

Sensorless Torque Estimation in Multidegree-of-Freedom Flexible Systems

Islam S. M. Khalil, E. D. Kunt, Asif Sabanovic
 Faculty of Engineering and Natural Science-Mechatronics Engineering
 Sabanci University
 Orhanli-Tuzla, 34956, Istanbul, Turkey
 Email: kahalil,edkunt@su.sabanciuniv.edu, asif@sabanciuniv.edu

Abstract This paper presents a sensorless torque estimation algorithm for multidegree-of-freedom flexible systems. The proposed algorithm makes it possible to estimate externally applied torques due to flexible system's interaction with the environment without taking any measurement from the system. The algorithm is based on modifying the disturbance observer in order to decouple the reacted torque waves out of the total disturbance on the actuator. Then reacted torque waves are used along with the actuator's current and velocity to estimate flexible system parameters, dynamics and the external torques or disturbances. Several experimental results are included in order to confirm the validity of the proposed torque estimation algorithm.

I. INTRODUCTION

Torque and force control have been used in wide variety of applications such as machining tasks, assembly and grasping. And typically these applications requires using a force sensor to provide the controller with a force feedback signal. However, force sensor has certain disadvantages and prevent realization of precise force control. Especially, sensor noise that causes degradation of control performance [1]. Therefore, sensorless force estimation plays an important role in the success of the force control process. Interaction force between the end effector and flexible objects is estimated using visual feedback instead of using force sensor [2]. By making a relationship between the end effector force and the deformation of the flexible object that can be visually detected. A comparison between the sensorless force control systems and force control system is performed in [1]. Where the reaction force is estimated by assuming that the internal force and other terms of the disturbance are identified precisely. Force is estimated and force error observer is designed in [3] using the velocity information without using the actual force sensor signal. Therefore, the associated strain gage problems are avoided, such as the narrow band width due to the natural frequency of the sensor. In this paper, actuator is used to estimate the externally applied forces or torques on a flexible multidegree-of-freedom system. Where the actuator's current and velocity are measured and disturbance is estimated. Surprisingly enough that disturbance which can be estimated from the actuator side contains two types of information. The first coupled information is related to the actuator's parameters such as the self varied-inertia torque and the actuator's torque ripple. The other coupled set of information is related to the

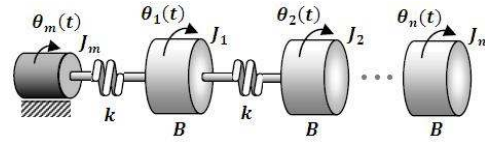


Fig. 1. Lumped flexible inertial system

plant, such as the reacted systems load and the externally applied torques or forces. Therefore, in this work the disturbance observer's structure is modified in order to decouple these information out of the estimated total disturbance. In addition, the reacted load from the plant is proved to contain plant's parameters, dynamics and externally applied disturbances.

A sensorless force estimation algorithm is then introduced based on two actuator measurements (current and velocity), considering the reacted mechanical load that can be estimated by the actuator's parameters as a natural feedback from the plant [4].

This paper is organized as follows. In section 2, reacted mechanical waves are investigated and proved to contain enough information about the plant. Such as plant's parameters, dynamics and disturbances. In addition, disturbance observer is modified in order to decouple the reacted mechanical waves out of the total disturbance. In section 3, the sensorless force estimation algorithm is introduced. Section 4 includes the experimental results. Finally, remarks and conclusion are included in section 5.

II. MECHANICAL WAVES ESTIMATION

A. Mechanical Waves Analysis

For a Multidegree-of-freedom flexible system with n lumped masses connected to an actuator as shown in Fig.1. the equations of motion are [5]

$$\begin{aligned} J_m \ddot{\theta}_m + B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1) &= \tau_m \\ &\vdots \\ J_n \ddot{\theta}_n - B(\dot{\theta}_{n-1} - \dot{\theta}_n) - k(\theta_{n-1} - \theta_n) &= \tau_{ext_n} \end{aligned} \quad (1)$$

where, J_m and J_i are the actuator inertia and the inertia of the i^{th} lumped mass. k and B are the uniform system's stiffness and damping coefficients. θ_m and θ_i are the actuator angular

position and the i^{th} lumped mass s position. τ_m and τ_{ext_i} are actuator s torque and the external torque applied on the i^{th} lumped mass, respectively. Putting the equations of motion together and solving for $B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1)$ we obtain

$$\begin{aligned}\tau_{ref} &\triangleq \sum_{i=1}^n J_i \ddot{\theta}_i - \sum_{i=1}^n \tau_{ext_i} \\ &\triangleq B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1)\end{aligned}\quad (2)$$

where τ_{ref} is the reelected torque wave from the dynamical system on the actuator. The previous equation shows that the reelected torque wave τ_{ref} carries enough information about the system s dynamics, parameters and external disturbances due to the system s interaction with the environment.

