs LAT, the probability of transmission for sensor $i$, conditioned on the event that $r_i = r$ is given by

$$\Pr\{\text{LAT Tx} \,|\, r_i = r\} = \int s_n(\gamma, r) f(\gamma \,|\, r)\, d\gamma \quad (37)$$

$$\lim_{n\to\infty} n \Pr\{\text{LAT Tx} \,|\, r_i = r\} \to x. \quad (38)$$

Therefore, for LAT control, the probability of transmission is independent of the radial distance. However, for the LIT case

$$\Pr\{\text{LIT Tx} \,|\, r_i = r\} = \int s_n(\gamma) f(\gamma \,|\, r)\, d\gamma \quad (39)$$

$$\lim_{n\to\infty} n \Pr\{\text{LIT Tx} \,|\, r_i = r\} \to \int \frac{g(\gamma) f(\gamma \,|\, r)}{f(\gamma)}\, d\gamma \quad (40)$$

which evidently depends on the radial distance. These facts are illustrated in Fig. 6. For the LIT control, the probability of transmission decreases with increase in radial distance. This is because all sensors use an identical transmission control independent of their location, whereas the path loss increases with distance. Therefore, sensors farthest from the mobile AP are less likely to transmit. Fig. 6 also shows the distribution of successful sensors versus radial distance. Using a similar argument, it can be shown that the probability of success of a sensor in LAT is independent of the radial distance. In LIT, the probability of success decreases with radial distance. The intuitive explanation for this observation is that, in LIT control, the transmitting sensors are unaware of the distance and, therefore, the transmission control does not compensate for the path loss.

Fig. 7 shows the variation of error exponent with code rate for the protocol described earlier in this section. It is clear from the plot that, though spreading code dependent transmission has a higher achievable rate, the spreading code independent transmission has a greater error exponent for low rate regimes.

## VI. Conclusion

We present in this work the design of random access and coding schemes for large scale sensor networks with mobile access points. This paper primarily focuses on sensor networks with one mobile access point. Sensor networks with multiple mobile access points have been analyzed in [24].

In this work, we have shown, for the physical layer using orthogonal CDMA, that O-ALOHA is asymptotically optimal and is close to the optimal centralized scheduler under the single packet capture model. Although we have used the channel gain and the sensor location as part of the transmission control, our framework, first established in [8], allows the incorporation of other side information, either in the form of probability distribution or in the form of actual measurements.

Although O-ALOHA is throughput optimal asymptotically, and it allows the power at each sensor to be arbitrarily small, we caution that O-ALOHA is not necessarily energy efficient. Specifically, O-ALOHA tends to over transmit for higher throughput. To this end, centralized scheduling for sensor networks [20] would have eliminated redundant transmissions while maintaining high throughput. Further results on robustness and sensitivity issues of O-ALOHA are presented in [25].

A new feature presented in this paper is the coded random access that combines coding, CDMA, and O-ALOHA. Although we have used random coding as a tool for characterizing the performance, the type of codes used by the network can be arbitrary. This opens to the possibility of using some of the most efficient codes such as the class of low-density parity check (LDPC) codes. However, the scenario that coded random access considered in this paper is applicable is restricted to the case that sensors are allow to collaborate. Specifically, the network as a whole must decide which codeword to transmit and the codebook must be distributed to the sensors. In general, this need not be the case, and there may be misinformed sensors present in the network. An information theoretic analysis of such networks is given in [26].

## APPENDIX

### A. *Proof of Theorem 2*

We known from Theorem 1 that if $s_{n,i}(\gamma) = q_i \min((dT/dF)(x/n), 1)$, then

$$\lambda_\infty(s_{n,i}(\cdot)) = e^{-xq_i} \sum_{k=1}^{\infty} \frac{(xq_i)^k}{k!} C_k(T(\cdot)) \tag{41}$$

where $q_i$ is the probability of choosing the $i$th spreading code. We are interested in the throughput of this system as the average number of transmitting sensors goes to infinity, that is $\lim_{x\to\infty} \lambda_\infty(s_{n,i}(\cdot))$. Hajek *et al.* in [17] have shown that the throughput of an infinite user system, where the received power is distributed with a roll-off $\delta$ is equal to $\beta^{-\delta}(\sin(\pi\delta)/\pi\delta)$. Therefore

$$\lim_{x\to\infty} \lambda_\infty(s_{n,i}(\cdot)) = \beta^{-\delta} \frac{\sin(\pi\delta)}{\pi\delta}. \tag{42}$$

