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Separability and Distillability of Multiparticle Quantum Systems

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We present a family of 3-qubit states to which any arbitrary state can be depolarized. We fully classify those states with respect to their separability and distillability properties. This provides a sufficient condition for nonseparability and distillability for arbitrary states. We generalize our results to N-particle states.

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Entanglement is an essential ingredient in most applications of quantum information. It arises when the state of a multiparticle system is nonseparable; that is, when it cannot be prepared locally by acting on the particles individually. Although in recent years there have been important steps towards the understanding of this feature of quantum mechanics, we do not know yet how to classify and quantify entanglement.

Several years ago, entanglement was thought to be directly connected to the violation of Bell-type inequalities [1]. However, Werner [2] introduced a family of mixed states describing a pair of two-level systems (qubits), the so-called Werner states (WS) which, despite being nonseparable, do not violate any of those inequalities [3]. This family is characterized by a single parameter, the fidelity F, which measures the overlap of WS with a Bell (maximally entangled) state. A WS with $F > 1/2$ is nonseparable, whereas if $F \leq 1/2$, it is separable. WS have played an essential role in our understanding of the quantum properties of two-qubit states [4]. First of all, any state of two qubits can be reduced to a WS by acting locally on each qubit (the so-called depolarization process) [5]. This automatically provides a sufficient criterion to determine if a given state is nonseparable [6,7]. On the other hand, Bennett *et al.* [5] showed how one can obtain WS of arbitrarily high fidelity out of many pairs with $F > 1/2$ by using local operations and classical communication. This process, called distillation (or purification), is one of the most important concepts in quantum information theory. When combined with teleportation [8], it allows one to convey secret information via quantum privacy amplification [9] or to send quantum information over noisy channels [8,10]. In the case of two qubits, the partial transposition [11] turned out to be a central tool in the classification of such systems, providing necessary and sufficient conditions for separability [6,7] and distillability [12].

The description of the entanglement and distillability properties of systems with more than two particles is still almost unexplored (see Refs. [13,14], however). In this Letter we provide a complete classification of a family of states of three-particle systems. These states are characterized in terms of four parameters and play the role of WS in such systems. In order to classify 3-qubit states with respect to their entanglement, we define five different classes. To display the distillability properties, we introduce a powerful purification procedure. We also generalize our results to systems of N qubits. Among other things, this allows us to give the necessary and sufficient separability and distillability conditions of mixtures of a maximally entangled state and the completely depolarized state [15,16]. Moreover, since all states can be depolarized to the form we analyze, our results automatically translate into sufficient conditions for nonseparability and distillability of general multiparticle systems. This paper is organized as follows: first we give a classification of entangled states of three qubits; then we analyze the separability and distillability properties of a certain class of three-qubit states, and then we generalize our results to more than three qubits.

1. Classification of states.—Let us consider three qubits A, B, and C. We classify their possible states according to whether they are separable or not with respect to the different qubits. In particular, according to whether they can be written in one or more of the following forms:

$$\rho = \sum_i |a_i\rangle_A \langle a_i| \otimes |b_i\rangle_B \langle b_i| \otimes |c_i\rangle_C \langle c_i| , \quad (1a)$$

$$\rho = \sum_i |a_i\rangle_A \langle a_i| \otimes |\varphi_i\rangle_{BC} \langle \varphi_i| , \quad (1b)$$

$$\rho = \sum_i |b_i\rangle_B \langle b_i| \otimes |\varphi_i\rangle_{AC} \langle \varphi_i| , \quad (1c)$$

$$\rho = \sum_i |c_i\rangle_C \langle c_i| \otimes |\varphi_i\rangle_{AB} \langle \varphi_i| . \quad (1d)$$

Here, $|a_i\rangle$, $|b_i\rangle$, and $|c_i\rangle$ are (unnormalized) states of systems A, B, and C, respectively, and $|\varphi_i\rangle$ are states of two systems. We have the following complete set of disjoint classes of states:

Class 1, fully inseparable states: Those are states that cannot be written in any of the above forms (1).

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An example is the GHZ state [17] $|\Psi_0^+\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$ [18], which is a maximally entangled state of the three qubits.