B. Disturbance estimation

Equation.1 represents the mechanical dynamics of the actuator without considering its parameters variation, that are given as follows [6]- [7]

$$\begin{aligned}J_m &= J_{m0} + \Delta J_m \\ k_t &= k_{t0} + \Delta k_t\end{aligned}\quad (3)$$

Where k_t is the actuator s torque constant, J_{m0} and k_{t0} are the nominal actuator s inertia and torque constant while ΔJ_m and Δk_t are the variation between the actual and nominal actuator s inertia and torque constant. Rewriting eq.1

$$(J_{m0} + \Delta J_m)\ddot{\theta}_m = (k_{t0} + \Delta k_t)i_m - \tau_{ref}\quad (4)$$

re-arranging the terms

$$J_{m0}\ddot{\theta}_m = k_{t0}i_m - \tau_{ref} - \Delta J_m\ddot{\theta}_m + \Delta k_t i_m\quad (5)$$

Where, $\Delta J_m\ddot{\theta}_m$ and $\Delta k_t i_m$ are the actuator s varied self-inertia torque and actuator s torque ripple. Therefore, the disturbance on the actuator side is

$$d = -\tau_{ref} - \underbrace{\Delta J_m\ddot{\theta}_m + \Delta k_t i_m}_{\text{disturbance}}\quad (6)$$

which indicates that the disturbance on the actuator side is composed of two components. The last two terms of the right hand side of eq.6 represents the best disturbance component that is related to the actuator parameters s variations. While the second disturbance component τ_{ref} is due to the attached system with this actuator. Therefore, disturbance d has to be estimated then reelected torque wave has to be decoupled out of it. From eq.5 the disturbance d can be computed as follows

$$d = J_{m0} \frac{d^2 \theta_m}{dt^2} - k_{t0} i_m\quad (7)$$

or estimated through a low pass with a corner frequency g_{dist}

$$\hat{d} = \frac{g_{dist}}{s + g_{dist}} [J_{m0} \ddot{\theta}_m - i_m k_{t0}]\quad (8)$$

Therefore, the estimation error is

$$\tilde{d} = \hat{d} - d\quad (9)$$

introducing eq.7 and eq.8 into eq.9

$$\tilde{d} = [J_{m0} \ddot{\theta}_m - i_m k_{t0}] \frac{g_{dist}}{s + g_{dist}} - J_m \ddot{\theta}_m + i_m k_t\quad (10)$$

multiplying eq.10 by $(s + g_{dist})$ and making the following definition

$$\xi \triangleq g_{dist} \Delta J \ddot{\theta}_m - s J_m \ddot{\theta}_m + g k_t i_m + s i_m k_t$$

we obtain the following differential equation

$$\frac{d}{dt} \tilde{d} + g_{dist} \tilde{d} = \xi\quad (11)$$

which describes the estimation error dynamics and has the following solution

$$\tilde{d}(t) = e^{-g_{dist}t} \int_0^t e^{g_{dist}\tau} \xi(\tau) d\tau + c e^{-g_{dist}t}\quad (12)$$

which indicates that the estimation error will exponentially decay, and the low pass corner frequency can be considered as the observer gain. In other words, changing the observer gain controls the speed of the estimation convergence.

$$\begin{aligned}t \rightarrow \infty &\Rightarrow \tilde{d} \rightarrow 0 \\ \tilde{d} \rightarrow 0 &\Rightarrow \hat{d} \rightarrow d\end{aligned}$$

The direct differentiation of the velocity signal can be avoided by using the following observer configuration to keep the noise amplification level as low as possible [8]- [9].

$$\hat{d} = \frac{g_{dist}}{s + g_{dist}} [J_{m0} \dot{\theta}_m + k_{t0} i_m] - g_{dist} J_{m0} \dot{\theta}_m\quad (13)$$