Taking limits with respect to $\delta$ and summing the throughputs across the $N$ spreading codes

$$\sum_{i=1}^{N} \lim_{\delta\to 0} \lambda_\infty(s_{n,i}(\cdot)) = N. \tag{43}$$

We have already seen that the best possible throughput for such a system using orthogonal spreading codes is $N$ (refer to Section II). Hence, this transmission control is optimal.

### B. *Proof of Theorem 3*

We know that, the transmission control is given by

$$s_n(\gamma) = \min\left(\frac{dT}{dF}\frac{x}{n}, 1\right). \tag{44}$$

Since $q$, the probability for choosing a spreading code is only a constant, it has been conveniently ignored. Since $\sup(dT/dF) < \infty, \exists m$ s.t $\forall n > m$

$$\frac{dT}{dF}\frac{x}{n} < 1. \tag{45}$$

Since we are concerned with rate of convergence as $n \to \infty$, we can write $s_n(\gamma) = (dT/dF)(x/n)$. Proceeding along the lines of the proof of Theorem 1

$$\lambda_n(x) = \sum_{k=1}^{\infty} \binom{n}{k} \left(1 - \frac{x}{n}\right)^{n-k} \frac{x^k}{n^k} C_k(T(\cdot)) \tag{46}$$

$$\lambda_\infty(x) = \sum_{k=1}^{\infty} \frac{x^k e^{-x}}{k!} C_k(T(\cdot)). \tag{47}$$

In order to proceed with the proof, we require the following lemma.

*Lemma 1:*

$$\lim_{n\to\infty} n \left( \binom{n}{k} \left(1 - \frac{x}{n}\right)^{n-k} \frac{x^k}{n^k} - \frac{x^k e^{-x}}{k!} \right) \tag{48}$$

$$= -\frac{x^2}{2} \left( \frac{x^k}{k!} - \frac{2x^{k-1}}{(k-1)!} + \frac{x^{k-2}}{(k-2)!} \right). \tag{49}$$

From Lemma 1, we can say that

$$\lim_{n\to\infty} n(\lambda_n(x) - \lambda_\infty(x)) \tag{50}$$

$$= \frac{e^{-x}x^2}{2} \sum_{k=1}^{\infty} \left( \frac{x^k}{k!} - \frac{2x^{k-1}}{(k-1)!} + \frac{x^{k-2}}{(k-2)!} \right) C_k(T(\cdot)) \tag{51}$$

$$= O\left( \frac{x^2}{2} \frac{d^2}{dx^2} \lambda_\infty(x) \right). \tag{52}$$

We now need to prove that the right-hand side (RHS) in (51) is bounded, in order to complete the proof of convergence rate. We shall, therefore, bound $|\lim_{n\to\infty} n(\lambda_n(x) - \lambda_\infty(x))|$

$$|\lim_{n\to\infty} n(\lambda_n(x) - \lambda_\infty(x))| \tag{53}$$

$$\le \frac{x^2 e^{-x}}{2} \sum_{k=0}^{\infty} |C_k(T(\cdot))| \left( \left| \frac{x^k}{k!} + \frac{x^{k-1}}{(k-1)!} + \frac{x^{k-2}}{(k-2)!} \right| \right) \tag{54}$$

$$\le \frac{x^2 e^{-x}}{2} \sum_{k=1}^{\infty} \left( \left| \frac{x^k}{k!} \right| + \left| \frac{x^{k-1}}{(k-1)!} \right| + \left| \frac{x^{k-2}}{(k-2)!} \right| \right) \tag{55}$$

$$= \frac{x^2 e^{-x}}{2} (4e^x) = 2x^2. \tag{56}$$

The inequality in (55) is due to the assumption that $\beta > 1$ and, therefore, $C_k(T(\cdot))$ is the probability of capture and can at most be equal to one. Hence, for finite $x$, the expression is bounded.