Class 2, 1-qubit biseparable states: Biseparable states with respect to qubit A are states that are separable with respect to the first qubit, but nonseparable with respect to the other two. That is, states that can be written in the form (1b) but not as (1c) or (1d). A trivial example would be a state $|0\rangle_A \otimes |\Phi^+\rangle_{BC}$, where $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ is a maximally entangled state of two qubits.

Class 3, 2-qubit biseparable states: Biseparable states with respect to qubits A and B are states that are separable with respect to the first qubit and second qubit, but nonseparable with respect to the third one. That is, states that can be written in the forms (1b) and (1c) but not as (1d). For examples, see below.

Class 4, 3-qubit biseparable states: Those are states that can be written as (1b), (1c), and (1d), but not as (1a). For an example, see Ref. [19].

Class 5, fully separable states: These are states that can be written in the form (1a). A trivial example is a product state $|0\rangle_A \otimes |0\rangle_B \otimes |0\rangle_C$.

One can also consider the process of distillation of multiparticle states and relate it to this classification. Assume that we have many trios (3 qubits) in the same state ρ and we can perform only local operations. Then, if the state ρ belongs to the class k it is clear that it is not possible to obtain one trio of a class $k' < k$ (for instance, one cannot convert a 2-qubit biseparable state into a 1-qubit biseparable state). In some cases, however, one may produce some maximally entangled states within one class: (i) Within the fully inseparable states, one may be able to distill a GHZ state; (ii) within the 1-qubit biseparable states, one may distill a $|\Phi^+\rangle$ state of two particles. The specific conditions under which this is possible are not known. On the other hand, if one has some extra entanglement, one may *activate* some of the states and change the corresponding class. For example, if we have a biseparable state with respect to particles A and B (class 3) whose density operator has a negative partial transpose [11] with respect to C and we have some extra states $|\Phi^+\rangle_{AB}$ at our disposal, then one can distill a GHZ state (class 1) [20]. This leads to the interesting result that even though particle A is disentangled from BC and B from AC, with entanglement between AB we can obtain a fully inseparable state. Note also that this way of activating some hidden entanglement is different from the one presented in Ref. [21]. It is also worth mentioning that it is not known whether the entanglement of the states in class 4 can be activated in any form.

2. Three-qubit systems.—Let us define the orthonormal GHZ-basis [17]

$$|\Psi_j^\pm\rangle \equiv \frac{1}{\sqrt{2}}\left(|j\rangle_{AB}\,|0\rangle_C \pm |(3-j)\rangle_{AB}\,|1\rangle_C\right), \quad (2)$$

where $|j\rangle_{AB} \equiv |j_1\rangle_A\,|j_2\rangle_B$ with $j = j_1 j_2$ in binary notation. For example, $|\Psi_0^\pm\rangle = \frac{1}{\sqrt{2}}(|000\rangle \pm |111\rangle)$ are standard GHZ states. We consider a family of three-qubit states of the form

$$\rho_3 = \sum_{\sigma=\pm} \lambda_0^\sigma |\Psi_0^\sigma\rangle\langle\Psi_0^\sigma| + \sum_{j=1}^{3} \lambda_j (|\Psi_j^+\rangle\langle\Psi_j^+| + |\Psi_j^-\rangle\langle\Psi_j^-|). \quad (3)$$

The λs are positive numbers and are restricted by $\mathrm{tr}(\rho_3) = 1$, and therefore the states are characterized by four parameters. We will assume that the labeling has been chosen so that $\Delta \equiv \lambda_0^+ - \lambda_0^- \geq 0$. By using random local operations one can convert any state to this form while keeping the values of $\lambda_0^\pm \equiv \langle\Psi_0^\pm|\rho|\Psi_0^\pm\rangle$ and $2\lambda_j \equiv \langle\Psi_j^+|\rho|\Psi_j^+\rangle + \langle\Psi_j^-|\rho|\Psi_j^-\rangle$ unchanged [22]. Thus, any state can be reduced to this form using this depolarization procedure. Note that one has the freedom to choose a local basis $\{|0\rangle, |1\rangle\}$ in A, B, and C. In this sense, the state $|\Psi_0^+\rangle$ is an arbitrary maximally entangled state.