C. Reelected Torque Wave Decoupling

Disturbance estimate obtained using eq.6, can be written as follows

$$\hat{\tau}_{ref} = -\hat{d} - \underbrace{\Delta J_m \ddot{\theta}_m + \Delta k_t i_m}_{\text{disturbance}}\quad (14)$$

which indicates that in order to decouple the reelected wave out of the estimated disturbance, the self varied-inertia torque and actuator s torque ripple have to be determined best. Keeping in mind that variation between the actual and nominal actuator s parameters are inherent properties of the actuator. In other words, they are independent to the plant connected with the actuator. Therefore, they can be estimated from the unloaded actuator and in this case the reelected torque wave is eliminated from equations of motion eq.1.

$$(J_{m0} + \Delta J_m) \ddot{\theta}_m + B \dot{\theta}_m = (k_{t0} + \Delta k_t) i_m\quad (15)$$

$$J_{m0} \ddot{\theta}_m = k_{t0} i_m + d_{par}\quad (16)$$

where d_{par} is the Actuator s parameters variation disturbance

$$d_{par} = \Delta k_t i_m - \Delta J_m \ddot{\theta}_m - B \dot{\theta}_m\quad (17)$$

that can be estimated using the actuator s current and velocity through a low pass filter as follows

$$\hat{d}_{par} = \frac{g_{dist}}{s + g_{dist}} [J_{m0} \dot{\theta}_m + i_m k_{t0}] - g_{dist} J_{m0} \dot{\theta}_m\quad (18)$$

eq.17 becomes

$$\hat{d}_{par} = -B \dot{\theta}_m + \Delta k_{t0} i_m - \Delta J_m \ddot{\theta}_m\quad (19)$$

where, \hat{d}_{par} is the estimated parameters s disturbance vector data point, while $\dot{\theta}_m$, $\ddot{\theta}_m$ and i_m are data points vectors of

$$\begin{bmatrix} \hat{k} \\ \hat{B} \end{bmatrix} = G^\dagger [\hat{\tau}_{ref}] \quad (30)$$

Where G^\dagger is the pseudo inverse of G , \hat{k} and \hat{B} are the estimates of the system's uniform stiffness and damping coefficients. The previous procedure is considered as an off-line parameters estimation experiment that is performed in a certain system's frequency range to estimate flexible system's uniform stiffness and damping.

B. Flexible Motion Estimation

Equation 26 is valid in narrow region of the system's frequency range. In addition, it estimates the motion of the flexible system when its rigidly behaving. Therefore, eq.26 is not enough to estimate the flexible system's motion, where the amplitude ratios between the masses are no longer unity and masses are no longer in phase.

Recalling eq.2 and replacing the actual parameters with the estimated ones

$$\hat{\tau}_{ref} = \hat{B}(\dot{\theta}_m - \dot{\theta}_1) + \hat{k}(\theta_m - \theta_1) \quad (31)$$

re-arranging the terms

$$\hat{B}\dot{\theta}_1 + \hat{k}\theta_1 = \hat{B}\dot{\theta}_o + \hat{k}\theta_o - \hat{\tau}_{ref} \quad (32)$$

solving the first order differential equation for $\theta_1(t)$ that has to be denoted as $\hat{\theta}_1(t)$ since it depends on observed variable $\hat{\tau}_{ref}$ and estimated parameters such as \hat{k} and \hat{B} .

$$\hat{\theta}_1(t) = e^{-\frac{\hat{k}}{\hat{B}}t} \int_0^t \beta e^{\frac{\hat{k}}{\hat{B}}\tau} d\tau + e^{-\frac{\hat{k}}{\hat{B}}t} c_1 \quad (33)$$

where

$$\beta \triangleq \frac{\alpha}{\hat{B}}$$

$$\alpha \triangleq \hat{B} \dot{\theta}_o + \hat{k} \theta_o - \hat{\tau}_{ref}$$

$\hat{\theta}_1(t)$ is the position estimate of the first mass and eq.33 is valid through the entire system's frequency range regardless to the frequency content of the forcing function.