*Proof of Lemma 1:* We shall use induction on $k$. Let

$$L(k) \triangleq n \left( \binom{n}{k} \left(1 - \frac{x}{n}\right)^{n-k} \left(\frac{x}{n}\right)^k - \left(\frac{x^k e^{-x}}{k!}\right) \right). \tag{57}$$

We need to show

$$\lim_{n\to\infty} L(k) = \frac{x^2 e^{-x}}{2}\left(\frac{x^k}{k!} - \frac{2x^{k-1}}{(k-1)!} + \frac{x^{k-2}}{(k-2)!}\right). \quad (58)$$

Consider $k = 1$

$$L(1) = \lim_{n\to\infty} n\left[n\left(1-\frac{x}{n}\right)^{n-1}\frac{x}{n} - xe^{-x}\right]$$
$$= x\lim_{n\to\infty} n\left[\left(1-\frac{x}{n}\right)^{n-1} - e^{-x}\right]. \quad (59)$$

Replacing $n$ by $(1/t)$

$$\lim_{n\to\infty} L(1) = x\lim_{t\to 0}\left[\frac{(1-xt)^{\frac{1}{t}-1} - e^{-x}}{t}\right].$$

Applying L'Hospital's rule

$$\lim_{n\to\infty} L(1) = x\lim_{t\to 0}\left[(1-xt)^{\frac{1}{t}-1}\left(-\frac{1}{t^2}\log(1-xt) - \left(\frac{1}{t}-1\right)\left(\frac{x}{1-xt}\right)\right)\right]$$
$$= x\lim_{t\to 0}(1-xt)^{\frac{1}{t}-1}\left[\lim_{t\to 0}\left(-\frac{1}{t^2}\log(1-xt)\right) - \lim_{t\to 0} x\left(\frac{1}{t}-1\right)\frac{1}{1-xt}\right]$$
$$= xe^{-x}\left[\lim_{t\to 0}\frac{-1}{t^2}\left(-xt - \frac{x^2t^2}{2} - \ldots\right) - x\left(\frac{1}{t}-1\right)(1+xt+\ldots)\right]$$

where we use Taylor series expansion for $\log(1-xt)$ and $(1)/(1-xt)$. Neglecting higher powers of $t$

$$\lim_{n\to\infty} L(1) = xe^{-x}\left(-\frac{x^2}{2} + x\right) = -\frac{x^2e^{-x}}{2}(x-2). \quad (60)$$

Assuming $\lim_{n\to\infty} L(k) = (x^2e^{-x})/(2)((x^k)/(k!)) - 2(x^{k-1})/((k-1)!) + (x^{k-2})/((k-2)!))$, we have

$$\frac{x^k}{k!}\left[\lim_{n\to\infty} n\left(\frac{n!}{(n-k)!}\left(1-\frac{x}{n}\right)^{n-k}\frac{x^k}{n^k} - e^{-x}\right)\right] = \frac{x^2e^{-x}}{2}\left(\frac{x^k}{k!} - 2\frac{x^{k-1}}{(k-1)!} + \frac{x^{k-2}}{(k-2)!}\right)$$

i.e.,

$$\lim_{n\to\infty} n\left(\frac{n!}{(n-k)!}\left(1-\frac{x}{n}\right)^{n-k}\frac{x^k}{n^k} - e^{-x}\right) = -e^{-x}\left(\frac{x^2}{2} - xk + k(k-1)\right). \quad (61)$$

Consider $\lim_{n\to\infty} L(k+1)$

$$\lim_{n\to\infty} L(k+1) = \lim_{n\to\infty} n\left[\binom{n}{k+1}\left(1-\frac{x}{n}\right)^{n-k-1}\times\frac{x^{k+1}}{n^{k+1}} - \frac{x^{k+1}e^{-x}}{(k+1)!}\right]$$
$$= \frac{x^{k+1}}{(k+1)!}\lim_{n\to\infty}\left[\frac{n!}{(n-k-1)!}\times\left(1-\frac{x}{n}\right)^{n-k-1}\frac{1}{n^k} - ne^{-x}\right]$$
$$= \frac{x^{k+1}}{(k+1)!}\left[\lim_{n\to\infty}\left(\frac{n-k}{n-x}\right)\times\lim_{n\to\infty} n\left\{\frac{n!}{(n-k)!}\left(1-\frac{x}{n}\right)^{n-k}\frac{x^k}{n^k} - e^{-x}\right\} - \lim_{n\to\infty}(k-x)\left(\frac{n}{n-x}\right)e^{-x}\right]. \quad (62)$$