We will need later on the conditions under which the operator ρ_3 has negative partial transpose [11] with respect to each qubit. One can readily check that

$$\begin{aligned} \rho_3^{T_A} &\geq 0 \quad \text{iff } \Delta \leq 2\lambda_2\,, \\ \rho_3^{T_B} &\geq 0 \quad \text{iff } \Delta \leq 2\lambda_1\,, \\ \rho_3^{T_C} &\geq 0 \quad \text{iff } \Delta \leq 2\lambda_3\,. \end{aligned} \quad (4)$$

2.1. Separability.—We start out by analyzing the separability properties of the states (3). We show: (i) ρ_3 can be written in the form (1b) iff $\rho_3^{T_A} \geq 0$ [and analogously for (1c) and (1d) with $\rho_3^{T_B} \geq 0$ and $\rho_3^{T_C} \geq 0$, respectively]. (ii) ρ_3 can be written as (1a) iff $\rho_3^{T_A}, \rho_3^{T_B}, \rho_3^{T_C} \geq 0$. These results give rise to the classification of the states given in Table I.

(i) If $\rho_3^{T_A} \geq 0$, then ρ_3 can be written in the form (1b) (the opposite is true given the fact that positive partial transposition is a necessary condition for separability [6]). The idea of our proof is to define an operator $\tilde{\rho}$ which can be written as (1b) and can be brought into the form (3) by local operations, which is sufficient to show the separability of ρ_3 since a separable operator is converted into a separable one by depolarization. We define

$$\tilde{\rho} = \rho_3 + \frac{\Delta}{2}\left(|\Psi_2^+\rangle\langle\Psi_2^+| - |\Psi_2^-\rangle\langle\Psi_2^-|\right). \quad (5)$$

TABLE I. Separability and distillability classification of ρ_3.

Positive operators	Class	Distillability
None	1	(GHZ) $|\Psi_0^+\rangle_{ABC}$
$\rho_3^{T_A}$	2	(pair) $|\Phi^+\rangle_{BC}$
$\rho_3^{T_A}, \rho_3^{T_B}$	3	activate with $|\Phi^+\rangle_{AB}$
All	5	$\cdots$

This operator is positive since $\Delta \leq 2\lambda_2$ [equivalently, $\rho_3^{T_A} \geq 0$, cf. (4)] and fulfills $\tilde{\rho}^{T_A} = \tilde{\rho}$. It has been shown in [23] that all states in $\mathcal{C}^2 \otimes \mathcal{C}^N$ which fulfill $\tilde{\rho}^{T_A} = \tilde{\rho}$ are separable, thus the last property ensures separability of particle A. Furthermore, the state $\tilde{\rho}$ can be depolarized to the state ρ_3 [22].

(ii) We show that if $\rho_3^{T_A}, \rho_3^{T_B}, \rho_3^{T_C} \geq 0$ then ρ_3 is fully separable (note again that the opposite is trivially true). Again, the idea is to define an operator $\hat{\rho}$ which can be depolarized into the form ρ_3 by using local operations and that is fully separable. Let $\hat{\rho}$ be a state of the form (3) with coefficients $\hat{\lambda}_0^{\pm} \equiv \lambda_0^{\pm}$, and $\hat{\lambda}_k^{\pm} \equiv \lambda_k \pm \Delta/2$ ($k = 1, 2, 3$). Clearly, $\hat{\rho}$ can be depolarized into ρ_3 [22]. We now rewrite $\hat{\rho}$ as follows:

$$\hat{\rho} = \frac{1}{2}\sum_{k=0}^{3}(\hat{\lambda}_k^+ + \hat{\lambda}_k^- - \Delta)(|\Psi_k^+\rangle\langle\Psi_k^+| + |\Psi_k^-\rangle\langle\Psi_k^-|) + \Delta\sum_{k=0}^{3}|\Psi_k^+\rangle\langle\Psi_k^+|. \tag{6}$$

Since $\rho_3^{T_A}, \rho_3^{T_B}, \rho_3^{T_C} \geq 0$, all coefficients in (6) are positive. The first term in (6) can be written as $\sum_{k=0}^{3}\frac{(\hat{\lambda}_k^+ + \hat{\lambda}_k^- - \Delta)}{2}(|k,0\rangle\langle k,0| + |(3-k),0\rangle\langle(3-k),0|)$ and is thus separable. Let us define $|\phi_0\rangle = |+++\rangle$, $|\phi_1\rangle = |+-\rangle$, $|\phi_2\rangle = |-+-\rangle$, and $|\phi_3\rangle = |-+\rangle$, where $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$. Using this, the second term in (6) can now be written as $\sum_{k=0}^{3}|\phi_k\rangle\langle\phi_k|$ and is thus also separable, which concludes the proof.