Recalling the flexible system's equation of motion and replacing the first mass position with its estimate we obtain

$$\hat{B} \dot{\theta}_2 + \hat{k} \theta_2 = J_1 \hat{\theta}_1 - \hat{B}(\dot{\theta}_o - \dot{\hat{\theta}}_1) - \hat{k}(\theta_o - \theta_1) + \hat{B} \hat{\theta}_1 + \hat{k} \hat{\theta}_1 \quad (34)$$

solving for $\hat{\theta}_2(t)$ we obtain

$$\hat{\theta}_2(t) = e^{-\frac{\hat{k}}{\hat{B}}t} \int_0^t \zeta e^{\frac{\hat{k}}{\hat{B}}\tau} d\tau + e^{-\frac{\hat{k}}{\hat{B}}t} c_2 \quad (35)$$

where

$$\zeta \triangleq \frac{\gamma}{\hat{B}}$$

$$\gamma \triangleq J_1 \hat{\theta}_1 - \hat{B}(\dot{\theta}_o - \dot{\hat{\theta}}_1) - \hat{k}(\theta_o - \theta_1) + \hat{B} \hat{\theta}_1 + \hat{k} \hat{\theta}_1$$

In general, the position estimate of the i^{th} lumped mass is

$$\hat{\theta}_i(t) = e^{-\frac{\hat{k}}{\hat{B}}t} \int_0^t \Omega e^{\frac{\hat{k}}{\hat{B}}\tau} d\tau + e^{-\frac{\hat{k}}{\hat{B}}t} c_i \quad (36)$$

where

$$\Omega \triangleq \frac{\Psi}{\hat{B}}$$

$$\Psi \triangleq g(J_{i-1}, \hat{\theta}_{i-1}, \dot{\hat{\theta}}_{i-1}, \hat{\theta}_{i-1}, \hat{k}, \hat{B})$$

eq.36 can be considered as a set of position observers that are recursively estimating positions of system's lumped masses. Surprisingly enough that if a proper differentiation tool is used, the velocity and the accelerations of each mass can be obtained without taking any single measurement from the flexible system's side.

C. External Torque Estimation

Since the flexible system's dynamics can be observed and the uniform parameters can be estimated, eq.2 can be used in order to estimate the external torques or disturbance due to the system's interaction with the environment.

$$\hat{\tau}_{ext} = \sum_{i=1}^n J_i \hat{\theta}_i - \hat{\tau}_{ref} \quad (37)$$

Where $\hat{\tau}_{ext}$ is the estimate of the external applied forces on

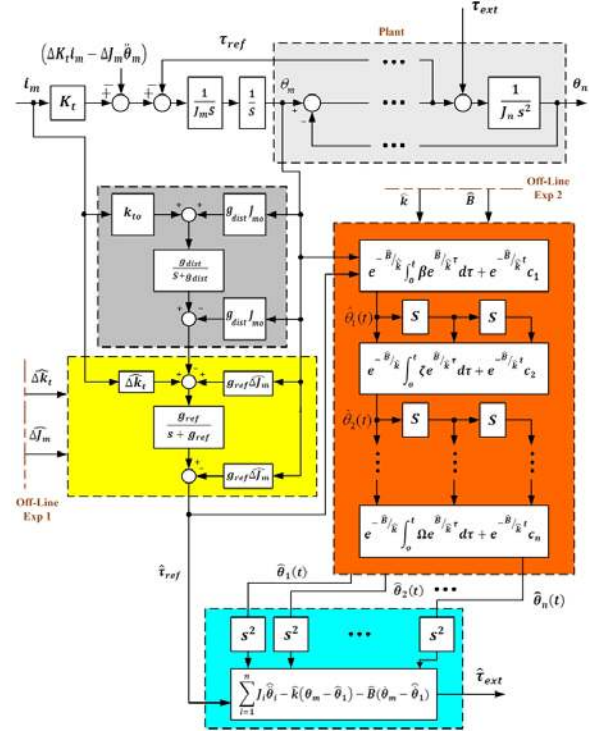


Fig. 3. External applied torque estimation

the plant. The sensorless torque estimation process is illustrated in Fig.3. Indeed, the word Sensorless is not precise, since one must sense or measure some variable to obtain some information as a basis of estimating the unknown variables. In this context the word sensorless refers to the flexible plant that is kept free from any measurement and only two measurement are required from the actuators side as shown in Fig.3. Where actuator's current and velocity are measured and used to estimate the total disturbance, then the reacted torque wave is decoupled out of this disturbance and used as an input for a chain of flexible motion observers that provide eq.37 with the necessary entries to estimate the external torque. The previous procedure requires performing a couple of off-line experiments based on the same actuators parameters current and velocity. The first experiment is performed to estimate the parameters variation disturbance in order to decouple the reacted torque wave, while the second experiment is performed in order to estimate the uniform system's parameters.