Substituting from (61)

$$\lim_{n\to\infty} L(k+1) = \frac{x^{k+1}e^{-x}}{(k+1)!}\left[-\frac{x^2}{2} + xk - k(k-1) - k + x\right] \quad (63)$$
$$= \frac{-x^2e^{-x}}{2}\left[\frac{x^{k+1}}{(k+1)!} - \frac{2x^k}{k!} + \frac{x^{k-1}}{(k-1)!}\right]. \quad (64)$$

Hence, proved.

### *C. Proof of Proposition 1*

We know that

$$\eta(x) = \frac{C_{\text{dep}}(x)}{C_{\text{ind}}(x)} \quad (65)$$
$$= \frac{\lambda_\infty(x)}{1-\left(1-\frac{\lambda_\infty(x)}{N}\right)^N}. \quad (66)$$

As $N\to\infty$

$$\eta(x) \to \frac{\lambda_\infty(x)}{1-e^{-\lambda_\infty(x)}}. \quad (67)$$

It is easily shown that if $\lambda_\infty(x) \geq 0$, then the RHS of (67) is a monotonically increasing function of $\lambda_\infty(x)$. Since, $\lim_{N\to\infty,\lambda_\infty(x)\to 0}\eta(x) = 1$, we can say that for all values of $x, \lim_{N\to\infty}\eta(x) \geq 1$. Hence, proved.

### *D. Proof of Proposition 2*

$$\lim_{R\to 0, N\to\infty}\alpha = \lim_{N\to\infty}\frac{\lim_{R\to 0}\mathcal{E}_{\text{ind}}\left(R,\frac{x}{N}\right)}{\lim_{R\to 0}\mathcal{E}_{\text{dep}}(R,x)} \quad (68)$$
$$= \lim_{N\to\infty}\frac{\log(P_{\text{ind}}\left(\frac{x}{N}\right)}{\log(P_{\text{dep}}(x))} \quad (69)$$
$$= \lim_{N\to\infty}\frac{N\log\left(1-\frac{\lambda_\infty\left(\frac{x}{N}\right)}{N}\right)}{\log\left(1-\frac{\lambda_\infty(x)}{N}\right)} \quad (70)$$
$$= \lim_{N\to\infty}\frac{N\lambda_\infty\left(\frac{x}{N}\right)}{\lambda_\infty(x)} \quad (71)$$

substituting $N$ by $(1/t)$

$$\lim_{R\to 0, t\to 0}\alpha = \lim_{t\to 0}\frac{\lambda_\infty(xt)}{t\lambda_\infty(x)} = \lim_{t\to 0}\frac{\lambda'_\infty(xt)}{\lambda_\infty(x)} \quad (72)$$
$$= \frac{\lim_{t\to 0}\frac{d}{dt}(\lambda_\infty(xt))}{\lambda_\infty(x)}. \quad (73)$$

We know that

$$\lambda_\infty(x) = \sum_{k=0}^{\infty}\frac{e^{-x}x^k}{k!}C_k(T(\cdot)). \quad (74)$$

Therefore

$$\lim_{R\to 0, N\to\infty} \alpha = \frac{xC_1(T(\cdot))}{\sum_{k=1}^{\infty} \frac{e^{-x}x^k}{k!} C_k(T(\cdot))}. \quad (75)$$

Since we have assumed that the threshold, $\beta > 1$, we can say that

$$C_1(T(\cdot)) \geq C_k(T(\cdot)) \quad \forall k. \quad (76)$$

This is because $C_1(T(\cdot))$ is the probability of capture in the absence of interference. Therefore

$$\lim_{R\to 0, N\to\infty} \alpha \geq x. \quad (77)$$


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