2.2. Distillability.—We turn now to analyze the distillability properties of ρ_3. We show that we can distill a maximally entangled state $|\Phi^+\rangle_{\alpha\beta}$ between α and β iff both $\rho_3^{T_\alpha}, \rho_3^{T_\beta}$ are not positive. This automatically means that if all three partial transposes are not positive, we can distill a GHZ state (since we can distill an entangled state between A and B and another between A and C and then connect them to produce a GHZ state [24]). Furthermore, from our previous analysis on separability we have that if any state belongs to class 3 the partial transpose with respect to the third particle is negative (otherwise, it would belong to class 5). As mentioned above, if we have that ρ^{T_C} is not positive but $\rho^{T_A}, \rho^{T_B} \geq 0$ and we have maximally entangled states between A and B at our disposal, then we can activate the entanglement between ABC and create a GHZ state [20] (see Table I).

In order to prove the statements concerning distillability, we just have to show that if $\rho_3^{T_B}, \rho_3^{T_C}$ are not positive then we can distill a maximally entangled state between B and C. That this condition is necessary follows from the fact that by local operations one cannot change the positivity of the partial transpose, and therefore if one is able to distill [21] (which gives rise to nonpositive partial transposes) one must start with nonpositive partial transposes. Let us consider first that we perform a projection measurement in A on the state $|+\rangle$; one can easily show that the remaining state of B and C is purifiable iff $\Delta/2 > \lambda_1 + \lambda_3$ (which corresponds to having a fidelity $F > 1/2$ between the resulting pair). It may happen that this condition is not satisfied even though $\Delta/2 > \lambda_1, \lambda_3$ [i.e., $\rho_3^{T_B}, \rho_3^{T_C}$ are not positive, cf. (4)]. In such a case, we can use the following purification procedure. The idea is to combine M trios in the same state ρ_3, perform a measurement, and obtain one trio with the same form (3) but in which the new Δ is exponentially amplified with respect to $\lambda_{1,3}$. In order to do that, we proceed as follows: We take M trios, and apply the operator $P = |00\cdots00\rangle\langle00\cdots00| + |10\cdots00\rangle\langle11\cdots11|$ in all three locations. This corresponds to measuring a POVM that contains P obtaining the outcome associated to P. The resulting state $P^{\otimes 3}\rho_3^{\otimes M}(P^\dagger)^{\otimes 3}$ has the first trio in an (unnormalized) state of the form (3) but with $\tilde{\Delta}/2 = (\Delta/2)^M$ and $\tilde{\lambda}_k = \lambda_k^M$. Given that $\Delta/2 > \lambda_1, \lambda_3$, for M sufficiently large we can always have $\tilde{\Delta}/2 > \tilde{\lambda}_1 + \tilde{\lambda}_3$. This implies that if we project A onto $|+\rangle$ in this new trio we will have a state between B and C with $F > 1/2$ and therefore that can be purified to a maximally entangled state.

3. Multiqubit systems.—We now generalize the above results to the case of $N \geq 3$ qubits. We will just quote the results here, since the corresponding proofs are similar to those of the three-qubit case (see [25]). We consider the family of states

$$\rho_N = \sum_{\sigma=\pm}\lambda_0^\sigma|\Psi_0^\sigma\rangle\langle\Psi_0^\sigma| + \sum_{j=1}^{2^{(N-1)}-1}\lambda_j(|\Psi_j^+\rangle\langle\Psi_j^+| + |\Psi_j^-\rangle\langle\Psi_j^-|). \tag{7}$$

with the GHZ basis

$$|\Psi_j^\pm\rangle \equiv \frac{1}{\sqrt{2}}(|j\rangle|0\rangle \pm |(2^{N-1} - j - 1)\rangle|1\rangle), \tag{8}$$

and j is again understood in binary notation. As before, using spin flip and phase-shift operations, one can depolarize any state of N qubits into this form [25]. We will denote as $A_1, A_2, \ldots, A_N$ the different qubits. One can readily check that the partial transpose of this operator with respect to the qubit A_N is positive iff $\Delta \equiv \lambda_0^+ - \lambda_0^- \leq 2\lambda_{2^{N-1}-1}$ and similarly for the rest of the qubits.