IV. EXPERIMENTAL RESULTS

The experimental setup consists of a multidegree-of-freedom inertial flexible system attached to an actuator that is used as a platform for measurements and estimations. Optical encoders are attached to each mass of the system in order to verify the performance of the positions observers. Table.I summarizes the parameters used in the following experiments

TABLE I
EXPERIMENTAL PARAMETERS

Parameter	Value	Parameter	Value
J_1	5152.9 gcm ²	J_3	6192.7 gcm ²
J_2	5152.9 gcm ²	f_1	1 rad/sec
f_2	2 rad/sec	f_3	3 rad/sec
f_4	4 rad/sec	k_{to}	40.6 mNm/A
k_b	235 rpm/v	J_{mo}	209 gcm ²
g_{dist}	100 rad/sec	g_{lpf}	100 rad/sec

A. Rigid Body Motion Estimation Experiment

This experiment is performed at the system's low frequency range that is achieved by filtering the forcing function to avoid exciting the system's flexible modes. Fig.4 shows the response of a 3 Dof flexible system, where all the amplitude ratios are unity and the masses are in phase. The results shown in Fig.4 indicates the validity of eq.26 in this frequency range. Where the estimated rigid body position is following the actual position of the rigid system. The frequency of

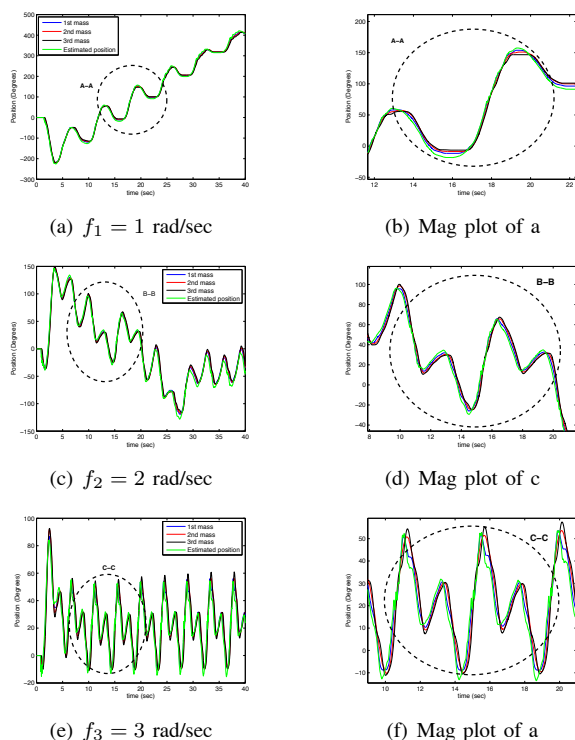


Fig. 4. Rigid body motion estimation

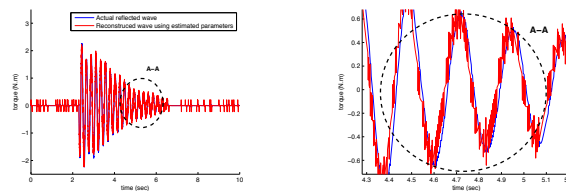
the forcing function is increased gradually in order to determine the frequency region at which eq.26 is valid. It turns out that eq.26 is valid below 3 rad/sec. Therefore, parameters estimation experiment has to be performed in this frequency range for this particular system as its based on eq.26.

B. Parameters Estimation Experiment

Equation 30 is used to estimate the flexible system's stiffness and damping. Therefore, estimate of the reacted torque wave is computed along with the data matrix G , which depends on the actuator's and rigid system's position. Table.II shows the experimental system's parameters obtained by Eq.30. In order to obtain more reliable results the experiment was repeated l times and the obtained

TABLE II
PARAMETERS ESTIMATION RESULTS

Par	1st Exp	2nd	3rd	4th	5th
\hat{k} KN/m	1.579	1.533	1.645	1.511	1.562
\hat{B} Nsec/m	0.088	0.087	0.088	0.089	0.089



(a) reacted and reconstructed
reconstructed torque wave

Fig. 5. Parameters estimation experiment

average values are

$$\hat{k}_{avg} = \frac{\sum_{i=1}^l k_i}{n} = \frac{30.9306}{20} = 1.54653 \text{ kN/m} \quad (38)$$

$$\hat{B}_{avg} = \frac{\sum_{i=1}^l B_i}{n} = \frac{1.6866}{20} = 0.08433 \text{ Nsec/m} \quad (39)$$

The difference between these parameters and the actual ones that are known before hand is less than 5 percent, that can be acceptable for certain applications. Fig.5 shows both the original reacted wave and the reconstructed one using the estimated parameters using Eq.27.

C. Flexible Motion Estimation Experiment

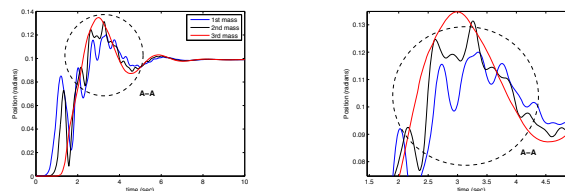


Fig. 6. Flexible Oscillation of a 3DOF Dynamical System

Using the recursive flexible motion observers eq.36, flexible motion can be estimated regardless to the frequency content of the forcing function. Fig.6 shows the response of the 3 Dof flexible system to an arbitrary forcing function that forces the system to flexibly oscillate. The position estimates of each lumped mass of the system is shown in Fig.7 and compared with the actual optical encoder measurement. The results show the validity of the proposed algorithm to estimate the motion of the multidegree-of-freedom flexible system.

D. Torque Estimation Experiment

Estimating the external torque applied on the flexible system using eq.37 requires estimating the flexible system's dynamics and the estimate of the reacted torque wave¹. Experimentally a sinusoidal torque disturbance is added to the flexible system and simply measured and compared with the estimated one. The frequency of the externally applied sinusoidal torque was varied between 1-5 rad/sec. Fig.8-a

¹Experimentally reacted torque wave is assumed to be equal to the disturbance estimated from the actuator, that cannot be generalized since the parameters variation disturbance can effect the results