3.1. Separability.—On one hand, we have that if $\rho_N^{T_{A_k}} \geq 0$ then it can be written in the form $\rho_N = \sum_i |a_i\rangle_{A_k}\langle a_i| \otimes |\varphi_i\rangle_{\text{rest}}\langle\varphi_i|$, and therefore ρ_N is separable with respect to particle A_k. On the other hand, if considering all possible partitions of the qubits in two sets it turns out that for each partition the partial transpose with respect to one of the sets is positive, then ρ_N is fully separable.

3.2. Distillability.—In order to analyze the distillability of a maximally entangled state between particles A_i

and A_k let us consider all possible partitions of the N qubits into two sets such that the particles A_i and A_k belong to different sets. If for all such partitions, the partial transposition with respect to one set is negative, then distillation is possible.

3.3. Example.—Finally, we will apply our results to the case in which we have a maximally entangled state of N particles mixed with the completely depolarized state

$$\rho(x) = x|\Psi_0^+\rangle\langle\Psi_0^+| + \frac{1-x}{2^N}1\,. \tag{9}$$

These states have been analyzed in the context of robustness of entanglement [16], NMR computation [15], and multiparticle purification [13]. In all these contexts bounds are given regarding the values of x for which $\rho(x)$ is separable or purifiable. For example, in Refs. [15,16] they show that in the case $N = 3$ if $x \leq 1/(3 + 6\sqrt{2}), 1/25$ then the state is separable, respectively. In Ref. [13] it is shown that for $N = 3$ if $x > 0.32\,263$ then $\rho(x)$ is distillable. Using our results we can state that $\rho(x)$ is fully nonseparable and distillable to a maximally entangled state iff $x > 1/(1 + 2^{N-1})$, and fully separable otherwise. Specializing this for the case $N = 3$ we obtain that for $x > 1/5$ it is nonseparable and distillable.

In summary, we have given a full characterization of the entanglement and distillability properties of a family of states of three qubits. These states play the role of Werner states in these systems since any state can be reduced to such a form by depolarization. Thus, our results provide sufficient conditions for nonseparability and distillability for general states. In particular, if a state ρ after depolarization belongs to the class k, then ρ must be in a class $k' \leq k$. We have generalized our results to an arbitrary number of particles.

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[20] One can show this by noting that the singlets allow one to teleport states between locations A and B. Thus, for all practical purposes we can consider a pair of qubits AB as a four-level system in which we can perform arbitrary operations. The situation is equivalent to that in which one has two-level systems entangled to four-level systems such that the density operator describing one pair acts on $\mathbb{C}^2 \otimes \mathbb{C}^4$ and has a negative partial transpose. It can be easily shown [W. Dür, J.I. Cirac, and M. Lewenstein (unpublished)] that in systems $2 \times N$ negative partial transpose is a necessary and sufficient condition for distillation, and therefore one can distill arbitrary states. Using again teleportation, one can end up with a GHZ state shared at A, B, and C.
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[22] By mixing we understand in the following that a certain operation is (randomly) performed with $p = \frac{1}{2}$, while with $p = \frac{1}{2}$ no operation is performed. It is easy to check that the following sequence of mixing operations is sufficient to make ρ diagonal in the basis (2) without changing the diagonal coefficients: In the first round we apply simultaneous spin flips at all three locations; in the second and third round we apply σ_z to systems B and C or A and C, respectively. Finally, one can depolarize the subspaces spanned by $\{|\Psi_j^\pm\rangle\}$ for each $j > 0$ by using random operations that change $|0\rangle_\alpha \rightarrow e^{i\phi_\alpha}|0\rangle_\alpha$ ($\alpha = A, B, C$) with $\phi_A + \phi_B + \phi_C = 2\pi$ (this condition ensures that $\lambda_0^\pm$ remains unchanged).
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