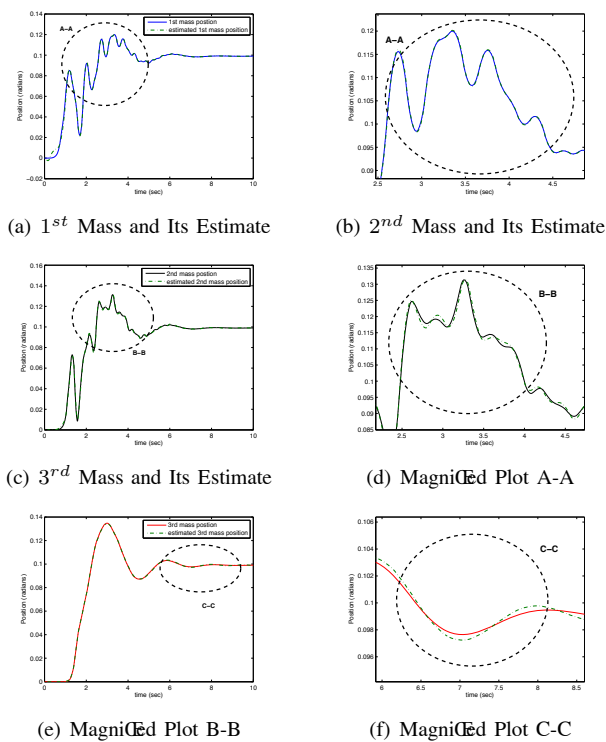


Fig. 7. Flexible body motion estimation experimental results

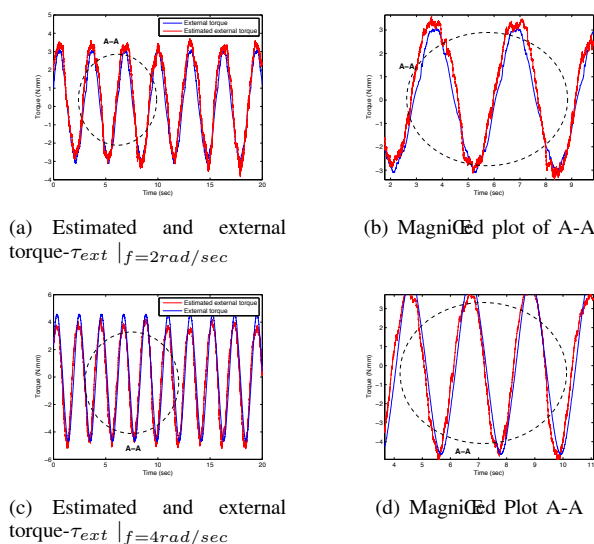


Fig. 8. External torque estimation

shows the estimated and actual torque of 2 rad/sec frequency, while Fig.8-c shows the same result when the external torque's frequency is 4 rad/sec.

V. CONCLUSION

This paper introduces an algorithm for estimating the externally applied disturbance due to flexible system's interaction with the environment. The proposed algorithm is treating the disturbances and system's flexibility differently. Disturbance is not only used as a step toward the accomplishment of robust motion control system but treated as coupled signal that contains information about sys-

tem's parameters, dynamics and external forces/torques. In addition, flexibility is treated as an efficient tool by shaping the control input to obtain certain behavior of the Multidegree-of-freedom system. In other words, making it possible to minimize number of generalized-coordinates describing flexible system's motion, that in turn makes it possible to determine system parameters easier as too many unknowns are dropped.

The re-excited mechanical waves are decoupled out of the total disturbance and proved to contain enough information about the dynamical system. Then, system is rigidly excited with a pre-filtered control input and rigid motion is estimated. Moreover, rigid system's position is used along with re-excited torque wave estimate to estimate the uniform system's stiffness and damping with less than 5 percent error when compared with the actual parameters that are known before hand. Then, the estimated parameters are used along with the re-excited mechanical waves and actuator parameters to design a chain of flexible motion observers that are recursively estimating flexible motion of each lumped mass of the system. Proper differentiation of the position estimates makes all the system's dynamics available. Eventually, the estimated dynamics along with the re-excited torque waves are used to estimate the externally applied torque. Experimental results show the validity of the algorithm that can be used in order to accomplish a sensorless force control assignments.

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REFERENCES

- [1] A. Kato and K. Ohnishi, Robust force sensorless control in motion control system, *f in AMC 06f*. Istanbul, Turkey: IEEE, 2006, pp. 165~170.
- [2] H. Huang, D. Sun, J. K. Mills, and W. J. Li, Visual-based impedance force control of three-dimensional cell injection system, *f in Proc. IEEE International Conference on Robotics and Automation 07*, Roma, Italy, Apr. 2007, pp. 4196~4201.
- [3] W. Iida and K. Ohnishi, Sensorless force control with force error observer, *f in ICIT 03f*. Maribor, Slovenia: IEEE, 2003, pp. 157~162.
- [4] W.J.O Connor, Wave-based analysis and control of lump-modeled flexible robots, *f in Proc. IEEE Transaction on robotics 07*, vol. 23, Apr. 2007, pp. 342~352.
- [5] I. S. M. Khalil and A. Sabanovic, Sensorless wave based control of flexible structures using actuator as a single platform for estimation and control, *f IREACO*, vol. 2, no. 1, pp. 83~90, Jan 2009.
- [6] K. Ohnishi, M. Shibata, and T. Murakami, Motion control for advanced mechatronics, *f in Proc. IEEE/ASME Transaction On Mechatronics 96*, vol. 1, Mar. 1996, pp. 56~67.
- [7] T. Murakami and K. Ohnishi, Robust and adaptive control strategies, *f in Proc. IFAC International Control for Intelligent Automation 92*, vol. 1, Italy, Oct. 1992, pp. 367~372.
- [8] , Observer-based motion control-application to robust control and parameters identification, *f in Proc. IEEE 93*, vol. 1, Jun. 1993, pp. 1~6.
- [9] T. Murakami, F. Yu, and K. Ohnishi, Torque sensorless control in multidegree-of-freedom manipulator, *f in Proc. IEEE Transaction On Industrial Electronics 07*, vol. 40, Apr. 1993, pp. 259~265.
- [10] T. Tsuji, M. Mizuochi, H. Nishi, and K. Ohnishi, A velocity measurement method for acceleration control, *f in Proc. IEEE 05*, vol. 1, Mar. 2005, pp. 1943~1947.